

Pre-Service Teachers' 3D Visualization Strategies

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Abstract

This study examines primary and early childhood pre-service teachers' strategies on a written task that promotes 3D geometric thinking and visualization processes. Visualization and conceptualisation of 3D objects are complex cognitive processes, and both require the development of students' abilities to decode and encode spatial information. The analysis of 289 pre-service teachers' written responses resulted in identifying students' difficulties in decoding and encoding visual information. The visual information dominated student thinking, and they found it hard to identify relationships between the 2D representation and the 3D mental construction of the solids. Most made incorrect claims regarding relative volumes. Neither spatial visualisation nor formula-driven computation provided adequate engagement with the task. Visualization and conceptualisation of 3D objects are complex cognitive processes, and pre-service teachers need to engage with a variety of learning activities to help them develop their abilities to decode and encode spatial information and, it is hoped, develop their 3D geometric thinking. However, from a learning approach perspective, the results indicate a dominant surface learning approach; this may arise from prior inadequate learning. The best lesson the student may get, therefore, from this task is not the mastery of a mathematical computation, but awareness of the importance of teaching design and aligned teaching methods.

Keywords

3D Geometric Thinking, Visualization, Pre-Service Teachers, Teacher Education

1. Introduction

A number of research studies have focused on children's thinking on three-dimensional (3D) solids. Most stud-

ies appear to be on the nets of solids (Mariotti, 1989; Stylianou et al., 1999; Cohen, 2003), on their plane representations (Cooper & Sweller, 1989; Ma et al., 2009), and on the constructions of the solids by unit cubes (Battista & Clements, 1996; Sack, 2013). The National Council of Teachers of Mathematics (2000) highlights the importance of the development of students' 3D geometric thinking. Students' 3D visualization capacity is directly related to their abilities to identify 3D objects and their properties, to represent and compare 3D objects based on their properties, and to calculate the surface area and the volume of the 3D objects. Developing children's ability to conceptualise 3D objects is an important part of geometry and spatial learning (Battista, 2007) teachers' ability to teach this is important. The focus of this study is on the investigation of primary and early childhood pre-service teachers' strategies on a written task that promotes 3D geometric thinking.

2. 3D Visualization Abilities

The development of visualization and the reasoning based on mental images, are associated with the improvement in geometric thinking (Hershkowitz, 1989; Wheatley, 1990; Duval, 2006). In our present study, visualization is considered as the kind of reasoning that is based on the use of visual space elements, either mental or physical (Gutierrez, 1996; Presmeg, 2006). Wheatley (1990) maintains that the process of visualization includes the construction of a mental image, its representation and, when necessary, its appropriate transformation. The extent to which this visualization process is considered to contribute to the development of mathematical thinking depends on whether it leads to abstraction and generalization. Lowrie (2012), analysing students' visual and spatial reasoning, highlights the importance of the process of decoding and encoding information of a given mathematical task. Students make sense of (internalise) the task by decoding the visual information and then encode this in producing their own mental representations. In the case of a 3D geometry task, the process of decoding seems to be related to a number of student capabilities. These include: identifying 3D shapes in various representations (3D or net); focusing on their components parts; investigating their properties; making relationships between different 3D objects based on their properties. On the other hand, the process of encoding includes students' capabilities to construct their own mental representations based on their decoding results (Pitulis & Christou, 2010).

3. Methodology

Two hundred and eighty nine (289) first year undergraduate primary and early childhood pre-service teachers from a regional Australian university participated in this study. The students are predominately female (80%), young (median age 19 years old), regional and remote (74%), first-in-family (64% university-wide), low socio-economic (31%); while low on self-identified disability (4%), Indigenous Aboriginal or Torres Strait Islander background (2%) and Non-English Speaking Background (1%). The students were enrolled in a Foundational Mathematics course that focuses on the development of students' mathematical knowledge for teaching. In Australia, primary and early childhood education students are expected to have a level of numeracy broadly equivalent to the top 30% of the population (AITSL, 2011). As such, they should acquire a solid understanding of basic mathematical concepts and the ways these are connected. The rationale behind the Foundational Mathematics course is that prospective teachers should know the mathematical topics and procedures they are going to teach. Furthermore, they should develop skills of mathematical reasoning and communication, and be able to demonstrate competence in evaluating and interpreting students' work (Ma, 1999; Hill et al., 2005; Mason, 2008). Moreover, they should be able to identify students' difficulties and misconceptions, and should be able to provide students with examples and models. Finally, they should be aware of problem-solving strategies, and be able to apply these in various problem-solving situations. It is important for students to be familiar with the use of hands-on and virtual manipulatives, and to present the mathematics concepts in a meaningful way.

The main topics of the Foundational Mathematics course are: problem solving, numeration systems, number theory, real numbers, pre-algebra, measurement, geometry, probability and statistics. Each of ten teaching weeks includes a one-hour lecture and a two-hour tutorial. In tutorials, students are engaged in hands-on and digital activities in order to develop their understandings regarding the various topics. An online component is also available for self-directed study. This comprises more than 100 online activities for each weekly topic, and allows students to practice skills extensively. Finally, each student must participate in a final two-hour written examination, which includes twenty-five questions covering the whole range of the teaching topics. In the present study, students' responses to two of the exam questions are analysed.

3.1. Tasks

In this study, students' responses to the following task, a task that test visualization and 3D geometric thinking, are analysed.

A rectangular piece of paper (Figure 1) can be rolled into a cylinder in two different directions. If there is no overlapping, which cylinder has the greater volume, the one with the long side of the rectangle as its height, or the one with the short side of the rectangle as its height, or will the volumes be the same?

The task addresses different types of 3D geometry thinking. Students were asked to visualise both cylinders based on their net, and to compare them in terms of their volumes. Conceptualising the representation of a 3D object is considered a crucial dimension of 3D geometric thinking. This task involves the ability to reconstruct the cylinders from the net and the ability to focus on the properties of the solids and compare the solids in terms of their properties. Volume, as the property of a solid, is related to 3D geometric thinking. Battista (2007) argues that students' ability to conceptualise the volume of a solid is not only related to the implementation of the volume formula and the numerical operations, but also to the visualization of the structure of the solid. In this exam task, the process of visualizing the structure of the solid is strongly related to the process of decoding and encoding visual and spatial information (Lowrie, 2012; Pittalis & Christou, 2010). Students need to decode the external representation of the rectangular sheet, focus on its properties, visualise the construction and build relationships between the two-dimensional (2D) representation and the 3D mental representations of the two cylinders. Considering Battista's (2007) assumption, we also take into account the students' correct responses to a second exam question, a question that involved just the numeric calculation of cylinder. This second task was to "Find the volume of a tin of beans of base radius 5 cm and height 14 cm".

The comparison of students' responses to those two tasks allows us to test whether just knowing the volume formula and associated numerical operations is, alone, sufficient to resolve such types of 3D tasks, or whether a visualization process is also required.

3.2. Analysis of the Data

The data consists of the 289 students' written responses to these two tasks. Analyse the responses to the first task were made, first, through coding the types of reasoning provided by each student. Then, by scrutinizing the data line by line, we identified aspects of students' geometric thinking, and we formed function categories. Finally, we re-examined students' responses to both tasks, looking for possible connections between students' ability to calculate correctly the volume of a cylinder and their ability to visualise two cylinders based on their nets and to compare them in term of their volume.

4. Results

The first level of analysis of the students' written responses to the first task resulted in the formation of two large categories: those responses that included a justification, and those that present only the answer without any kind of reasoning. 81 students answered the task without providing any reasoning, while 208 students justified their answer. Four students left the task unanswered. Importantly, 81 students provided no sign of working out or investigation; answers were limited to one sentence responses. Those responses are distributed into three subcategories.

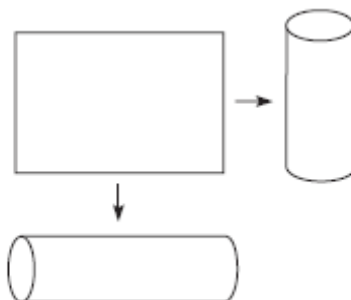


Figure 1. A rectangular piece of paper that can be rolled into a cylinder in two different directions.

- 65 students declared that the two cylinders have the same volume. Representative answers were: “The volumes will be the same” and “Both cylinders will have the same volume”. However, in just two cases, the answers revealed some kind of uncertainty, as students stated that, “the volumes should be the same”.
- Only ten students answered that the cylinder with the short side of the rectangle as its height would have the greater volume.
- Six students answered that the cylinder with the long side of the rectangle as its height would have the greater volume. Students’ responses followed the same pattern, copying the wording from the question, as for example, “the one with the long side”, “The greatest volume will have the cylinder with the longer side as its height”, and “volumes will be different, the cylinder which has the long side of the rectangle as its height will be greater”.

It is interesting at this point to compare students’ answers to the first task with their corresponding answers to the second one. Checking corresponding students’ answers to the second task, nearly 50% of the students from each subcategory were able to recall the volume formula and applied it correctly. The rest of the students could neither recall the volume formula of a cylinder nor apply it properly. **Table 1** shows the distribution of the three subcategories of students’ responses to the first task and the corresponding number of correct responses to the second task.

Students who provided their reasoning for the first tasks fall into two broad subcategories: (a) those who tried to justify their answer based on the application of the volume formula of the cylinder; and (b) those who based their reasoning on the identification and interpretation of solid’s properties. In other words, students’ justifications for the first task included an algebraic approach or a written description of their attempts to visualise the construction of the cylinders and make connections between the component elements of the solid and its properties. The written description approach is similar to the process of coding and decoding of visual information based on external representation (Lowrie, 2012).

A total number of 58 students justified their answers to the first task by using the cylinder volume formula. Of these, 13 students tried to apply the volume formula, and ended up claiming that the volume of both cylinders was the same. Nearly half of the 13 seemed to successfully recall the formula of the cylinder’s volume in the second task. In those cases, students correctly recalled the formula, and took two arbitrary values for the length and the width of the rectangle and tried to apply the formula for each cylinder. However, instead of conceptualising the two dimensions of the rectangle as the circumferences of the bases, and consequently calculating the radius of each base, they assumed that the radius of each base is half the length of the each side of the rectangle. They, therefore, partially applied the formula, and concluded that the volumes were the same.

On the other hand, 12 students unsuccessfully applied the formula, and concluded that the cylinder with the long side as its height would have the greater volume. Although eight of those students seemed to be aware of the formula in the second task, they were not able to apply it to the first task. This group of students also substituted the length and the width of the rectangle for arbitrary given values, but they were misled by either an incorrect calculation of the radius of the bases or other miscalculations.

Finally, 33 out of the 58 students argued that the cylinder with the sort side as its height would have a greater volume. They based their reasoning by giving values to the dimensions of the rectangle and successfully applying the formula. However, many students seemed to find difficulty in calculating the radius of the base. In some cases, they halved the circumference or they gave arbitrary but proportional values. It is worth mentioning that 30 of those students had also given a correct answer to the second task. **Table 2** presents the frequency of the different responses based on the algebraic approach to the first task and the corresponding number of correct answers in second task.

Students’ responses that included a visualization approach to the first task also resulted in different outcomes. The majority of those students, 122 out of 150, argued that the volumes of the two cylinders were the same.

Table 1. Distribution of students’ responses to the first task without any reasoning and correct answers in the second task.

First Task Responses without reasoning		Second Task Correct calculations
Same size	65	39
Short side as height	10	5
Long side as height	6	4

Table 2. Distribution of students who applied an algebraic approach to the first task and correctly answered the second task.

First task responses—Algebraic approach		Second task correct calculations
Same size	13	5
Short side as height	33	30
Long side as height	12	8

From the total of 150 students who provided a description in their response, 19 stated that the cylinder with the short side as its height would have the greater volume, while nine students claimed that the cylinder with the long side as its height would have the greater volume.

Some interesting issues about students' geometric thinking emerged from the analysis of the reasoning by those students who considered the two volumes to be the same. Of these, 87 of 122 relied on the visual information of the task, and based their answer on the invariance of the surface area. In particular, those students argued that, for example, "the volumes will be the same as the paper still has the same dimensions whichever way it is rolled". Similarly, others students justified the equivalence thus: "the volumes will be equal to one another because the piece of paper doesn't vary in size" and "the volumes will be the same as both shapes are made using the same size of paper". Moreover, others specified that "the volumes will be the same as the surface area does not change", "the volumes will be the same as they have the same area" and "the area is the same for both cylinders". Finally, a small number of students argued that "both cylinders have the same net" or "cylinders have the size". The students who focused on the visual information of the task failed to conceptualise either the construction of the two solids or the differences in their properties (Lowrie, 2012). This was despite the fact that 47 of those students were able to recall, and correctly apply, the cylinder's volume formula in the second task.

There were, on the other hand, a relatively small number of students who managed to visualise the construction of the two cylinders, although they responded that the volumes would have the same measure. In particular, 35 students justified their answer based on the mental construction of the two cylinders, though informally. They visualised the two solids and described them as, for example, "slim and long" and "short and wide", or "one will be tall and skinny while the other is shorter and wider". Based on an intuitive approach, they failed to conceptualise the impact of the variance of the height and the circumference of the base to the variance of the volume of the cylinder. Typical responses include: "The volumes will be the same because the cylinder with the long side as the height will be taller but the one with the short side as the height will have a bigger opening at the top. They will even out to be the same"; "the volumes will be the same because what the short side lacks in height it makes up for with a greater radius". Interestingly, only 10 of these 35 students did not respond correctly in second task.

Nineteen students who argued correctly that the "shorter and fatter cylinder will be larger in volume" successfully interpreted the relation between the variance of the height and the circumference of the base, and the change in the volume of the cylinder. One of students' responded: "... the larger side becomes the circle which is the largest contributor to volume size". It is worth mentioning that 11 of those 19 students also used the volume formula to verify their visualizations.

Finally, nine students responded that the "cylinder with the greater volume is the one with the long side of the rectangle as height". Those students also managed to visualise the two cylinders, but focused only on the length of the height: "if the height is a larger number which it would be in this case the volume will be larger". **Table 3** presents students' responses to the first task that included an attempt of visualising the two cylinders and comparing mentally the produced cylinders. The corresponding number of correct students' responses to the second task is also provided.

5. Discussion

Summarising the results of the analysis of students' responses to the 3D task, it is apparent that the visual information provided to students misled them and dominated their thinking. Students could not analyse or decode the information, and therefore could not build relationships between the 2D representation and the 3D mental construction of the solids. A total of 200 of the 289 students responded that the volume of the two cylinders will be the same, and only 122 provided evidence of their attempts to visualise and make sense of the spatial informa-

Table 3. Distribution of students' visualization responses in first task and correct answers in the 2nd task.

First Task responses—Visualization strategies		Second Task Correct calculations	
Same size	Surface area/paper/net	87	47
	Solids mental representation	35	25
	Short side as height	19	11
	Long side as height	9	4

tion provided. A relatively small number of students managed to decode the information and to built relationships between the 2D representation elements and the 3D construction of the solids.

On the other hand, the acquisition of the volume formula and the numerical calculations also proved not to provide an adequate approach. Only 33 students managed to respond correctly by implementing the formula, though 178 students could calculate the volume of a cylinder, as evidenced in their responses to the second task. This finding is consistent with Battista's (2007) claims that the implementation of a formula and the numerical operation are not enough for conceptualising a 3D spatial task.

This study adds to the literature that visualization and conceptualisation of 3D objects are complex cognitive processes. Students need to be engaged with a variety of learning activities to develop their abilities to decode and encode spatial information and consequently develop their 3D geometric thinking (Lowrie, 2012; Pittalis & Christou, 2010). How student engage these learning activities then becomes important. If this study is examined from a learning approaches perspective (e.g. Biggs, 1987), a key question be posed by these results is whether the learning approaches of the students effected the way these students approached the specific test task. Furthermore, once might ask whether the approach taken by students over time has been adequate to allow the development of appropriate visual and spatial reasoning. From a pre-service teacher perspective, understanding the importance of aligned teaching methods, in order to improve visual and spatial reasoning, should be considered over and above individual cognitive factors that could limit or reduce the ability to improve the process of decoding and encoding information of a given mathematical task.

The fact that surface learning can be identified among the student group, as indicated by the forms of operation (versus analysis) recorded, indicates that apart from individuals presage factors, it is likely that either teaching or assessment methods have been, at some point, inadequate (Biggs et al., 2001). To this point, Biggs et al. (2001) further indicates that "...the most effective way of ensuring high quality teaching and learning is for teachers to take responsibility for ensuring that assessment and other contextual elements in the teaching and learning system are constructively aligned to promote deep approaches to learning".

6. Conclusion

Student responses to the 3D visualization task have highlighted several conceptual approaches. Importantly, neither spatial visualization nor formula-driven computation provided adequate class-wide engagement with the task. This emphasizes the notion that visualization and conceptualization of 3D objects are complex cognitive processes. Pre-service teachers need to master such conceptualization in order to teach children. One possible approach to achieve this is to ensure that students are engaged with a variety of learning activities to help them develop their abilities to decode and encode spatial information and, consequently, improve their 3D geometric thinking. While this may simply be viewed as a teaching and training challenge, examining the issue from a learning approach perspective offers further insight. The results indicate a surface learning approach dominantly at work here. This may arise from prior inadequate learning. Seen from this perspective, the best lesson the pre-service teachers may get from this task is not the mastery of a mathematical computation, but the awareness of the importance of teaching design and of the aligned teaching methods.

References

- AITSL (2011). *Accreditation of Initial Teacher Education Programs in Australia*. Carlton South: Mceedcya.
- Battista, M. T., & Clements, D. H. (1996). Students' Understanding of Three-Dimensional Rectangular Arrays of Cubes. *Journal for Research in Mathematics Education*, 27, 258-292. <http://dx.doi.org/10.2307/749365>
- Battista, M. T. (2007). The Development of Geometric and Spatial Thinking. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 843-908). Charlotte, NC: Information Age.

- Biggs, J. B. (1987). *The Study Process Questionnaire (SPQ): Manual*. Hawthorn: Australian Council for Educational Research.
- Biggs, J., Kember, D., & Leung, D. Y. (2001). The Revised Two-Factor Study Process Questionnaire: R-SPQ-2F. *The British Journal Of Educational Psychology*, 71, 133-149. <http://dx.doi.org/10.1348/000709901158433>
- Cohen, N. (2003). Curved Solid Nets. In N. Paterman, B. J. Doughery, & J. Zillox (Eds.), *Proceedings of the 27th International Conference of Psychology in Mathematics Education*, Vol. 2, 229-236.
- Cooper, M., & Sweller, J. (1989). Secondary School Students' Representations of Solids. *Journal for Research in Mathematics Education*, 20, 202-212. <http://dx.doi.org/10.2307/749283>
- Duval, R. (2006). A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics. *Educational Studies in Mathematics*, 61, 103-131. <http://dx.doi.org/10.1007/s10649-006-0400-z>
- Gutierrez, A. (1996). Visualization in 3-Dimensional Geometry: In Search of a Framework. In L. Puig, & A. Gutierrez (Eds.), *Proceedings of the 20th International Conference for the Psychology of Mathematics Education*, Vol. 1, 3-19.
- Hershkowitz, R. (1989). Visualization in Geometry—Two Sides of the Coin. *Focus on Learning Problems in Mathematics*, 11, 61-76.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement. *American Educational Research Journal*, 42, 371-406. <http://dx.doi.org/10.3102/00028312042002371>
- Lowrie, T. (2012). Visual and Spatial Reasoning: The Changing Form of Mathematics Representation and Communication. In B. Kaur, & T. T. Lam (Eds.), *Reasoning, Communication and Connections in Mathematics, Yearbook 2012, Association of Mathematics Teachers* (pp. 149-168). Singapore: Word Scientific Publishing. http://dx.doi.org/10.1142/9789814405430_0008
- Ma, H. L., Wu, D. B., Chen, J. W., & Hsieh, K. J. (2009). Mitchelmore's Development Stages of the Right Rectangular Prisms of Elementary School Students in Taiwan. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 57-64). Thessaloniki: PME.
- Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Mariotti, M. A. (1989). Mental Images: Some Problems Related to the Development of Solids. In G. Vergnaud, J. Rogalski, & M. Artique (Eds.), *Proceedings of the 13rd International Conference for the Psychology of Mathematics Education* (Vol. 2, pp. 258-265). Paris: PME.
- Mason, J. (2008). PCK and Beyond. In P. Sullivan, & S. Wilson (Eds.), *Knowledge and Beliefs in Mathematics Teaching and Teaching Development* (Vol. 1, pp. 301-322). Rotterdam/Taipe: Sense Publishers.
- Pittalis, M., & Christou, C. (2010). Types of Reasoning in 3D Geometry Thinking and Their Relations with Spatial Ability. *Educational Studies in Mathematics*, 75, 191-212. <http://dx.doi.org/10.1007/s10649-010-9251-8>
- Presmeg, N. (2006). Research on Visualization in Learning and Teaching Mathematics. In A. Gutiérrez, & P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future* (pp. 205-236). Rotterdam: Sense.
- Sack, J. J. (2013). Development of a Top-View Numeric Coding Teaching-Learning Trajectory within an Elementary Grades 3-D Visualization Design Research Project. *The Journal of Mathematical Behavior*, 32, 183-196. <http://dx.doi.org/10.1016/j.jmathb.2013.02.006>
- Stylianou, D. A., Leikin, R., & Silver, E. A. (1999). Exploring Students' Solution Strategies in Solving a Spatial Visualization Problem Involving Nets. In O. Zaslavsky (Ed.), *Proceedings of the 23rd International Conference for the Psychology of Mathematics Education* (Vol. 4, pp. 241-248). Haifa: PME.
- Wheatley, G. H. (1990). Spatial Sense and Mathematics Learning. *Arithmetic Teacher*, 37, 10-11.