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Random Attractor Family for the Kirchhoff Equation of Higher Order with White Noise

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Abstract

The existence of random attractor family for a class of nonlinear high-order Kirchhoff equation stochastic dynamical systems with white noise is studied. The Ornstein-Uhlenbeck process and the weak solution of the equation are used to deal with the stochastic terms. The equation is transformed into a general stochastic equation. The bounded stochastic absorption set is obtained by estimating the solution of the equation and the existence of the random attractor family is obtained by isomorphic mapping method. Temper random compact sets of random attractor family are obtained.

Keywords

Stochastic Dynamical System, White Noise, Random Attractor Family, Wiener Process, Ornstein-Uhlenbeck Process

1. Introduction

In this paper, we study the random attractor family of solutions to the strongly damped stochastic Kirchhoff equation with white noise:

$$u_{tt} + M\left(\left\|D^m u\right\|^2\right)(-\Delta)^m u + \beta(-\Delta)^m u_t + g(x,u) = q(x)\dot{W}, \tag{1.1}$$

with the Dirichlet boundary condition

$$u(x,t) = 0, \frac{\partial^i u}{\partial v^i} = 0, i = 1,2,\cdots,m-1, x \in \partial\Omega, t > 0, \tag{1.2}$$

and the initial value conditions

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x), x \in \Omega \subset R^n, \tag{1.3}$$

where $m > 1$ is a positive integer; $\beta > 0$ is a constant; Ω is a bounded region with smooth boundary in R^n. Δ is the Laplacian with respect to the variable to variable $x \in \Omega$. M is a general real-valued function; $g(x,u)$ is a non-linear and non-local source term. W is derivative of a one-dimensional two-valued Wiener process $W(t)$ and $q(x)\dot{W}$ formally describes white noise.

B.L. Guo and X.K. Pu described in detail the related concepts and theories of infinite dimensional stochastic dynamical systems, and discussed in detail the existence and uniqueness, attractor and inertial manifold of some nonlinear evolution equations and wave equation solutions in [1] [2].

D.H. Cai and X.M. Fan [3], considered the dissipative KDV equation with multiplicative noise.

$$\mathrm{d}u = \left(au_{xxxx} + u_{xx} + \beta uu_{xx} + ru\right)\mathrm{d}t = f(x)\mathrm{d}t + bu\mathrm{d}W(t), x \in D, t > 0. \quad (1.4)$$

By transforming the equation into a stochastic KDV-type equation without white noise, the existence of stochastic attractors for dynamic systems determined by the original equation is proved by discussing the dynamic absorptivity and asymptotic property determined by the new equation.

Yin *et al.* [4] have mainly studied the dissipative Hamiltonian amplitude modulated wave instability equation with multiplicative white noise.

$$\mathrm{d}u_t + \alpha u_t \mathrm{d}t - \beta u_{xt}\mathrm{d}t - \gamma u_{xx}\mathrm{d}t + iu_x + f\left(|u|^2\right)u\mathrm{d}t = u \cdot \mathrm{d}W(t). \quad (1.5)$$

Stochastic dynamic system has compact random attractors in space $E_0 = H_1 \times L^2$.

Xu *et al.* [5] studied the non-autonomous stochastic wave equation with dispersion and dissipation terms.

$$u_{tt} - \Delta u - \alpha\Delta u_t - \beta u_{tt} + h(u)u_t + \lambda u + f(x,u) = g(x,t)u + \varepsilon u \cdot \frac{\mathrm{d}W}{\mathrm{d}t}. \quad (1.6)$$

The existence of random attractors for non-autonomous stochastic wave equations with product white noise is obtained by using the uniform estimation of solutions and the technique of decomposing solutions in a region.

Lin *et al.* [6] studied the existence of stochastic attractors for higher order nonlinear strongly damped Kirchhoff equation.

$$\mathrm{d}u_t + \left[(-\Delta)^m u_t + \phi\left(\left\|D^m u\right\|^2\right)(-\Delta)^m u + g(u)\right]\mathrm{d}t = q(x)\mathrm{d}W(t), x \in \Omega, m > 1. \quad (1.7)$$

The O-U process is mainly used to deal with the stochastic terms, and the existence of stochastic attractors is obtained.

Qin *et al.* [7] studied random attractors for the Kirchhoff-type suspension bridge equations with Strong Damping and white noises.

$$u_{tt} + \Delta^2 u + \Delta^2 u_t + \left(p - |\nabla u|^2\right)\Delta u + bu^+ + f(u) = q(x)\dot{W}. \quad (1.8)$$

Kirchhoff stress term $\left(p - |\nabla u|^2\right)\Delta u$ and dissipation term bu^+ are treated. It is assumed that the non-linear term $f(u)$ satisfies the growth and dissipation conditions.

For more relevant studies, it can be referred to references in [8]-[13].

On the basis of some random attractors of Kirchhoff equation with white noise studied by predecessors, the existence and uniqueness of solutions of stochastic higher-order Kirchhoff equation with strong damping of white noise, nonlinear and non-local source terms and the existence of attractors of stochastic Kirchhoff equation are discussed. This paper is organized as follows. In Section 2, some basic assumptions and basic concepts related to random attractor for general random dynamical system are presented. Section 3 deals with random term and proof the existence of random attractor family by using the isomorphism mapping method.

2. Preliminaries

In this section, some symbols are made and assumption Kirchhoff Stress term $M(s)$ satisfying condition (a) and Nonlinear term $g(x,u)$ satisfies condition (b). In addition, some basic definitions of stochastic dynamical systems are also introduced.

For narrative convenience, we introduce the following symbols:

$$D=\nabla, H_0^m(\Omega)=H^m(\Omega)\cap H_0^1(\Omega), H=L^2(\Omega),$$

$$H_0^{m+k}(\Omega)=H^{m+k}(\Omega)\cap H_0^1(\Omega),$$

$$E_k=H_0^{m+k}(\Omega)\times H_0^k(\Omega),(k=0,1,2,\cdots,m).$$

And definition

$$(y_1,y_2)=\left(\nabla^{m+k}u_1,\nabla^{m+k}u_2\right)+\left(\nabla^k v_1,\nabla^k v_2\right),\forall y_i=(u_i,v_i)\in E_k, i=1,2. \tag{2.1}$$

Kirchhoff Stress term $M(s)$ satisfies condition (a):

a) $M(s)$ is locally bounded and measurable, $M(s)\in C^2(\Omega)$ and $1+\varepsilon\le\sigma_0\le M(s)\le\sigma_1$ where σ_1,σ_2 is a constant;

Nonlinear term $g(x,u)$ satisfies condition (b):

b) Let $g(x,u)$ be nonnegative nonlocal bounded and measurable, $g(x,u)\in C^2(\Omega)$, $g(x,u)\le a(x)\left(1+|u|^P\right)$, $0<a(x)$ and $a(x)\in C^1$;

Here are some basics about random attractors.

Let $\left(B\left(R^+\right)\times F\times B(X), B_k(w)\subset D(w)\right)$ be a probabilistic space and define a family of transformation $\{\theta_t, t\in R\}$ preserving measures and ergodicity:

$$\theta_t w(\cdot)=w(\cdot+t)-w(t), \tag{2.2}$$

then $\left(\Omega,F,P,(\theta_t)\right)_{t\in R}$ is an ergodic metric dynamical system.

Let $\left(X,\|\cdot\|_X\right)$ be a complete separable metric space and $B(X)$ be a Borel σ-algebra on X.

Definition 2.1. ([7]) Let $\left(\Omega,F,P,(\theta_t)\right)_{t\in R}$ is a metric dynamic system, suppose that the mapping

$$S:R^+\times\Omega\times X\to X,\ (t,w,x)\mapsto S(t,w,x), \tag{2.3}$$

is $\left(B\left(R^+\right)\times F\times B(X)\right), B(X)$-measurable mapping and satisfies the following properties:

1) The mapping $S(t,w):=S(t,w,\cdot)$ satisfies

$$S(0,w)=id,\ S(t+s,w)=S(t,\theta_s w)\circ S(s,w); \tag{2.4}$$

2) $(t,w,x)\mapsto S(t,w,x)$ is continuous, for any $w\in\Omega$.

Then S is a continuous stochastic dynamical system on $(\Omega,F,P,(\theta_t))_{t\in R}$.

Definition 2.2. ([7]) It is said that the random set $B(w)\subset X$ is tempered, for $w\in\Omega,\beta\geq 0$, we have

$$\lim_{|s|\to\infty}\inf \mathrm{e}^{-\beta s} d\left(B\left(\theta_{-s}w\right)\right)=0 \tag{2.5}$$

where $d(B)=\sup_{x\in B}\|x\|_X$, for any $x\in X$.

Definition 2.3. ([7]) Note that $D(w)$ is the set of all random sets on X, and random set $B_k(w)$ is called the absorption set on $D(w)$. If for any $B_k(w)\subset D(w)$ and $P-a.e.w\in\Omega$, there exists $T_B(w)>0$ such that

$$S(t,\theta_{-t}\omega)\left(B(\theta_{-t}\omega)\right)\subset B_0(\omega). \tag{2.6}$$

Definition 2.4. ([7]) Random set $A(w)$ called the random attractor of continuous stochastic dynamical systems $S(t)$ on X, if random set $A(w)$ satisfies the following conditions:

1) $A(w)$ is a random compact set;

2) $A(w)$ is the invariant set $D(w)$, that is, for any $t>0$ $S(t,w)A(w)=A(\theta_t w)$;

3) $A(w)$ attracts all the set on $D(w)$, that is, for any $B(w)\subset D(w)$ and $P-a.e.w\in\Omega$, with the following limit:

$$\lim_{t\to\infty} d\left(S(t,\theta_{-t}w)\left(B(\theta_{-t}w)\right),A(w)\right)=0, \tag{2.7}$$

where $d(A,B)=\sup_{x\in A}\inf_{y\in B}\|x-y\|_H$ is Hausdorff half distance. (where $A,B\subseteq H$).

Definition 2.5. ([7]) Let random set $B_k(w)\subset D(w)$ be a random absorbing set of stochastic dynamical system $(S(t,w))_{t>0}$, and the random set $B_k(w)$ satisfy:

1) Random set $B_k(w)$ is a closed set on Hilbert space X.

2) For $P-a.e.w\in\Omega$, random set $B_k(w)$ satisfies the following asymptotic compactness conditions: for any sequence $x_n\in S(t_n,\theta_{-t_n}w)B_0(\theta_{-t_n}w)$, there is an convergence subsequence in space X, when $t_n\to+\infty$, Then stochastic dynamical system $(S(t,w))_{t>0}$ has a unique global attractor.

$$A_k(\omega)=\bigcap_{\tau\geq t_k(w)}\overline{\bigcup_{t\geq\tau} S(t,\theta_{-t}\omega)B_0(\theta_{-t}\omega)}. \tag{2.8}$$

The Ornstein-Uhlenbeck process [7] is given as following.

Let $z(\theta_t w)=-\alpha\int_{-\infty}^{0}\mathrm{e}^{\alpha t}\theta_t w(\tau)\mathrm{d}\tau$, where $t\in R$. For any $t\geq 0$, the stochastic process $z(\theta_t w)$ satisfies the Ito equation

$$\mathrm{d}z+\alpha z\mathrm{d}t=\mathrm{d}W(t). \tag{2.9}$$

According to the nature of O-U process, there exists a probability measure P, θ_t-invariant set, and the above stochastic process

$$z(\theta_t\omega)=-\alpha\int_{-\infty}^{0}\mathrm{e}^{\alpha\tau}\theta_t\omega(\tau)\mathrm{d}\tau. \tag{2.10}$$

satisfies the following properties

1) The mapping $S \to z(\theta_s \omega)$ is a continuous mapping, for any given $\omega \in \Omega_0$;

2) The random variable $\|z(\omega)\|$ is tempered;

3) There exist a tempered set $r(\omega) > 0$, such that

$$\|z(\theta_t w)\| + \|z(\theta_t w)\|^2 \le r(\theta_t w) \le r(w) e^{\frac{\alpha}{2} t}; \tag{2.11}$$

4) $$\lim_{t\to\infty} \frac{1}{t} \int_0^t |z(\theta_\tau \omega)|^2 \, d\tau = \frac{1}{2\alpha}; \tag{2.12}$$

5) $$\lim_{t\to\infty} \frac{1}{t} \int_0^t |z(\theta_\tau \omega)| \, d\tau = \frac{1}{\sqrt{\pi\alpha}}. \tag{2.13}$$

3. The Existence of Random Attractor Family

In this section, we consider the existence of random attractor family. To deal with the random term we need to transform the problem (1.1) - (1.3) into a general stochastic problem. It is proved that there exists a bounded stochastic absorption set for stochastic dynamical systems. The stochastic dynamical system exists stochastic attractor family and a slowly increasing stochastic compact set.

For convenience, Equation (1.1) - (1.3) can be transformed into

$$\begin{cases} du = u_t dt \\ du_t + \left[M\left(\left\| A^{\frac{m}{2}} \right\|^2 \right) A^m u + \beta A^m u_t + g(x,u) \right] dt = q(x) dW(t), t \in [0, +\infty], \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x) \end{cases} \tag{3.1}$$

where $A = -\Delta$.

Let $\phi = (u, y)^{\mathrm{T}}$, $y = u_t + \varepsilon u$. Then the problem (3.1) can be simplified to:

$$\begin{cases} d\phi + L\phi dt = F(\theta_t \omega, \phi) \\ \phi_0(\omega) = (u_0, u_1 + \varepsilon u_0)^{\mathrm{T}} \end{cases} \tag{3.2}$$

where

$$\phi = \begin{pmatrix} u \\ y \end{pmatrix}, L = \begin{pmatrix} \varepsilon I & -I \\ \left(M\left(\left\| A^{\frac{m}{2}} u \right\|^2 - \beta\varepsilon \right) A^m + \varepsilon^2 \right) I & (\beta A^m - \varepsilon) I \end{pmatrix},$$

$$F(\theta_t \omega, \phi) = \begin{pmatrix} 0 \\ -g(x,u) + q(x) dW(t) \end{pmatrix}.$$

Let $v = y - q(x)\delta(\theta_t \omega)$, then the question (3.2) can be written as:

$$\begin{cases} d\varphi + L\varphi dt = \bar{F}(\theta_t \omega, \varphi) \\ \varphi_0(\omega) = (u_0, u_1 + \varepsilon u_0 - q(x)\delta(\theta_t \omega))^{\mathrm{T}'} \end{cases} \tag{3.3}$$

where

$$\varphi=\begin{pmatrix}u\\ v\end{pmatrix}, L=\begin{pmatrix}\varepsilon I & -I\\ \left(M\left(\left\|A^{\frac{m}{2}}u\right\|^{2}-\beta\varepsilon\right)A^{m}+\varepsilon^{2}\right)I & \left(\beta A^{m}-\varepsilon\right)I\end{pmatrix},$$

$$\bar{F}(\theta_t\omega,\varphi)=\begin{pmatrix}q(x)\delta(\theta_t\omega)\\ -g(x,u)-\left(\beta A^{m}+\varepsilon+1\right)q(x)\delta(\theta_t\omega)\end{pmatrix}.$$

Lemma 3.1. Let $E_k=H_0^{m+k}(\Omega)\times H_0^{k}(\Omega)$ for any $y=(y_1,y_2)^{\mathrm{T}}\in E_k$ $(k=1,2,\cdots,m)$, if $0<\varepsilon\le\frac{1}{\beta-1}$,

$$(Ly,y)_{E_k}\ge k_1\|y\|_{E_k}^{2}+k_2\left\|\nabla^{m+k}y_2\right\|^{2}, \tag{3.4}$$

where $k_1=\min\left\{\frac{\beta\varepsilon+\varepsilon-\varepsilon^{2}\lambda_1^{-m}}{2\beta},\frac{\beta\lambda_1^{m}-\beta\varepsilon^{2}-2\varepsilon}{2}\right\}, k_2=\frac{\beta(1-\beta\varepsilon+\varepsilon)}{2\beta}$.

Proof: For any $y=(y_1,y_2)^{\mathrm{T}}$, according to hypothesis (a), we have

$$\begin{aligned}
(Ly,y)_{E_k}&=\left(\nabla^{m+k}(\varepsilon y_1-y_2),\nabla^{m+k}y_1\right)+\Bigg(\nabla^{k}M\left(\left\|A^{\frac{m}{2}}u\right\|^{2}\right)A^{m}y_1\\
&\qquad-\beta\varepsilon A^{m}y_1+\varepsilon^{2}y_1+\beta A^{m}y_2-\varepsilon y_2,\nabla^{k}y_2\Bigg)\\
&=\varepsilon\left\|\nabla^{m+k}y_1\right\|^{2}-(\nabla^{m+k}y_1,\nabla^{m+k}y_2)+M\left(\left\|A^{\frac{m}{2}}u\right\|^{2}\right)\left(\nabla^{m+k}y_1,\nabla^{m+k}y_2\right)\\
&\quad-\beta\varepsilon\left(\nabla^{m+k}y_1,\nabla^{m+k}y_2\right)+\varepsilon^{2}\left(\nabla^{k}y_1,\nabla^{k}y_2\right)\\
&\quad+\beta\left(\nabla^{m+k}y_2,\nabla^{m+k}y_2\right)-\varepsilon\left(\nabla^{k}y_2,\nabla^{k}y_2\right)\\
&=\varepsilon\left\|\nabla^{m+k}y_1\right\|^{2}-\left(\nabla^{m+k}y_1,\nabla^{m+k}y_2\right)+\left(M\left(\left\|A^{\frac{m}{2}}u\right\|^{2}\right)-\varepsilon\right)\left(\nabla^{m+k}y_1,\nabla^{m+k}y_2\right)\\
&\quad+(\varepsilon-\beta\varepsilon)\left(\nabla^{m+k}y_1,\nabla^{m+k}y_2\right)+\varepsilon^{2}\left(\nabla^{k}y_1,\nabla^{k}y_2\right)\\
&\quad+\beta\left(\nabla^{m+k}y_2,\nabla^{m+k}y_2\right)-\varepsilon\left(\nabla^{k}y_2,\nabla^{k}y_2\right)\\
&\ge\varepsilon\left\|\nabla^{m+k}y_1\right\|^{2}-(\beta\varepsilon-\varepsilon)\left(\nabla^{m+k}y_1,\nabla^{m+k}y_2\right)+\varepsilon^{2}\left(\nabla^{k}y_1,\nabla^{k}y_2\right)\\
&\quad+\beta\left(\nabla^{m+k}y_2,\nabla^{m+k}y_2\right)-\varepsilon\left(\nabla^{k}y_2,\nabla^{k}y_2\right)\\
&\ge\varepsilon\left\|\nabla^{m+k}y_1\right\|^{2}-\frac{(\beta\varepsilon-\varepsilon)}{2\beta}\left\|\nabla^{m+k}y_1\right\|^{2}-\frac{\beta(\beta-1)\varepsilon}{2}\left\|\nabla^{m+k}y_2\right\|^{2}\\
&\quad-\frac{\varepsilon^{2}}{2\beta}\left\|\nabla^{k}y_1\right\|^{2}-\frac{\beta\varepsilon^{2}}{2}\left\|\nabla^{k}y_2\right\|+\beta\left\|\nabla^{m+k}y_2\right\|^{2}-\varepsilon\left\|\nabla^{k}y_2\right\|^{2}\\
&=\frac{2\beta\varepsilon-\beta\varepsilon+\varepsilon}{2\beta}\left\|\nabla^{m+k}y_1\right\|^{2}-\frac{\beta-\beta^{2}\varepsilon+\beta\varepsilon}{2\beta}\left\|\nabla^{m+k}y_2\right\|^{2}-\frac{\varepsilon^{2}}{2\beta}\left\|\nabla^{k}y_1\right\|^{2}\\
&\quad+\frac{\beta}{2}\left\|\nabla^{m+k}y_2\right\|^{2}-\frac{\beta\varepsilon^{2}+2\varepsilon}{2}\left\|\nabla^{k}y_2\right\|\\
&\ge\frac{\beta\varepsilon+\varepsilon}{2\beta}\left\|\nabla^{m+k}y_1\right\|^{2}+\frac{\beta(1-\beta\varepsilon+\varepsilon)}{2\beta}\left\|\nabla^{m+k}y_2\right\|^{2}-\frac{\varepsilon^{2}\lambda_1^{-m}}{2\beta}\left\|\nabla^{m+k}y_1\right\|^{2}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{\beta\lambda_1^m-\beta\varepsilon^2-2\varepsilon}{2}\left\|\nabla^k y_2\right\|^2\\
&=\frac{\beta\varepsilon+\varepsilon-\varepsilon^2\lambda_1^{-m}}{2\beta}\left\|\nabla^{m+k}y_1\right\|^2+\frac{\beta(1-\beta\varepsilon+\varepsilon)}{2\beta}\left\|\nabla^{m+k}y_2\right\|^2\\
&+\frac{\beta\lambda_1^m-\beta\varepsilon^2-2\varepsilon}{2}\left\|\nabla^k y_2\right\|^2\\
&\geq k_1\left(\left\|\nabla^{m+k}y_1\right\|^2+\left\|\nabla^k y_2\right\|^2\right)+k_2\left\|\nabla^{m+k}y_2\right\|^2\\
&\geq k_1\|y\|_{E_k}^2+k_2\left\|\nabla^{m+k}y_2\right\|^2.
\end{aligned}
\tag{3.5}
$$

where $k_1=\min\left\{\frac{\beta\varepsilon+\varepsilon-\varepsilon^2\lambda_1^{-m}}{2\beta},\frac{\beta\lambda_1^m-\beta\varepsilon^2-2\varepsilon}{2}\right\},k_2=\frac{\beta(1-\beta\varepsilon+\varepsilon)}{2\beta}$.

Lemma 3.1 is proved.

Lemma 3.2. Let ϕ be a solution of the problem (3.2), then there exists a bounded random compact set $\bar{B}_{0k}(\omega)\in D(E_k)$, so that for any random set $\bar{B}_{0k}(\omega)\in D(E_k)$, there exists a random variable $T_{B_k(\omega)}>0$, such that

$$
\phi(t,\theta_t\omega)B_k(\theta_{-t}\omega)\subset\bar{B}_{0k}(\omega),\forall t\geq T_{B_k(\omega)},\omega\in\Omega. \tag{3.6}
$$

Proof: Let φ be a solution of the problem (3.3), by taking the inner product of two sides of the Equation (3.3) is obtained by using $\varphi=(u,v)^{\mathrm{T}}\in E_k$,

$$
\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\varphi\|_{E_k}^2+(L\varphi,\varphi)_{E_k}=\left(\bar{F}(\theta_t\omega,\varphi),\varphi\right). \tag{3.7}
$$

From Lemma 1, we have

$$
(L\varphi,\varphi)\geq k_1\|\varphi\|_{E_k}^2+k_2\left\|\nabla^{m+k}v\right\|^2. \tag{3.8}
$$

According to the inner product defined on E_k.

$$
\begin{aligned}
\left(\bar{F}(\theta_t\omega,\varphi),\varphi\right)=&\left(\nabla^{m+k}q(x)\delta(\theta_t\omega),\nabla^{m+k}u\right)+\left(\nabla^k\left(-g(x,u)\right.\right.\\
&\left.\left.+\left(\varepsilon+1-\beta A^m\right)q(x)\delta(\theta_t\omega)\right),\nabla^k v\right).
\end{aligned}
\tag{3.9}
$$

According to Holder inequality, Young inequality and Poincare inequality, we have

$$
\left(\nabla^{m+k}q(x)\delta(\theta_t\omega),\nabla^{m+k}u\right)\leq\frac{\lambda_1^{-m}}{2\varepsilon}\left\|A^{m+\frac{m}{2}}q(x)\delta(\theta_t\omega)\right\|^2+\frac{\varepsilon}{2}\left\|\nabla^{m+k}u\right\|^2. \tag{3.10}
$$

$$
\begin{aligned}
&\left(\nabla^{m+k}\left(1-\beta A^m\right)q(x)\delta(\theta_t\omega),\nabla^k v\right)\\
&\leq\frac{1}{2\varepsilon}\left(\left\|\nabla^k q(x)\delta(\theta_t\omega)\right\|^2+\beta^2\left\|A^{m+\frac{m}{2}}q(x)\right\|^2\right)\left|\delta(\theta_t\omega)\right|^2+\frac{\varepsilon\lambda_1^{-m}}{2}\left\|\nabla^{m+k}v\right\|^2.
\end{aligned}
\tag{3.11}
$$

$$
\left(\nabla^k\varepsilon q(x)\delta(\theta_t\omega),\nabla^k v\right)\leq\frac{\varepsilon}{2}\left\|\nabla^k q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2+\frac{\varepsilon\lambda_1^{-m}}{2}\left\|\nabla^{m+k}v\right\|^2. \tag{3.12}
$$

According to hypothesis (b), we have

$$\begin{aligned}
&\left(-\nabla^k g(x,u),\nabla^k v\right)\\
&\le \left|g(x,u),\nabla^{2k}v\right| \le \int_\Omega \left|a(x)\left(1+|u|^p\right)\nabla^{2k}v\right|\mathrm{d}x\\
&\le \|a(x)\|_\infty \int_\Omega \left(1+|u|^p\,\mathrm{d}x\right)^{\frac{1}{2}}\left\|\nabla^{2k}v\right\| \le C_1\left\|\nabla^{2k}v\right\|\\
&\le \frac{\beta\varepsilon^2\lambda_1^{-(m-k)}}{2}\left\|\nabla^{m+k}v\right\|^2+\frac{C_1^2}{2\beta\varepsilon^2}.
\end{aligned} \tag{3.13}$$

Combining (3.8)-(3.13) yields, we have

$$\begin{aligned}
&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\varphi\|_{E_k}^2+k_1\|\varphi\|_{E_k}^2+k_2\left\|\nabla^{m+k}v\right\|^2\\
&\le \frac{\lambda_1^{-m}}{2\varepsilon}\left\|A^{m+\frac{m}{2}}q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2+\frac{\varepsilon}{2}\left\|\nabla^{m+k}u\right\|^2+\frac{\varepsilon\lambda_1^{-m}}{2}\left\|\nabla^{m+k}v\right\|^2\\
&\quad+\frac{1}{2\varepsilon}\left\|\nabla^k q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2+\frac{\beta^2}{2\varepsilon}\left\|A^{m+\frac{m}{2}}q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2+\frac{\varepsilon\lambda_1^{-m}}{2}\left\|\nabla^{m+k}v\right\|^2\\
&\quad+\frac{\varepsilon}{2}\left\|\nabla^k q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2+\frac{\beta\varepsilon^2\lambda_1^{-(m-k)}}{2}\left\|\nabla^{m+k}v\right\|^2+\frac{C_1^2}{2\beta\varepsilon^2}.
\end{aligned} \tag{3.14}$$

Then

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\|\varphi\|_{E_k}^2+2k_1\|\varphi\|_{E_k}^2+\left(2k_2-2\varepsilon\lambda_1^{-m}-\beta\varepsilon^2\lambda_1^{-(m-k)}\right)\left\|\nabla^{m+k}v\right\|^2\\
&\le \varepsilon\left\|\nabla^{m+k}u\right\|^2+\frac{C_1^2}{2\beta\varepsilon^2}+\frac{\beta^2}{\varepsilon}\left\|A^{m+\frac{m}{2}}q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2\\
&\quad+\frac{\lambda_1^{-m}}{\varepsilon}\left\|A^{m+\frac{m}{2}}q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2+\left(\varepsilon+\frac{1}{\varepsilon}\right)\left\|\nabla^k q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2.
\end{aligned} \tag{3.15}$$

Taking $\eta=2k_1$,

$$\begin{aligned}
P_1=&\frac{\beta^2}{\varepsilon}\left\|A^{m+\frac{m}{2}}q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2+\frac{\lambda_1^{-m}}{\varepsilon}\left\|A^{m+\frac{m}{2}}q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2\\
&+\left(\varepsilon+\frac{1}{\varepsilon}\right)\left\|\nabla^k q(x)\right\|^2\left|\delta(\theta_t\omega)\right|^2.
\end{aligned}$$

we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\varphi\|_{E_k}^2+\eta\|\varphi\|_{E_k}^2\le C_2+P_1\left\|\delta(\theta_t\omega)\right\|^2. \tag{3.16}$$

By the Gronwall inequality, $P-a.e.\omega\in\Omega$ then

$$\|\varphi(t,\omega)\|_{E_k}^2\le \mathrm{e}^{-\eta t}\|\varphi_0(\omega)\|_{E_k}^2+\int_0^t \mathrm{e}^{-\eta(t-r)}\left(C_2+P_1\left\|\delta(\theta_r\omega)\right\|^2\right)\mathrm{d}r. \tag{3.17}$$

And because $\delta(\theta_t\omega)$ is tempered, and $\delta(\theta_t\omega)$ is continuous about *t*, so according to reference [7], we can get a temper random variable $r_1:\Omega\to R^+$, so that for any $t\in R,\omega\in\Omega$, we have

$$\left|\delta(\theta_t\omega)\right|^2\le r_1(\theta_t\omega)\le \mathrm{e}^{\frac{\eta}{2}t}r_1(\omega). \tag{3.18}$$

Replace ω in Equation (3.17) with $\theta_{-t}\omega$, we can obtain that

$$\left\|\varphi\left(t,\theta_{-t}\omega\right)\right\|_{E_k}^2 \le \mathrm{e}^{-\eta t}\left\|\varphi_0\left(\theta_{-t}\omega\right)\right\|_{E_k}^2 + \int_0^t \mathrm{e}^{-\eta(t-r)}\left(C_2 + p_1\left|\delta\left(\theta_t\omega\right)\right|^2\right)\mathrm{d}r, \quad (3.19)$$

where ($r-t=\tau$)

$$\begin{aligned}&\int_0^t \mathrm{e}^{-\eta(t-r)}\left(C_2 + p_1\left|\delta\left(\theta_{r-t}\omega\right)\right|^2\right)\mathrm{d}r\\&=\int_{-t}^0 \mathrm{e}^{\eta\tau}\left(C_2 + p_1\left|\delta\left(\theta_\tau\omega\right)\right|^2\right)\mathrm{d}r \le \frac{C_2}{\eta}+\frac{2}{\eta}p_1 r_1\left(\omega\right).\end{aligned} \quad (3.20)$$

Because $\varphi_0\left(\theta_{-t}\omega\right)\in B_{ok}\left(\theta_{-t}\omega\right)$ is also temper, and $\left|\delta\left(\theta_{-t}\omega\right)\right|$ is also tempered, so we can let

$$R_0^2\left(\omega\right)=\frac{C_2}{\eta}+\frac{2}{\eta}p_1 r_1\left(\omega\right). \quad (3.21)$$

Then $R_0^2\left(\omega\right)$ is also temper, $\hat{B}_{0k}=\left\{\varphi\in E_k:\left\|\varphi\right\|_{E_k}\le R_0\left(\omega\right)\right\}$ is a random absorb set, and because of

$$\begin{aligned}&\bar{S}\left(t,\theta_t\omega\right)\varphi_0\left(\theta_{-t}\omega\right)\\&=\varphi\left(t,\theta_{-t}\omega\right)\left(\varphi_0\left(\theta_{-t}\omega\right)+\left(0,q\left(x\right)\delta\left(\theta_{-t}\omega\right)\right)^{\mathrm{T}}\right)-\left(0,q\left(x\right)\delta\left(\theta_{-t}\omega\right)\right)^{\mathrm{T}}.\end{aligned} \quad (3.22)$$

So let

$$\hat{B}_{0k}\left(\omega\right)=\left\{\varphi\in E_k:\left\|\varphi\right\|_{E_k}\le R_0\left(w\right)+\left\|\nabla^k q\left(x\right)\delta\left(w\right)\right\|=\bar{R}_0\left(w\right)\right\}. \quad (3.23)$$

then $\hat{B}_{0k}\left(\omega\right)$ is a random absorb set of $\varphi\left(t,\omega\right)$, and $\hat{B}_{0k}\left(\omega\right)\in D\left(E_k\right)$.

Thus, the whole proof is proved.

Lemma 3.3. When $k=m$, for any $B_m\left(\omega\right)\in D\left(E_m\right)$, Let $\phi\left(t\right)$ is a solution of the Equation (3.2) with the be initial value $\phi_0=\left(u_0,u_1+\varepsilon u_0\right)^{\mathrm{T}}\in B_m$, it can be decompose $\phi=\phi_1+\phi_2$, where ϕ_1,ϕ_2 satisfy

$$\begin{cases}\mathrm{d}\phi_1+L\phi_1\mathrm{d}t=0\\ \phi_{10}=\phi_0=\left(u_0,u_1+\varepsilon u_0\right)^{\mathrm{T}}\end{cases} \quad (3.24)$$

$$\begin{cases}\mathrm{d}\phi_2+L\phi_2\mathrm{d}t=F\left(\omega,\phi\right)\\ \phi_{20}=0\end{cases} \quad (3.25)$$

Then $\left\|\phi_1\left(t,\theta_t\omega\right)\right\|_{E_m}^2\to 0,\left(t\to\infty\right)$, for any $\phi_0\left(\theta_t\omega\right)\in B_m\left(\theta_{-t}\omega\right)$, there exist a temper random radius $R_1\left(\omega\right)$, such that

$$\left\|\phi_2\left(t,\theta_{-t}\omega\right)\right\|_{E_m}^2\le R_1\left(\omega\right). \quad (3.26)$$

Proof: Let $\varphi=\varphi_1+\varphi_2=\left(u_1,u_{1t}+\varepsilon u_1\right)^{\mathrm{T}}+\left(u_2,u_{2t}+\varepsilon u_2-q\left(x\right)\delta\left(\theta_t\omega\right)\right)^{\mathrm{T}}$ is a solution of Equation (3.3), then according to the Equation (3.24) and (3.25), we can see that φ_1,φ_2 meet separately

$$\begin{cases}\mathrm{d}\varphi_1+L\varphi_1\mathrm{d}t=0\\ \varphi_{10}=\varphi_0=\left(u_0,u_1+\varepsilon u_0\right)^{\mathrm{T}}\end{cases} \quad (3.27)$$

$$\begin{cases}\mathrm{d}\varphi_2+L\varphi_2\mathrm{d}t=F\left(\omega,\varphi\right)\\ \varphi_{20}=0\end{cases} \quad (3.28)$$

By taking the inner product of equation within E_m, we have

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\varphi_1\|_{E_m}^2+\left(L\varphi_1,\varphi_1\right)_{E_m}=0. \tag{3.29}$$

According to lemma 1 and Gronwall inequality,

$$\left\|\varphi_1\left(t,\omega\right)\right\|_{E_m}^2\le \mathrm{e}^{-2k_1t}\left\|\varphi_0\left(\omega\right)\right\|_{E_m}^2. \tag{3.30}$$

Replacing ω by $\theta_{-t}\omega$ in (3.30), because $\delta\left(\theta_{-t}\omega\right)\in B_m$ is tempered, then

$$\left\|\varphi_1\left(t,\theta_{-t}\omega\right)\right\|_{E_m}^2\le \mathrm{e}^{-2k_1t}\left\|\varphi_0\left(\theta_{-t}\omega\right)\right\|_{E_m}^2\to 0,\forall\varphi_0\left(\theta_{-t}\omega\right)\in B_m. \tag{3.31}$$

Taking inner product (3.30) with $\varphi_2=\left(u_2,u_{2t}+\varepsilon u_2-q\left(x\right)\delta\left(\theta_t\omega\right)\right)^{\mathrm{T}}$ in E_m and from Lemma 1 and Lemma 2, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\varphi_2\|_{E_m}^2+\eta\|\varphi_2\|_{E_m}^2\le C_3+P_2\left|\delta\left(\theta_t\omega\right)\right|^2. \tag{3.32}$$

where $\eta=2k_1,P_2=\frac{\beta^2}{\varepsilon}\left\|A^{\frac{3m}{2}}q\left(x\right)\right\|^2+\frac{\lambda_1^{-m}}{\varepsilon}\left\|A^{\frac{3m}{2}}q\left(x\right)\right\|^2+\left(\varepsilon+\frac{1}{\varepsilon}\right)\left\|\nabla^k q\left(x\right)\right\|^2$.

Replacing ω by $\theta_{-t}\omega$ in (3.32) and from Gronwall inequality, we have

$$\begin{aligned}\left\|\phi_2\left(t,\theta_{-t}\omega\right)\right\|_{E_m}^2&\le \mathrm{e}^{-\eta t}\left\|\phi_{20}\left(\theta_{-t}\omega\right)\right\|_{E_m}^2+\int_0^t \mathrm{e}^{-\eta(t-r)}\left(C_2+P_1\left|\delta\left(\theta_{r-t}\omega\right)\right|^2\right)\mathrm{d}r\\&\le\frac{C_3}{\eta}+\frac{2}{\eta}P_2r_1\left(\omega\right).\end{aligned} \tag{3.33}$$

So there is exist a temper random radius

$$R_1^2\left(\omega\right)\le\frac{C_3}{\eta}+\frac{2}{\eta}P_2r_1\left(\omega\right). \tag{3.34}$$

For any $\omega\in\Omega$,

$$\left\|\varphi_2\left(t,\theta_{-t}\omega\right)\right\|_{E_m}\le R_1\left(\omega\right). \tag{3.35}$$

This completes the proof of Lemma 3.3.

Lemma 3.4. The Stochastic Dynamic System $\left\{S\left(t,\omega\right),t\ge 0\right\}$, while $t=0$, $P-a.e.\omega\in\Omega$ determined by Equation (3.2) has a compact attracting set $K\left(\omega\right)\subset E_k$.

Proof: Let $K\left(\omega\right)$ be a closed ball with radius $R_1\left(\omega\right)$ in space E_k. According to the embedding relation $E_k\subset E_0$, then $K\left(\omega\right)$ is a compact set in E_k. for any temper random set $B_k\left(\omega\right)$, for any $\forall\varphi\left(t,\theta_t\omega\right)\in B_k$, according to Lemma 3.1, $\varphi_2=\varphi-\varphi_1\in K\left(\omega\right)$, so for any $\forall t\ge T_{B_k(\omega)}>0$, we have

$$\begin{aligned}&d_{E_k}\left(S\left(t,\theta_{-t}\omega\right)B_k\left(\theta_{-t}\omega\right),K\left(\omega\right)\right)\\&=\inf_{\vartheta(t)\in K(\omega)}\left\|\varphi\left(t,\theta_{-t}\omega\right)-\vartheta\left(t\right)\right\|_{E_k}^2\le\left\|\varphi\left(t,\theta_{-t}\omega\right)\right\|_{E_k}^2\\&\le \mathrm{e}^{-\eta t}\left\|\varphi_0\left(\theta_{-t}\omega\right)\right\|_{E_k}^2\to 0,\left(t\to\infty\right)\end{aligned} \tag{3.36}$$

So, the whole proof is complete.

According to Lemma 3.1 - Lemma 3.4, there are the following theorems.

Theorem 3.1. Random dynamical system $\left\{S\left(t,\omega\right),t\ge 0\right\}$ has a family of random attractors $A_k\left(\omega\right)\subset K\left(\omega\right)\subset E_k,\omega\in\Omega$, and there exists a slowly increasing random set $K\left(\Omega\right)$,

$$A_k(\omega)=\bigcap_{t\geq 0}\overline{\bigcup_{\tau\geq t} S(t,\theta_{-t}\omega)K(\theta_{-t}\omega)} \tag{3.37}$$

And $S(t,\omega)A_k(\omega)=A_k(\theta_t\omega)$.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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