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Erratum to "Manifolds with Bakry-Emery Ricci Curvature Bounded Below", Advances in Pure Mathematics, Vol. 6 (2016), 754-764

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The original online version of this article (Issa Allassane Kaboye, Bazanfaré Mahaman (2016) Manifolds with Bakry-Emery Ricci Curvature bounded below 6, 754-764. http://dx.doi.org/10.4236/apm.2016.611061 unfortunately contains a mistake. The author wishes to correct the errors.

***Lemma* 3.5.** *Let* $(M, g, e^{-f}dvolg)$ *be a complete smooth metric measure space with* $Ric_f \geq 0$; *fix* $p \in M$; *if there exists c so that* $|f(x)| \leq c$ *then for* $R \geq r > 0$

$$\frac{Vol_f(B(p,R))}{Vol_f(B(p,r))} \leq e^{4c}\left(\frac{R}{r}\right)^n$$

Proof

Let x be a point in M and let $\gamma:[0,r] \to M$ be a minimal geodesic joining p to x and $(e_i(t))_{1\leq i\leq n-1}$ be a parallel orthonormal vector fields along γ. Set $Y_i(t) = \frac{t}{r}e_i(t)$.

By the second variation formula we have:

$$m(r) = \Delta r \leq \sum_{i=1}^{n-1} I(Y_i, Y_i)$$

$$= \int_0^r \left(\sum_{i=1}^{n-1} \|Y_i'(t)\|^2 - \langle R(Y_i(t),\gamma'(t))\gamma'(t), Y_i(t)\rangle\right)\mathrm{d}t$$

$$\leq \frac{1}{r^2}\int_0^r \left(n-1-t^2 Ric(\gamma'(t))\right)dt$$

$$\frac{n-1}{r} + \int_0^r \frac{t^2}{r^2} Hess(f)(\gamma',\gamma')\mathrm{d}t$$

$$\frac{n-1}{r}+\int_0^r \frac{t^2}{r^2}\left(f\circ\gamma\right)'' \mathrm{d}t$$
$$=\frac{n-1}{r}+\frac{1}{r^2}\int_0^r \frac{\mathrm{d}}{\mathrm{d}t}\left(t^2\left(f\circ\gamma\right)'(t)\right)\mathrm{d}t-\frac{2}{r^2}\int_0^r t\left(f\circ\gamma\right)'(t)\mathrm{d}t$$
$$=\frac{n-1}{r}+\partial_r f-\frac{2}{r}f(x)+\frac{2}{r^2}\int_0^r\left(f\circ\gamma\right)(t)\mathrm{d}t$$

Hence

$$m_f(r)=\Delta r-\partial_r f=\frac{\partial}{\partial r}\left(\ln\left(A_f(r,\theta)\right)\right)$$
$$\le\frac{n-1}{r}+\partial_r f-\frac{2}{r}f(x)+\frac{2}{r^2}\int_0^r\left(f\circ\gamma\right)(t)\mathrm{d}t$$

For all positive reals r and s, integrating this relation we have:

$$\int_r^s m_f(t)\mathrm{d}t=\ln\left(\frac{A_f(s,\theta)}{A_f(r,\theta)}\right)\le\left(\frac{s}{r}\right)^{n-1}+2\int_r^s\left(\frac{1}{t^2}\int_0^t f(u)\mathrm{d}u-\frac{1}{t}f(t)\right)\mathrm{d}t$$
$$=\ln\left(\frac{s}{r}\right)^{n-1}-\left(\frac{2}{t}\int_0^t f\mathrm{d}t\right)\Bigg|_r^s+2\int_r^s\frac{1}{t}f\mathrm{d}t-2\int_r^s\frac{1}{t}f\mathrm{d}t$$
$$=\ln\left(\frac{s}{r}\right)^{n-1}-\left(\frac{2}{t}\int_0^t f\mathrm{d}t\right)\Bigg|_r^s\le\ln\left(\frac{s}{r}\right)^{n-1}+4c$$

Therefore we have $r^{n-1}A_f(s,\theta)\le e^{4c}A_f(r,\theta)s^{n-1}$. Hence

$$\int_0^R\int_{S^{n-1}}r^{n-1}A_f(s,\theta)\mathrm{d}\theta\mathrm{d}r\le e^{4c}\int_0^R\int_{S^{n-1}}A_f(r,\theta)\mathrm{d}\theta\mathrm{d}r$$

which implies

$$\frac{R^n}{n}\int_{S^{n-1}}A_f(s,\theta)\mathrm{d}\theta\le e^{4c}s^{n-1}\int_0^R\int_{S^{n-1}}A_f(r,\theta)\mathrm{d}\theta\mathrm{d}r=e^{4c}s^{n-1}vol_f\left(B(p,R)\right)$$

and integrating from 0 to R' with respect to s we obtain the conclusion.