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Young Children's Representations of Addition in Problem Solving

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Abstract

Representation is crucial for both learning and solving mathematics problems. This paper explores young children's representations (aged 6 years old) in addition activities in Malaysia. Six children were purposely selected representative of different ability groups. First, a pre-test was administered. Then, the children practiced using multiple representations to solve addition problems after being introduced to the addition concept concretely. Following the children's success in utilizing multiple representations to solve practice tasks, the final problems were posed. Analysis of the children's artefacts, in combination with the children's behaviours and informal interviews revealed insight into their understanding of the addition concepts.

Keywords

Young Children, Addition, Representations, Understanding, Problem Solving

1. Introduction

Representation is a key feature in learning mathematics and solving problems. In many mathematics classrooms, children are often introduced to a variety of representations. Mathematics teachers frequently use multiple representations including concrete materials, virtual manipulatives, drawings, pictures, and written and spoken symbols during instructions (Ahmad, Tarmizi, & Nawawi, 2010; Elia, Gagatsis, & Demetriou, 2007; Polly, Margerison, & Piel, 2014). As a consequence of being exposed to such learning approaches, students utilized various types of representation in learning and problem solving (Carruthers & Worthington, 2005; Manches & O'Malley, 2016; Rosli, Goldsby, & Capraro, 2015). The positive effects of using multiple representations in learning as highlighted by previous researches include motivational benefit and deeper understanding of concepts (Ainsworth, 1999), besides the fact that multiple representations facilitate problem solutions (Lesh, Post, & Behr, 1987).

2. Research Background

Researchers emphasize the importance of developing children's arithmetic skills since the early years of instruction (Gelman & Gallistel, 1978; Patel & Canobi, 2010). Skills learned in the early years of schools are important knowledge to be used in many aspects of this 21st century life as well as for future life and learning. One of the first fundamental arithmetic concepts that young children learn in school is addition. In Malaysia, young children aged 6 years old (often known as "preschool" children) are expected to understand the addition concepts, and later solve problems using the operation (Kementerian Pelajaran Malaysia, 2010). This basic skill is one of the crucial foundations needed for later mathematical learning (e.g. learning of subtraction and multiplication).

Counting forms a basis for addition skills (Butterworth, 2005). It is through the counting experiences (i.e. that involve children to combine sets of collection and count them to find the sum) that children build understanding of the addition concept. Children will then gradually be able to build on concepts and skills from meaningful counting activities (Eisenhardt et al., 2014). They find totals using a variety of strategies including count all, count on, and recall strategies. Carpenter and Moser (1982) suggested ways that children may use for counting all after having already constructed two groups of objects is by either; 1) by physically combining the sets to get the total, or 2) counting all the objects without physically moving the objects together.

Former researches provide evidence for the positive connection between representations, understanding, and problem solving. When students generate representation/representations themselves, they are focused on the structure of the problems as they constructed representations, which promotes the students' use of metacognitive skills, thus contributed to understanding that later supported the solution performance (Manches & O'Malley, 2016; Terwel, van Oers, van Dijk, & van den Eeden, 2009). As previous researchers highlighted, although multiple representations facilitated problem solving (Elia et al., 2007), students still struggle in utilizing it (Yang & Huang, 2004), thus they tend to utilize only a single form of representation (Beyranevand, 2010). In addition to a variety of representations frequently used by participants in previous study (i.e. including concrete materials, drawings, spoken and written symbols), the present study offered children with the opportunity to use digital cameras to create visual pictures more easily and quickly (than making drawings).

3. The Study

The purpose of this particular study was to explore the ways children represented their understanding of addition. The participants were six "preschool" children (i.e. of the same class), and this study was conducted at a public "preschool" in Malacca, Malaysia during the first term of school. These children were purposely selected to represent differing levels of early mathematical achievement. The initial selection was based on the teacher's recommendation and later confirmed by the pre-test (that focused on counting skills). The pre-test also includes several addition and subtraction items (so as to identify any existing knowledge about addition). The researcher worked only with this group of children throughout the study, while the remainder of the class continued their lesson with the teacher. The focus group consisting of six children are considered sufficient as representative of the three mathematical achievements as other children being pre-tested early in the study demonstrated similar mathematical act and behaviours. Although not included in the data presented in this paper, it is important to note that all children considerably improved their basic number skills following the exploration of the addition concepts.

Findings concerning understanding of the addition concept represented by young children (aged 6 years old) as inferred via the creation of various representation forms were reported. Based on the constructivist theory of learning, the children were provided with the learning environments that encouraged the exploration of concept using multiple representations. By prompting the children to make their own sense of addition, the children were actively exploring and constructing their own understanding of the problem.

Prior to the administration of this study, the children had been practicing on counting skills, but had not yet received any formal instruction on basic mathematical operations. The researcher acted as the teacher to the group of six children over a period of five weeks. Initially, the children were introduced to the addition process through modelling with concrete materials. Later, the children practiced using a variety of representations (i.e. including various physical objects, drawings, and written symbols) to solve simple addition problems, with the help of the researcher. Then the children were requested to work independently to solve similar addition problems using various forms of representations previously explored. The data for this paper is drawn from both the practice and the final tasks. The practice problems required the children to find the sums of familiar objects found in daily life (i.e. balls, fruits, and eggs). In the final tasks, the children were invited to find the total number of legs for different types of animals (refer **Table 1**). However, the complexity of the problems for the final tasks was varied according to each child's performance in the pre-test.

Qaisya, Rozy, Ali, and Amy obtained the least scores in the pre-test and were identified as being *perceptual counter,* meaning that they needed concrete items

Table 1. The problem story for the final task.

Task	Challenge level	Problem question (Find the total number of legs for the given animals)
Problem A	High	2 tigers and a chicken
Problem B	Average	An elephant and a tiger
Problem C	Low	2 elephants

to perform counting. However, Qaisya and Rozy demonstrated better counting ability (i.e. counted to a higher number with more confidence) compared to both Ali and Amy. Norman and Nadia were identified as *figurative counters*, meaning they could count efficiently in the absence of actual items. Additionally, Norman mastered the number knowledge (i.e. number facts to 10), while Nadia even performed mental counting. The proficient counters (Norman and Nadia) were challenged to solve addition with more addends (i.e. three addends), while the less able counters (Qaisya, Rozy, Amy, and Ali) were requested to solve addition with only two addends. Despite the fact that Problem B and C involved similar number of addends, Qaisya and Rozy had to find sums of legs for different types of animal, while Ali and Amy were given animals of similar type.

4. Data Sources and Analysis

Data collection included the children's artefacts, observations, conversations with the children, field notes, audio, and video recording. Initially, the children's work (i.e. in the form of drawing, written work, and physical constructions) were analysed and categorized into different forms of representations. Analysis of the children's artefacts exhibited various ways of demonstrating the addition concept. The data were summarised and organised in the form of a table comprising each child's pre-test score, representations and accompanied talk, as well as events and behaviours. All these data indicated the children's thinking related to the concept of addition. The table allowed a child's representation form to be linked across various data sources, thus providing details concerning each child's representations. Additionally, the table permitted for comparison of representation forms and thinking among different children. The representations, in combination with the processes involved (identified through observations and conversations with the children) gave the information of the children's thinking regarding addition.

5. Findings

This paper reports only the data from Norman, Qaisya, and Amy, which are considered to be representative of the range of findings for the entire abilities group.

Specifically, this study aims to answer the following research question:

1) In what ways do young children (aged five and six years) represent their understanding of addition?

5.1. Addition as Separate Addends

The children exhibited many examples of addition as comprising of two distinct groups of objects. Each time after representing the first addends either concretely or with drawings, the children constantly created a space before they continue representing the second addends, suggestive of addition as two distinct groups. Likewise, their photographs showed the same idea of addition.

5.1.1. Concrete Materials

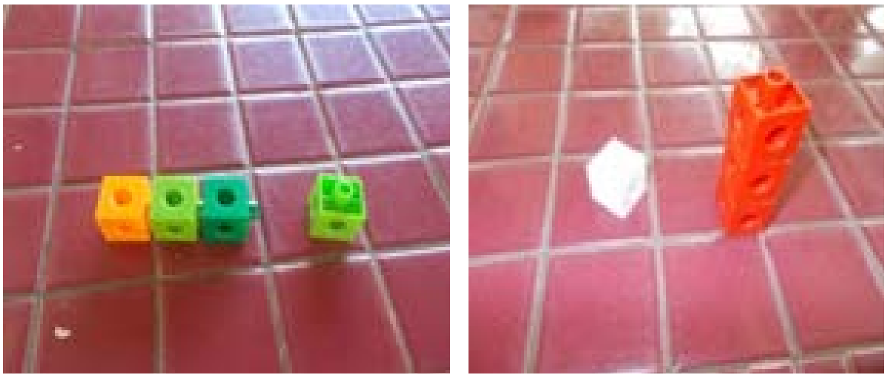
In an early task when representing addition with concrete materials, all the children were observed doing similar steps. They represented the first addend, and then continued representing the second addend. Interestingly, when given an addition problem of "3 + 1", the children came out with different ways of presenting the quantity when exhibiting the first addend. As in **Figure 1(a)**, Qaisya formed a horizontal line with the cubes, while Amy stacked the cubes vertically (refer **Figure 1(b)**). They continued modelling the second addend by putting one cube as a separate unit from the previous construction. Obviously, all children pointed (or touched) the cubes and counted them without first putting the cubes together. By doing so, they identified addition as comprising of two groups of objects, and counted all the objects in both groups to get the total.

Using similar addends (3 and 1), the researcher requested the children to use other physical materials including the children's fingers, pegs, and coins. All the children repeated the similar steps as they did previously with the cubes. They had no difficulty in manipulating various physical objects for the addition situation given, but none of them ever combined the addends prior to counting the total. The children continued practicing addition with various addends but with totals no larger than 10. The children managed to successfully represent the addition (i.e. addition as two groups of objects) without any assistance.

5.1.2. Drawings

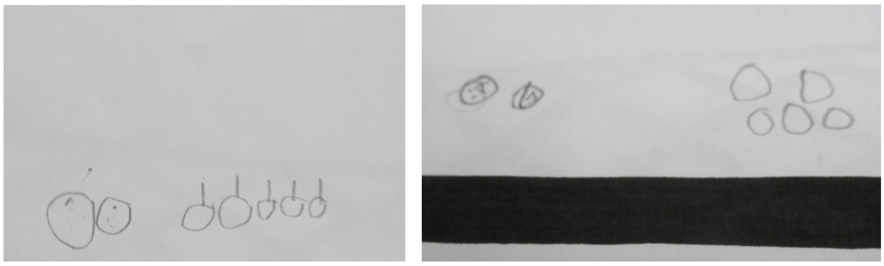
Similar to the way they worked with concrete materials, the children's drawings for the addition solution remained as two distinct groups without any marks and signs combining the groups. At the beginning of the study, when asked to make drawings, all children produced detailed drawings.

As in **Figure 2**, Amy and Norman as well as majority of the children drew the sets of oranges separately from the sets of apples. Obviously, the drawings (as can be seen in **Figure 2(a)** and **Figure 2(b)**) had a gap between the two sets of fruits. In addition to creating a space to differentiate between the two groups of



(a) (b)

Figure 1. Examples of the children's construction of addition using cubes showing different ways of representing the quantity of the first addend. (a) Three cubes (in a line) by Qaisya. (b) Three cubes (in a pile) by Amy.



(a) (b)

Figure 2. Examples of drawings with gaps between the addends and adding details to differentiate between different types of fruits. (a) Inclusion of details to both fruits to differentiate between the two types of fruits (by Norman). (b) Inclusion of details to only one type of fruit to differentiate between two fruits (by Amy).

objects, the children added details to their drawings for similar intention. As in Figure 2(a), Norman clearly secured some space between the two types of fruits. Additionally, he added details to both fruits to differentiate oranges from apples. Similar to Norman's intention, Amy made a large gap between her different fruits. She chose to add details to only one type of fruit (i.e. orange) as the first addend, making the other circles (although without details on them) easily recognizable as representing the other type of fruit (i.e. apple) as the second addend. In later drawings, the totals as exhibited in the children's drawings remained as two distinct groups.

5.2. Photographing Addition

In this task termed "My Addition Story", the children were required to take photos and created their own addition story based on the photos they captured. The selection of subjects photographed for the addition task varied among the children. Norman chose items he had been working with previously. Norman used the coins as his subject, while Amy selected the picture cards (i.e. ice cream picture cards) recently added by the researcher.

When photographing pictures for the addition story, majority of the children first manipulated the objects and later captured what they had created. They began by constructing the first addends, followed by creating the second addends, and finally photographed their work that exhibited the totals (that also display the addends simultaneously).

Obviously, the children's photographs of the totals were demonstrated as two separate groups (as in Figure 3). Despite the power of digital camera that allowed the children to take a great number of photos without any difficulty, Norman captured only a single photo. It seemed that the single photograph he captured functioned to show the totals as two addends. In contrast, Amy produced more photographs than Norman to describe her addition situation. Despite producing three photographs, Amy still did not combine the objects in her final photograph when showing the total. She simply relocated the "ice creams" by putting the groups of "ice creams" closer to ensure that all of them fit within

(a)

(b)

Figure 3. Photographing the totals as two separate groups. (a) Norman's photograph: 4 coins and 2 coins make 6 coins. (b) Amy's photograph: 4 ice creams and 2 ice creams make 6 ice creams.

the camera frame. Although she moved the groups of "ice creams" closer, they were still identified as two distinct groups since there was a clear gap between the first and the second row of "ice creams".

6. Discussion

The children in this study seem to display the acquisition of basic addition principles involving addition as containing two distinct groups and counting all the objects to get the total (Carpenter & Moser, 1982; Gray & Tall, 1994). However the act of combining the sets as a single set prior to counting all the constructed objects (Gray & Tall, 1994) was rarely observed in this study. Regardless of representing with concrete materials, drawings or making gestures, majority of the children showed little to no evidence of addition as *combining* the groups.

The struggle that the children faced when demonstrating addition as found by (Polly et al., 2014) may offer an explanation for the limited representation of addition as *combining* two groups found in this study. When comparing two different tasks involving the use of concrete materials, the researcher reported that the kindergarten children were far better able to perform addition tasks that required representation of the addends and calculating the answer (i.e. 5 + 3 = ?) than to demonstrate the addition processes (i.e. show 3 + 1 = 4). Although both tasks required representation of the addends together with the answer, the latter required demonstration of the whole addition processes (3 + 1 = 4), thus was a more challenging task for the children. Since the children in the current study were just beginning to explore the addition concept, it was not surprising that demonstrating the abstract concept was also challenging to them compared to only manipulating the concrete materials to find the answer. If the children in the Polly et al. (2014) study struggled demonstrating addition as *combining* despite being previously taught addition and have been in kindergarten for 6 months, possibly the children in this study who recently explored addition with only little support for several weeks too, found demonstrating addition as problematic.

The absence of addition as *combining* repeatedly occurred in another task given by Polly et al. (2014) whereby when invited to create their addition stories given all the three numbers (i.e. 3 + 2 = 5), some children included all the numbers in their stories but without any action of joining the sets. Somewhat similar to the children in this study who were given concrete materials to work out various addition problems, they made no physical actions suggesting *combining* objects in the two groups together. Likewise, the children's objects in the final photographs (i.e. for "My Addition Story" task) remained as separated groups. This is in contrast to the children in Manches and O'Malley (2016) study that actively made various actions on manipulatives including grouping objects, swapping over groups, and moving objects when solving additive composition task.

Neither did the children in the present study *combined* their drawings. This is in contrast with children in other studies who commonly produced "dynamic representations" (Carruthers & Worthington, 2005) to represent action, movement, and relationship (Changsri, 2015; Papandreou, 2014). Drawings produced by children in other studies contained various marks including arrows, lines, and circles. However, the children in the current study merely drew objects to represent quantities with no additional marks to represent actions on the objects they drew. The fact that the children were more familiar with drawings as an art subject rather than as a means to represent mathematical thinking might have influenced them to only add details to drawing and produced static drawings rather than dynamic representations.

Possible explanation for the lack of the *combining* act as observed when adding with both concrete materials and drawings may be due to the term used for this study. The term "problem" used in various tasks in the present study might have led the children to focus more on finding the answer. Since the concern was towards the total, they thought that it is not necessary for them to clearly show their understanding of the addition concept in reaching the solution, resulting in not demonstrating the *combining* act explicitly.

The instruction to solve posed problems without specifically including the instruction to show the addition concept (that the children themselves understand) did not fully exhibit the children's thinking of addition, even if they possibly demonstrate the addition ideas in their solutions (i.e. addition as *combining*). Should the instruction to find the total was followed by an explicit request to show their understanding of the addition concept, it is likely that the children might show their attempt in *combining* the addends to get the totals. As observed in an addition task given by (Changsri, 2015: p. 64), the instruction to "talk about how to calculate 8 + 3" initiated the children to produce pictures that clearly represent the ideas of addition as *combining*. Some children produced arrows, while others drew circles to *combine* the block diagrams. The plus sign was also used in both drawings of blocks and number sentence to highlight the idea of *combining* both addends, in addition to finding the answer. Furthermore, the children expressed the concept of addition as "increasing" by counting one by one on a picture, resulting from an instruction that clearly requested the children to express their thinking of addition.

7. Limitations of the Study

Although this study accounted for all levels of mathematical achievement (which was found lacking in the previous research), the participants involved only a small number of children. Furthermore, the study was only conducted in one school setting which might have influenced the findings of the study. Each of these limitations minimizes the extent of generalizations that were made from the findings of this study.

8. Implications, Conclusion and Recommendations

A "culture" of multiple representations should be created in the classroom by the teacher. Instructions should involve a variety of representation rather than emphasizing on a particular representation form. Students must be given enough opportunity to practice a variety of representation forms—both the production of multiple representations and the internalization of mathematical ideas through social activity involving various representation forms. With the teachers' encouragement and support, students will be able to internalize as well as integrate multiple representations into their cognitive structure and use the representations as communication tool.

Despite the children's success in utilizing a variety of representations to solve various addition problems, the findings demonstrated that the children are still in the early stage of understanding the addition concept. The findings from this study support the idea that the addition concept is largely developed from the children's informal knowledge of addition (i.e. counting collections). Thus, children may require guide and support from the teacher. Additionally, it is necessary to provide children with enough practices to develop their understanding. Further research is necessary to determine both effective teaching pedagogies and learning experiences that will develop young students' ability in utilizing multiple representations for learning and problem solving purposes. Furthermore, the diversity of responses from one single classroom calls for learning experiences that could cater for individual differences rather than a single approach that often times was expected to fit all students' needs.

References

Ahmad, A., Tarmizi, R. A., & Nawawi, M. (2010). Visual Representations in Mathematical

Word Problem Solving among Form Four Students in Malacca. *Procedia-Social and Behavioral Sciences, 8*, 356-361. https://doi.org/10.1016/j.sbspro.2010.12.050

Ainsworth, S. (1999). The Functions of Multiple Representations. *Computers & education, 33,* 131-152. https://doi.org/10.1016/S0360-1315(99)00029-9

Beyranevand, M. L. (2010). *Investigating Mathematics Students' Use of Multiple Representations when Solving Linear Equations with One Unknown.* Unpublished Doctoral Dissertation, Lowell: University of Massachusetts.

Butterworth, B. (2005). The Development of Arithmetical Abilities. *Journal of Child Psychology and Psychiatry, 46,* 3-18. https://doi.org/10.1111/j.1469-7610.2004.00374.x

Carpenter, T. P., & Moser, J. M. (1982). The Development of Addition and Subtraction Problem-Solving Skills. In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), *Addition and Subtraction: A Cognitive Perspective* (pp. 9-24). Hillsdale, NJ: Lawrence Erlbaurn Associates.

Carruthers, E., & Worthington, M. (2005). Making Sense of Mathematical Graphics: The Development of Understanding Abstract Symbolism. *European Early Childhood Education Research Journal, 13,* 57-79. https://doi.org/10.1080/13502930585209561

Changsri, N. (2015). First Grade Students' Mathematical Ideas of Addition in the Context of Lesson Study and Open Approach. *Conference Proceeding of ICMI Study 23: Primary Mathematics Study on Whole Numbers*, 60-67.

Eisenhardt, S., Fisher, M. H., Thomas, J., Schack, E. O., Tassell, J., & Yoder, M. (2014). Is It Counting, or Is It Adding? *Teaching Children Mathematics, 20,* 498-507. https://doi.org/10.5951/teacchilmath.20.8.0498

Elia, I., Gagatsis, A., & Demetriou, A. (2007). The Effects of Different Modes of Representation on the Solution of One-Step Additive Problems. *Learning and Instruction, 17,* 658-672. https://doi.org/10.1016/j.learninstruc.2007.09.011

Gelman, R., & Gallistel, C. (1978). *Young Children's Understanding of Numbers*. Cambridge, MA: Harvard University Press.

Gray, E. M., & Tall, D. O. (1994). Duality, Ambiguity, and Flexibility: A "Proceptual" View of Simple Arithmetic. *Journal for Research in Mathematics Education, 25,* 116-140. https://doi.org/10.2307/749505

Kementerian Pelajaran Malaysia (2010). *Dokumen Standard Kurikulum Prasekolah.* Kuala Lumpur: Kementerian Pelajaran Malaysia.

Lesh, R., Post, T., & Behr, M. (1987). Representations and Translations among Representations in Mathematics Learning and Problem Solving. *Problems of Representation in the Teaching and Learning of Mathematics, 21,* 33-40.

Manches, A., & O'Malley, C. (2016). The Effects of Physical Manipulatives on Children's Numerical Strategies. *Cognition and Instruction, 34,* 27-50. https://doi.org/10.1080/07370008.2015.1124882

Papandreou, M. (2014). Communicating and Thinking through Drawing Activity in Early Childhood. *Journal of Research in Childhood Education, 28,* 85-100. https://doi.org/10.1080/02568543.2013.851131

Patel, P., & Canobi, K. H. (2010). The Role of Number Words in Preschoolers' Addition Concepts and Problem-Solving Procedures. *Educational Psychology, 30,* 107-124. https://doi.org/10.1080/01443410903473597

Polly, D., Margerison, A., & Piel, J. A. (2014). Kindergarten Teachers' Orientations to Teacher-Centered and Student-Centered Pedagogies and Their Influence on Their Students' Understanding of Addition. *Journal of Research in Childhood Education, 28,* 1-17. https://doi.org/10.1080/02568543.2013.822949

Rosli, R., Goldsby, D., & Capraro, M. M. (2015). Using Manipulatives in Solving and Posing Mathematical Problems. *Creative Education, 6,* 1718-1725. https://doi.org/10.4236/ce.2015.616173

Terwel, J., van Oers, B., van Dijk, I., & van den Eeden, P. (2009). Are Representations to Be Provided or Generated in Primary Mathematics Education? Effects on Transfer. *Educational research and Evaluation, 15,* 25-44. https://doi.org/10.1080/13803610802481265

Yang, D. C., & Huang, F. Y. (2004). Relationships among Computational Performance, Pictorial Representation, Symbolic Representation and Number Sense of Sixth-Grade Students in Taiwan. *Educational Studies, 30,* 373-389. https://doi.org/10.1080/0305569042000310318