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Properties of Suborbits of the Dihedral Group D_n Acting on Ordered Subsets

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Abstract

A group action on a set is a process of developing an algebraic structure through a relation defined by the permutations in the group and the elements of the set. The process suppresses most of the group properties, emphasizing the permutation aspect, so that the algebraic structure has a wider application among other algebras. Such structures not only reveal connections between different areas in Mathematics but also make use of results in one area to suggest conjectures and also prove results in a related area. The structure (G, X) is a transitive permutation group G acting on the set X. Investigations on the properties associated with various groups acting on various sets have formed a subject of recent study. A lot of investigations have been done on the action of the symmetric group S_n on various sets, with regard to rank, suborbits and subdegrees. However, the action of the dihedral group has not been thoroughly worked on. This study aims at investigating the properties of suborbits of the dihedral group D_n acting on ordered subsets of $X = \{1, 2, \cdots, n\}$. The action of D_n on $X^{\{r\}}$, the set of all ordered r-element subsets of X, has been shown to be transitive if and only if $n = 3$. The number of self-paired suborbits of D_n acting on $X^{\{r\}}$ has been determined, amongst other properties. Some of the results have been used to determine graphical properties of associated suborbital graphs, which also reflect some group theoretic properties. It has also been proved that when $G = D_n$ acts on ordered adjacent vertices of G, the number of self-paired suborbits is $n + 1$ if n is odd and $n + 2$ if n is even. The study has also revealed a conjecture that gives a formula for computing the self-paired suborbits of the action of D_n on its ordered adjacent vertices. Properties of suborbits are significant as they form a link between group theory and graph theory.

Keywords

Dihedral Group, Rank, Transitive Action, Subdegrees, Suborbit

1. Introduction

Previous investigations on rank, suborbits and subdegrees have taken into account the symmetric group S_n acting on various subsets of $X = \{1, 2, \cdots, n\}$. The rank and subdegrees of S_n on 2-element subsets were shown to be 3 and 1, 2, $\binom{n}{2}$ respectively [1]. The study was generalized to the action of S_n on $X^{(r)}$, unordered r-element subsets of $X = \{1, 2, \cdots, n\}$ where it was established that the rank is $r + 1$ if $n \geq 2r$, and the sudegrees are; 1, $r\binom{n-r}{r-1}, \binom{r}{2}\binom{n-r}{r-2}, \cdots, \binom{n-1}{r}$.

It was also proved that the suborbits of S_n acting on $X^{(r)}$ are self-paired [2]. Similarly, the action of S_n on ordered r-element subsets of $X = \{1, 2, \cdots, n\}$ was discussed. The action was shown to be transitive, the rank of S_n on $X^{[3]}$ as 34 for $n \geq 6$, and the rank of S_n on $X^{[2]}$ as 7 for $n \geq 4$. Properties of suborbital graphs were also examined in this action [3]. The action of the dihedral group D_n on $X^{(r)}$ has also been considered where the action was proved transitive for values of $r = 1$ and $r = n - 1$ [4].

The study of suborbits of the dihedral group acting on ordered subsets has revealed some interesting properties which translate to properties of associated suborbital graphs. This has seen a clear connection between Group Theory and Graph Theory, which has realized the artistic value in Mathematics. Section 2 outlines some of the notations and preliminary definitions which have been used in the investigations. Section 3 discusses the rank, subdegrees and suborbits of D_n on $X^{[r]}$. Some properties of suborbits have also been discussed in this Section. Section 4 has seen the application of properties of suborbits in construction of associated suborbital graphs. Section 5 examines suborbits of D_n on its ordered adjacent vertices.

The dihedral group D_n consists of all the symmetries of a regular n-sided polygon. The group is of order $2n$, constituted by n rotations and n reflections.

2. Notations and Preliminary Definitions

Notation 2.1

Throughout this paper, G is the dihedral group D_n, $X^{[r]}$ is the set of all ordered r-element subsets of $X = \{1, 2, \cdots, n\}$, and $n\,P_r$ is n permutation r.

Definition 2.2 (Group action) [5]

Let X be a non-empty set. The group action of G on X is a relation on the pair (G, X) such that gx is a unique image of every x in X under g in G. The relation satisfies the algebraic laws of identity and associativity. Namely,

- $Ix = x$, for all $x \in X$, where $I \in G$
- $g_1(g_2 x) = (g_1 g_2) x$, for all g_1, g_2 in G and x in X

Definition 2.3 (Orbit of an element) (see [5]: p. 31)

A group action partitions the set into disjoint equivalence classes known as G-orbits. The orbit of each x in X is the subset of X, $Orb_G(x) = \{gx \mid g \in G\}$, which contains all the images of x under every g in G. The group G is transitive

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if given any pair of elements x_i, x_j in X, there exists g in G such that $gx_i = x_j$. Thus G is transitive if and only if there is exactly one orbit.

Definition 2.4 (Stabilizer of x, G_x)

Let G be a group acting transitively on a set X. The stabilizer of x in X is the set of all elements g in G such that $gx = x$. The set is denoted by $G_x = \{g \in G : gx = x, \text{ for a fixed } x \text{ in } X\}$. The G_x-orbits on X, $\Delta_0, \Delta_1, \cdots, \Delta_{m-1}$ are known as suborbits of G. The rank of G is m and the cardinalities, $|\Delta_i| (i = 0, 1, \cdots, m-1)$, are the subdegrees of G. It was established in [6] that both m and the cardinalities of the suborbits are independent of the choice of x in X.

Definition 2.5

Let G act transitively on a set X and let Δ be an orbit of G_x on X. Define $\Delta^* = \{gx | g \in G, x \in g\Delta\}$. Then Δ^* is also an orbit of G_x and is called the G_x-orbit paired with Δ. If $\Delta = \Delta^*$, then Δ is said to be self-paired [7].

Theorem 2.6 (see [8])

Let G act transitively on a set X, and suppose $g \in G$. The number of self-paired suborbits of G is given by

$$\frac{1}{|G|} \sum_{g \in G} |Fix(g^2)|.$$

Theorem 2.7 (see [9])

Let G act transitively on X and let G_x be the stabilizer of the point $x \in X$. Let $\Delta_i, i = 0, 1, \cdots, m-1$ be the orbits of G_x on X. If $O_i \subseteq X \times X$, $i = 0, 1, \cdots, m-1$, is the suborbital corresponding to Δ_i, then G is primitive if and only if each non-trivial suborbital graph Γ_i is connected.

Theorem 2.8 (Orbit-Stabilizer Theorem [10]: p. 72)

Let G be a group acting on a finite set X with x in X. The size of the orbit of x in G is the index $|G : stab_G(x)|$. Thus, $|orb_G(x)| = |G : stab_G(x)|$.

Theorem 2.9 (Cauchy-Frobenius Lemma [11]: p. 98)

Suppose G is a group acting on a finite set X. The number of G-orbits on X is given by $\frac{1}{|G|} \sum |Fix(g)|$ where $|Fix(g)|$ denotes the number of elements in X fixed by g in G.

3. Main Results

3.1. Rank, Subdegrees and Suborbits of $G = D_n$ on $X^{[r]}$

The action of G on $X^{[r]}$ is defined by the rule; $g[x_1, x_2, \cdots, x_r] = [gx_1, gx_2, \cdots, gx_r]$ for all g in G and $[x_1, x_2, \cdots, x_r]$ in $X^{[r]}$. The set $X^{[r]}$ consists of all permutations of r elements from $X = \{1, 2, \cdots, n\}$, and its cardinality is $n P_r$.

Theorem 3.1.1

The action of $G = D_n$ on $X^{[r]}$ is transitive if and only if $n = 3$.

Proof:

Let $g \in G$ and $[x_1, x_2, \cdots, x_r]$ in $X^{[r]}$. Now, g fixes an ordered r-element subset if and only if $g[x_1, x_2, \cdots, x_r] = [x_1, x_2, \cdots, x_r]$ so that $gx_1 = x_1, gx_2 = x_2, \cdots, gx_r = x_r$. This is possible only if g is the identity. It follows

that the number of elements in $X^{[r]}$ fixed by g is $n\,P_r$. Using Theorem 2.9, the number of G-orbits on $X^{[r]}$ is;

$$\frac{1}{2n}\left(\frac{n!}{(n-r)!}\right) = \frac{(n-1)!}{2(n-r)!}, \quad r \leq n$$

$$= \frac{1}{2}(n-1)(n-2)\cdots(n-r+1).$$

The least number of G-orbits on $X^{[r]}$ is realized when $n = r$, where the number is $\dfrac{(n-1)!}{2}$. If the action is transitive, then $\dfrac{(n-1)!}{2} = 1$, $\Rightarrow n = 3$. Conversely, if $n = 3$, then $\dfrac{(n-1)!}{2} = 1$, and the action is transitive.

Example 3.1.2

The set $X^{[2]}$ consists of all ordered 2-element subsets of $X = \{1, 2, \cdots, n\}$. Hence, its cardinality is $n\,P_2 = \dfrac{n!}{(n-2)!}$.

Let $[x, y]$ be in $X^{[2]}$. Now, g in G fixes an ordered pair, $[x, y]$, if and only if $g[x, y] = [x, y]$, so that $gx = x$ and $gy = y$. When n is odd, this is possible only if g is the identity. It follows that, $\sum_{g \in G} |Fix(g)| = n\,P_2 = \dfrac{n!}{(n-2)!}$. Using Theorem 2.9, the number of G-orbits on $X^{[2]}$ is $\dfrac{1}{2n}\left(\dfrac{n!}{(n-2)!}\right)$. Since $n = 3$, the number of G-orbits is 1 and $G = D_3$ acts transitively on $X^{[2]}$.

When n is even, g fixes an ordered pair if g is the identity or a reflection through a diagonal. If g is the identity, $|Fix(g)|$ is $(n\,P_2)$. Since each reflection fixes 2 elements of $X^{[2]}$ and the number of such reflections is $n/2$, then $|Fix(g)| = n$. The number of G-orbits on $X^{[2]}$ is;

$$\frac{1}{2n}\left(\frac{n!}{(n-2)!} + n\right) = \frac{n}{2}$$

For transitivity,

$$\frac{n}{2} = 1, \quad \Rightarrow n = 2.$$

But, $n \geq 3$ for G to be defined. It follows that the action is intransitive.

Theorem 3.1.3

The rank of $G = D_3$ on $X^{[r]}$ is 6 and each suborbit contains 1 element.

Proof:

Let the group $G = D_3$ act on $X^{[r]}$ and $[1, 2, \cdots, r] \in X^{[r]}$. Since the action is transitive, $|\mathrm{Orb}_G[1, 2, \cdots, r]| = |X^{[r]}| = n\,P_r$. By Theorem 2.8, $|\mathrm{Orb}_G[1, 2, \cdots, r]| = |G : G_{[1,2,\cdots,r]}|$. But $|G_{[1,2,\cdots,r]}| = 1$, by Theorem 3.1.1. Thus, $|X^{[r]}| = |G| = 6$. Now, g in $G_{[1,2,\cdots,r]}$ acts by fixing each element of $X^{[r]}$ in its own $G_{[1,2,\cdots,r]}$-orbit. Since $|X^{[r]}| = 6$ then the rank of G on $X^{[r]}$ is 6.

Clearly, the subdegrees of G are; 1, 1, 1, 1, 1, 1.

Example 3.1.4

The rank of $G = D_3$ on $X^{[3]}$ is 6 and each suborbit is of length 1.

Since the action is transitive, $\left|\text{Orb}_G\left[1,2,3\right]\right| = \left|X^{[3]}\right| = 6$. From Theorem 2.8, $\left|\text{Orb}_G\left[1,2,3\right]\right| = \left|G : G_{[1,2,3]}\right|$. It follows that the size $\left|G_{[1,2,3]}\right|$ is 1. Now, the action of g in $G_{[1,2,3]}$ on $X^{[3]}$ fixes each of the 6 elements in its orbit. The 6 suborbits of G are:
$$\Delta_0 = \left\{\left[1,2,3\right]\right\}, \quad \Delta_1 = \left\{\left[1,3,2\right]\right\}, \quad \Delta_2 = \left\{\left[3,1,2\right]\right\}, \quad \Delta_3 = \left\{\left[3,2,1\right]\right\}, \quad \Delta_4 = \left\{\left[2,3,1\right]\right\},$$
$\Delta_5 = \left\{\left[2,1,3\right]\right\}$. Clearly, the subdegrees of G acting on $X^{[3]}$ are: 1, 1, 1, 1, 1, 1.

3.2. Some Properties of suborbits of D_3 on $X^{[r]}$

Some properties of suborbits of D_3 acting on $X^{[r]}$ have been discussed in the following Theorems.

Theorem 3.2.1

The number of self-paired suborbits of D_3 on $X^{[r]}$ is 4.

Proof:

The number of self-paired suborbits is determined by the fixed point set of g^2, by Theorem 2.6.

If $r = 2$, then g^2 fixes an ordered set of 2 elements if g is the identity or g is a reflection. If g is the identity, then $\left|Fix\left(g^2\right)\right| = 3\,P_2 = 6$. If g is a reflection, then $\left|Fix\left(g^2\right)\right| = 3\,P_2 = 6$. Since the number of reflections is 3, then the number of self-paired suborbits is; $\dfrac{1}{\left|D_3\right|}\sum_{g \in D_3}\left|Fix\left(g^2\right)\right| = \dfrac{1}{6}\left\{6 + 3(6)\right\} = 4$. The 4 self-paired suborbits of D_3 on $X^{[2]}$ are; $\Delta_0 = \left[1,2\right]$, $\Delta_1 = \left[2,1\right]$, $\Delta_2 = \left[1,3\right]$ and $\Delta_5 = \left[3,2\right]$. By Definition 2.5, $g\left[\Delta_0,\Delta_1\right] = \left[\Delta_1,\Delta_0\right]$ when $g = (12)$, $g\left[\Delta_0,\Delta_2 = \Delta_2,\Delta_0\right]$ when $g = (23)$ and $g\left[\Delta_0,\Delta_5\right] = \left[\Delta_5,\Delta_0\right]$ when $g = (13)$.

If $r = 3$, then g^2 fixes an ordered set of 3 elements if g is the identity or a reflection. Computation of self-paired suborbits is similar to the case when $r = 2$. The 4 self-paired suborbits of D_3 on $X^{[3]}$ are; $\Delta_0 = \left[1,2,3\right]$, $\Delta_1 = \left[1,3,2\right]$, $\Delta_3 = \left[3,2,1\right]$ and $\Delta_5 = \left[2,1,3\right]$. Clearly, g can be chosen accordingly so that; $g\left[\Delta_0,\Delta_1\right] = \left[\Delta_1,\Delta_0\right]$ when $g = (23)$, $g\left[\Delta_0,\Delta_3\right] = \left[\Delta_3,\Delta_0\right]$ when $g = (13)$ and $g\left[\Delta_0,\Delta_5\right] = \left[\Delta_5,\Delta_0\right]$ when $g = (12)$.

Theorem 3.2.2

Let $G = D_3$ act transitively on $X^{[r]}$. If $\Delta_i = \left[x_1,x_2,\cdots,x_r\right]$ is a $G_{[1,2,\cdots,r]}$-orbit on $X^{[r]}$, where $x_i \in \left\{1,2,\cdots,r\right\}$, then Δ_i is self-paired if and only if the permutation $g\left[1,2,\cdots,r\right] = \left[x_1,x_2,\cdots,x_r\right]$ is such that $g^2 = 1$.

Proof:

If Δ_i is self-paired, then there exists g in D_3 such that $g\left[\Delta_0,\Delta_i\right] = \left[\Delta_i,\Delta_0\right]$, by Theorem 2.5. Considering $\Delta_0 = \left[1,2,\cdots,r\right]$, $g(1) = x_1, g(2) = x_2, \cdots, g(r) = x_r$ and $g(x_1) = 1, g(x_2) = 2, \cdots, g(x_r) = r$. This implies that $g(1) = x_1$ and $g(x_1) = 1$, $\Rightarrow x_1 = g^{-1}(1)$. Thus, $g(1) = g^{-1}(1)$. Hence, $g^2 = 1$. Conversely, if $g^2 = 1$, then $g = g^{-1}$. Now, g is such that $g\left[1,2,\cdots,r\right] = \left[x_1,x_2,\cdots,x_r\right]$ and $g\left[x_1,x_2,\cdots,x_r\right] = \left[1,2,\cdots,r\right]$, $\Rightarrow g\left[\Delta_0,\Delta_i\right] = \left[\Delta_i,\Delta_0\right]$. Hence, Δ_i is self-paired.

Theorem 3.2.3

Suppose $G = D_3$ is transitive on $X^{[r]}$. Then Δ_i and Δ_j are paired suborbits of G if

and only if the permutations g and h, in the maps $g\Delta_i = \Delta_0$ and $h\Delta_j = \Delta_0$, are inverses of each other.

Proof:

Suppose Δ_i and Δ_j are paired suborbits of G. Then there exists g and h in G such that $g\Delta_i = \Delta_0$ and $h\Delta_j = \Delta_0$. There exists x_i in Δ_i and y_i in Δ_j such that $gx_i = 1$ and $hy_i = 1$. By Definition 2.5, $g(1) = y_i$ and $h(1) = x_i$. It follows that, $gh(1) = 1$ and $hg(1) = 1$. Hence, g and h are inverses of each other. Conversely, if g and h are inverses of each other, then g maps x_i to 1 and 1 to y_i. Similarly, h maps y_i to 1 and 1 to x_i. Hence, Δ_i and Δ_j are paired suborbits.

Example 3.2.4

Let $G = D_3$ act on $X^{[2]}$. Then $\Delta_3 = [3, 1]$ and $\Delta_4 = [2, 3]$ are paired suborbits of G. Clearly, $g\Delta_3 = \Delta_0$ and $h\Delta_4 = \Delta_0$, when $g = (123)$ and $h = (132)$, where g and h are inverses of each other.

4. Suborbital Graphs of $G = D_3$ Acting on $X^{[r]}$

Let $\Delta = [x_1, \cdots, x_r]$ be a suborbit of G on $X^{[r]}$, where $x_i \in \{1, 2, \cdots, n\}$. Then the suborbital O corresponding to Δ is given by;
$O = \left\{ \left(g[1, 2, \cdots, r], g[x_1, x_2, \cdots, x_r] \right) \middle| g \in G, [x_1, x_2, \cdots, x_r] \in \Delta \right\}$. The graph Γ corresponding to suborbital O is formed by considering $X^{[r]}$ as the vertex set and drawing an edge from $[c_1, c_2, \cdots, c_r]$ to $[d_1, d_2, \cdots, d_r]$ if and only if $\left([c_1, c_2, \cdots, c_r], [d_1, d_2, \cdots, d_r] \right) \in O$. Now, the suborbital graph corresponding to a self-paired suborbit Δ_i has an edge from $C = [c_1, c_2, \cdots, c_r]$ to $D = [d_1, d_2, \cdots, d_r]$ only if $|C \cap D| = 1$. The graph corresponding to a paired suborbit Δ_i has an edge from $C = [c_1, c_2, \cdots, c_r]$ to $D = [d_1, d_2, \cdots, d_r]$ only if $|C \cap D| = 0$.

Theorem 4.1

All suborbital graphs corresponding to the action of D_3 on $X^{[r]}$ are disconnected.

Proof:

Let Γ_i be the suborbital graph corresponding to a self-paired suborbit Δ_i. Suppose $[c_1, c_2, \cdots, c_r]$ is a point on the vertex set, $X^{[r]}$. Then there is an edge from $S = [c_1, c_2, \cdots, c_r]$ to $T = [d_1, d_2, \cdots, d_r]$ if the corresponding coordinates, c_i and d_i, are identical. The 3 coordinates can be rearranged in 3! ways. Each coordinate can take the same position in 1/3(3!) = 2 possibilities. Thus, an edge joining exactly 2 vertices is a connected component and the graph is disconnected. If Δ_i is a paired suborbit, then the corresponding graph has an edge from $S = [c_1, c_2, \cdots, c_r]$ to $T = [d_1, d_2, \cdots, d_r]$ if none of the corresponding coordinates, c_i and d_i, are identical. The number of such rearrangements is 1/2(3!) = 3. It follows that a path joining exactly 3 vertices is a connected component. Hence, the graph is disconnected.

Theorem 4.2

The action of $G = D_3$ on $X^{[r]}$ is imprimitive.

Proof:

Suppose Γ_i is a suborbital graph corresponding to a suborbit, Δ_i, of G on $X^{[3]}$.

Then, from Theorem 4.1, the graph is disconnected. From Theorem 2.7, the action of D_3 on $X^{\{r\}}$ is imprimitive.

Theorem 4.3

Let $G = D_3$ act on $X^{\{r\}}$. Suppose Δ_i is a self-paired suborbit of G. Then the number of connected components of the suborbital graph Γ_i corresponding to Δ_i is 3. If Δ_i is a paired suborbit of G, then the number of connected components in the corresponding graph is 2.

Proof:

Let Γ_i be the suborbital graph corresponding to the self-paired subotbit Δ_i. From Theorem 4.1, the connected component is an edge with exactly 2 vertices. The number of connected components is

$$N(\Gamma_i) = \frac{\text{Number of vertices in } \Gamma_i}{2} = 3.$$

If Δ_i is a paired suborbit, then the corresponding graph is a path joining exactly 3 vertices. It follows, $N(\Gamma_i) = 6/3 = 2$.

5. Suborbits of $G = D_n$ Acting on Ordered Adjacent Vertices of G

Let $G = D_n$ and S be the set of all ordered pairs of adjacent vertices of G. The action of G on S is defined in Section 3.1, since S is a subset of $X^{\{2\}}$.

Theorem 5.1

The action of G on S is transitive.

Proof:

Let $[x, y] \in S$, $g \in G$ and G act on S. Now, g fixes $[x, y]$ only if g is the identity. It follows that $|Fix(g)| = |S| = 2n$. Using Theorem 2.9, the number of G-orbits on S is 1. Hence, the action is transitive.

Theorem 5.2

The rank of G on S is $2n$ and each suborbit contains 1 element.

Proof:

Using Theorem 2.8, the size $\left| \text{Orb}_G [x, y] \right| = \left| G : G_{[x,y]} \right|$, for any element $[x, y]$ in S. Since the action is transitive, $\left| \text{Orb}_G [x, y] \right| = |S| = 2n$. It follows that $\left| G_{[x,y]} \right| = 1$. Now, $G_{[x, y]}$ acts on S by fixing each of the $2n$ elements in its own orbit. Therefore, the rank of G on S is $2n$ and each suborbit contains 1 element.

Theorem 5.3

The number of self-paired suborbits of $G = D_n$ acting on S is $n + 1$ when n is odd and $n + 2$ when n is even.

Proof:

If n is odd, then by Theorem 2.6, g^2 fixes $[x, y]$ in S, if g is the identity or g is a reflection. If g is the identity, then $\left| Fix(g^2) \right| = 2n$. From each reflection, the number $\left| Fix(g^2) \right| = 2n$. Since the number of reflections is n, then

$\sum_{g \in G} \left| Fix(g^2) \right| = 2n + n(2n)$ and the number of self-paired suborbits is

$\frac{1}{2n} \left(2n + 2n^2 \right) = n + 1$.

If n is even, then g^2 fixes $[x, y]$ in S if g is the identity, or g is a reflection, or g is a rotation of $180°$. From each possibility, $\left| Fix\left(g^2\right) \right| = 2n$. But the number of reflections is n and hence, $\sum_{g \in G} \left| Fix\left(g^2\right) \right| = 2n + 2n^2 + 2n$. The number of self-paired suborbits is then; $\frac{1}{2n}\left(4n + 2n^2\right) = n + 2$.

The following conjecture was revealed in the process of the investigations;

If n is odd, then the $n + 1$ self-paired suborbits of $G = D_n$ on S are given by; $\Delta_0 = [1, 2]$, $\Delta_1 = [1, n]$, $\Delta_i = [i, i-1]\left(i = 2, 3, \cdots, n\right)$. If n is even, then the $n + 2$ self-paired suborbits of D_n acting on S are given by; $\Delta_0 = [1, 2]$, $\Delta_1 = [1, n]$, $\Delta_2 = \left[\frac{n+2}{2}, \frac{n+4}{2}\right]$, $\Delta_{i+1} = [i, i-1]\left(i = 2, 3, \cdots, n\right)$.

6. Conclusion

Let $G = D_3$ act transitively on $X^{\{r\}}$. It has been established that the number of self-paired suborbits is 4. The graph Γ_i corresponding to a self-paired suborbit Δ_i is determined by g in the map $g[\Delta_i, \Delta_0] = [\Delta_0, \Delta_i]$. If g fixes k in Δ_0, then the corresponding graph Γ_i has an edge from $C = [c_1, c_2, c_3]$ to $D = [d_1, d_2, d_3]$ if and only if the kth coordinate of C is identical to the kth coordinate of D. The results could be used to investigate other properties of suborbital graphs associated with the action of D_3 on ordered subsets.

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