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The Effect of Activity-Based Teaching on Remedying the Probability-Related Misconceptions: A Cross-Age Comparison

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The aim of this paper is to compare the effect of activity-based teaching on remedying probability-related misconceptions of students at different grades. Thus, a cross-sectional/age study was conducted with a total of 74 students in 6th-8th grades. Experimental instructions were given to all the groups three times/ week, 40 min/session, for 2 weeks. Students' progress was examined by pre-test and post-test measurements. The results of the analysis showed that, as a result of the intervention, all graders' post-test scores regarding all the concepts (PC: *Probability Comparison*, E: *Equiprobability* and R: *Representativeness*) showed a significant increase when compared to pre-test scores. It was found out that this increase did not create a significant difference based on age in PC concept, but that in 8th grade students, it showed a significant difference in E and R concepts compared to 6th graders. On the other hand, it was also assessed that the increases observed between 7th and 8th graders with regard to E and R concepts were not significant. In summary, the implemented intervention can be suggested to have different effects depending on age and the concept.

Keywords: Activity-Based Teaching; Misconception; Probability; Cross-Age

Introduction

The term "probability learning" is associated with a specific experimental paradigm within which a person is presented with a succession of trials on each of which one of two outcomes is possible and, on each trial, is required to predict which will happen before showing the outcome (Greer, 2001). Probability starts at early ages (3 - 5 ages) by comparing two events quantitatively, and at these ages children are unlikely to calculate the probability of an event. However, they may also reflect on some events and make some unintentional predictions, such as "highly probable" or "hardly probable", "or not equally probable". Fischbein (1975) suggested that "when, without special instructions, the probabilities of the responses approximate the probabilities of the events, it is possible to assume that the subject possesses a particular intuition of chance and probability". Children, as they grow older, tend to carry out evaluations based on more scientific and numerical calculations, by putting aside intutional evaluations in calculating and comparing the probability of events. It is believed that during the transformation of these evaluations, strategies employed in teaching probability concepts are important. Activity-based strategies must be used, especially during phases when they first confront probability concepts in formal education. In this sense, Shaughnessy (1977) emphasizes that employing activity-based teaching is important in providing meaningful learning and in remedying the students' misconceptions of probability.

Misconceptions and Probability

One of the most significant factors impeding the comprehension of a subject is misconception. Misconception is defined as perceptions or conceptions which are far from the meaning agreed upon by the experts (Zembat, 2008) or as the perceptions that are diverging from the view of experts in a field or subject (Hammer, 1996). For the past 30 years or so, scholars have studied students' misconceptions regarding mathematics. Studies have shown that students' conceptions of scientific issues are often not in line with accepted scientific thinking; that is, they have misconceptions regarding various notions. Fast (2001), Gürbüz (2007) and Shaughnessy (1977) suggested that some misconceptions in probability stem from the nature of the subject or the pupils' prior theoretical probability knowledge. Hammer (1996) pointed out that misconceptions affected students' perceptions and understandings. Misconceptions make it difficult to understand the subject of probability. Indeed, many studies were conducted about the misconceptions in teaching mathematics in general, and specifically in the subject of probability (Barnes, 1998; Batanero & Serrano, 1999; Bezzina, 2004; Dooren, Bock, Depaepe, Janssens, & Verschaffel, 2003; Fast, 1997, 2001; Fischbein, Nello, & Marino, 1991; Fischbein & Schnarch, 1997; Garfield & Ahlgren, 1988; Kahneman & Tversky, 1972; Lecoutre, 1992;

Mevarech, 1983; Morsanyi, Primi, Chiesi, & Handley, 2009; Polaki, 2002; Shaughnessy, 1977).

Some Types of Misconceptions

Representativeness Heuristic

When people make guesses about the outcomes of an experiment, they decide about the representativeness of these outcomes by examining them in certain ways. For example, when people are asked about the set of outcomes that can be obtained in an experiment of rolling a die several times successively, they're inclined to think that the chances of heads and tails are equal and the distribution is random (Amir & Williams, 1999; Shaughnessy, 1977). Similarly, Tversky and Kahneman (2003) states that "*after observing a long run of red on the roulette wheel, most people erroneously believe that black is now due, presumably because the occurrence of black will result in a more representative sequence than the occurrence of an additional red*" (p. 207). They also claim that people expect that a sequence of events generated by a random process will represent the essential characteristics of that process even when the sequence is short. In considering tosses of a coin, for example, people regard the sequence HTHTTH (H: Head; T: Tail) to be more likely than the sequence HHHTTT, which does not appear random, and also HHHTTT more likely than the sequence HHHHTH, which does not represent the fairness of the coin.

Positive and Negative Recency

Positive recency misconception is the belief that the outcome obtained from successive experiments will re-occur in future trials. Negative recency misconception is, on the other side, the belief that the outcome obtained from successive experiments will not occur in future trials. For example, in an experiment of tossing a coin successively five times and obtaining 5 times heads repeatedly, the belief that the next trial will also result in a heads is another example for positive recency effect, just as the belief that the next trial will result in a tails is known as the negative recency effect.

Equiprobability Bias

This is the belief that the probabilities of the outcomes of an experiment are equal although, in fact, they are not. Jun (2000) showed a different approach and put the equiprobability bias in three categories. These categories are: a) *thinking that each n different probable outcomes have* 50% *probability* b) *thinking that each outcome has a probability of* 1/*n* c) *thinking that the probabilities of n possible outcomes are all equal if they, in fact, have similar probabilities*. For example, a random selection is made in a farm containing 100 sheep and 250 goats. Thinking that the probabilities of choosing a sheep or a goat are equal is an example of Type (a). In another experiment of rolling together two dice designed as (123 456) and (222 333); thinking that the sum of possible outcomes would be equal is an example of Type (b). In another experiment of randomly selecting a student from a class of 20 boys and 25 girls, thinking that the probabilites of the outcome to be a girl or boy to be equal is an example of Type (c).

Activity-Based Teaching (ABT)

Activities are defined as tools that help in creating links between mathematical structures, increasing mathematical power, and constructing mathematical knowledge and visual illustrations of verbal knowledge (Moyer, Bolyard, & Spikell, 2002). Piaget (1952) claims that activities should be used in teaching mathematics due to the fact that mentally immature students cannot understand mathematical concepts. The activities that are designed to concretize and present the mathematical expressions clearly help students think creatively and develop their worlds of imagination (Thompson, 1992). Through activities, students will have the opportunity to learn in a more flexible environment, in collaboration with their peers and will be engaged in active learning. In parallel, Shaw (1999) states that he agrees with the educators who claim that students should not be passive when they build the knowledge. Using activities during the learning process necessitates active participation of students.

In an activity-based learning process, students move from an understanding of getting knowledge directly towards reaching knowledge, discussing to internalize this knowledge and constructing new knowledge through this discussion. This argumentation and constructing process helps students learn together, express their ideas easily, explain and justify their reasoning, and develop mathematical language. Having a learning environment where students argue with each other about valid arguments in mathematics could be the core of mathematics teaching (Sfard et al., 1998). Thus, it can be said that students' argumentation plays an important role in the occurrence of such positive effects in the learning process of probability concepts. Aspinwall and Shaw (2000) state that the activities allow students to make productive arguments about the concepts such as *data*, *chance and fair* and help in developing students' intuitions.

This process offers a learning environment where students construct the knowledge by sharing their ideas with each other. It can be said that through such a learning environment, students correct each other's mistakes with the help of their friends and with the help of the instructor. This interaction among students shows the importance of activity-based teaching in these environments. It can also be stated that through argumentation between instructor-student and student-student, the instructor had the opportunity to learn about the students' thinking and misconceptions. In sum, by presenting a flexible and reliable learning environment through active participation of students, activity-based teaching creates a practice-based discussion environment, contributes to students' math language and reasoning skills and helps students overcome misconceptions.

Literature Review Regarding Probability (Cross-Age Comparison)

Since prehistoric times, people have faced random physical events, e.g. unpredictable natural events and games of chance, but the birth of probability theory and its turning into a branch of mathematics did not occur until the middle of the 17th century. Probability as a subject started to appear in school curricula after the 19th century and since then, cognitive psychologists and mathematics educators have examined students' misconceptions concerning probability in different age groups (Batanero & Serrano, 1999; Bezzina, 2004; Dooren et al., 2003; Fast, 2001; Fischbein et al., 1991; Fischbein & Schnarch, 1997; Garfield, & Ahlgren, 1988; Gürbüz, Birgin, & Çatlıoğlu, 2012; Kahneman & Tversky, 1972; Konold et al., 1993; Lecoutre, 1992; Morsanyi et al., 2009; Offenbach, 1964; Pratt, 2000; Watson & Kelly, 2004; Watson & Moritz, 2002; Weir, 1962). For example, Weir (1962) did a study to reveal how three different instructional practices based on the same material affected pupils' learning in age groups 5 - 7 and 9 - 13. As a result of the analysis, it was determined that: a) younger pupils preferred tips and encouragement more than older pupils, b) older pupils changed their initial answers after receiving tips more than younger pupils, c) different instructional practices had no effect on pupils' selection of situations or choices on which hints were given, d) it was harder in older pupils to overcome prejudices. So, it was concluded that the pupils' prejudices or prior knowledge played an important role in their decisions about chance or probability concepts. Offenbach (1964), who conducted a study in order to determine the effect of carrot and stick (or reward and punishment) on a total 60 students' (30 of preschoolers and 30 of 4th graders) guessing more frequent event, found that the correct guessing ratios of both preschoolers and 4th-graders in all groups were almost equal, and the difference was only in their strategies. As the age increased, students were observed to make rule-based predictions. Fischbein et al. (1991) found out that the more students' learning level increases, the more the percentage of correct answers increases. However, it was also found that as learning level increases, concept mistakes also increase, yet in some it decreases. Fischbein and Schnarch (1997), who explored the changes in misconceptions of 5th, 6th, 7th, 9th, and 11th graders and college students who had not been educated in probability, reported that an increase in the students' education level variably decreased or increased or did not change their misconceptions about some concepts. Batanero and Serrano (1999) conducted a study with 277 pupils aged between 14 and 17 in Spain in order to investigate how the meaning given to the concept of "randomness" by the pupils changed with age. It was revealed that age was not important in understanding the concept of "randomness"; it is a hard concept to understand and, in order to master it well, it is essential to understand many other probability concepts, such as sample space, probability of an event, probability comparisons and so on. Watson and Moritz (2002) conducted a study to investigate the development of pupils in answering questions regarding the probability of a single event, compound events and conditional events. As a result of the evaluations, when the ratio of groups' correct answers to the questions related to conditional probability was compared, it was found that the percentage of correct answers increased with the level of education. However, no correlation was found between the level of education and the ratio of correct answers to the questions related to the probability of complex events. Dooren et al. (2003), who compared misconceptions in 10th and 12th graders, implied that there was no significant difference between the groups despite an increase in the level of education decreasing their misconceptions. Watson and Kelly (2004), who used a test based on a spinner divided into two identical parts (50 - 50) to determine 3rd, 5th, 7th and 9th graders' understanding of statistical variation in a chance setting, identified that there was a steady increase in conceptual development in the whole process at 3rd, 5th, and 9th grades, but not for 7th grade. Gürbüz et al. (2012), who compared the probability-related misconceptions of 540 pupils in 5th-8th grades, found that the percentage of correct answers increased when the level of education increased, whereas the misconceptions about the concept of *compound events I* decreased, the percentage of correct answers decreased and the misconceptions about the concept of *compound events II* increased. It was also found that in concepts of *probability of an event* and *probability comparisons*, as the level of education increased, both the percentage of correct answers and the misconceptions increased.

Probability concepts are widely used in decision-making processes related to uncertain situations we encounter in our daily lives. In spite of this importance, due to several reasons, probability concepts are not being taught as effectively in Turkey as it is in many other countries. The most important reason that the subject of probability is not taught effectively is the existence of this subject-related misconception. The reviewed literature showed that students' errors or misconceptions varied depending on age and level of education. This study aims at comparing and evaluating the effect of activity-based teaching on remedying the misconceptions of students at different grades (6, 7, 8) and ages (12 - 14) regarding some concepts (*Probability Comprasions-PC*, *Representativeness-R*, *Equiprobability-E*) in probability subject.

Method

Research Design

To determine students' conception in relation to their grade and understanding, cross-age and longitudinal studies are generally used. Despite the fact that the cross-age research involves different cohorts of students, it is more applicable than the longitudinal study when time is limited (Abraham, Williamson, & Westbrook, 1994). In these types of studies, monitored groups are few but detailed and comprehensive knowledge can be obtained. Also, cross-age studies do provide an opportunity to observe shifts in concept development as a consequence of students' maturity, an increase in intellectual development, and further learning.

Participants

This study was conducted with a total of 74 pupils (aged 12 - 14) studying in a primary school in the Southeastern Region of Turkey. The students participating in the study generally comefrom low- or middle-level socio-economic classes (based on the opinions of the school principal and teachers). The school of study is located in the province center. **Table 1** shows the grades, ages and class sizes of students in the study group.

Procedure

All of the student groups in the sample had previously been given a formal education in the subject of probability. Before the teaching intervention, the Misconception Test (MT) was

Table 1.
The distribution of the students in the study group according to grade and age.

Grade	6th grade	7th grade	8th grade
Age	12	13	14
Class Size (n)	23	24	27
(%)	31.08	32.43	36.48

administered to all groups as a pre-test. All groups were encouraged to answer all questions. The subject of probability was instructed in all groups (6, 7 and 8) with the same strategy (*activity-based teaching*) and by the same instructor. The implementations were carried out with 3- or 4-student groups. Thus, more communication and discussion took place. The six sessions, each lasting 40 minutes, were planned for the instructions of the concepts. During these sessions, conceptual questions that were designed to stretch students' thinking to a higher level were asked to the students. These included questions such as "*What strategy did you use to obtain you're answer?*", "*Why, or Why not?*" After the intervention, the MT was administered to all groups as a post-test.

Data Collection

In order to collect data, students' answers to pre- and post-tests related to each concept and the argumentation among the students in groups were used. Students' answers to pre- and post-tests were taken as a basis in order to determine whether their misconceptions regarding the concepts Probability Comparisons (PC), Representativeness (R) and Equiprobability (E) were remedied or not.

Instrumentation

The MT consisting of 12 questions (sample questions are presented in Appendix) was a two-tier question that consisted of a multiple-choice portion and an open-ended response. Some of the questions were developed by the researchers, and some of them were developed with the help of related literature (Baker & Chick, 2007; Fischbein et al., 1991; Kahneman & Tversky, 1972; Nilsson, 2009; Tatsis, Kafoussi & Skoumpourdi, 2008; Watson & Kelly, 2004). This test measures students' conceptions of three principal concepts, each of which involved 4 questions. The validity of the test was confirmed by two mathematics teachers and two mathematics educators. Furthermore, the pilot test was performed with 120 6th, 7th and 8th graders who did not participate in the real study. The administration of the pilot study took one class-hour (40 minutes). The pilot study revealed that questions on probability subject were understandable and clear for all grade levels. In this study, the Kuder-Richardson formula 20 (KR-20) reliability coefficient of the instrument was found as 0.85.

Activities and Materials

The activities used throughout the process of intervention were implemented in the same format and with the same strategies at all grade levels. The details of these activities are presented below:

One of the activities undertaken during the intervention process was the **"Which Spinner?"** activity. This activity was carried out using spinners A and B (see **Figure 1**). Spinner A has 4 identical red parts and 2 identical green parts. Among these six identical parts, two parts were numbered as 2 and four parts were numbered as 5. On the other hand, Spinner B has 2 identical red parts and 4 identical green parts. These identical parts are numbered as 1, 2, 3, 4, 5, and 6 respectively. Before and after the spinners were turned, the teacher asked the students questions such as, "*Which of the spinners is more likely to stop in a red area?*", "*What is the relationship between the probabilities of spinner A to stop in a red area and spinner B to stop in a green area?, Why?*", "*Are the probabilities of both spinners A and B to stop in the area numbered as 5, equal?*" The teacher tried to obtain the full participation of the students. The students turned the spinners with a variety of designs as shown in **Figure 2** many times, and the teacher helped to deepen the discussion environment by directing them in similar questions as shown above.

Another activity was the **"Which Number"** activity. In this activity, a material which had 16 tip up parts numbered from 1 to 16 respectively and each part had an area of 1 m^2 was used as given in **Figure 2**. Different amounts and frequency of points or chocolates were hidden in some places of this material. Some places were left without any reward. The scores written on the parts of the material, the number and frequency of chocolates and blank parts on the materials were written on the board, but these amounts were changed at the beginning of each activity. The groups were asked different questions about these materials. For example "*A randomly opened box will be more likely to be full or empty?*", "*In a randomly opened box, will the content more likely be a chocolate, or a score?*", "*What is the*

Figure 1.
Materials and reflections from the teaching process.

Figure 2.
Materials and reflections from the teaching process.

probability of a randomly opened box to be empty?" In this process, a group receives whatever comes out from the randomly opened box, either a chocolate, or a score. However, if an empty box is chosen, then this group is left out from the activity.

Another activity used in this process is the activity of **"Rolling Dice"**. For this activity, the researchers brought the class several dice designed in different forms (for example; 123 456, 111 444, 44 6666, or 123 455). The instructor first distributed these dice to the students and wanted them to do many experiments (50, 70, 100), to note the results of these experiments and to discuss these records. Then, the teacher transferred the records of all groups onto the board and enabled them to see the results of their experiments. The groups were asked to discuss these results and their ratios among themselves and by asking questions such as, "*Compare the ratios of the results obtained from the* 123 456 *die with those obtained from the* 123 455 *die*", "*Compare the ratios of the results obtained from the* 111 444 *die with those obtained from the* 44 6666 *die.*"

The last activity was **"Deal or No Deal?"** activity. After a short question-answer episode, the content of the activity was briefly explained to the students. If we want to explain the content of the activity here briefly, there are 10 boxes labeled with numbers between (1 - 10), as in **Figure 3**, and different amounts of money (1 TL, 5 TL, 10 TL, 10 TL, 20 TL, 20 TL, 20 TL, 50 TL, 100 TL) were put in these boxes. The amounts of money and their frequencies were altered at the beginning of every game. These amounts of money were put in an ascending order on the board in a way all students can see, and the frequency of each amount was indicated on the side. When the implementation process initiated, students competed to participate. Although all the class participated in the process, 10 students competed on the board beside the boxes, and one student took part as the single contestant. While all boxes were closed, the contestant picked the number of the box he wanted to be opened and then wanted the help of his/her friends on predicting the amount of money inside the box. His/her friends tried to uncover the contestant's thoughts by asking questions such as, "Why this box?" After obtaining the thoughts of all the class, the contestant decided that either the box would be opened, or it would be changed. The teacher organizing this process as a conductor asked students several questions before each box was opened, for example, "*Which amount of TL has the highest probability as an outcome?*", "*Which amount of TL has the lowest probability as an outcome?*", "*What's the probability of picking* 100 *TL*", "*Which amounts of TL have the same probability?*", "*Do you think the skills of the player have anything to do with winning the game? Why?*" A similar process went on until all boxes had been opened, and from time to time, by considering the amounts of money in the unopened boxes, different offers were made to the contestant. If the contestant accepted the offer, the game ended, but if not, the game continued until all boxes were opened. The contestants beside the boxes and the single contestant were changed every time and all the class was given the opportunity to participate in the activity.

Figure 3.
Materials and reflections from the teaching process.

Data Analysis

In analyzing the data, students' answers were classified in regard to the levels in **Table 2**. Since two external mathematics educators who had experience in analyzing qualitative data initially categorized the data separately, they discussed the consistency of the categorization. There was high agreement, approximately 90%, in most of the categorization. All disagreements were resolved by negotiation. The assessment test consisted of two phases *(1st phase multiple choice, 2nd phase open-ended)*, and, therefore, the assessment criteria also consisted of two phases. In this paper, the following symbolism is used for indicating the grade to which the quoted subjects belong: G6 means Grade 6, just as G followed by 7 or 8 indicates the respective grade.

Each group's total score was calculated and inputted into SPSS, and statistical comparisons were made in terms of the misconception level of groups. Two-way repeated measures ANOVA was employed to compare pre-test scores with post-test scores in each concept in MT.

Results and Discussion

Results of pre-test and post-test of the groups regarding MT were presented in **Table 3** and **Figure 4**. One-way ANOVA was carried out in comparing the pre- and post-test results of the groups.

The one-way ANOVA test was used to compare groups' scores regarding MT in pretest. As shown in **Table 4**, a significant difference was found between groups' pre-test scores related to PC [F (2 - 68) = 8.693, $p < 0.01$], and it was revealed that this difference was (Mean difference = 0.42572, $p < 0.01$) age;

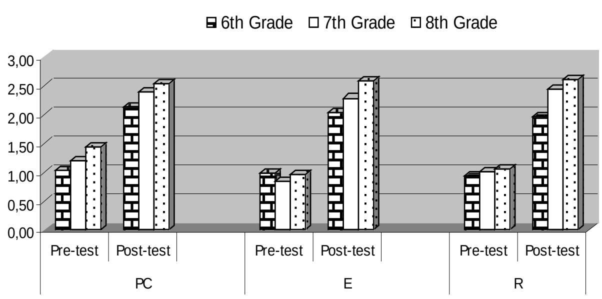

Figure 4.
Pre-test and Post-test scores of the groups on PC, E, and R concepts.

Table 2.
The rubric developed and used for MT and students' sample responses to them.

<table>
<tr><th>Comprehension Levels</th><th>Explanation</th><th>SCORE</th><th>Assessment Criteria 1st Phase –2nd Phase</th><th>Sample Response</th></tr>
<tr><td rowspan="2">Correct Justification</td><td rowspan="2">Answers that encompass all aspects of the valid justification</td><td>3</td><td>Correct Answer
Correct Justification</td><td>
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>1</td><td>(1,1)</td><td>(1,2)</td><td>(1,3)</td><td>(1,4)</td><td><u>(1,5)</u></td><td>(1,6)</td></tr>
<tr><td>2</td><td>(2,1)</td><td>(2,2)</td><td>(2,3)</td><td><u>(2,4)</u></td><td>(2,5)</td><td>(2,6)</td></tr>
<tr><td>3</td><td>(3,1)</td><td>(3,2)</td><td><u>(3,3)</u></td><td>(3,4)</td><td>(3,5)</td><td>(3,6)</td></tr>
<tr><td>4</td><td>(4,1)</td><td><u>(4,2)</u></td><td>(4,3)</td><td>(4,4)</td><td>(4,5)</td><td>(4,6)</td></tr>
<tr><td>5</td><td><u>(5,1)</u></td><td>(5,2)</td><td>(5,3)</td><td>(5,4)</td><td>(5,5)</td><td>(5,6)</td></tr>
<tr><td>6</td><td>(6,1)</td><td>(6,2)</td><td>(6,3)</td><td>(6,4)</td><td>(6,5)</td><td>(6,6)</td></tr>
</table>
PC 2: In the table above, there are 2 outcomes for Musa to win the game whereas there are 5 outcomes for Meryem to win the game. Thus, Meryem's chance to win the game is higher.
E1: When two dice are rolled together, the sums "2" and "12" are obtained one time. For 2 the outcome is to be (1,1), and for 12 the outcome is to be (6,6). So, Choice c (see Table below).
<table>
<tr><th>+</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>1</td><td><u>2</u></td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr>
<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td><u>12</u></td></tr>
</table>
</td></tr>
<tr><td>2</td><td>Incorrect Answer-Correct Justification</td><td>R1: Choice a is the correct answer. The number of elements in the sample universe of an experiment is 2. The chances of heads and tails are equal in the next toss; therefore, the outcome may be both.
R3: As distributon of outcomes is more random, choice c is more realistic.</td></tr>
<tr><td rowspan="2">Partially Correct Justification</td><td rowspan="2">Answers that do not encompass all aspects of the valid justification</td><td>2</td><td>Correct Answer - Partially Correct Justification</td><td>E4: Choice c is the correct answer. Because the probabilities of stopping at yellow and blue colors are equal in both cases.
PC 1: Choice b, because there are more green balls in the basket.</td></tr>
<tr><td>1</td><td>Incorrect Answer-Partially Correct Justification</td><td>R3: The answer is choice a. According to me, the probability of each letter on the dice is 1/6, and therefore the probability of each letter is equal.
PC 4: There are equal numbers of red-coloured parts in both spinners. But, in spinner B, all red-coloured parts are together. Choice b.</td></tr>
<tr><td rowspan="2">Wrong Justification</td><td rowspan="2">Answers that contain incorrect knowledge</td><td>1</td><td>Correct Answer-Wrong Justification</td><td>PC 2: Meryem's chance to win is = 6/9, and Musa's chance is = 3/9. So, Meryem's chance is higher.
E4: Since 3 people spun Spinner A and 5 spun Spinner B, choice c is correct.</td></tr>
<tr><td>0</td><td>Incorrect Answer-Wrong Justification</td><td>E1: Chances of 2 is = 2/14 and chances of 12 is = 12/14 so the sum will more likely be 12.
R1: Choice b, in this toss, tail comes, because all toss before are head.</td></tr>
<tr><td rowspan="3">No Justification</td><td rowspan="3">Correct, incorrect or blank answers with no justifications written.</td><td>1</td><td>Correct Answer–No Justification</td><td>…</td></tr>
<tr><td>0</td><td>Incorrect Answer–No Justification</td><td>…</td></tr>
<tr><td>0</td><td>No Answer–No Justification</td><td>…</td></tr>
</table>

PC: Probability Comparisons, R: Representativeness, E: Equiprobability.

c) between 8th and 6th graders, while it was (Mean difference = 0.26042, $p < 0.05$) between 8th and 7th graders. However, no significant difference was found between groups' pre-test scores E [$F(2 - 68) = 2.040$, $p > 0.05$] ve R [$F(2 - 68) = 0.943$, $p > .05$]. Therefore, it can be said that all groups had the same level of misconceptions in E and R concepts prior to the instructional process. Moreover, when the pretest results of the groups are compared, it can be seen that the number of misconceptions changes significantly depending on age related to the concept PC. It could be noted that: a) there is better understanding of this concept depending on age; b) students have to use this concept throughout their growing by teachers possess affluent knowledge of this concept; d) an effective presentation of this concept in all levels was effective in the creating of such a picture related to PC concept. It could also be maintained that in E and R concepts, less use of these concepts in daily life and teachers' lack of knowledge related to these concepts played a role.

As can be observed in **Figure 4** and **Table 3**, as a result of intervention, the number of answers with misconceptions in all groups related to PC, E and R concepts was lessened.

Table 5 illustrates that intervention created a siginificant effect in remedying misconceptions related to PC, E and R concepts in all class levels.

It could be understood from **Table 5** that as a result of intervention, misconceptions related to PC concept in 7th grade students (Mean = 1.19) were decreased more compared to 6th grade (Mean = 1.10) and 8th grade students (Mean = 1.08). It is also shown that this effect was higher in 8th graders (Mean = 1.61) related to E concept compared to 7th (Mean = 1.43) and

Table 3.
Descriptive statistics.

Concept	Measure	Grade 6		Grade 7		Grade 8	
		M	SD	M	SD	M	SD
PC	Pre-test	1.03	0.35	10.19	0.34	1.45	0.35
	Post-test	2.14	0.31	2.39	0.32	2.54	0.29
	Improvement*	1.10	0.52	1.19	0.50	1.08	0.37
E	Pre-test	0.98	0.33	0.85	0.37	0.97	0.32
	Post-test	2.03	0.32	2.29	0.29	2.59	0.33
	Improvement*	1.04	0.37	1.43	0.46	1.61	0.46
R	Pre-test	0.94	0.41	1.00	0.48	1.06	0.38
	Post-test	1.97	0.21	2.44	0.38	2.61	0.44
	Improvement*	1.03	0.33	1.44	0.66	1.55	0.62

Table 4.
The comparison of Pre-test scores of groups on MT related with the all concepts using ANOVA.

Measure		Sum of Squares	df	Mean Square	F	p	Difference
PC	Between Groups	2.172	2	1.086	8.693	0.000	8th grade > 6th grade
	Within Groups	8.494	68	0.125			8th grade > 7th grade
	Total	10.665	70				
E	Between Groups	0.269	2	0.135	1.121	0.332	no significance
	Within Groups	8.164	68	0.120			
	Total	8.433	70				
R	Between Groups	0.161	2	0.080	0.436	0.648	no significance
	Within Groups	12.526	68	0.184			
	Total	12.687	70				

Table 5.
Groups' paired t-test results.

Grade	Pre-test-Post-test	Mean Difference	SD	t	df	p
6th grade	PC	1.10870	0.52671	10.095	22	0.000
	E	1.04348	0.37426	13.371	22	0.000
	R	1.03261	0.33966	14.580	22	0.000
7th grade	PC	1.19792	0.50529	11.614	23	0.000
	E	1.43750	0.46186	15.248	23	0.000
	R	1.44792	0.66340	10.692	23	0.000
8th grade	PC	1.08333	0.37349	14.210	23	0.000
	E	1.61458	0.46026	17.185	23	0.000
	R	1.55208	0.62978	12.073	23	0.000

6th graders (Mean = 1.04), that related to R concept, it was higher in 8th graders (Mean = 1.55) compared to 7th graders (Mean = 1.44) and 6th graders (Mean = 1.03). Results of one-way ANOVA and Tukey HSD test applied to assess if the intervention had a class-level significant effect in remedying misconceptions related to PC, E and R concepts are shown below.

Table 6 shows that the intervention created a significant difference in terms of remedying groups' misconeptions related to all concepts but that this effect was not significant in PC [F (2 - 68) = 0.368, p > 0.05] concept depending on age but it was significant in E [F (2 - 68) = 10.568, p < 0.01] and R [F (2 - 68) = 5.505, p < 0.01] concepts depending on age. According to Tukey HSD results, as shown in **Table 7**, misconceptions related to E concept were overcome more significantly by 8th [mean difference = 0.57111, p < 0.01] and 7th [mean difference = 0.39402, p < 0.01] graders than 6th graders, but this effect was not significant in 8th graders compared to 7th [mean difference = 0.17708, p > 0.05] graders. Similarly, misconceptions related to R concept were overcome more significantly by 8th [mean difference = 0.51947, p < 0.01] and 7th [mean difference = 0.41531, p < 0.01] graders than 6th graders, but this effect was not significant in 8th graders compared to 7th [mean difference = 0.10417, p > 0.05] graders. It could be said the fact that in removal of groups' misconceptions at similar level in PC concept, students had a certain level of knowledge related to this concept, and understanding of this concept does not necessitate much theoretical knowledge were effective. It could also be stated that in removal of groups' misconceptions at different levels in E and R concepts, in addition to students' age, their knowledge about other probability concepts in understanding these concepts and the need for deeper logical reasoning were effective. It is specifically important to say that in addition to the intervention conducted, age also played a role in removal of groups' misconceptions related to E and R concepts. In other words, it could be inferred that as age level increases, the intervention led to a greater decrease in misconceptions related to E and R concepts. Gürbüz et al. (2012) reached similar findings.

When the answers of participants were examined, it was found that students had different justifications for their right or wrong answers in pre- and post-test. Since the research group consists of 74 students in total, all papers in pre-test and post-test could be examined in detail. However, since it was not possible to transfer all answers by students to the study, some sample student answers were given.

For example regarding PC 1, some students were found to give incorrect answers in pre-test either because of building a relationship with general chance factor and favourite color concept or because of concentrating on the location of the balls in the basket. For example, "We can say nothing unless we know the favourite colors of the person who choses the balls (G7)", "I can't comment on it because it depends on chance (G6;G7)", "Green, because they're at the bottom and when the basket is mixed they will be at the top (G6)". Such approaches of students who gave wrong or misconceptional answers to question PC 1 are in line with the student approaches in the studies of Gürbüz (2007; 2010), Gürbüz, Çatlıoğlu, Birgin and Erdem (2010) and Jones et al. (1997). In addition, as a result of the effect of intervention, more correct answers were observed in all grade levels. Explanations related to question PC 1 made by some of the students in post-test are as follows: "Green because the number of green balls in the basket is the highest (G6; G7;G8)", "Choice b, since the number of green balls in the basket is higher than others, the probability of getting green is the highest. Numerically, P(G) = 4/9 (G7;G8)".

It was found that, regarding question PC 2, some of these students used general chance factor in probability subject, while another group used the perception that the probabilities will be the same and a small portion gave illogical justifications in pre-test. "The families of Musa and Meryem should be checked; whoever was born in a luckier family will win (G7)", "Musa and Meryem have the same chance. Because there's only one 3 and one 6 on the die (G6;G7;G8)", "3 + 6 = 9:2 = 4,5 both have the same chance to win (G6)". It should be noted here that, as students' ages and level of education increased they tended to answer the question by relating it to chance factor. Amir and Williams (1999), Baker and Chick (2007), Batanero and Serrano (1999), Fischbein et al. (1991), Lecoutre (1992) and Nilsson (2007) reported similar conclusions in their studies. Some of the students who gave misconceptional answers also used some probable outcomes as reported by Amir and Williams (1999). For example, "Musa is my favourite because

Table 6.
The comparison of post-test scores of groups on MT related with all concepts using ANOVA.

Variable		Sum of Squares	df	Mean Square	F	p
PC	Between Groups	0.173	2	0.087	0.388	0.680
	Within Groups	15.184	68	0.223		
	Total	15.357	70			
E	Between Groups	3.997	2	1.999	10.568	0.000
	Within Groups	12.860	68	0.189		
	Total	16.857	70			
R	Between Groups	3.527	2	1.764	5.505	0.006
	Within Groups	21.783	68	0.320		
	Total	25.310	70			

Table 7.
Tukey HSD results.

Variable	(I) grup	(J) grup	Mean Difference (I-J)	Std. Error	p
E Improvement	7th grade	6th grade	0.39402*	0.12690	0.008
	8th grade	6th grade	0.57111*	0.12690	0.000
		7th grade	0.17708	0.12554	0.341
R Improvement	7th grade	6th grade	0.41531*	0.16515	0.038
	8th grade	6th grade	0.51947*	0.16515	0.007
		7th grade	0.10417	0.16338	0.800

when the outcomes are (1,2) and (2,1), their sum would be three. On the other hand, in order to obtain six, there's only (3,3) (G7)", "Musa and Meryem have the same chance. Because for 3, (1,2), (2,1) and for 6, (1,5), (5,1) (G7;G8)". Such misconceptional answers from students can be argued to stem from students' lack of sufficient knowledge in sample space concept. In parallel, Baker and Chick (2007), Bezzina (2004), Chernoff (2009), Fischbein et al. (1991), Gürbüz (2007; 2010), Keren (1984), Nilsson (2007) and Polaki (2002) showed in their studies that students' knowledge about sample space concept played an important role in their answers to questions related to probability subject. Moreover, few students gave misconceptional answers by considering gender factor as reported by Amir and Williams (1999). For example, "Musa is the favourite. Becuase, males are more lucky in this kind of chance games" (G6;G7). In question PC 2, it's understood that students' justifications are effected from their individual learning, experiences, cultures and beliefs. Amir and Williams (1999), Fischbein et al. (1991), Sharma (2006) and Shaughnessy (1993) reported similar results in their studies. When students' answered PC 2 question in the post-test, it could be seen that a great number of answers that did not make sense were corrected. For example, "Choice b, because 6 is (G6;G7),","Meryem is more advantegous as the probability of total score to be 6 is high (G6;G7), "Choice b, because cases where Musa will win are (1,2) ve (2,1) but cases where Meryem will win are (1,5), (5,1), (2,4), (4,2) and (3,3). Since Meryem has more cases to win, she is more advantegous (G7;G8)".

Many students realized the importance of size in solving question PC 4, but they gave answers containing misconceptions because they concentrated on only one aspect of size in the pre-test. For example, some of the students gave answers such as, "Choice b, because on spinner B the red color covered more than half of the shape (G6;G7)", "The probability of spinner B to stop on a red color is higher because there are more red areas (G7;G8)". On the other hand, there were answers to question PC 4 contrary to mathematical logic such as "We can not decide at what colors the spinners will stop because we don't know at what speed they are turned (G6)", "When looking at the direction of the arrow, both have a high chance to stop at red color (G7)". However, though some mistakes made related to PC 4 question in the pre-test were repeated by a few students, most students gave correct answers in the post-test. For example, "Choice B, since red colours are gathered on spinner B (G6;G7)", "The probability of stopping in a red area on both spinners are equal (G6;G7;G8)", "As the areas covered by red colour on both spinners are equal, choosing A or B does not change my chance to win. P(A) = P(B) = 6/12 (G6;G7;G8)".

It was found that, in question E 1, some of the students couldn't reason probabilistically, and they couldn't think or know that there was no number greater than 6 on the faces of a traditional die in the pre-test. For example, "12 is more likely because 12 is greater than 2 (G6;G7)", "For the sum to be 2, the dice should be (1,1); for the sum to be 12, the dice should be (8,4), (6,6), (7,5), (9,3), (10,2), (11,1). Therefore the probability of 12 is higher (G7;G8)". Polaki (2002) names this type of thinking as subjective probabilistic thinking, and, according to him, the students reflecting at this level can not give logical or mathematical answers. On the contrary, Bezzina (2004) pointed out that the students falsely believe that a "six" is the most difficult score to obtain when a die is rolled at random. It could be said that students gave more correct answers to E 1 question in the post-test. For example, "Choice C, for the sum to be 2, the dice should be (1,1); for the sum to be 12, the dice should be (6,6), thus, the probability of the total to be 2 or 12 is equal(G6;G7;G8)", "When a die is rolled, in total, the minimum will be 2 and maximum will be 12 and there is only one case where each of these total numbers could be gathered (G7;G8), so choice C is true".

Students gave answers containing misconceptions to question E 4 by using their intuitions and informal strategies in the pre-test. For example, "I would choose spinner B because more people turn it (G6;G7;G8)", "3 people are turning spinner A, 5 people are turning spinner B therefore in order to equalize the number of people turning each spinner, I would choose spinner A (G6;G7)". It could be argued that the students giving such kinds of justifications have "Outcome Approach" misconception. Likewise, according to Jun (2000) and Konold (1989), students who have this kind of misconception make decisions considering the results of previous outcomes. It is possible to see that students gave more correct answers in the post-test. For example, "Since the probability of A spinner to stop on yellow and of B spinner to stop on blue color are equal, it does not make a difference (G6;G7;G8)", "Both the yellow and blue areas cover half of the circle, that is why, the answer is C (G6;G7;G8)", "Choice C, the probability of A spinner to stop on yellow and of B spinner to stop on blue color is 50 % (G6; G7;G8)".

In question R 1, some of the students mostly showed misconceptions of positive and negative recency in the pre-test. For example, "The first four outcomes were heads. So, the fifth toss would more likely be tails (Negative recency) (G6;G7; G8)", "The successive outcomes were heads. So, the next toss will more likely result in heads (Positive recency) (G6;G7; G8)". It is expressed that children' gender could be guessed through the same approaches. For instance, Kahneman and Tversky (1972) stated that for a family with 6 children, it's believed that the order of the genders will more likely be MFFMFM (M:Male; F:Female) instead of MMMMMM or MMMFFF. It's possible to find similar results in the study by Fast (1997). However, students in the post-test learnt first tosses would not affect later tosses. For example, "The probability of getting heads and tails are the same because there are two faces of the coin, being heads and tails (G6;G7;G8)", "The outcome in the previous toss does not affect the later one, thus Choice C. (G6;G7;G8)", "Since getting heads and tails are independent events, the probability of getting either one is the same (G7;G8)".

In question R 3, only a few students at all levels gave justification in the pre-test. This concept is hard to understand and in order to understand it first other concepts (Sample Space, The Probability of an Event, Probability Comparison) should be well comprehended. Some students showed the misconception of representativeness heuristic in this question. For example, some students stated that "Choice a, because it's more realistic that all faces come as an outcome equal times (G6;G7;G8)", "All the outcomes have equal probability, so choice a (G6;G7;G8)", "Choice c, in which the same order exists, is more probable (G6;G7)".Batanero and Serrano (1999), Shaughnessy (1977) and Gürbüz and Birgin (2012) obtained results in their studies similar to these. Batanero and Serrano (1999) found that smaller children dealt more with runs, or whether heads and tails come out in an order; whereas, older students focused more on the number of heads and tails. Though it was possible to see some answers given in the post-test similar to answers with misconceptions in the pre-test, it was observed that more correct answers were given in all grade levels. For example, "Choice b, because when this die is tossed it is not realistic to get both faces equally (G6;G7; G8)", "Since the distribution is more randomly, b is more logical (G7;G8)", "As more regular outcomes are gathered successively c is a more correct anwers (G6;G7;G8)".

In summary, it can be argued that with the implemented intervention, all graders' scores increased and the number of misconceptions decreased. On the other hand, while there was not any singnificant relationship between age and remedying misconceptions in groups related to PC concept, it could generally be noted that, depending on age, the intervention helped students remedy more misconceptions in E and R concepts.

Conclusions and Implications

When the pre-test results are examined, a significant difference was observed between groups' misconceptions related to PC concept, and this difference was found between 8th and 6th graders and 8th and 7th graders. On the other hand, no significant difference was found among groups related to E and R concepts. However, as a result of the intervention, all graders' post-test scores regarding all the concepts showed a significant increase when compared to pre-test scores. While there was no any singnificant relationship between age and remedying misconceptions in groups related to PC concept, it could generally be noted that, depending on age, the intervention helped students overcome more misconceptions in E and R concepts. It was determined that the intervention did not make a significant difference according to age in remedying misconceptions related to PC concepts in groups, but that age had a significant effect on overcoming misconceptions (more in 7th and 8th graders than 6th graders) in E and R concepts. To summarize, it can be suggested that the implemented intervention has different effects depending on age and the concept. Activity-based teaching, which contributes to remedying misconceptions and provides learning relevant to real life, should be performed in the subject of mathematics.

The number of justifications related to the concepts in MT was found to increase with the increase of age when participants' answers were examined. This could be attributed to the development of mathematical reasoning and language in addition to age factor. As a matter of fact, in learning probability concepts, Fischbein et al. (1991), Offenbach (1964), Watson and Moritz (2002), and Way (2003) emphasized age, Erdem (2011), Lamprianou and Lamprianou (2003), Memnun (2008), Offenbach (1965) and Olson (2007) focused on mathematical reasoning, Ford and Kuhs (1991), Gibbs and Orton (1994), Kazıma (2006) and Tatsis et al. (2008) mentioned the effect of language development. In this sense, for further research, the relationship between age and language development, and between age and mathematical reasoning should be studied.

Argumentation is an important process both in learning mathematical concepts and in analyzing the nature of activity within mathematics classrooms (Sfard et al., 1998). This process must be consciously made use of in the teaching of concepts that require a deeper thinking, such as probability concepts. Activities used in this study contributed to the discussion process being more consciously and productively executed. It revealed that during the process, groups consisting of more aged members had more effective discussions. It is believed that the effective discussion process was effective in the development of aged groups' learning of some probability concepts. Because of this reason, discussion environments must be created in maths teaching starting from early ages.

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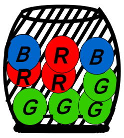
Appendix

Some Assessment Items

PC 1



On the balls, "R" represents red; "B" represents blue and "G" represents green respectively.

There are 4 green, 3 red and 2 blue, a total of 9 balls in this basket. When you close your eyes and pick out a ball after mixing all the balls in the basket, which color will this ball most likely be? Why?

a) Blue
b) Green
c) Red

PC 2

Musa and Meryem play with a pair of dice. If the sum of the points is 3, Musa is the winner. If the sum of the points is 6, Meryem is the winner. Which of the following answers seems to you to be the correct one? Why?

a) Musa is the favourite
b) Meryem is the favourite
c) Musa and Meryem have the same chance

E 1

When two dice are rolled at the same time, which outcome is more likely for the sum of the numbers on the upper faces of the dice, 2 or 12? Why?

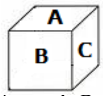
a) 2 is more likely
b) 12 is more likely
c) The probabilities of 2 or 12 are equal.

R 1

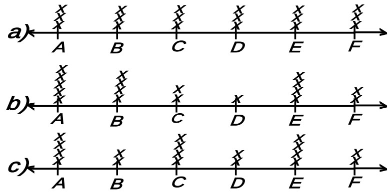
A coin is tossed four times and the results are HHHH. What is more likely for the next toss, Heads or Tails? Why? (T = Tails, H= Heads)

a) H is more likely
b) T is more likely
c) Both have equal chance

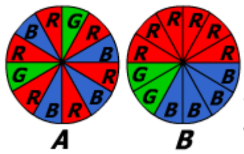
R 3



A die on whose sides the letters A, B, C, D, E, and F are written is tossed 18 times. Which of the following results is more realistical? Why?



PC 4

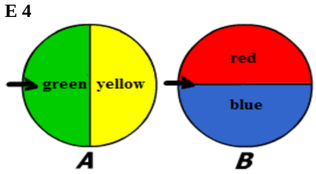


On the spinners, R represents red; B representsblue and G represents green

A and B are two spinners. When these spinners are turned at the same time, which one is more likely to stop at red? Why?

a) Spinner A
b) Spinner B
c) Both spinners have an equal chance

E 4



The rule of the game dictates that when the spinners above are turned, if they stop on yellow or blue areas an MP3 player is won by the player. There are 50 players in the competition and the first 3 players turned the spinner A and it stopped each time on the yellow area. The next 5 people turned spinner B and it stopped each time on the blue area. If it were you, which spinner would you choose? Why?

a) Spinner A
b) Spinner B
c) Both Spinners A or B are the same