Local Solutions to a Class of Parabolic System Related to the P-Laplacian

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Abstract

In this paper, the existence and uniqueness of local solutions to the initial and boundary value problem of a class of parabolic system related to the p-Laplacian are studied. The regularization method is used to construct a sequence of approximation solutions, with the help of monotone iteration technique, then we get the existence of solution of a regularized system. By the use of a standard limiting process, the existence of the local solutions of the system is obtained. Finally, the uniqueness of the solution is also proven.

Keywords

Existence, Uniqueness, Evolution, P-Laplacian, Parabolic System

1. Introduction

The objective of this paper is to study the existence and uniqueness of local solutions to the initial and boundary value problem of the parabolic system

$$u_{it} - div\left(\left|\nabla u_i\right|^{p_i-2}\nabla u_i\right) = f_i\left(x,t,u_1,u_2,u_3\right), \left(x,t\right) \in \Omega_T, \tag{1.1}$$

$$u_i\left(x,0\right) = u_{i0}\left(x\right), x \in \Omega, \tag{1.2}$$

$$u_i\left(x,t\right) = 0, \left(x,t\right) \in \partial\Omega \times \left(0,T\right), \tag{1.3}$$

where $p_i > 2, i = 1,2,3, \quad \Omega_T = \Omega \times \left(0,T\right), \Omega \subset R^n$ is a bounded domain with smooth boundary $\partial\Omega$. The conditions of f_i and u_{i0} will be given later.

System (1.1) is popular applied in non-Newtonian fluids [1] [2] and nonlinear filtration [3], etc. In the non-Newtonian fluids theory, $p_i \left(i=1,2,3\right)$ are all characteristic quantity of the medium. Media with $p_i > 2 \left(i=1,2,3\right)$ are called dilatant fluids and those with $p_i < 2 \left(i=1,2,3\right)$ are called pseudoplastics. If $p_i = 2 \left(i=1,2,3\right)$, they

are Newtonian fluids.

Some authors have studied the global finiteness of the solutions (see [4] [5]) and blow-up properties of the solutions (see [6]) with various boundary conditions to the systems of evolutionary Laplacian equations. Zhao [7] and Wei-Gao [8] studied the existence and blow-up property of the solutions to a single equation and the systems of two equations. We found that the method of [8] can be extended to the general systems of n equations. For the sake of simplicity, this paper only makes a detailed discussion on $n = 3$. Since the system is coupled with nonlinear terms, it is in general difficult to study the system. In this paper, we consider some special cases by stating some methods of regularization to construct a sequence of approximation solutions with the help of monotone iteration technique and obtain the existence of solutions to a regularized system of equations. Then we obtain the existence of solutions to the system (1.1)-(1.3) by a standard limiting process. Systems (1.1) degenerates when $u_i = 0$ or $\nabla u_i = 0$. In general, there would be no classical solutions and hence we have to study the generalized solutions to the problem (1.1)-(1.3).

The definition of generalized solutions in this work is the following.

Definition 1.1. *Function* $u = (u_1, u_2, u_3)$ *is called a generalized solution of the system* (1.1)-(1.3) *if* $u_i \in L^\infty(\Omega_T) \cap L^{p_i}\left(0, T; W_0^{1, p_i}(\Omega)\right)$, $u_{it} \in L^2(\Omega_T), i = 1, 2, 3,$ *and satisfies*

$$\iint_{\Omega_T} \left(-u_i \varphi_{it} + |\nabla u_i|^{p_i - 2} \nabla u_i \nabla \varphi_i\right) \mathrm{d}x\mathrm{d}t - \int_\Omega u_{i0} \varphi_i(x, 0) \mathrm{d}x$$
$$= \iint_{\Omega_T} f_i(x, t, u_1, u_2, u_3) \varphi_i \mathrm{d}x\mathrm{d}t, \tag{1.4}$$

for any $\varphi_i \in C^1\left(\overline{\Omega_T}\right), \varphi_i(x, T) = 0, \varphi_i(x, t) = 0,$ *for* $(x, t) \in \partial\Omega \times (0, T), i = 1, 2, 3.$

Equations (4) implies that

$$\int_0^t \int_\Omega \left(-u_i \varphi_{it} + |\nabla u_i|^{p_i - 2} \nabla u_i \nabla \varphi_i\right) \mathrm{d}x\mathrm{d}t + \int_\Omega u_i(x, t) \varphi_i(x, t) \mathrm{d}x - \int_\Omega u_{i0} \varphi_i(x, 0) \mathrm{d}x$$
$$= \int_0^t \int_\Omega f_i(x, t, u_1, u_2, u_3) \varphi_i \mathrm{d}x\mathrm{d}t, \quad a.e. \quad t \in (0, T). \tag{1.5}$$

The followings are the constrains to the nonlinear functions $f_i, i = 1, 2, 3,$ involved in this paper.

Definition 1.2. *A function* $f = f(u_1, u_2, u_3)$ *is said to be quasimonotone nondecreasing* (*resp., nonincreasing*) *if for fixed* $u_j (j \neq i),$ f *is nondecreasing* (*resp., nonincreasing*) *in* $u_i, (i = 1, 2, 3).$

Our main existence result is following:

Theorem 1.3. *If there exist nonnegative functions* $f_i(x, t, u_1, u_2, u_3) \in C\left(\overline{\Omega} \times [0, T] \times R^3\right)$ *which are quasimonotonically nondecreasing for* $u_1,$ $u_2,$ $u_3 (i = 1, 2, 3),$ *and a nonnegative function* $g(s) \in C^1(R)$ *such that*

$$|f_i(x, t, u_1, u_2, u_3)| \leq \min\{g(u_1), g(u_2), g(u_3)\} \text{ and } u_{i0} \in L^\infty(\Omega) \cap W_0^{1, p_i}(\Omega). \tag{1.6}$$

Then there exists a constant $T' \in [0, T]$ *such that the system* (1.1)-(1.3) *has a solution* $u = (u_1, u_2, u_3)$ *in the sence of Definition* 1.1 *with* T *replaced by* T'.

In Theorem 1.3, we just obtain the existence of local solution. As known to all, when the system degenerates into an equation, as long as some order of growth conditions is

added on f, we can find the global solution, which is the main result of [7]. The existence of the global solution of (1.1)-(1.3) remains to be further studied.

On the other hand, similar to [8], we made the assumption of monotonicity to f_i. From the current point of view, the condition is relatively strong. It is well worth studying how to reduce f_i monotonicity requirements of the system (1.1)-(1.3).

2. Proof of Theorem 1.3

To prove the theorem, we consider the following regularized problem

$$u_{it} - div\left(\left(|\nabla u_i|^2 + \varepsilon\right)^{\frac{p_i-2}{2}} \nabla u_i\right) = f_{i\varepsilon}(x,t,u_1,u_2,u_3), (x,t) \in \Omega_T, \tag{2.1}$$

$$u_i(x,0) = u_{i0\varepsilon}(x), x \in \Omega, \tag{2.2}$$

$$u_i(x,t) = 0, (x,t) \in \partial\Omega \times (0,T), \tag{2.3}$$

where $f_{i\varepsilon} \in C^1\left(\bar{\Omega} \times [0,T] \times R^3\right)$, $f_{i\varepsilon}$ are quasimonotone nondecreasing and $f_{i\varepsilon} \to f_i$ uniformly on bounded subsets of $\bar{\Omega} \times [0,T] \times R^3$, also

$$\left|f_{i\varepsilon}(x,t,u_1,u_2,u_3)\right| \le \min\left\{g(u_1), g(u_2), g(u_3)\right\}, \tag{2.4}$$

$u_{i0\varepsilon} \in C_0^1(\Omega), |u_{i0\varepsilon}|_{L^{p_i}} \le |u_{i0}|_{L^{p_i}}, |\nabla u_{i0\varepsilon}|_{L^{p_i}} \le C|\nabla u_{i0}|_{L^{p_i}}, u_{i0\varepsilon} \to u_{i0}$ strongly in $W_0^{1,p_i}(\Omega)$.

Lemma 2.1. *The regularized problem* (2.1)-(2.3) *has a generalized solution.*

Proof. Starting from a suitable initial iteration $\left(u_{1\varepsilon}^{(0)}, u_{2\varepsilon}^{(0)}, u_{3\varepsilon}^{(0)}\right)$, we construct a sequence $\left\{\left(u_{1\varepsilon}^{(k)}, u_{2\varepsilon}^{(k)}, u_{3\varepsilon}^{(k)}\right)\right\}$ from the iteration process

$$u_{i\varepsilon t}^{(k)} - div\left(\left(\left|\nabla u_{i\varepsilon}^{(k)}\right|^2 + \varepsilon\right)^{\frac{p_i-2}{2}} \nabla u_{i\varepsilon}^{(k)}\right) = f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}, u_{3\varepsilon}^{(k-1)}\right), (x,t) \in \Omega_T, \tag{2.5}$$

$$u_{i\varepsilon}^{(k)}(x,0) = u_{i0\varepsilon}(x), x \in \Omega, \tag{2.6}$$

$$u_{i\varepsilon}^{(k)}(x,t) = 0, (x,t) \in \partial\Omega \times (0,T), \tag{2.7}$$

where $i = 1,2,3$. It is clear that for each $k = 1,2,\cdots$, the above system consists of three nondegenerated and uncoupled initial boundary-value problems.

By classical results (see [9]) for fixed ε and k the problem (2.5)-(2.7) has a classical solution $\left\{\left(u_{1\varepsilon}^{(k)}, u_{2\varepsilon}^{(k)}, u_{3\varepsilon}^{(k)}\right)\right\}$ if $\left\{\left(u_{1\varepsilon}^{(k-1)}, u_{2\varepsilon}^{(k-1)}, u_{3\varepsilon}^{(k-1)}\right)\right\}$ is smooth.

To ensure that this sequence converges to a solution of (2.1)-(2.3), it is necessary to choose a suitable initial iteration. The choice of this function depends on the type of quasimonotone property of (f_1, f_2, f_3). In the following, we establish the monotone property of the sequence.

Set $\bar{u}_{i\varepsilon}^{(0)}(x,t) = \sup_{\Omega_T} \left\{u_{i0\varepsilon}(x)\right\}, i = 1,2,3$. Let $\bar{u}_{i\varepsilon}^{(1)}$ be a classical solution of the following problem.

$$\bar{u}_{i\varepsilon t}^{(1)} - div\left(\left(\left|\nabla \bar{u}_{i\varepsilon}^{(1)}\right|^2 + \varepsilon\right)^{\frac{p_i-2}{2}} \nabla \bar{u}_{i\varepsilon}^{(1)}\right) = f_{i\varepsilon}\left(x,t,\bar{u}_{1\varepsilon}^{(0)}, \bar{u}_{2\varepsilon}^{(0)}, \bar{u}_{3\varepsilon}^{(0)}\right), (x,t) \in \Omega_T, \tag{2.8}$$

$$\bar{u}_{i\varepsilon}^{(1)}(x,0) = u_{i0\varepsilon} \le \bar{u}_{i\varepsilon}^{(0)}, x \in \Omega, \tag{2.9}$$

$$\bar{u}_{i\varepsilon}^{(1)}(x,t) = 0, (x,t) \in \partial\Omega \times (0,T), \tag{2.10}$$

By $f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(0)},u_{2\varepsilon}^{(0)},u_{3\varepsilon}^{(0)}\right) \le f_{i\varepsilon}\left(x,t,\bar{u}_{1\varepsilon}^{(0)},\bar{u}_{2\varepsilon}^{(0)},\bar{u}_{3\varepsilon}^{(0)}\right)$ and the comparison theorem (see [10]), we have that

$$\bar{u}_{1\varepsilon}^{(1)} \le \bar{u}_{1\varepsilon}^{(0)}, \bar{u}_{2\varepsilon}^{(1)} \le \bar{u}_{2\varepsilon}^{(0)} \text{ and } \bar{u}_{3\varepsilon}^{(1)} \le \bar{u}_{3\varepsilon}^{(0)}. \tag{2.11}$$

Hence by the quasimonotone nondecreasing property of $f_{i\varepsilon}$, we have

$$\begin{aligned}
f_{i\varepsilon}\left(x,t,\bar{u}_{1\varepsilon}^{(1)},\bar{u}_{2\varepsilon}^{(1)},\bar{u}_{3\varepsilon}^{(1)}\right) &\le f_{i\varepsilon}\left(x,t,\bar{u}_{1\varepsilon}^{(0)},\bar{u}_{2\varepsilon}^{(1)},\bar{u}_{3\varepsilon}^{(1)}\right) \\
&\le f_{i\varepsilon}\left(x,t,\bar{u}_{1\varepsilon}^{(0)},\bar{u}_{2\varepsilon}^{(0)},\bar{u}_{3\varepsilon}^{(1)}\right) \le f_{i\varepsilon}\left(x,t,\bar{u}_{1\varepsilon}^{(0)},\bar{u}_{2\varepsilon}^{(0)},\bar{u}_{3\varepsilon}^{(0)}\right)
\end{aligned} \tag{2.12}$$

for $i = 1,2,3$.

Using the same argument as above, we can obtain a classical solution $\left(\bar{u}_{1\varepsilon}^{(2)}, \bar{u}_{2\varepsilon}^{(2)}, \bar{u}_{3\varepsilon}^{(2)}\right)$ of the problem

$$\bar{u}_{i\varepsilon t}^{(2)} - div\left(\left(\left|\nabla\bar{u}_{i\varepsilon}^{(2)}\right|^2 + \varepsilon\right)^{\frac{p_i-2}{2}}\nabla\bar{u}_{i\varepsilon}^{(2)}\right) = f_{i\varepsilon}\left(x,t,\bar{u}_{1\varepsilon}^{(1)},\bar{u}_{2\varepsilon}^{(1)},\bar{u}_{3\varepsilon}^{(1)}\right), (x,t) \in \Omega_T, \tag{2.13}$$

$$\bar{u}_{i\varepsilon}^{(2)}(x,0) = u_{i0\varepsilon}, x \in \Omega, \tag{2.14}$$

$$\bar{u}_{i\varepsilon}^{(2)}(x,t) = 0, (x,t) \in \partial\Omega \times (0,T), \tag{2.15}$$

for $i = 1,2,3$.

By the comparison theorem, we have

$$\bar{u}_{1\varepsilon}^{(2)} \le \bar{u}_{1\varepsilon}^{(1)}, \bar{u}_{2\varepsilon}^{(2)} \le \bar{u}_{2\varepsilon}^{(1)} \text{ and } \bar{u}_{3\varepsilon}^{(2)} \le \bar{u}_{3\varepsilon}^{(1)}. \tag{2.16}$$

By induction method, we obtain a nonincreasing sequence of smooth functions

$$\bar{u}_{i\varepsilon}^{(0)} \ge \bar{u}_{i\varepsilon}^{(1)} \ge \bar{u}_{i\varepsilon}^{(2)} \ge \cdots \ge \bar{u}_{i\varepsilon}^{(k)} \ge \cdots \tag{2.17}$$

In a similar way, by setting $\underline{u}_{i\varepsilon}^{(0)}(x,t) = \inf_{\Omega_T}\left\{u_{i0\varepsilon}(x)\right\}, i = 1,2,3,$ we can get a solution $\left(\underline{u}_{1\varepsilon}^{(1)}, \underline{u}_{2\varepsilon}^{(1)}, \underline{u}_{3\varepsilon}^{(1)}\right)$ of

$$\underline{u}_{i\varepsilon t}^{(1)} - div\left(\left(\left|\nabla\underline{u}_{i\varepsilon}^{(1)}\right|^2 + \varepsilon\right)^{\frac{p_i-2}{2}}\nabla\underline{u}_{i\varepsilon}^{(1)}\right) = f_{i\varepsilon}\left(x,t,\underline{u}_{1\varepsilon}^{(0)},\underline{u}_{2\varepsilon}^{(0)},\underline{u}_{3\varepsilon}^{(0)}\right), (x,t) \in \Omega_T, \tag{2.18}$$

$$\underline{u}_{i\varepsilon}^{(1)}(x,0) = u_{i0\varepsilon} \ge \underline{u}_{i\varepsilon}^{(0)}, x \in \Omega, \tag{2.19}$$

$$\underline{u}_{i\varepsilon}^{(1)}(x,t) = 0, (x,t) \in \partial\Omega \times (0,T), \tag{2.20}$$

with

$$\underline{u}_{1\varepsilon}^{(1)} \ge \underline{u}_{1\varepsilon}^{(0)}, \underline{u}_{2\varepsilon}^{(1)} \ge \underline{u}_{2\varepsilon}^{(0)} \text{ and } \underline{u}_{3\varepsilon}^{(1)} \ge \underline{u}_{3\varepsilon}^{(0)} \tag{2.21}$$

In the same way as above, we obtain a nondecreasing sequence of smooth functions

$$\underline{u}_{i\varepsilon}^{(0)} \le \underline{u}_{i\varepsilon}^{(1)} \le \underline{u}_{i\varepsilon}^{(2)} \le \cdots \le \underline{u}_{i\varepsilon}^{(k)} \le \cdots \tag{2.22}$$

It is obvious that $\underline{u}_{i\varepsilon}^{(0)} \le \bar{u}_{i\varepsilon}^{(0)}$. By induction method, we may assume that $\underline{u}_{i\varepsilon}^{(k)} \le \bar{u}_{i\varepsilon}^{(k)}$. Since $f_{i\varepsilon}$ is quasimonotone nondecreasing, we have

$$f_{i\varepsilon}\left(x,t,\underline{u}_{1\varepsilon}^{(k)},\underline{u}_{2\varepsilon}^{(k)},\underline{u}_{3\varepsilon}^{(k)}\right) \le f_{i\varepsilon}\left(x,t,\overline{u}_{1\varepsilon}^{(k)},\underline{u}_{2\varepsilon}^{(k)},\underline{u}_{3\varepsilon}^{(k)}\right)$$
$$\le f_{i\varepsilon}\left(x,t,\overline{u}_{1\varepsilon}^{(k)},\overline{u}_{2\varepsilon}^{(k)},\underline{u}_{3\varepsilon}^{(k)}\right) \le f_{i\varepsilon}\left(x,t,\overline{u}_{1\varepsilon}^{(k)},\overline{u}_{2\varepsilon}^{(k)},\overline{u}_{3\varepsilon}^{(k)}\right) \tag{2.23}$$

for $i = 1,2,3$.

$$\underline{u}_{i\varepsilon t}^{(k+1)} - div\left(\left(\left|\nabla \underline{u}_{i\varepsilon}^{(k+1)}\right|^2 + \varepsilon\right)^{\frac{p_i-2}{2}} \nabla \underline{u}_{i\varepsilon}^{(k+1)}\right) = f_{i\varepsilon}\left(x,t,\underline{u}_{1\varepsilon}^{(k)},\underline{u}_{2\varepsilon}^{(k)},\underline{u}_{3\varepsilon}^{(k)}\right), (x,t) \in \Omega_T, \quad (2.24)$$

$$\overline{u}_{i\varepsilon t}^{(k+1)} - div\left(\left(\left|\nabla \overline{u}_{i\varepsilon}^{(k+1)}\right|^2 + \varepsilon\right)^{\frac{p_i-2}{2}} \nabla \overline{u}_{i\varepsilon}^{(k+1)}\right) = f_{i\varepsilon}\left(x,t,\overline{u}_{1\varepsilon}^{(k)},\overline{u}_{2\varepsilon}^{(k)},\overline{u}_{3\varepsilon}^{(k)}\right), (x,t) \in \Omega_T, \quad (2.25)$$

$$\underline{u}_{i\varepsilon}^{(k+1)}(x,0) = u_{i0\varepsilon} = \overline{u}_{i\varepsilon}^{(k+1)}(x,0), x \in \Omega, \tag{2.26}$$

$$\underline{u}_{i\varepsilon}^{(k+1)}(x,t) = 0 = \overline{u}_{i\varepsilon}^{(k+1)}(x,t), (x,t) \in \partial\Omega \times (0,T), \tag{2.27}$$

By the comparison principle, we have $\underline{u}_{i\varepsilon}^{(k+1)} \le \overline{u}_{i\varepsilon}^{(k+1)}$. Therefore

$$\underline{u}_{i\varepsilon}^{(0)} \le \underline{u}_{i\varepsilon}^{(1)} \le \cdots \le \underline{u}_{i\varepsilon}^{(k)} \le \underline{u}_{i\varepsilon}^{(k+1)} \le \overline{u}_{i\varepsilon}^{(k+1)} \le \overline{u}_{i\varepsilon}^{(k)} \le \cdots \le \overline{u}_{i\varepsilon}^{(1)} \le \overline{u}_{i\varepsilon}^{(0)}. \tag{2.28}$$

Taking $u_{i\varepsilon}^{(k)} = \underline{u}_{i\varepsilon}^{(k)} (i = 1,2,3)$, we get a nondecreasing bounded sequence $\left\{u_{i\varepsilon}^{(k)}\right\}_{k=1}^{\infty} (i = 1,2,3)$. Hence there exist functions $u_{i\varepsilon} (i = 1,2,3)$ such that

$$\lim_{k\to\infty} u_{i\varepsilon}^{(k)} = u_{i\varepsilon}, a.e. \text{ in } \Omega_T. \tag{2.29}$$

By the continuity of $f_{i\varepsilon} (i = 1,2,3)$, we have

$$\lim_{k\to\infty} f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(k)},u_{2\varepsilon}^{(k)},u_{3\varepsilon}^{(k)}\right) = f_{i\varepsilon}\left(x,t,u_{1\varepsilon},u_{2\varepsilon},u_{3\varepsilon}\right) a.e. \text{ in } \Omega_T. \tag{2.30}$$

We now prove that there exist $T' \in (0,T]$ and a constant M (independent of k and ε) such that for all k, we have

$$\left|u_{i\varepsilon}^{(k)}\right|_{L^{\infty}(\Omega_{T'})} \le M, i = 1,2,3. \tag{2.31}$$

Let $v_i^{\pm}(t)(i = 1,2,3)$ be the solutions of the ordinary differential equations

$$\frac{dv_i^{\pm}}{dt} = \pm g(v_i), v_i^{\pm}(0) = \pm \left|u_{i0}\right|_{L^{\infty}(\Omega)}, (i = 1,2,3). \tag{2.32}$$

By standard results in [11], there exist $T_i \in (0,T), i = 1,2,3$, such that $v_i^{\pm}$ exists on $[0,T_i]$ with T_i depends only on $\left|u_{i0}\right|_{L^{\infty}(\Omega)}$. By the comparison theorem

$$\left|u_{i\varepsilon}^{(k)}(x,t)\right| \le \max\left\{v_i^+(t), -v_i^-(t)\right\}, (i = 1,2,3). \tag{2.33}$$

Setting $T' = \frac{1}{3}\min\{T_1,T_2,T_3\}, M = \max\left\{v_i^+(T'), -v_i^-(T')\right\}$, we obtain (2.31).

We now claim that $u_{i\varepsilon}^{(k)} \rightharpoonup u_{i\varepsilon}$, as $k \to \infty$, in $L^{p_i}\left(0,T';W_0^{1,p_i}(\Omega)\right)(i = 1,2,3)$, where $\rightharpoonup$ stands for weak convergence, $i = 1,2,3$.

Multiplying (2.5) by $u_{i\varepsilon}^{(k)}$ and integrating over $\Omega_{T'}$, we obtain that

$$\iint_{\Omega_{T'}}\left[u_{i\varepsilon t}^{(k)}u_{i\varepsilon}^{(k)}-div\left(\left(\left|\nabla u_{i\varepsilon}^{(k)}\right|^2+\varepsilon\right)^{\frac{p_i-2}{2}}\nabla u_{i\varepsilon}^{(k)}\right)u_{i\varepsilon}^{(k)}\right]\mathrm{d}x\mathrm{d}t$$
$$=\iint_{\Omega_{T'}}f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)},u_{3\varepsilon}^{(k-1)}\right)u_{i\varepsilon}^{(k)}\mathrm{d}x\mathrm{d}t. \tag{2.34}$$

Furthermore

$$\int_{\Omega}\int_0^{T'}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(u_{i\varepsilon t}^{(k)}\right)^2\mathrm{d}t\mathrm{d}x+\iint_{\Omega_{T'}}\left(\left|\nabla u_{i\varepsilon}^{(k)}\right|^2+\varepsilon\right)^{\frac{p_i-2}{2}}\left|\nabla u_{i\varepsilon}^{(k)}\right|^2\mathrm{d}x\mathrm{d}t$$
$$=\iint_{\Omega_{T'}}f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)},u_{3\varepsilon}^{(k-1)}\right)u_{i\varepsilon}^{(k)}\mathrm{d}x\mathrm{d}t, i.e. \tag{2.35}$$

$$\iint_{\Omega_{T'}}\left(\left|\nabla u_{i\varepsilon}^{(k)}\right|^2+\varepsilon\right)^{\frac{p_i-2}{2}}\left|\nabla u_{i\varepsilon}^{(k)}\right|^2\mathrm{d}x\mathrm{d}t$$
$$=\iint_{\Omega_{T'}}f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)},u_{3\varepsilon}^{(k-1)}\right)u_{i\varepsilon}^{(k)}\mathrm{d}x\mathrm{d}t$$
$$-\frac{1}{2}\int_{\Omega}\left[\left(u_{i\varepsilon}^{(k)}\left(x,T'\right)\right)^2-\left(u_{i\varepsilon}^{(k)}\left(x,0\right)\right)^2\right]\mathrm{d}x. \tag{2.36}$$

By (2.12) and the property of $f_{i\varepsilon}$

$$\left|\nabla u_{i\varepsilon}^{(k)}\right|_{L^{p_i}(\Omega_{T'})}^{p_i}\le\iint_{\Omega_{T'}}\left(\left|\nabla u_{i\varepsilon}^{(k)}\right|^2+\varepsilon\right)^{\frac{p_i-2}{2}}\left|\nabla u_{i\varepsilon}^{(k)}\right|^2\mathrm{d}x\mathrm{d}t\le C, \tag{2.37}$$

where C is a constant independent of ε and k.

Multiplying (2.5) by $u_{i\varepsilon t}^{(k)}$ and integrating over $\Omega_{T'}$, we have

$$\iint_{\Omega_{T'}}\left[\left(u_{i\varepsilon t}^{(k)}\right)^2-div\left(\left(\left|\nabla u_{i\varepsilon}^{(k)}\right|^2+\varepsilon\right)^{\frac{p_i-2}{2}}\nabla u_{i\varepsilon}^{(k)}\right)u_{i\varepsilon t}^{(k)}\right]\mathrm{d}x\mathrm{d}t$$
$$=\iint_{\Omega_{T'}}f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)},u_{3\varepsilon}^{(k-1)}\right)u_{i\varepsilon t}^{(k)}\mathrm{d}x\mathrm{d}t. \tag{2.38}$$

By Cauchy inequality and integrating by parts, we obtain

$$\iint_{\Omega_{T'}}\left(u_{i\varepsilon t}^{(k)}\right)^2\mathrm{d}x\mathrm{d}t=-\iint_{\Omega_{T'}}\left(\left(\left|\nabla u_{i\varepsilon}^{(k)}\right|^2+\varepsilon\right)^{\frac{p_i-2}{2}}\nabla u_{i\varepsilon}^{(k)}\right)\nabla u_{i\varepsilon t}^{(k)}\mathrm{d}x\mathrm{d}t$$
$$+\iint_{\Omega_{T'}}f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)},u_{3\varepsilon}^{(k-1)}\right)u_{i\varepsilon t}^{(k)}\mathrm{d}x\mathrm{d}t$$
$$\le-\int_{\Omega}\int_0^{T'}\frac{\mathrm{d}}{\mathrm{d}t}\left(\left|\nabla u_{i\varepsilon}^{(k)}\right|^{p_i}\right)\mathrm{d}t\mathrm{d}x+\frac{1}{2}\iint_{\Omega_{T'}}\left(u_{i\varepsilon t}^{(k)}\right)^2\mathrm{d}x\mathrm{d}t \tag{2.39}$$
$$+\frac{1}{2}\iint_{\Omega_{T'}}f_{i\varepsilon}^2\left(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)},u_{3\varepsilon}^{(k-1)}\right)\mathrm{d}x\mathrm{d}t$$
$$=-\int_{\Omega}\left(\left|\nabla u_{i\varepsilon}^{(k)}\left(x,T'\right)\right|^{p_i}-\left|\nabla u_{i0\varepsilon}^{(k)}\right|^{p_i}\right)\mathrm{d}x+\frac{1}{2}\iint_{\Omega_{T'}}\left(u_{i\varepsilon t}^{(k)}\right)^2\mathrm{d}x\mathrm{d}t$$
$$+\frac{1}{2}\iint_{\Omega_{T'}}f_{i\varepsilon}^2\left(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)},u_{3\varepsilon}^{(k-1)}\right)\mathrm{d}x\mathrm{d}t.$$

Hence

$$\iint_{\Omega_{T'}}\left(u_{i\varepsilon t}^{(k)}\right)^2\mathrm{d}x\mathrm{d}t\le C_1\left(\int_{\Omega}\left(\left|\nabla u_{i0\varepsilon}^{(k)}\right|^{p_i}\right)\mathrm{d}x+\iint_{\Omega_{T'}}f_{i\varepsilon}^2\left(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)},u_{3\varepsilon}^{(k-1)}\right)\mathrm{d}x\mathrm{d}t\right)\le C. \tag{2.40}$$

By (2.37) and (2.40), we obtain that there exists a subsequence of $u_{i\varepsilon}^{(k)}$ converging weakly in the following sense as $j \to \infty$.

$$\nabla u_{i\varepsilon}^{(kj)} \rightharpoonup \nabla u_{i\varepsilon}, \text{ in } L^{p_i}\left(\Omega_{T'}\right) \tag{2.41}$$

$$\left|\nabla u_{i\varepsilon}^{(kj)}\right|^{p_i-2} u_{i\varepsilon xl}^{(kj)} \rightharpoonup w_{i\varepsilon xl}, \text{ in } L^{\frac{p_i}{p_i-1}}\left(\Omega_{T'}\right), \text{ for some } w_{i\varepsilon xl} \tag{2.42}$$

$$u_{i\varepsilon t}^{(kj)} \rightharpoonup u_{i\varepsilon t}, \text{ in } L^2\left(\Omega_{T'}\right) \tag{2.43}$$

where $\rightharpoonup$ stands for weak convergence, $i = 1, 2, 3$.

From (2.29), (2.30), (2.37), (2.40) and the uniqueness of the weak limits, we have that, as $k \to \infty$,

$$\nabla u_{i\varepsilon}^{(k)} \rightharpoonup \nabla u_{i\varepsilon}, \quad \text{in } L^{p_i}\left(\Omega_{T'}\right) \tag{2.44}$$

$$u_{i\varepsilon}^{(k)} \to u_{i\varepsilon}, f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(k)},u_{2\varepsilon}^{(k)},u_{3\varepsilon}^{(k)}\right) \to f_{i\varepsilon}\left(x,t,u_{1\varepsilon},u_{2\varepsilon},u_{3\varepsilon}\right) \quad a.e. \text{ in } \Omega_{T'} \tag{2.45}$$

$$u_{i\varepsilon t}^{(k)} \rightharpoonup u_{i\varepsilon t}, \quad \text{in } L^2\left(\Omega_{T'}\right) \tag{2.46}$$

We now claim that $w_{i\varepsilon xl} = \left|\nabla u_{i\varepsilon}\right|^{p_i-2} u_{i\varepsilon xl}, i = 1, 2, 3$.

Multiplying (2.5) by $\left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon}\right)\phi_i$ and integrating over $\Omega_{T'}$ with $\phi \in C_0^1\left(\Omega_{T'}\right), \phi_i \geq 0$, we get

$$\iint_{\Omega_{T'}} \left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon}\right)\phi_i u_{i\varepsilon t}^{(kj)} dxdt + \iint_{\Omega_{T'}} \left(\nabla u_{i\varepsilon}^{(kj)} - \nabla u_{i\varepsilon}\right)\phi_i \left(\left|\nabla u_{i\varepsilon}^{(kj)}\right|^2 + \varepsilon\right)^{\frac{p_i-2}{2}} \nabla u_{i\varepsilon}^{(kj)} dxdt$$

$$+ \iint_{\Omega_{T'}} \left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon}\right)\nabla \phi_i \left(\left|\nabla u_{i\varepsilon}^{(kj)}\right|^2 + \varepsilon\right)^{\frac{p_i-2}{2}} \nabla u_{i\varepsilon}^{(kj)} dxdt \tag{2.47}$$

$$= \iint_{\Omega_{T'}} \left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon}\right)\phi_i f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)},u_{3\varepsilon}^{(k-1)}\right) dxdt.$$

Hence

$$\iint_{\Omega_{T'}} \left(\nabla u_{i\varepsilon}^{(kj)} - \nabla u_{i\varepsilon}\right)\phi_i \left(\left|\nabla u_{i\varepsilon}^{(kj)}\right|^2 + \varepsilon\right)^{\frac{p_i-2}{2}} \nabla u_{i\varepsilon}^{(kj)} dxdt$$

$$= \iint_{\Omega_{T'}} \left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon}\right)\phi_i f_{i\varepsilon}\left(x,t,u_{1\varepsilon}^{(k-1)},u_{2\varepsilon}^{(k-1)},u_{3\varepsilon}^{(k-1)}\right) dxdt - \iint_{\Omega_{T'}} \left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon}\right)\phi_i u_{i\varepsilon t}^{(kj)} dxdt \tag{2.48}$$

$$- \iint_{\Omega_{T'}} \left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon}\right)\nabla \phi_i \left(\left|\nabla u_{i\varepsilon}^{(kj)}\right|^2 + \varepsilon\right)^{\frac{p_i-2}{2}} \nabla u_{i\varepsilon}^{(kj)} dxdt.$$

Since the three terms on the right hand side of the above equality converge to 0 as $j \to \infty$. This yields that

$$\lim_{j\to\infty} \iint_{\Omega_{T'}} \left(\nabla u_{i\varepsilon}^{(kj)} - \nabla u_{i\varepsilon}\right)\phi_i \left|\nabla u_{i\varepsilon}^{(kj)}\right|^{p_i-2} \nabla u_{i\varepsilon}^{(kj)} dxdt = 0. \tag{2.49}$$

On the other hand, since $\nabla u_{i\varepsilon} \in L^{p_i}\left(\Omega_{T'}\right)$, we have that

$$\lim_{j\to\infty} \iint_{\Omega_{T'}} \left(\nabla u_{i\varepsilon}^{(kj)} - \nabla u_{i\varepsilon}\right)\phi_i \left|\nabla u_{i\varepsilon}\right|^{p_i-2} \nabla u_{i\varepsilon} dxdt = 0. \tag{2.50}$$

Note that

$$\int_0^1 \left| \nabla \left(s u_{i\varepsilon}^{(kj)} \right) + (1-s) u_{i\varepsilon} \right|^{p_i-2} \mathrm{d}s \left| \nabla \left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon} \right) \right|^2$$

$$\leq \left(\left| \nabla u_{i\varepsilon}^{(kj)} \right|^{p_i-2} \nabla u_{i\varepsilon}^{(kj)} - \left| \nabla u_{i\varepsilon} \right|^{p_i-2} \nabla u_{i\varepsilon} \right) \left(\nabla u_{i\varepsilon}^{(kj)} - \nabla u_{i\varepsilon} \right). \tag{2.51}$$

Following (2.50) and (2.51), we have

$$\lim_{j\to\infty} \iint_{\Omega_{T'}} \phi_i \left(\int_0^1 \left| \nabla \left(s u_{i\varepsilon}^{(kj)} + (1-s) u_{i\varepsilon} \right) \right|^{p_i-2} \mathrm{d}s \right) \left| \nabla \left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon} \right) \right|^2 \mathrm{d}x\mathrm{d}t = 0. \tag{2.52}$$

Since

$$\iint_{\Omega_{T'}} \int_0^1 \left| \nabla \left(s u_{i\varepsilon}^{(kj)} + (1-s) u_{i\varepsilon} \right) \right|^{p_i-2} \mathrm{d}s\mathrm{d}x\mathrm{d}t \leq C \tag{2.53}$$

and

$$\left| \left| \nabla u_{i\varepsilon}^{(kj)} \right|^{p_i-2} u_{i\varepsilon xl}^{(kj)} - \left| \nabla u_{i\varepsilon} \right|^{p_i-2} u_{i\varepsilon xl} \right|$$

$$= \left| \int_0^1 \frac{\mathrm{d}}{\mathrm{d}s} \left\{ \left| s\nabla u_{i\varepsilon}^{(kj)} + (1-s) \nabla u_{i\varepsilon} \right|^{p_i-2} \left(s u_{i\varepsilon xl}^{(kj)} + (1-s) u_{i\varepsilon xl} \right) \right\} \mathrm{d}s \right|$$

$$\leq \left| \int_0^1 \left| s\nabla u_{i\varepsilon}^{(kj)} + (1-s) \nabla u_{i\varepsilon} \right|^{p_i-2} \left(u_{i\varepsilon xl}^{(kj)} - u_{i\varepsilon xl} \right) \mathrm{d}s \right| \tag{2.54}$$

$$+ \left| \int_0^1 (p_i-2) \left| s\nabla u_{i\varepsilon}^{(kj)} + (1-s) \nabla u_{i\varepsilon} \right|^{p_i-4} \left(s u_{i\varepsilon xl}^{(kj)} + (1-s) u_{i\varepsilon xl} \right) \left(u_{i\varepsilon xl}^{(kj)} - u_{i\varepsilon xl} \right) \mathrm{d}s \right|$$

$$\leq C \left| \nabla \left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon} \right) \right| \int_0^1 \left| s\nabla u_{i\varepsilon}^{(kj)} + (1-s) \nabla u_{i\varepsilon} \right|^{p_i-2} \mathrm{d}s,$$

by Hölder inequality, we have

$$\left| \iint_{\Omega_{T'}} \left(\left| \nabla u_{i\varepsilon}^{(kj)} \right|^{p_i-2} u_{i\varepsilon xl}^{(kj)} - \left| \nabla u_{i\varepsilon} \right|^{p_i-2} u_{i\varepsilon xl} \right) \phi_i \mathrm{d}x\mathrm{d}t \right|$$

$$\leq C \left(\iint_{\Omega_{T'}} \phi_i \int_0^1 \left| \nabla \left(s u_{i\varepsilon}^{(kj)} + (1-s) u_{i\varepsilon} \right) \right|^{p_i-2} \mathrm{d}s \left| \nabla \left(u_{i\varepsilon}^{(kj)} - u_{i\varepsilon} \right) \right|^2 \mathrm{d}x\mathrm{d}t \right)^{\frac{1}{2}} \tag{2.55}$$

$$\times \left(\iint_{\Omega_{T'}} \phi_i \int_0^1 \left| \nabla \left(s u_{i\varepsilon}^{(kj)} + (1-s) u_{i\varepsilon} \right) \right|^{p_i-2} \mathrm{d}s\mathrm{d}x\mathrm{d}t \right)^{\frac{1}{2}} \to 0, j \to \infty.$$

i.e.,

$$\iint_{\Omega_{T'}} \left(w_{i\varepsilon xl} - \left| \nabla u_{i\varepsilon} \right|^{p_i-2} u_{i\varepsilon xl} \right) \phi_i \mathrm{d}x\mathrm{d}t, \text{ for any } \phi_i. \tag{2.56}$$

Hence

$$w_{i\varepsilon xl} = \left| \nabla u_{i\varepsilon} \right|^{p_i-2} u_{i\varepsilon xl}, i = 1, 2, 3. \tag{2.57}$$

This proves that any weak convergence subsequence of $\left| \nabla u_{i\varepsilon}^{(k)} \right|^{p_i-2} u_{i\varepsilon xl}^{(k)}, i = 1, 2, 3$ will have $w_{i\varepsilon xl}$ as its weak limit and hence by a standard argument, we have that as $k \to \infty$,

$$\left| \nabla u_{i\varepsilon}^{(k)} \right|^{p_i-2} u_{i\varepsilon xl}^{(k)} \rightharpoonup \left| \nabla u_{i\varepsilon} \right|^{p_i-2} u_{i\varepsilon xl}, \quad \text{in } L^{\frac{p_i}{p_i-1}}(\Omega_{T'}), i = 1, 2, 3. \tag{2.58}$$

Combining the above results, we have proved that $u_\varepsilon = (u_{1\varepsilon}, u_{2\varepsilon}, u_{3\varepsilon}), (x,t) \in \Omega_{T'}$ is a generalized solution of (2.1)-(2.3).

Proof of theorem 1.3.

Since $u_{i\varepsilon}$ satisfy similar estimates as (2.31), (2.37) and (2.40), combining the property of $f_{i\varepsilon}$, we know that there are functions $u_i \in L^{p_i}\left(0, T'; W_0^{1,p_i}(\Omega)\right)(i=1,2,3)$ (as $\varepsilon \to 0$) such that for some subsequence of $(u_{1\varepsilon}, u_{2\varepsilon}, u_{3\varepsilon})$, denoted again by $u_{i\varepsilon}$,

$$\nabla u_{i\varepsilon} \rightharpoonup \nabla u_i, \quad \text{in } L^{p_i}\left(\Omega_{T'}\right) \tag{2.59}$$

$$u_{i\varepsilon} \to u_i, f_{i\varepsilon}\left(x, t, u_{1\varepsilon}, u_{2\varepsilon}, u_{3\varepsilon}\right) \to f_i\left(x, t, u_1, u_2, u_3\right) \quad a.e. \text{ in } \Omega_{T'} \tag{2.60}$$

$$u_{i\varepsilon t} \rightharpoonup u_{it}, \quad \text{in } L^2\left(\Omega_{T'}\right) \tag{2.61}$$

$$\left|\nabla u_{i\varepsilon}\right|^{p_i-2} u_{i\varepsilon xl} \rightharpoonup w_{ixl}, \quad \text{in } L^{\frac{p_i}{p_i-1}}\left(\Omega_{T'}\right) \tag{2.62}$$

In a similar way as above, we prove that $w_{ixl} = \left|\nabla u_i\right|^{p_i-2} u_{ixl}, i=1,2,3$.

By a standard limiting process, we obtain that $u = (u_1, u_2, u_3)$ satisfies the initial and boundary value conditions and the integrating expression. Thus $u = (u_1, u_2, u_3)$ is a generalized solution of (1.1)-(1.3).

3. Uniqueness Result to the Solution of the System

We now prove the uniqueness result to the solution of the system.

Theorem 3.1. *Assume* $f = (f_1, f_2, f_3)$ *is Lipschitz continuous in* (u_1, u_2, u_3), *then the solution of* (1.1)-(1.3) *is unique.*

Proof. Assume that $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ are two solutions of (1.1)-(1.3). Let $w_i = u_i - v_i, i = 1, 2, 3$, then following (1.5),

$$\int_0^t \int_\Omega \left(-u_i w_{it} + \left|\nabla u_i\right|^{p_i-2} \nabla u_i \nabla w_i\right) dx dt + \int_\Omega u_i(x, t) w_i(x, t) dx - \int_\Omega u_{i0} w_i(x, 0) dx$$
$$= \int_0^t \int_\Omega f_i(x, t, u_1, u_2, u_3) w_i dx dt, \quad a.e. \quad t \in (0, T). \tag{3.1}$$

$$\int_0^t \int_\Omega \left(-v_i w_{it} + \left|\nabla v_i\right|^{p_i-2} \nabla v_i \nabla w_i\right) dx dt + \int_\Omega v_i(x, t) w_i(x, t) dx - \int_\Omega v_{i0} w_i(x, 0) dx$$
$$= \int_0^t \int_\Omega f_i(x, t, v_1, v_2, v_3) w_i dx dt, \quad a.e. \quad t \in (0, T). \tag{3.2}$$

By (3.1) subtracting (3.2), we get

$$\frac{1}{2} \int_\Omega (u_i - v_i)^2 dx = -\int_0^t \int_\Omega \left(\left|\nabla u_i\right|^{p_i-2} \nabla u_i - \left|\nabla v_i\right|^{p_i-2} \nabla v_i\right) \nabla(u_i - v_i) dx dt$$

$$+ \int_0^t \int_\Omega \left(f_i(x, t, u_1, u_2, u_3) - f_i(x, t, v_1, v_2, v_3)\right)(u_i - v_i) dx dt. \tag{3.3}$$

$$\leq \int_0^t \int_\Omega \left(f_i(x, t, u_1, u_2, u_3) - f_i(x, t, v_1, v_2, v_3)\right)(u_i - v_i) dx dt.$$

By the inequality (3.3) and the Lipschitz condition, a simple calculation shows that

$$\int_\Omega \left(\left|u_1 - v_1\right|^2 + \left|u_2 - v_2\right|^2 + \left|u_3 - v_3\right|^2\right) dx$$

$$\leq 2K \int_0^t \int_\Omega \left(\left|u_1 - v_1\right| + \left|u_2 - v_2\right| + \left|u_3 - v_3\right|\right)^2 dx dt \tag{3.4}$$

$$\leq 6K \int_0^t \int_\Omega \left(\left|u_1 - v_1\right|^2 + \left|u_2 - v_2\right|^2 + \left|u_3 - v_3\right|^2\right) dx dt$$

Setting $F(t) = \int_0^t \int_\Omega \left(\left|u_1 - v_1\right|^2 + \left|u_2 - v_2\right|^2 + \left|u_3 - v_3\right|^2\right) dx dt$, then (3.4) can be written as $F'(t) \leq 6KF(t)$. Since $F(0) \equiv 0$, by a standard argument, we have $F(t) \equiv 0$, and

hence $u_i \equiv v_i \left(i = 1, 2, 3 \right)$.

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