

Reviewing the Learning Process through Creative Puzzle Solving

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Abstract

Human beings are at a continuous learning process at various levels and with different motivations during their whole lifetime. Puzzle solving may beneficially be applied to increase the motivation, enhance the mastering apprehension, promote the creative processes, expand the ability to engage and solve miscellaneous challenges from various viewpoints, and hence lead to an improved learning process and problem solving capability. That is, the application of puzzles may lead to better learning and increased knowledge in general, stimulating the reasoning process and the apprehension of the need for both creativity and hard work. Thus, teachers of both students and teachers may find it beneficial to utilize the art of puzzle solving. Typically, the puzzles are very suitable for and mostly used in mathematics and natural science classes. Nevertheless, the puzzles are in general also applicable for any type of class. The aim of this study is to examine and discuss the learning process through applying creative puzzle solving as a teaching tool. These aspects are illustrated through a review of several selected puzzle examples.

Keywords

Problem Solving, Puzzle, Review, Motivation, Education, Teaching, Learning Process, Creativity, Creative, Reasoning, Math, Science

1. Introduction

As we human beings go through life we encounter various puzzles, some more intriguing and memorable than others. Some puzzles may be long forgotten while others may still continue to thrill and excite us and our acquaintances over and over again. Before moving on we need to elaborate a bit around what type of

puzzles we are discussing here, although an exact definition will not be attempted in this respect as such a classification may be rather hard and still not correct.

The puzzles discussed in this context cover a large field with few or none constraints. Nevertheless, there are some common denominators for many of these puzzles, although with some exceptions too. A main denominator is the out-of-the-box thinking and solutions. A second one is that most of the puzzles, or nuts as some like to call them, *may* be solved in a very short time, even immediately or within a few seconds, and still they *may* not be solved during weeks or months even if the solution is just below the surface and begging to be discovered so to speak. The second one then introduces or leads to a third common denominator, which we may denote as the aha-experience, i.e. the sudden realization of how truly simple or beautiful the solution really is when discovering it for the very first time. One may then even exclaim “Aaahh I should have seen this immediately!” In principle, the term ‘*should*’ may be discussed in this respect, but clearly, the term ‘*could*’ would nevertheless be quite fitting. As a result from this, a fourth common denominator is that when you, even finally, have found a solution, you are totally convinced that you really have found the solution. And you know that even if someone would tell you another solution, you would still claim you had a valid and beautiful solution. As we will see later, some of the very good puzzles may have two or even several solutions.

As to continue with our common puzzle denominators, and as it would be beneficial, we should first have a look on some typical puzzles. Examples of selected puzzles for this purpose may be:

- *The Hole Digging*. 5 men dig 4 holes in 3 days. How long time does one man use to dig half a hole?
- *The Equation*. Solve the expression: $(x - a)(x - b)(x - c) \dots (x - z) = ?$
- *The Triangle and the Square*. [An equilateral triangle of 3 matches is made in advance.] Make 4 identical equilateral triangles as this one (same size also) out of a total of 6 matches. [A square of 4 matches is made in advance.] Make 6 identical squares as this one (same size also) out of a total of 9 matches.
- *The 9 Dots*. [A square of 9 dots is made on paper in advance, where there are three rows consisting of three dots each with equal distances between all dots, i.e. there will be one dot in the middle of the square surrounded by eight dots on the sides of the square.] Draw 4 straight lines without lifting the pencil where you are striking all the 9 dots in the figure.
- *The Cake Division - The Cake Sharing - The Cake Nut - The Nut Cake*. [Preferably carried out with three real cakes with topping.] Cut the cake in eight identical pieces using only three one-motion straight cuts with a knife.
- *The Moat*. Get over the moat in a safe way using two logs where each log is just too short. [See figure and description in the following.]
- *The Earth and the Marble*. A cord or a rope is placed tightly around the Earth at equator. If this cord is prolonged with 6 m and lifted above the ground at

the same height around the whole equator, how high above the ground will the cord be situated? Will it be possible to press a thin sheet of paper under the cord? And what if we have a small marble instead of the Earth, and also prolong this cord with 6 m, what will be the height (above the marble's surface) of the cord now?

- *The Paper Sheet Folding.* Fold a paper sheet 100 times. Press out all air between the paper sheets. Put the paper pile on the table in front of you. Guess how far above the table does the paper pile reach?

The last puzzle given in the list above, i.e. the paper sheet folding, is not really a puzzle like the others treated here, still it is included as this problem or challenge with its solution presents a nice teaching opportunity with a very intriguing and thought-provoking solution.

With background in the second and third common denominator, a fifth one may be deduced. That is, many of these puzzles might be solved just as fast or even faster by small kids as by grown-ups and highly educated persons. One may then start to wonder why a puzzle may be solved within seconds by a small kid while a very intellectual and educated professor may use several weeks or ending up not solving the puzzle at all. A sixth common denominator follows directly from the fifth one, i.e. many of the puzzles may be solved without any or only a small amount of scholarly background and education. Hence, at all levels, teachers of both students and teachers wanting to further improve and enhance their education skills may find it beneficial to employ the art of puzzle solving in their classrooms.

The objective of this study is to address and discuss the learning process through applying creative puzzle solving as a teaching tool. First, motivation and mastering are treated. Thereafter, development and learning aspects are considered. Furthermore, the art of puzzles in the learning process is illuminated. In addition, several selected puzzle examples are presented and discussed. Finally, some empirical results are given and discussed.

By shedding light on these puzzles, there is a hope and a goal that more teachers and lecturers will start to utilize and exploit puzzle solving during their classes, with increased motivation, mastering, engagement, development and learning abilities among both students and teachers as a result. Increased problem solving capabilities and promotion of creative thinking may then walk hand in hand. Moreover, we should not forget the sheer joy and playful aspects, of course.

2. Motivation and Mastering

Motivation may be crucial to the actual quality of a work task being carried out, including the learning process and performance among pupils and students. Hence, motivation will also be important for the mastering of the learning process, i.e. the higher motivation the higher mastering. Furthermore, increased mastering will often lead to increased motivation, i.e. a positive and

self-strengthening circle mechanism.

Pupils and students who feel that they are not performing sufficiently or satisfactorily in ordinary school subjects, may often become interested in and master the puzzle types described within this work. Think about that pupil who managed to solve that puzzle no one else in the class was able to solve. The pupil will remember that and carry the positive mastering feeling within himself or herself for a long time if not for the rest of his or her life. That is, the puzzles may contribute to give our pupils positive memories for the rest of their lives. With mastering follows motivation, which hence may have a positive rub-off effect on the whole learning process. The pupils may then start to comprehend an increased interest for the more ordinary school subjects. Various aspects of motivation and mastering are discussed by [Imsen \(1993\)](#).

When students and others are battling puzzles with eagerness, there is normally an *intrinsic (inner) motivation* driving them. Hence, the puzzles themselves are the driving force and motivation for the interest, contrary to an *extrinsic (outer) motivation* where external factors like e.g. grades are the main driving force and motivation. With background in today's development towards a more competitive society with increased pressure to achieve good grades, risk of unemployment, insecurity, etc., many teachers would desire an increased inner motivation among their students. That is, students who take pleasure in the learning process itself, and who do not only view the learning as a means to obtain good grades and certificates, credits, student and university places, and a secure job. Nevertheless, extrinsically motivated students will also often find the puzzles to be very interesting, both as the puzzles themselves and as a remedy to improve their learning and their grades. In this respect we may note the following words by [Polya \(1988\)](#): "The space devoted by popular newspapers and magazines to crossword puzzles and other riddles seems to show that people spend some time in solving unpractical problems. Behind the desire to solve this or that problem that confers no material advantage, there may be a deeper curiosity, a desire to understand the ways and means, the motives and procedures, of solution." Thus, we human beings like to puzzle with puzzles, which hence may be exploited in the teaching. Inner motivation is amongst others also treated by Skemp through the psychology of learning mathematics ([Skemp, 1993](#)).

The interest for puzzles among students and others may be viewed in light of the two upper levels in Maslow's hierarchy of needs, often visualized as Maslow's pyramid of needs, i.e. need for recognition and positive self-esteem (second top level) and need for self-actualization (uppermost top level), the latter one including e.g. vitality, creativity, self-sufficiency, authenticity, playfulness and more fundamental needs in Maslow's hierarchy of needs are physiological needs (air, water, homeostasis, sex and sleep), need for safety and security, and need for love and social belonging.

During puzzle solving many pupils and students will experience an increased

positive self-image and hence also self-esteem, e.g. with respect to themselves, fellow students, teachers, parents, siblings and friends, as they are able to solve the puzzles. Imagine that pupil who managed to solve that puzzle no one else managed to solve. So simple and evident solution, but only she saw it, and which she will remember for a long time. The pupils will of course not be able to solve all the puzzles each time, but all of them will normally be able to understand the solution when they see it and exclaim “aha!” as an eye-opener. Thus, the pupils are learning in an entertaining and easy way. Hence, the puzzles with all their creative and challenging sides, may also contribute to satisfy the pupils need for recognition and positive self-esteem and ultimately self-actualization, i.e. the top levels in Maslow’s hierarchy of needs.

Pupils and students will also attack the puzzles as they quite simply want to solve them, i.e. an achievement motive is present. A puzzle or nut is a challenge, readily somewhat different from the other tasks the students are facing during their routine schooling.

Atkinson (1957) has expressed the total achievement motivation T_a as the sum of the desire to achieve success T_s (motivation to achieve) and the fear for failure T_{-f} (motivation to avoid failure) (Atkinson, 1957; Imsen, 1993):

$$T_a = T_s + T_{-f} \quad (1)$$

The larger motive to achieve (which often will be the case with puzzles, i.e. inner motivation), the larger total achievement motivation. The motivation to avoid failure, i.e. the fear for failure, will decrease or weaken the total achievement motivation. Most students will be rather unsure if they will manage to solve the puzzle to be presented, however, they also know that they may have a fair chance to actually be able to solve the upcoming puzzle, i.e. one may in principle assess the probability to achieve or failure to be about equal, i.e. 50%/50%.

Notwithstanding, *the actual fear* for failure will not be so large. *It is inherently in the nature of the puzzles that it is allowed to not be able to solve them.* A student does not lose face, neither with respect to himself nor fellow students, if he does not manage to solve the puzzle. Thus, the student will not feel so much resistance versus trying to solve the puzzle, even if he knows that there is a fairly high probability for him not being able to solve the puzzle. There is no shame associated with not being able to solve a task one knows many will not manage to solve. Nevertheless, the student does also know that he may very well be able to find the solution of the puzzle. Among other things, this is part of the common characteristics for many of the puzzles presented herein. That is, the puzzle solution is obvious when you have seen the solution (hopefully by solving it yourself). However, to see or discover the solution may take only a few seconds or several hours, days, weeks and even months, i.e. very intriguing and fascinating.

Furthermore, Atkinson (1957) is dividing the motivation to achieve T_s in three factors (Atkinson, 1957; Imsen, 1993):

$$T_s = M_s P_s I_s = M_s P_s (1 - P_s) \quad (2)$$

where M_s is a fundamental mastering motive (motive to achieve success), P_s is a person's subjective assessment of the probability to succeed (subjective probability of success) and I_s is a person's subjective assessment of the value of succeeding (subjective incentive value of success).

We may here note that the subjective incentive value of success I_s may be written as $1 - P_s$. That is, the subjective assessment of the value of succeeding (I_s) increases with increasing degree of difficulty of the tasks and thus decreasing P_s . The motivation to achieve (T_s) will then be at its largest when $P_s = 0.5$, which is deduced from differentiating $T_s = M_s P_s (1 - P_s)$ with respect to P_s and setting $dT_s/dP_s = 0$ and thereby obtaining $P_s = 0.5$. Hence, medium hard or difficult tasks give the highest motivation according to this motivation theory. Bearing in mind the fifty-fifty chance mentioned above for achieve or failure with solving the puzzles, all should be well suited for having a high motivation for succeeding with the puzzle solving. Naturally, this should not be taken totally literally as each individual is different, and in any case the rough estimate for a fifty-fifty chance for solving a puzzle is just that, i.e. a rough estimate.

Within this theory we may also speak of an inverse mastering motive, that is, a motive to avoid failure M_{-f} . Pupils with an anxious predisposition (high M_{-f}) will often start battling tasks which may seem to be way too difficult for them as it is not humiliating to fail when no one is expecting you to succeed. Medium difficult tasks on the other hand appear humiliating and lead to anxiety. Easy tasks, which an anxious pupil is confident to master, will not result in any particular anxiety. For further elaborations and details on these issues it is referred to the works by [Atkinson \(1957\)](#) and [Imsen \(1993\)](#). Moreover, for a developmental perspective on expectancy-value theory of achievement motivation it is referred to the study by [Wigfield \(1994\)](#), while [Graham & Weiner \(1996\)](#) give a review of the theories and principles of motivation. The puzzles, however, will normally not provoke any anxiety reactions as most pupils like this kind of exercises and the lack of humiliation associated with them. A physicist's perspective on motivating students to learn science is given by [Silverman \(2015\)](#).

Many of the puzzles are paradoxically both hard and easy to solve at the same time, i.e. the puzzles are rather special. Thus, the puzzles have something to offer all types of pupils and students, both students dominated by desire for achieving success and students dominated by fear for failure. All pupils and students are offered the opportunity to feel that they master, from actually solving the puzzle to understanding the point(s) of the puzzle even if they did not solve the puzzle in the first place. The aha-experience is important. Increased mastering leads to increased motivation. Subsequently, with increased motivation follows increased learning and then again increased mastering, i.e. an example of a good circle.

3. Development and Learning

So-called *schemes* are treated by Piaget ([Imsen, 1993](#)) and Skemp ([Skemp, 1993](#)).

A newborn baby has some few inherited schemes, e.g. the sucking reflex (sensory motory schemes). Gradually, the child develops new schemes. With respect to our puzzle solving it is in particular the *cognitive schemes* which are of special interest, i.e. schemes where thinking (reflection and reasoning) plays an important role. Learning is change of schemes, i.e. change of the *conceptual structures* one has formed. One may supplement and complete this with that learning may also be an expansion of existing structures (Fossland, 1994). Puzzle solving often involves that one has to think outside the customary and habitual (and maybe learnt?) paths. Hence, one has to change one's schemes, i.e. learning and gaining knowledge.

Constructivism is a particular learning model where the key idea is that knowledge is actively being constructed (Fossland, 1994). Among others, Sjøberg (1992) has advocated such a learning model in his book "Naturfagenes didaktikk, Fra vitenskap til skolefag" ("The didactics of natural sciences, From science to school subject"). Fossland (1994) is summarizing his description of constructivism in the following four paragraphs:

- Knowledge has to be actively constructed, i.e. and can not be received passively. Piaget states that the knowledge is being arranged in schemes, which hence is being put together in larger structures (Imsen, 1993; RVO, 1980).
- The pupils are not empty boxes prior to the teaching, i.e. the pupils have previous knowledge. The psychologist Ausubel stated this rather clearly (Driver, 1993; RVO, 1980; Sjøberg, 1992): "If I am forced to reduce all pedagogical psychology into only one principle, I want to say this: The by far the most important factor determining what the pupil will learn, is what the pupil already know. Get hold of that, and teach thereafter."
- Learning (adaption) occurs when new knowledge is related to old knowledge (assimilation) and old knowledge is adapted new knowledge (accommodation). It is referred to e.g. Driver (1993), Imsen (1993) and Sjøberg (1992) for Piaget's adaption theory. Fossland (1994) emphasizes that Piaget's focus on learning as a change of existing structures needs to be completed with that learning may also be an expansion of existing structures. Ausubel introduces the concept *advance organizers* for the creation of a *cognitive bridge* between new information and already existing knowledge structures. New subject matter is adjusted to the previous knowledge of the pupils by a teacher who is repeating relevant matter and emphasizing the connections. The learning will then become meaningful, as opposed to mechanical memorizing and repetitions (Imsen, 1993).
- Learning is socially influenced, i.e. the learning is not an isolated process. Communication with other human beings, culture and language are influencing. For example, the everyday usage and meaning of words like e.g. "force" and "work" play a role.

The puzzles may also contribute to formation of new conceptual structures. The newly gained knowledge may involve both small and large structural

changes, depending on how much of the previous knowledge of the pupils which is challenged.

Piaget divides the intellectual development in different stages (Imsen, 1993; Sjøberg, 1992):

- The sensory motory period (about 0 - 2 years)
- The preoperational period (about 2 - 7 years)
- The concrete operational period (about 7 - 11 years)
- The formal operational period (from about 11 years)

However, a series of investigations show that many young people between 13 - 17 years do not think formal operational (Imsen, 1993). Most of the puzzles presented here, as a basis, are fairly concrete or specific. That is, the puzzles may also be utilized in the primary school. Nevertheless, the nature of the puzzles makes them highly suitable also in youth school, secondary school, college and university. With other words, *the puzzles are pretty age-independent*. In addition, many of the puzzles may be explored more in depth, e.g. with calculations with symbols and algebra, i.e. requiring a more formal operational thinking. Thus, the puzzles require and promote both a concrete operational and a formal operational thinking.

The puzzles are strongly anchored to the visual perception. Often one may simply *see* the solution. To be able to create *images* in different contexts has always been very important in the learning process. One example may be the atomic theory. If one could not have an image of how these atoms look like (even if it may be far from the actual “reality”), it would have been much worse to preserve the knowledge in memory, utilize the knowledge or apply the knowledge for further learning. Hence, images may then contribute to an improved learning. Kaufmann states that, concerning new and unknown matter, support through visual conceptions may lead to better results in learning (Imsen, 1993). As well known, the puzzles do contain unknown elements. It is referred to the work by Imsen (1993) for more information about the significance of images, also versus the language, in the learning process.

We may for now end this section with the following words of wisdom: “*The more we know the more we know we don’t know. And the more we want to know... and that’s the whole fun of it, scientific research included!*”, representing a modified extrapolation from a quote by Aristotle, and which in our context was written as a general idea when through research attempting to develop new thermal insulation materials (Jelle et al., 2010), and thereafter also employed for the materials science research pathways and opportunities for building integrated photovoltaics (Jelle, 2016a).

Focusing on that attitude, one may find substantial amounts of motivation in the *unknown*. Thus, a development and learning path attempting at transforming the unknown into the known, and hence in that process discovering that the unknown is increasing considerably more than the known, may be a thrilling and rewarding path to follow.

4. The Art of Puzzles in the Learning Process

The nature of the puzzles makes them applicable in any class among students. Nevertheless, many teachers will put them into use during math and various science sessions, which may also of course be due to the background and interest of the specific teachers. The terms *mathematical puzzles* and *math nuts* are already well-known, and in addition to the motivational aspects, the employment of puzzles may also be grounded by didactical considerations in mathematics. A comprehensive examination of the psychology of learning mathematics is performed by Skemp (1993), whereas problem solving with methods and attitudes is treated by Polya (1988).

In mathematics there may be a need for substantial repetitive tasks, but nevertheless the math should not be pervaded by routine as the students will then perceive that as boring and hence lose their motivation. We may cite directly from Davis (1989): "... teaching mathematics as a collection of routine procedures to be learned by rote is to make class lessons extremely dull. This dullness is not inevitable. Treated for what it really is - an exploration of a rich world of possibilities and a persistent challenge - mathematics can be exciting, and mathematics classes can be fun." Children are playful by nature, hence we also need play in mathematics and in learning in general. The puzzles may play a welcome break and variation, and an instructive and interesting addition, in the general teaching. Thus, as these puzzles promote out-of-the-box and creative thinking, they may also make the whole learning process more interesting and fun.

Creativity and to find original solutions and solution methods are very important in mathematics and in the learning process in general, also including many other subjects than mathematics, and furthermore in scientific research. We may again cite directly from Davis (1989): "In some mathematics subjects, originality is the very thing we are trying to teach. Hence, if we eliminate originality, we destroy our main goal."

Davis is also advocating the need for recognizing originality among our pupils, and gives as an example a pupil who had invented his or her own method to perform subtractions, in a way that neither the teacher nor his or her acquaintances had ever seen before. The example reproduced in the following with $64 - 28 = 36$ demonstrates that it is possible "to take eight from four" in a rather elegant way (Davis, 1989):

$$\begin{array}{r}
 64 \\
 -28 \\
 -4 \\
 40 \\
 36
 \end{array}
 \quad (3)$$

When applying puzzles in the teaching we will with time discover many original initiatives from our pupils and students. It is very important to handle these initiatives in a good way. We have to show our students that we appreciate their

alternative solution methods, and that the path to the solution is just as important as the solution itself.

Fantasy, creativity and originality are a few keywords for the puzzles, which are also given as keyword goals in various subject curricula. Development within many scientific research fields may be attributed to a creative mentality and way of thinking. Naturally, it is important to emphasize how crucial e.g. fantasy, creativity and originality are to scientific research, or should we say to the art of science, for both students and the teachers, besides hard work of course. We will later see that the puzzles may contain all these aspects, e.g. both creativity and hard work. In this respect, the old saying that creativity is 1% inspiration and 99% perspiration should also be noted (Rowlands, 2011).

For more information concerning creativity related to science, mathematics, innovation and education it is referred to the available literature, e.g. the studies by Barrow (2010), Escultura (2012), Hadzigeorgiou et al. (2012), Lin (2011), Longshaw (2009), Pisanu & Menapace (2014), Schmidt (2011), Shaheen (2010), and Trnova & Trna (2014).

To guess or follow a hunch is often important in the learning process and in the foremost scientific research fields. Polya has been one of the leading spokesmen for this guessing in mathematics (Sinicrope, 1995). And to guess or to try to guess is an important feature in puzzle solving. When attempting to guess, the brain and its thoughts are forced, provoked and stimulated to seek for various solutions, including out-of-the-box thinking, and an evident solution may suddenly appear.

The puzzles include many exercise types, and within one single puzzle there may also be many variants, depending on how far in depth one wants to explore. The solution of a puzzle may often be seen immediately, i.e. in such cases the given puzzle will normally represent a *single step exercise* (and that single step may even be very short).

By taking a closer look at many of the puzzles, we will often discover many new aspects within these puzzles, i.e. which hence may be conceived as a special form of miscellaneous variations of a *multistep exercise*. Thus, the puzzles contain elements of both single step and multistep problem solving aspects. Some puzzles have a clear and specific *task definition*, while other puzzles may have a more open task definition. More open task definitions give more room for the students' own evaluations at different levels.

Normally, one would employ analytical and inductive methods to solve the puzzles, as a contrast to synthesis and deduction. However, when a solution has been found one may deduce many interesting aspects from the puzzle, its solving process and solution.

The students are exploring. They are trying, failing, rejecting the tried and failed solving strategy, and hence trying again. This trial and error experience is important for the students' learning process. They have to try to find alternative methods for reaching a solution. An insect like e.g. a fly may hit the same closed

window glass pane over and over again, not discovering that the neighbour window is open. An obstacle may be overcome by going around it instead of going through it. “*Do not forget that human superiority consists in going around an obstacle that cannot be overcome directly...*” (Polya, 1988).

Puzzle solving or cracking may invite to a more *heuristic* teaching method, contrary to the common from-teacher-to-student tuition with the teacher in a lecturer role. In a heuristic mindset the student is more of an explorer and researcher, i.e. not only a passive recipient of knowledge. By this reasoning the student as the active part is the ideal, even if it may require more of the precious time allocated for lessons, which in turn may lead us to the pedagogical “*learning by doing*” philosophy by John Dewey, i.e. “learn to know by doing and to do by knowing”.

Young is arguing that in the long run one will save most time by applying a heuristic method, and refers to a striking example: “*Before a child can walk time would be saved by carrying it across the room, but in the end it gets about by learning to walk*” (Young, 1924). Polya is stating further: “It is emphasized that all sorts of problems, especially practical problems, and even puzzles, are within the scope of heuristic” (Polya, 1988). For more thoughts and discussions concerning methods of solving it is referred to the studies by Polya (1988) and Young (1924).

Students will often perceive these puzzle exercises as more practical tasks. We may then be reminded by Bernal’s words: “*Practice without theory is blind, theory without practice is sterile*” (Bernal, 1954; 1978).

Polya (1988) addresses the aspect of solving mathematical exercises within his famous and well-acclaimed book “How to Solve It”. Nevertheless, even if the original foundation in this book is mathematics, very much if its core message and in addition several more detailed philosophies may be used in general as problem solving strategies. Polya states a procedure for problem solving, which consists of a series of questions the problem solver are supposed to ask himself or herself, e.g. “what is the unknown?”, “what are the data?”, “what is the condition?”, “have you seen it before?”, “do you know a related problem?”, “could you imagine a more accessible related problem?”, “a more general problem?”, “a more special problem?”, “did you use all the data?”, “did you use the whole condition?”, “have you taken into account all essential notions involved in the problem?”, “can you see clearly that the step is correct?”, “can you prove that it is correct?”, “can you check the result?”, “can you check the argument?”, “can you derive the result differently?”, “can you see it at a glance?”, “can you use the result, or the method, for some other problem?”, etc.

All of these questions are more or less obvious, but it is not just as obvious that we will actually ask ourselves these questions so thoroughly each time we are confronting various obstacles during our problem solving. Hence, it is important to have a conscious awareness of these questions, which may also often be applied beneficially when solving the discussed puzzles herein and similar

puzzles, even when including the more single step exercise puzzles.

Several aspects of problem solving are discussed by Polya (1988), and he does also make use of several good analogies. Among them is the term or concept “*stepping stone*”. If you are going to cross a river or a creek you may do that by getting to a stone in the middle of the river. Analogously, in mathematics it is often easier to first solve a more familiar problem (e.g. by introducing a new unknown), which then may help you towards a solution of the original problem. Polya (1988) takes this analogy further: “A primitive man wishes to cross a creek; but he cannot do so in the usual way because the water has risen overnight. Thus, the crossing becomes the object of a problem; “crossing the creek” is the x of this primitive problem. The man may recall that he has crossed some other creek by walking along a fallen tree. He looks around for a suitable fallen tree which becomes his new unknown, his y . He cannot find any suitable tree but there are plenty of trees standing along the creek; he wishes that one of them would fall. Could he make a tree fall across the creek? There is a great idea and there is a new unknown; by what means could he tilt the tree over the creek?” Hence, this primitive man “may become the inventor of the bridge and of the axe” (Polya, 1988).

Even if there are some large differences, do not these “*stepping stones*” and the “*fallen tree*” (or axed-down tree) also bear a close resemblance to Ausubel’s “*advance organizers*” and “*cognitive bridge*” as mentioned earlier? To solve a more familiar problem may also help with puzzle solving, but it is not a typical characteristic particularly for the puzzles. It is however a general characteristic or feature for all problem and puzzle solving. And it is a general characteristic that puzzle solving may promote ideas and inspiration for problem solving in general.

Polya (1988) discusses further that the students need to learn to struggle with problems: “Teaching to solve problems is education of the will. Solving problems which are not too easy for him, the student learns to persevere through success, to appreciate small advances, to wait for the essential idea, to concentrate with all his might when it appears. If the student had no opportunity in school to familiarize himself with the varying emotions of the struggle for the solution his mathematical education failed in the most vital point.”

We may also find very similar ideas in e.g. an earlier Norwegian national curriculum (KUF, 1993): “The most important of all pedagogical tasks is to communicate and pass on to children and young people that they at all times are developing, so they gain confidence in their own abilities. A good teacher also enhances their perseverance for enduring exertion and strain, so they will not at once give way if they do not succeed at first attempt. A teacher is therefore promotor, guide, conversation partner and director” (translated from Norwegian).

The puzzles do contain many of the above mentioned varieties, including both hard work and perseverance (even if a solution may also appear very fast), the

art of going around obstacles instead of facing them head-on, and to think and go outside the accustomed paths, i.e. so-called out-of-the-box thinking.

Importantly, experience and familiarity with these puzzles and their solving process and solutions, may increase the motivation and mastering apprehension, expand the ability to engage and solve miscellaneous challenges from various viewpoints, and may hence ultimately improve the learning process and enhance the problem solving capabilities, and thus also lead to increased knowledge in general.

5. Puzzle Examples

5.1. The Hole Digging

Puzzle description: 5 men dig 4 holes in 3 days. How long time does one man use to dig half a hole?

The solution here is that there is no such thing as half a hole, as half a hole is still a full and complete hole. The philosophy is that it is important to read the task description carefully before starting to solve it, that is to think before acting. That is very important in many problem solving cases including scientific research, and also in various everyday tasks.

You may also say that this puzzle may show the difference between a physicist and a mathematician, where the mathematician is presumed to start calculating at once the exact fraction, whereas the physicist knows that when digging half a hole it will still be a complete hole, although the mathematicians will probably not agree in such a prejudiced assumption.

From the problem solving procedure of [Polya \(1988\)](#) we have: “*Examine the solution obtained. Can you check the result? Can you check the argument? Can you derive the result differently? Can you see it at a glance? Can you use the result, or the method, for some other problem?*” If we check what it means to dig half a hole, many of us will probably react and maybe catch the point at an early stage. That is, it may be wise to think before starting a lot of calculations. [Polya \(1988\)](#) states clearly: “*You have to understand the problem. What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? Draw a figure.*”

If we had asked ourselves a few of these questions, then many more of us would have discovered much earlier that we can not dig half a hole as it is still a full and complete hole.

In this respect, we also see the well-known power of first drawing a sketch when attempting to solve a challenge. Hence, here we might have discovered the solution after just drawing a simple circle, i.e. the hole.

5.2. The Equation

Puzzle description: Solve the expression: $(x - a)(x - b)(x - c) \cdot \dots \cdot (x - z) = ?$

Naturally, when presented in a large audience, it is of course important that the solution is not spoken out loudly when people start to discover it. This exercise is a good example of a puzzle where the solution may be discovered immediately or after some hard (and rather tedious) work by writing up all the brackets until the end, where most people discover that the whole expression is identical to zero as they reach the special bracket $(x - x)$ which of course is zero, hence making the whole expression zero. Also noteworthy, is that some students in fact manage to write up all brackets, including the $(x - x)$ bracket, without disclosing the solution.

Thus, this example may then be related to scientific research where both immediate discoveries and inventions, and hard and painstakingly work, are essential ingredients and success factors. Again, the old saying that creativity is 1% inspiration and 99% perspiration should be noted (Rowlands, 2011).

We may let Polya (1988) tell us a story: “*Yet signs may also be deceptive. I once abandoned a certain path for lack of signs, but a man who came after me and followed that path a little farther made an important discovery—to my great annoyance and long-lasting regret. He not only had more perseverance than I did but he also read correctly a certain sign which I had failed to notice.*”

As in the above puzzle it is important to have perseverance, do not give up and go a bit further, even if it at times may seem futile and pointless, and to keep the eyes and mind open so one does not miss the solution when it appears in front of you.

5.3. The Triangle and the Square

Puzzle description (see also Figure 1): The Triangle: [An equilateral triangle of 3 matches is made in advance.] Make 4 identical equilateral triangles as this one (same size also) out of a total of 6 matches. The Square: [A square of 4 matches is made in advance.] Make 6 identical squares as this one (same size also) out of a total of 9 matches.

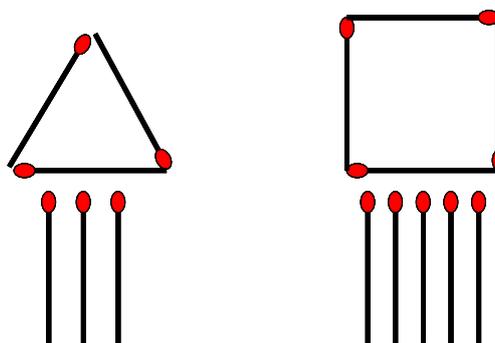


Figure 1. The puzzle description is as follows: The Triangle (left): Make 4 identical equilateral triangles as the one shown above (same size also) out of a total of 6 matches. The Square (right): Make 6 identical squares as the one shown above (same size also) out of a total of 9 matches.

Occasionally, the triangle and the square puzzles may be given as puzzles to be solved by using a pencil. However, the triangle puzzle will then involve an extra challenge as soon will be evident. Solving either the triangle or the square first, and then either with matches or a pencil, may make it either easier or more difficult to solve the other one.

The solutions are depicted in **Figure 2**, illustrating two different ways to achieve a solution by thinking in three dimensions. The triangle and the square puzzles represent puzzles with out-of-the-box solutions, as most people try to solve these ones by keeping themselves restricted to two dimensions, i.e. to the table or paper surface. Furthermore, both the triangle puzzle and the square puzzle may be solved in a few seconds or after several weeks or months (or never).

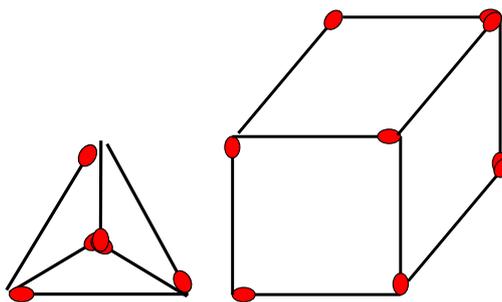


Figure 2. Solutions of the triangle and the square puzzles, which both are solved by thinking in three dimensions in two different ways, i.e. in real three dimensions for the triangle thus constructing a pyramid with four triangles, whereas a perspective drawing of matches on a flat two dimensional surface makes up an image of a three dimensional cube with six square surfaces.

We may again cite from **Polya (1988)**: “*Have you seen it before? Or have you seen the same problem in a slightly different form? Do you know a related problem? Do you know a theorem that could be useful? Look at the unknown! And try to think of a familiar problem having the same or a similar unknown. Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?*”

To see similarities and dissimilarities between tasks, in this case the different three dimensional aspects with the triangle and the square puzzles, may often lead us on to the path to the solution.

5.4. The 9 Dots

Puzzle description (see also Figure 3): [A square of 9 dots is made on paper in advance, where there are three rows consisting of three dots each with equal distances between all dots, i.e. there will be one dot in the middle of the square surrounded by eight dots on the sides of the square.] Draw 4 straight lines without lifting the pencil where you are striking all the 9 dots in the figure.

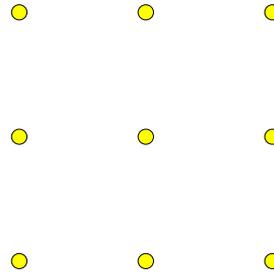


Figure 3. The puzzle description is as follows: Draw 4 straight lines without lifting the pencil where you are striking all the 9 dots as shown above.

The 9 dots puzzle represents a very good example of a puzzle with an out-of-the-box solution. The whole clue is to not let yourself be limited to only the paths within the nine dots, i.e. square-minded, but on the contrary break loose and draw outside the square (“box”) formed by the dots, as shown in the solution in **Figure 4**.

Do also note that this puzzle may be given on small or medium-sized square paper note (e.g. a post-it note) filling out all of the note, i.e. making it even harder to think out-of-the-box, i.e. greater boundaries to cross, both physically and mentally.

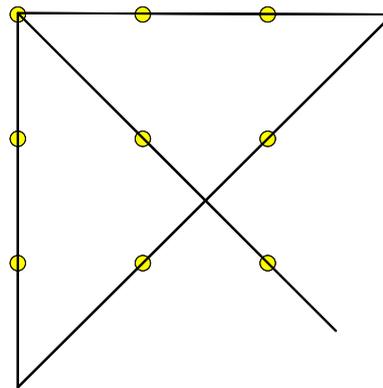


Figure 4. Solution of the 9 dots puzzle, which obviously represents an out-of-the-box solution.

5.5. The Cake Division

Puzzle description (see also Figure 5): [Preferably carried out with three real cakes with topping.] Cut the cake in eight identical pieces using only three one-motion straight cuts with a knife.

The cake division puzzle, also called the cake sharing, the cake nut or the nut cake, is indeed a very fun, instructive and rewarding puzzle to carry out in a large gathering with people working in teams competing with each other using real cakes with topping. Again, the puzzle solving involves out-of-the-box thinking, in addition to practical thinking and skills.

When solving the cake division puzzle on paper, i.e. not using real cakes, the difference of presenting the puzzle as in **Figure 5** or **Figure 6** should be noted,

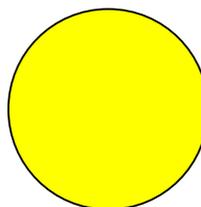
the latter one making it considerably easier to solve the puzzle, i.e. with respect to out-of-the-box thinking the box has already been shown in **Figure 6**.

Normally, the solutions will be discovered in the order given in **Figure 7**, if not it will be the lecturer's responsibility to accommodate that this will be the visible order among the participants. Finally, in order to be able to solve the last (known) solution, it is important that the lecturer has hidden a long sword in the back of the room which the victorious team can borrow to cut the third and last cake.

The difference of the three solutions when the cakes have topping should be noted, and these differences will be extra visible when real cakes with topping are used. The cake division and sharing may also become quite messy, and very fun indeed. It has been observed, though, that some adults have in fact become very irritated, even angry, because they were picturing themselves with rather bad cake pieces in the end, especially the bottom cake pieces from solution 1 as these people desperately wanted their toppings (**Figure 7**).

Solution 1 is the official solution found in the key book, whereas solution 2 was invented by a student in secondary school during one of the author's classes. Naturally, others may also of course have found the same solution before this student. Solution 3 was invented by another student in secondary school during another one of the author's classes, in fact by a student who studied for the culinary profession, i.e. he had obviously handled a very long knife before.

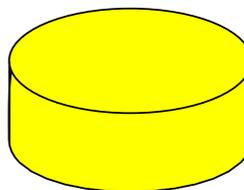
The Cake Nut - A Nut Cake ?



 x 3 = 8 identical cake pieces

Figure 5. The puzzle description is as follows: Cut the cake as shown above in eight identical pieces using only three one-motion straight cuts with a knife.

The Cake Nut - A Nut Cake ?



 x 3 = 8 identical cake pieces

Figure 6. The puzzle description is as follows: Cut the cake as shown above in eight identical pieces using only three one-motion straight cuts with a knife. This 3-dimensional cake drawing is not supposed to be shown before the first solution has been found.

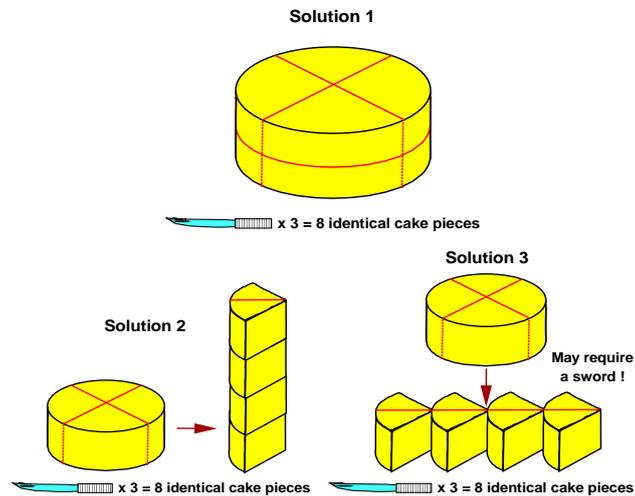


Figure 7. The three known solutions of the cake nut. Normally people, alone and in larger groups, will find the solutions in the sequence solution 1, solution 2 and finally solution 3.

5.6. The Moat

Puzzle description (see also Figure 8): Get over the moat in a safe way using two logs where each log is just too short. [See figure and description in the following.]

The moat puzzle may be solved in a matter of seconds, or several days or weeks. In a large audience the moat puzzle usually triggers many creative solutions, all of them forbidden though, and the lecturer should make sure that persons knowing the real solution do not spoil the fun for the others, e.g. by instructing all not to tell the solution if they are 100% sure they have got the right solution.

To the story goes that the author himself never got the chance to solve this puzzle as he and a classmate stood in front of the blackboard when the teacher presented the puzzle for them, and my classmate drew immediately the solution as she knew it from before. Lost opportunities, now I will never know if I would have used seconds, hours or weeks to solve this puzzle.

One crucial aspect with the moat puzzle is that it illustrates very convincingly the importance of looking at the problem from different angles or viewpoints (also literally in this case), as depicted in **Figure 9**. To try other

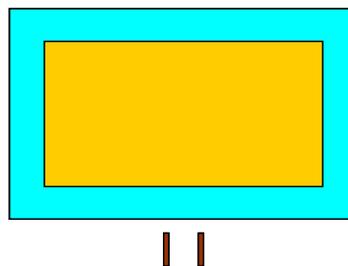


Figure 8. The puzzle description is as follows: Get over the moat as shown above in a safe way using two logs where each log is just too short.

attack points and solving strategies are also very crucial to scientific research and exploration.

Applying some of the earlier mentioned problem solving strategies, and questions to ask oneself, as given by [Polya \(1988\)](#) may also be very helpful when attempting to solve puzzles like this one. For example, when solving geometrical math exercises in youth and secondary school, many of these ones may often be solved much faster (or in fact solved at all) when turning around the paper sheet with the math exercises and looking at the geometrical figures from different angles.

The solution of the moat puzzle, which is proven by applying the Pythagorean theorem, is depicted in [Figure 10](#). Note also the difference depending on what side of the moat you are situated on, i.e. when placing the two logs.

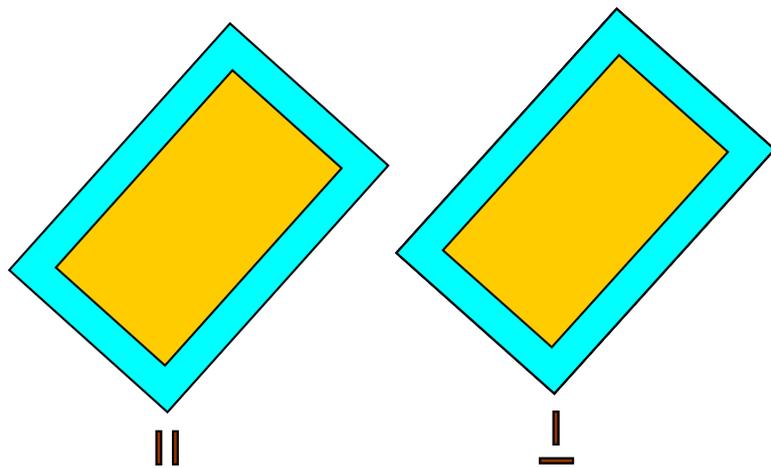


Figure 9. The puzzle description is as follows: Get over the moat as shown above in a safe way using two logs where each log is just too short. These two different perspective figures are not supposed to be shown before most people have had the opportunity to try to solve the puzzle based on the original figure as depicted in [Figure 8](#).

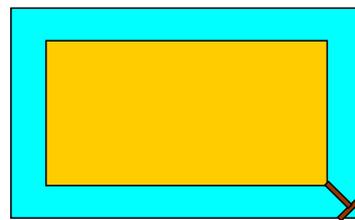


Figure 10. Solution of the moat puzzle, which is proven by applying the Pythagorean theorem.

5.7. The Earth and the Marble

Puzzle description (see also [Figure 11](#)): A cord or a rope is placed tightly around the Earth at equator. If this cord is prolonged with 6 m and lifted above the ground at the same height around the whole equator, how high above the ground will the cord be situated? Will it be possible to press a thin sheet of paper under the cord? And what if we have a small marble instead of the Earth, and

also prolong this cord with 6 m, what will be the height (above the marble’s surface) of the cord now?

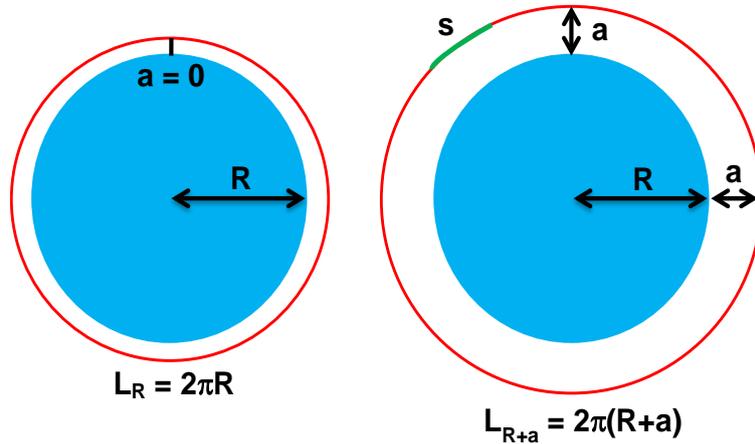


Figure 11. The puzzle description is as follows: A cord or a rope is placed tightly around the Earth at equator. If this cord is prolonged with 6 m and lifted above the ground at the same height around the whole equator, how high above the ground will the cord be situated?

Denoting the Earth’s radius, or the marble’s radius, as R , the cord prolongation as s , and the height above the surface as a , we may write the circumference in two ways and thus equalise these two expressions as follows (see **Figure 11**):

$$L_{R+a} = 2\pi(R + a) = 2\pi R + s \tag{4}$$

which hence gives the following solution:

$$a = s / (2\pi) \tag{5}$$

That is, the height a is only dependent on the cord prolongation s , thus the height a is independent of the radius R so it does not matter if it is the Earth or a small marble. If $s = 6$ m we obtain $a = (6 \text{ m}) / (2\pi) \approx 0.95 \text{ m}$.

Many students will protest wildly first time they see this result as it goes against their common sense. In addition to the surprising answer, this puzzle is valuable in demonstrating the strength of calculation with symbols in mathematics.

In order to be completely sure about the solution we may again listen to the words of **Polya (1988)**: “*Carrying out your plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?*” And we should also make our students listen to the very same words, and to follow them.

5.8. The Paper Sheet Folding

Puzzle description (see also Figure 12): Fold a paper sheet 100 times. Press out all air between the paper sheets. Put the paper pile on the table in front of you. Guess how far above the table the paper pile will reach?

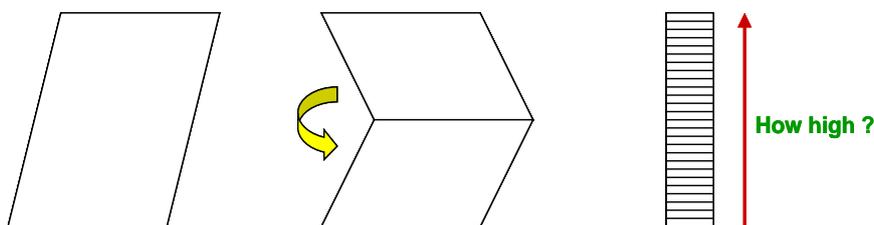


Figure 12. The puzzle description is as follows: Fold the paper sheet as shown above 100 times. Press out all air between the paper sheets. Put the paper pile on the table in front of you. Guess how far above the table the paper pile will reach?

As already mentioned earlier the paper sheet folding, is not really a puzzle like the others treated here. Nevertheless, as this task with its solution presents a nice teaching opportunity with a very intriguing and thought-provoking solution, it is included herein. In fact, it has also been used in lectures for illustrating some specific scientific research fields, where one may utilize mathematical and physical functional expressions increasing or decreasing rapidly with specific experimental parameters when attempting to manufacture and tailor-make new materials. A specific example of this is the utilization of the so-called Knudsen effect and equation when exploring the possibilities to make nano insulation materials (NIM) with very low thermal conductivity (Gao et al., 2013; Gao et al., 2014; Gao et al., 2015; Jelle et al., 2010; Jelle, 2011; Jelle et al., 2011; Jelle et al., 2014; Jelle, 2016b; Sandberg et al., 2013).

The paper sheet folding task works very nice for larger audiences, e.g. in a classroom or auditorium. When presenting this task for larger audiences, normally there will be one who has heard about this before and speaks out a very large value at some stage during the questioning and answering/guessing interaction with the class/audience (typical to the moon as a similar exercise has that as solution). However, with an experienced lecturer this represents no problem, on the contrary rather an opportunity to make the audience burst out in laughter when you tell the culprit to go to the fool's corner for having suggested such a ridiculous large distance. Now and then someone actually calculates a very large value (which may or may not be correct), as the lecturer you have to smoothly brush aside and disguise these attempts at the moment and keep the focus on the normal guessing activity in the audience. The poor laughing stocks will nevertheless soon regain their honour.

A typical (and good) scenario is that the audience starts guessing many rather low or small distances. The audience will commonly guess values from a few centimeters to a few hundred meters, with some brave ones going as high as a few kilometers, and then the moon guy of course. When the guessing is ebbing out the lecturer may start with the common accepted high value among the audience (e.g. a few kilometers) and tell that the distance is not large enough, and then continue with ever increasing distances telling repetitively that this distance is still not large enough, while continuously giving references the audience may relate to, e.g. the moon, Mars, the Sun, Jupiter, Pluto, the nearest star outside the

solar system (Proxima Centauri, i.e. Alpha Centauri C), the outskirts of our galaxy the Milky Way, the nearest major galaxy (Andromeda), the outskirts of our galaxy cluster, the outskirts of our super galaxy cluster, and finally, at the edge of our known Universe, i.e. a tremendous and inconceivable large distance for a thin paper sheet folded only 100 times. And the lecturer may add to an astonished and laughing audience "... and you may wish I was there now..." (at or even beyond the edge of the Universe).

The actual solution is $t \times 2^{100}$, i.e. the thickness t of the paper sheet multiplied with 2 up in hundred, that is, a doubling of the paper sheet thickness a hundred times. With a thickness of 0.1 mm the actual distance or height of the paper tile is about 1.27×10^{26} m, which may be compared to the distance to the edge of the Universe of about 1.23×10^{26} m assuming a spherical expansion with the velocity of light (approximately 3.00×10^8 m/s) from a singularity and outwards during 13×10^9 years after Big Bang, i.e. the radius of the Universe. Note that this calculation of the radius of the Universe is for illustration purposes only, e.g. as the Universe may be viewed as located on the surface of an expanding balloon, hence giving other dimensions and distances. As a last remark, with respect to out-of-the-box thinking and solutions, it is clear that the paper folding has an extreme out-of-the-universe solution.

5.9. Other Puzzle Examples

Naturally, many other puzzles and nuts than those presented herein exist. Within this context it was only room enough for presenting a few selected examples, hopefully illustrating the power, versatility and pure magic of these and similar puzzles.

A further extension and more comprehensive version of this work is planned, which will cover many other puzzles in addition to the ones already covered herein. As a prelude to the anticipated forthcoming work, and as inspiration and to facilitate an interested reader's yearning and quest for new puzzles, the following other puzzle example names are given: *The chess board and the grains*, *The number between 1 and 1000*, *The series sum*, *Think of a number*, *The equation with the 5's*, *The ten coins*, *The rope cutter*, *The road to freedom*, *The train trip*, *The bunker*, *Coffee and milk*, *Ice, sand and water*, *Drop in a liquid*, *The electrical lamp circuit*, *Gasoline and the exhaust gas*, and *The human electrons*.

Miscellaneous puzzles, including some of the above, may be found in the literature, e.g. in the works by Benestad et al. (1994ab), Hag (1992), Jelle (1995b), Kordemsky (1990), Kvam et al. (1989), NCTM (1993), Riiser (1990) and Schackt (1988). Thus, this section may be ended with the following homage and modification of William Shakespeare's famous words from Hamlet, i.e. *To be cracking nuts or to become cracking nuts...*

6. Empirical Results and Discussion

The puzzles covered within this study and miscellaneous other puzzles have

been used as an extra ingredient at several occasions and different teaching situations, e.g. for pupils and students in primary school, in secondary school, at university, at seminars and workshops, and at scientific conferences, hence including both classroom and lecture situations.

For example, during a seminar when a music band in the break between lectures has just been singing the stanza “You lift me high”, and you are up next with a lecture about how to make very small nanopores amount to very much (i.e. substantially increased thermal resistance), you can not resist the temptation to ask how high above the stage floor a paper pile will reach after folding a paper sheet 100 times (see the earlier described paper sheet folding puzzle). And when finally revealing the tremendous distance, i.e. beyond the edge of the Universe, you are attempting to sing the very same stanza with a singing voice only a mother could love.

In general, at all levels and situations, the presented puzzles have been received with curiosity and enthusiasm. However, it will be too extensive to go into all the experiences and empirical results gained from these within the context of the work discussed herein.

Nevertheless, a short excerpt from a study conducted during math classes in a secondary school class will be given here (Jelle, 1995b). Several of the puzzles were also utilized at the same time in a natural sciences class, where the emphasis of the educational study was on the art to dare to make a fool of oneself in physics and to learn from those experiences (Jelle, 1995a).

The general impression was that a large majority of the students liked these puzzles very much. An anonymous survey among the students did also confirm this general impression. Out of a total of 21 students in the class, 19 students (90%) wanted to continue with the puzzles, whereas 2 students (10%) did not. 12 students (55%) wanted to use more time on the puzzles, 8 students (36%) were of the opinion that the actual time usage was appropriate, while 2 students (9%) wanted to use less time on the puzzles (percentages calculated from 22 students as one student had selected two choices).

When the students were asked about the desired frequency for the puzzle solving or nut cracking, 9 students (39%) wanted puzzle solving each class lesson, 12 students (52%) wanted puzzle solving once per week, 2 students (9%) wanted puzzle solving once per month, and no students (0%) did not want any puzzles at all (percentages calculated from 23 students as two students had selected two choices).

The students in this class emphasized variation, relaxation, recreation, entertaining, enjoyable, outside ordinary mathematics, *think beyond normal borders*, and *learning to think differently*, as keywords for the puzzles. The comments “think beyond normal borders” and “learning to think differently” should especially be noted, e.g. with regard to research on developing new thermal insulation materials, “which requires that we may have to *think thoughts not yet thought of*” (Jelle et al., 2010). Thus, we may also be reminded about the follow-

ing quote from Albert Einstein (various versions exist): “*Imagination is more important than knowledge. Knowledge is limited. Imagination encircles the world.*” (Longshaw, 2009).

Some students are also mentioning that the puzzles are using of the time allocated for the ordinary mathematics tuition (the written survey forced all the students to write at least minimum one positive and one negative thing about the puzzles). In spite of this last fact, and the increasing pressure on students with respect to grades and curriculum, almost all of the students in this class wanted to continue with the puzzles.

Furthermore, a majority of the students did also want to spend more time on the puzzles. For further details from this specific class and the survey it is referred to the study by Jelle (1995b).

7. Conclusion

Applying puzzles in the teaching may increase the motivation, enhance the mastering apprehension, promote the creative processes, expand the ability to engage and solve miscellaneous challenges from various viewpoints, and hence lead to an improved learning process and problem solving capability. Teachers of both students and teachers may therefore find it beneficial to employ the art of puzzle solving in their classrooms. Several selected puzzle examples are presented and discussed to illustrate these aspects, also with background in theories and investigations on motivation, mastering, development and learning.

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