

Relation Contractive Selfmaps Involving Cauchy Sequences

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Abstract

We obtain two generalizations of a known theorem of A. Alam and M. Imdad (J. Fixed Point Theory Appl. 17 (2015) 693-702) showing that some standard proofs can be obtained involving only Cauchy sequences of the successive approximations. Suitable examples prove the effective generalization of our results in metric spaces not necessarily complete.

Keywords

Cauchy Sequence, \mathcal{A} -Self-Closed Relation, Relation Contraction, Relation Preserving

1. Introduction

Fixed point theorems involving contraction conditions under preserving relations are known in literature (cf. [1]-[11]). These theorems involve usual sequences of successive approximations in complete metric spaces as assumed in the paper of Alam and Imdad [12]. Historically speaking, it is well known that any real number is the sup (resp., inf) of Cauchy increasing (resp., decreasing) sequence of rational numbers. Hence the study of Cauchy sequences is interesting per se; indeed in other contexts such concept is introduced, for instance, in metric spaces and related recent generalizations known like b -metric spaces [13] [14] [15], partial metric spaces [8], Yoneda spaces [9] [16] and modular metric spaces [17] (we also suggest the good survey [18] as further deepening). Our aim is to prove that extensive theorems can be obtained by considering only Cauchy sequences in metric spaces not necessarily complete. Here fixed point theorems involving Cauchy sequences of Jungck type [6] [19] are not considered in order to make compact the results involving one only contraction.

2. Preliminaries

We start with some known definitions [1].

Definition 1. Let X be a nonempty set and \mathfrak{R} be a binary relation (eventually partial) defined on X . A sequence $\{x_n\}$ of X is called \mathfrak{R} -preserving if $(x_n, x_{n+1}) \in \mathfrak{R}$ for every $n = 0, 1, 2, \dots$.

From now on we consider such binary relations and we write simply $\mathfrak{R} \subseteq X^2$.

Definition 2. Let (X, d) be a nonempty metric space and $\mathfrak{R} \subseteq X^2$ is called d -self-closed if whenever $\{x_n\}$ is \mathfrak{R} -preserving and converging to a point $x \in X$, then there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that either $(x_{n(k)}, x) \in \mathfrak{R}$ or $(x, x_{n(k)}) \in \mathfrak{R}$ for every $k = 0, 1, 2, \dots$.

Definition 3. (cf. [20]). Let X be a nonempty set and $\mathfrak{R} \subseteq X^2$. For $x, y \in X$, a \mathfrak{R} -path of length k (where $k = 0, 1, 2, \dots$) in X from x to y is a finite sequence $\{z_0, z_1, z_2, \dots, z_k\}$, $1 \leq k$, of points of X satisfying the following conditions:

- 1) $z_0 = x$ and $z_k = y$;
- 2) $(z_i, z_{i+1}) \in \mathfrak{R}$ for each $i = 0, \dots, k-1$.

Notice that a path of length k involves $k+1$ elements of X , although they are not necessarily distinct. In [19], generalizing a famous theorem of [7], the following theorem was established.

Theorem 1. Let (X, \leq) be a partially ordered set and there exists a metric $d: X \times X \rightarrow [0, +\infty)$. Let T be a selfmap of X such that

- 1) T is monotone non-decreasing;
- 2) there exists a point $x_0 \in X$ such that $x_0 \leq T(x_0)$;
- 3) if $\{x_n\}$ is a non-decreasing Cauchy sequence in X , then $\{x_n\}$ converges to $z \in X$ and $T(x_n) \leq z$ for every n ;
- 4) there exists $c \in [0, 1)$ such that $d(T(x), T(y)) \leq c \cdot d(x, y)$ for all $(x, y) \in X^2$ with $x \geq y$;

then T has a fixed point $w \in X$ such that $x_0 \leq w$.

In [1], generalizing many theorems contained in the references therein cited, the following theorem was established (not including the case T continuous which we consider later).

Theorem 2. Let (X, d) be a complete metric space, $\mathfrak{R} \subseteq X^2$ and T be a selfmap of X such that

- 1) there exists in X a point x_0 such that $(x_0, Tx_0) \in \mathfrak{R}$;
- 2) \mathfrak{R} is T -closed, that is $(x, y) \in \mathfrak{R}$ implies $(Tx, Ty) \in \mathfrak{R}$;
- 3) \mathfrak{R} is d -self-closed;
- 4) there exists $c \in [0, 1)$ such that $d(T(x), T(y)) \leq c \cdot d(x, y)$ for all pair $(x, y) \in \mathfrak{R}$.

Then T has a fixed point. Moreover, if there exists a \mathfrak{R} -path from x to y for all $x, y \in X$, then this fixed point is unique.

3. Unification of Theorems 1 and 2

Now we unify Theorem 1 and 2 with the following:

Theorem 3. Let (X, d) be a metric space, $\mathfrak{R} \subseteq X^2$ and T be a selfmap of X . Suppose that

- 1) there exists in X a point x_0 such that $(x_0, Tx_0) \in \mathfrak{R}$;
 - 2) \mathfrak{R} is T -closed;
 - 3) for any sequence $\{y_n\}$ \mathfrak{R} -preserving, $n = 0, 1, 2, \dots$, which is Cauchy and converging to a point $y \in X$, there exists a subsequence $\{y_{n(k)}\}$ of $\{y_n\}$ such that either $(y_{n(k)}, y) \in \mathfrak{R}$ or $(y, y_{n(k)}) \in \mathfrak{R}$ for every $k = 0, 1, 2, \dots$;
 - 4) there exists $c \in [0, 1]$ such that $d(Tx, Ty) \leq c \cdot d(x, y)$ for all $(x, y) \in \mathfrak{R}$.
- Then T has a fixed point z in X and there exists a sequence $\{z_k\}$ such that either $(Tz_k, z) \in \mathfrak{R}$ or $(z, Tz_k) \in \mathfrak{R}$ for every $k = 0, 1, 2, \dots$. Moreover,
- 5) if there exists a \mathfrak{R} -path from x to y for all $x, y \in X$, then this fixed point is unique.

Proof. Let $x_0 \neq Tx_0$ otherwise the thesis is trivial. Put $h = d(x_0, Tx_0) > 0$ and $y_n = T^n(x_0)$ for every $n = 0, 1, 2, \dots$, so we have

$$\begin{aligned} T^0 x_0 = x_0 = y_0, T^1 x_0 = Tx_0 = Ty_0 = y_1, T^2 x_0 = Ty_1 = y_2, \dots, \\ Ty_n = y_{n+1} \quad \text{for } n = 0, 1, \dots \end{aligned}$$

Because of properties 1) and 2), the sequence $\{y_n\}$ is \mathfrak{R} -preserving. In virtue of property 4), we have that

$$d(y_{n+1}, y_n) = d(T^{n+1}x_0, T^n x_0) \leq c^n \cdot h \quad \text{for } n = 1, 2, \dots$$

and hence $\{y_n\}$ is a Cauchy sequence. By property 3), $\{y_n\}$ converges to a point z in X and there exists a subsequence $\{y_{n(k)}\}$ of $\{y_n\}$ such that either $(y_{n(k)}, z) \in \mathfrak{R}$ or $(z, y_{n(k)}) \in \mathfrak{R}$ for all $k = 0, 1, 2, \dots$. This implies that

$$d(y_{n(k)+1}, Tz) = d(Ty_{n(k)}, Tz) \leq c \cdot d(y_{n(k)}, z)$$

by property 4) and passing $k \rightarrow \infty$, we have $d(z, Tz) = 0$ and therefore z is a fixed point of T . By setting $z_k = y_{n(k)}$ for every $k = 0, 1, 2, \dots$, we have $(Tz_k, z) \in \mathfrak{R}$ if $(z_k, z) \in \mathfrak{R}$ or $(z, Tz_k) \in \mathfrak{R}$ if $(z, z_k) \in \mathfrak{R}$ for every $k = 0, 1, 2, \dots$ because of property 2). If property 5) holds, then it is a routine to prove that the fixed point is unique (cf., e.g. [2]).

Remark 1. Theorem 1 is generalized from Theorem 3 by defining the non-decreasing order “ \leq ” as relation $R \subseteq X^2$. Theorem 2 is generalized from Theorem 3 because the condition 3) of Theorem 2, i.e. Definition 2, is restricted only to Cauchy sequences and moreover the hypothesis that X is complete does not appear in Theorem 3 as well.

In the following example Theorem 3, inspired to Example 1 of [19], holds while Theorem 2 is not applicable.

Example 1. Let $X = \{(x, x) : x \in (-1, 1]\} \subseteq R^2$ endowed with the Euclidean metric d . Then (X, d) is a non-complete metric space and define \mathfrak{R} as $((x, x), (y, y)) \in \mathfrak{R}$ if $x \leq y$ and $x \neq 0$, $y \neq 0$, $((x, x), (0, 0)) \in \mathfrak{R}$ and $((0, 0), (x, x)) \in \mathfrak{R}$ if $x \in (-1, 1]$. Let $c \in [0, 1]$ and define $T : X \rightarrow X$ as $T((x, x)) = (cx, cx)$ if $x < 0$ and $f((x, x)) = (0, 0)$ if $x \geq 0$. It is immediate to verify that property 1) holds since $((0, 0), (0, 0)) = ((0, 0), (T0, T0)) \in \mathfrak{R}$,

moreover properties 2) and 3) hold trivially. Additionally we have that

$$d(T(x, x), T(y, y)) = d((kx, kx), (ky, ky)) = 2^{1/2} \cdot c \cdot |x - y| \\ = c \cdot d((x, x), (y, y)) \quad \text{if } x < y < 0,$$

$$d(T(x, x), T(y, y)) = d((kx, kx), (0, 0)) = 2^{1/2} \cdot c \cdot x \\ = k \cdot d((x, x), (y, y)) \quad \text{if } x \leq 0, y > 0,$$

$$d(T(x, x), T(y, y)) = d((0, 0), (0, 0)) = 0 < c \cdot d((x, x), (y, y)) \quad \text{if } 0 < x \leq y,$$

thus property 4) holds. Also property 5) holds because there exists at least an \mathfrak{R} -path of length 2, i.e. $((x, x), (0, 0)) \in \mathfrak{R}$ and $((0, 0), (y, y)) \in \mathfrak{R}$, joining two any points $(x, x), (y, y)$ of X . Indeed $(0, 0)$ is the unique fixed point of T but Theorem 2 is not applicable because X is not complete.

Remark 2. If 5) does not hold, Theorem 3 does not guarantee the uniqueness of the fixed point as proved in the following example:

Example 2. Let $X = [0, 1] - \{1/2\}$ be endowed with metric $d(x, y) = |x - y|$ for all $x, y \in X$. Define $\mathfrak{R} \subseteq X^2$ as follows: $(x, y) \in \mathfrak{R}$ if for all $x, y \in X$ such that $0 \leq x \leq y < 1/2$ or $1/2 < x \leq y \leq 1$. Then X is a metric space with the partially defined binary relation \mathfrak{R} . Define $T : X \rightarrow X$ as $Tx = x/2$ if $0 \leq x < 1/2$ and $Tx = (x + 1)/2$ if $1/2 < x \leq 1$. Then property 1) holds because $1/2 < x_0 \leq Tx_0$ if $x_0 \in (1/2, 1]$. The property 2) holds because T is strictly increasing in both intervals $[0, 1/2)$ and $(1/2, 1]$. The property 3) holds because it is enough to take strictly increasing sequences in $(1/2, 1]$. Property 4) holds also for $c = 1/2$. Property 5) fails because if $x \in [0, 1/2]$ and $y \in (1/2, 1]$, for any finite \mathfrak{R} -path of length k , $\{z_0, z_1, z_2, \dots, z_k\}$, there exists at least certainly some $m \in \{0, 1, \dots, k - 1\}$ such that $z_m \in [0, 1/2)$ and $z_{m+1} \in (1/2, 1]$, hence $(z_m, z_{m+1}) \in \mathfrak{R}$. Note that T has two fixed points which are 0 and 1.

Remark 3. Theorem 2 is not applicable to Example 2 because X is not complete.

4. Relation Contractions and Continuous Selfmapss

In [19] the following theorem appears:

Theorem 4. Let (X, \leq) be a nonempty partially ordered set and there exists a metric $d : X \times X \rightarrow [0, +\infty)$. Let T be a selfmap of X such that

- 1) there exists a point $x_0 \in X$ such that $x_0 \leq T(x_0)$;
- 2) T is continuous and non-decreasing;
- 3) if $\{x_n\}$ is a non-decreasing Cauchy sequence in X , then $\{Tx_n\}$ converges to a point $z \in X$;
- 4) there exists $c \in [0, 1]$ such that $d(T(x), T(y)) \leq c \cdot d(x, y)$ for all $(x, y) \in X^2$ with $x \geq y$. Then T has a fixed point.

In the case T is assumed continuous, Theorem 2 becomes [1]:

Theorem 5. Let (X, d) be a complete metric space, $\mathfrak{R} \subseteq X^2$ and T be a selfmap of X such that

- 1) there exists at least a point $(x_0, Tx_0) \in \mathfrak{R}$;

- 2) \mathfrak{R} is T -closed;
- 3) T is continuous;
- 4) there exists $c \in [0, 1]$ such that $d(T(x), T(y)) \leq c \cdot d(x, y)$ for all $(x, y) \in \mathfrak{R}$.

Then T has a fixed point.

Now we unify Theorems 4 and 5 with the following:

Theorem 6. Let (X, d) be a metric space, $\mathfrak{R} \subseteq X^2$ and T be a selfmap of X . Suppose that

- 1) there exists in X a point x_0 such that $(x_0, Tx_0) \in \mathfrak{R}$;
- 2a) \mathfrak{R} is T -closed;
- 2b) T continuous;
- 3) if x_n is a \mathfrak{R} -preserving Cauchy sequence in X , then Tx_n converges to a point $z \in X$;
- 4) there exists $c \in [0, 1]$ such that $d(Tx, Ty) \leq c \cdot d(x, y)$ for all $(x, y) \in \mathfrak{R}$.

Then T has a fixed point in X .

Proof. As in the proof of Theorem 3, let $x_0 \neq Tx_0$, $h = d(x_0, Tx_0) > 0$ and $y_n = T^n(x_0)$ for every $n = 0, 1, 2, \dots$. Because of properties (1) and (2.1), the sequence $\{y_n\}$ is \mathfrak{R} -preserving. In virtue of property 4), we have that

$$d(y_{n+1}, y_n) = d(T^{n+1}x_0, T^n x_0) \leq c^n \cdot h \quad \text{for } n = 1, 2, \dots$$

and hence $\{y_n\}$ is a Cauchy sequence. By property 3), $\{Ty_n\}$ converges to a point z and therefore $\{TTy_n\} = \{Ty_{n+1}\}$ converges to Tz because of property 2.2), thus $z = Tz$ because of the uniqueness of the limit.

Remark 4. Theorem 4 is generalized from Theorem 6 by defining the non-decreasing order “ \leq ” as relation $\mathfrak{R} \subseteq X^2$. Theorem 5 is generalized from Theorem 6 because if $\{x_n\}$ is a \mathfrak{R} -preserving Cauchy sequence in X , the completeness of X and the continuity of T assure that Tx_n converges to a point of X , i.e. the property 3) of Theorem 6 holds. The following example shows Theorem 5 is not applicable but Theorem 6 holds.

Example 3. Let $X = [0, 1] - 1/5$ with the metric $d(x, y) = |x - y|$ for all $x, y \in X$. Define $\mathfrak{R} \subseteq X^2$ as follows: $(x, y) \in \mathfrak{R}$ if $x \leq y$ for all $x, y \in X$. Define $T: X \rightarrow X$ as $Tx = (x + 4)/5$ for any $x \in X$. Obviously T is continuous in X and \mathfrak{R} is T -closed. If $\{x_n\}$ is a \mathfrak{R} -preserving (that is monotone non-decreasing) Cauchy sequence in X , then Tx_n is a monotone non-decreasing bounded sequence and hence converging to a point $z \in X$, thus properties 1), 2), 3) hold. Too property 4) holds because it is enough to assume $k = 1/5$. (X, d) is a metric space not complete, so Theorem 5 is not applicable while all the assumptions of Theorem 6 (or Theorem 4) are satisfied and 1 is the (unique) fixed point of T .

Remark 5. The uniqueness of the fixed point can be guaranteed from several additional properties of the relation \mathfrak{R} (cf. [1] [3] [4] [7] [8] [10] [19]) which we do not examine here. The following example, borrowed from [1], shows that the continuity of T in Theorem 6 is necessary.

Example 4. Consider $X = [0, 2]$ equipped with usual metric $d(x, y) = |x - y|$

for all $x, y \in X$. (X, d) is a complete metric space. Define $\mathfrak{R} \subseteq X^2$ as

$$\mathfrak{R} = \{(0,0), (0,1), (1,0), (1,1), (0,2)\}$$

and $T: X \rightarrow X$ as $T0 = 1/4$, $T(x) = 0$ if $0 < x \leq 1$, $Tx = 1$ if $1 < x \leq 2$. \mathfrak{R} is T -closed but T is not continuous. Consider any \mathfrak{R} -preserving sequence $\{x_n\}$, then $(x_n, x_{n+1}) \in \mathfrak{R} - \{(0,2)\}$ for all $n = 0, 1, 2, \dots$. Hence $x_n = 0$ or $x_n = 1$ for all $n = 0, 1, 2, \dots$. If $\{x_n\}$ is a \mathfrak{R} -preserving Cauchy sequence in X , then we have definitively $x_n = 0$ (resp., $x_n = 1$), i.e. there exists some suitable integer m such that $x_n = 0$ (resp., $x_n = 1$) for every integer $n > m$, which implies that $Tx_n = 1/4$ (resp., $Tx_n = 0$) for all $n > m$. Further

$$d(T0, T0) = d(T1, T1) = 0,$$

$$d(T1, T0) = d(T0, T1) = 1/4 \leq c \cdot 1 = cd(1, 0),$$

$$d(T0, T2) = 3/4 \leq c \cdot 2 = c \cdot d(0, 2),$$

where $c = 1/2$. Thus all the hypothesis of Theorem 6 hold except property 2.2 but T has no fixed points.

5. Conclusions

We have generalized fixed point theorems for theoretic-relation contractions about continuous selfmaps of metric spaces. Suitable examples prove the effective generalization of our results in metric spaces not necessarily complete.

Future studies shall be necessary for establishing extensions of the results here presented, essentially common fixed point theorems involving Cauchy sequences of Jungck type [6] [19] under a generalized condition of weak commutativity of two selfmaps such as, the weak compatibility (cf., e.g. [21]).

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Conflicts of Interest

The authors declare that they have no conflict of interest.

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