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Ideal Convergence in Generalized Topological Molecular Lattices

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Abstract

The convergence theory of ideals in generalized topological molecular lattices is studied. Some properties of this kind of convergence are investigated. Finally, the relations between convergence theories of both molecular nets and ideals in GTMLs are discussed together with the GT_2 separation axiom.

Keywords

Generalized Topological Molecular Lattice, Continuous GOH, Convergence of Molecular Nets, Convergence of Ideals

1. Introduction

After Wang [1] introduced the theory of topological molecular lattices or TMLs for short, several authors established various kinds of convergence theory in TMLs by using a corresponding concept of remote neighborhoods (see e.g. [2], [3]-[5]). The theory of remote neighborhood has been established first by Wang [1] as a dual notion of Pu and Liu's theory of the quasi-coincident neighborhoods in fuzzy topology [6] [7].

In [8], we introduced a generalization of Wang's topological molecular lattice called generalized topological molecular lattice or briefly GTML and studied the convergence theory of molecular nets by using the concept of generalized remote neighborhoods in these spaces.

In this paper, we aim to study the convergence of ideals in GTMLs and investigate the relations among this notion and that of molecular nets. Moreover, we state the relations with other defined topological notions in GTMLs such as generalized order homomorphism or GOH for short.

The paper is organized as follows. In Section 2, we will review some useful concepts in the paper. In Section 3, we will study the convergence in GTMLs in terms of ideals and investigate some properties of such conver-

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gence. Furthermore, we show the relations between convergence of ideals and the continuity of GOHs. In Section 4, we will discuss the relations between convergence of molecular nets and convergence of ideals in TMLs. Finally, Section 5 presents our conclusions.

2. Preliminaries

This section is devoted to recall some useful concepts which is required in the sequel. Let L be a complete lattice with the smallest element $\perp$ and the largest element $\top$, an element $a \in L$ is said to be a *molecule* (some time called *co-prime* or *join-irreducible*) if for $b, c \in L, a \le b \vee c$ then $a \le b$ or $a \le c$. The set of all molecules in L is denoted by $M(L)$. The subset $B \subseteq L$ is called a *minimal family* of a [1], if the following two conditions are hold:

a) $a = \vee B$.

b) If $D \subseteq L$ and $a \le \vee D$, then $\forall b \in B, \exists d \in D$ such that $d \ge b$.

The greatest minimal family of a is denoted by $\beta(a)$ while $\beta^*(a) = \beta(a) \cap M(L)$.

Throughout this paper, the entry $L(M)$ denotes a molecular lattice, that is a lattice L and the set of its molecules M. For a non empty subset I of a complete lattice L, I is said to be an *ideal* [9], if it satisfies the following conditions:

a) For $a \in I, x \in L$ and $x \le a \Rightarrow x \in I$.

b) For all $a, b \in I \Rightarrow a \vee b \in I$.

c) $\top \notin I$.

Definition 2.1 [8] *Let $L(M)$ be a molecular lattice. A subfamily $\eta \subset L$ is said to be a generalized closed topology, or briefly, generalized co-topology, if*

(T_1) η is closed under arbitrary intersections;

(T_2) $\top \in \eta$.

The pair $(L(M), \eta)$ is called a generalized topological molecular lattice, or briefly, GTML.

Definition 2.2 [8] *Let $(L(M), \eta)$ be a GTML, $a \in M, F \in \eta$, and $a \nleq F$. Then F is said to be a generalized remote neighborhood of a. The set of all generalized remote neighborhoods of a will be denoted by $\eta(a)$.*

Recall that according to the definition of ideals, the family $\eta(a)$ is not necessary be an ideal in GTMLs while the family $\eta^*(a) = \{F \in \eta(a) : \forall H \in \eta(a), F \vee H \in \eta(a)\} \subset \eta(a)$ satisfies the ideal conditions.

For a GTML $(L(M), \eta)$ and $A \in L$, the intersection of all η-elements containing A is called the generalized closure of A and denoted by A^-. that is,

$$A^- = \wedge\{F \in \eta : A \le F\}$$

Definition 2.3 [8] *Let $(L(M), \eta)$ be a GTML, $a \in M, A \in L$, then a is said to be an adherence point of A, if for all $F \in \eta(a)$, we have $A \nleq F$.*

It is clear that a is an adherence point of A if and only if $a \le A^-$.

Definition 2.4 [1] *Let L_1 and L_2 be complete lattices. A mapping $f : L_1 \to L_2$ is said to be a generalized order homomorphism or GOH for short if*

a) $f(a) = \perp$ if and only if $a = \perp$.

b) f is join preserving, *i.e*; $f(\vee_j a_j) = \vee_j f(a_j)$.

c) $f^{\vdash}$ is join preserving, where $\forall b \in L_2$, $f^{\vdash}(b) = \{a \in L_1 : f(a) \le b\}$.

Definition 2.5 [8] *Let $(L_1(M_1), \eta_1)$ and $(L_2(M_2), \eta_2)$ be GTMLs and $f : L_1 \to L_2$ be a GOH, then f is called*:

1) continuous GOH, if for every $H \in \eta_2$, we have $f^{\vdash}(H) \in \eta_1$.

2) continuous at a molecule $a \in M_1$, if for every $H \in \eta_2(f(a))$, we have $(f^{\vdash}(H))^- \in \eta_1(a)$.

For a directed set D and $A \in L$, the mapping $S : D \to M$ is called a molecular net and denoted by $S = \{S(n) : n \in D\}$. The molecular net S is said to be in A, if $S(n) \le A, \forall n \in D$.

The molecular net S is said to be:

1) *eventually* in A if there exists $n_0 \in D$ such that $\forall n \in D, n \ge n_0$, we have $S(n) \le A$.

2) *frequently* in A if for all $n \in D$ there exists $n_0 \in D$ such that $n \ge n_0, S(n_0) \le A$.

Definition 2.6 [8] *Let $(L(M), \eta)$ be a GTML, $S = \{S(n) : n \in D\}$ be a molecular net and $a \in M$, then*:

1) a is called a limit point of S, if $\forall F \in \eta(a), S(n) \nleq F$ eventually true, and denoted by $S \to a$. The join

of all limit points of S will be denoted by $limS$.
i.e, $limS = \vee\{x \in M : S \to x\}$.

2) a is called a cluster point of S, if $\forall F \in \eta(a), S(n) \not\leq F$ frequently true, and denoted by $S \infty a$. The join of all cluster points of S will be denoted by $cluS$.
i.e, $cluS = \vee\{x \in M : S \infty x\}$.

Definition 2.7 [8] *Let $(L(M),\eta)$ be a GTML, then $(L(M),\eta)$ is said to be a GT$_2$, if $\forall a,b \in M, a \wedge b = \bot$, there exists $H \in \eta^*(a)$ and $F \in \eta^*(b)$ such that $H \vee F = \top$.*

3. Convergence of Ideals in GTMLs

The aim of this section is to study the convergence in GTMLs in terms of ideals and investigate some properties of such convergence. Furthermore, we show the relations between convergence of ideals and the continuity of GOHs.

For the sake of convenience and no confusion, throughout this section and forwards, we restrict the attention of generalized remote neighborhoods of an element a in GTMLs into the set $\eta^*(a)$ instead of $\eta(a)$.

Definition 3.1 *Let $(L(M),\eta)$ be a GTML, $I \subset L$ be an ideal of L and $a \in M$, then*

1) a is said to be a limit point of I if $\eta^*(a) \subseteq I$, denoted by $I \to a$. In this case, we say that I converges to a.

The join of all limit points of I will be denoted by $limI$.

2) a is said to be a cluster point of I if $\forall F \in \eta^*(a)$ and $\forall A \in I$, we have $F \vee A \neq \top$, denoted by $I \infty a$. In this case, we say that I accumulates to a.

The join of all cluster points of I will be denoted by $cluI$.

As a consequence, we obtain the following proposition:

Proposition 1 *Let $(L(M),\eta)$ be a GTML, I and J be ideals of L with $I \subset J$ and $a,b \in M$. Then we have*:

1) $I \to a \Rightarrow J \to a$.
2) $J \infty a \Rightarrow I \infty a$.
3) $I \to a, b \leq a \Rightarrow I \to b$.
4) $I \infty a, b \leq a \Rightarrow I \infty b$.

Proof.

1) Let $I \to a$, then $\forall F \in \eta^*(a), F \in I \subset J$. Thus, $F \in J$ and hence $\eta^*(a) \subseteq J$.
Therefore, we have $J \to a$.

2) Let $J \infty a$, then $\forall F \in \eta^*(a)$ and $\forall A \in J$, we have $F \vee A \neq \top$.
Since $I \subset J$, then $B \in I \Rightarrow B \in J$ and hence $\forall F \in \eta^*(a)$ and $\forall B \in I$, $F \vee B \neq \top$.
Therefore, we have $I \infty a$.

3) Let $I \to a$, then $\eta^*(a) \subseteq I$. Since $b \leq a$, then we get $\eta^*(b) \subseteq \eta^*(a)$. So, $\eta^*(b) \subseteq I$.
Therefore, we have $I \to b$.

4) Let $I \infty a$, then $\forall F \in \eta^*(a)$ and $\forall A \in I$, $F \vee A \neq \top$. But $\eta^*(b) \subseteq \eta^*(a)$, then $\forall H \in \eta^*(b)$ and $\forall A \in I$, we have $H \vee A \neq \top$. Therefore, we have $I \infty b$. □

Theorem 2 *Let $(L(M),\eta)$ be a GTML, I be an ideal of L and $a \in M$, then*

1) $I \to a$ if and only if $a \leq limI$.
2) $I \infty a$ if and only if $a \leq cluI$.

Proof.

1) Let $I \to a$, by the definition of $limI = \vee\{x \in M : I \to x\}$, it is clear that $a \leq limI$.

Conversely, let $a \leq limI$ and $F \in \eta^*(a)$, then $a \not\leq F$ and hence $limI \not\leq F$. So, there exists $b \in M$ such that $I \to b, b \not\leq F$, then $F \in \eta^*(b)$. Thus, we have $\eta^*(a) \subseteq \eta^*(b)$ but $\eta^*(b) \subseteq I$, hence $\eta^*(a) \subseteq I$. Therefore, $I \to a$.

2) Let $I \infty a$, then similarly to 1), $a \leq cluI$ is clear.

Now, let $a \leq cluI$ and $F \in \eta^*(a)$, than $a \not\leq F$ and hence $cluI \not\leq F$. So, there exists $b \in M$ such that $I \infty b, b \not\leq F$, then $F \in \eta^*(b)$. Thus, for all $A \in I, F \vee A \neq \top$, also, we have $\eta^*(a) \subseteq \eta^*(b)$. So, $\forall F \in \eta^*(a), \forall A \in I, F \vee A \neq \top$. Therefore, $I \infty a$. □

Corollary 1 *Let $(L(M),\eta)$ be a GTML, I be an ideal of L and $a \in M$, then*

1) $I \to a$ if and only if $\forall b \in \beta^*(a), I \to b$.

2) $I\infty a$ if and only if $\forall b\in\beta^*(a), I\infty b$.

Theorem 3 *Let* $(L(M),\eta)$ *be a GTML,* $A\in L$, *and* $a\in M$, *then* $a\le A^-$ *if and only if there exists an ideal I in L such that* $A\notin I$ *and* $I\to a$.

Proof. $(\Rightarrow)$ Since $a\le A^-$, then *a* is an adherence point of *A*, *i.e*; $\forall F\in\eta^*(a), A\not\le F$.

Put $I_{\eta^*(a)}=\{B\in L:\exists F\in\eta^*(a), B\le F\}$, then $I_{\eta^*(a)}$ is an ideal and clearly that $A\notin I_{\eta^*(a)}$ also, we have $\eta^*(a)\subseteq I_{\eta^*(a)}$ which implies $I_{\eta^*(a)}\to a$.

$(\Leftarrow)$ Let $I\to a$, then $\forall F\in\eta^*(a)$, we have $\eta(a)^*\subseteq I$, *i.e*; $F\in I$. Since $A\notin I$, then $A\not\le F$. So, by **Definition 2.3**, *a* is an adherence point of *A* and hence $a\le A^-$. □

Lemma 1 *Let* $(L_1(M_1),\eta_1)$ *and* $(L_2(M_2),\eta_2)$ *be GTMLs,* $f:L_1\to L_2$ *be a GOH, and I be an ideal in* L_1. *Then the set*

$$f(I)=\{B\in L_2:\exists A\in I, s.t, \forall a\in M_1, a\not\le A\Rightarrow f(a)\not\le B\}$$

is an ideal in L_2.

Proof. It is easily to check the conditions of ideals. □

Theorem 4 *Let* $(L_1(M_1),\eta_1)$ *and* $(L_2(M_2),\eta_2)$ *be GTMLs,* $f:L_1\to L_2$ *be a continuous GOH at* $a\in M_1$ *and I be an ideal in* L_1. *If* $I\to a$, *then* $f(I)\to f(a)$.

Proof. Let *f* be a continuous GOH at $a\in M_1$ and *I* be an ideal in L_1 with $I\to a$, then $\forall H\in\eta_2^*(f(a))$, we have $(f^{\vdash}(H))^-\in\eta_1^*(a)\subseteq I$. Hence, we get that $f^{\vdash}(H)\in I$ and for every $a\not\le f^{\vdash}(H)\Rightarrow f(a)\not\le H$. so, $H\in f(I)$ which implies that $\eta_2^*(f(a))\subseteq f(I)$.

Therefore, $f(I)\to f(a)$. □

Theorem 5 *Let* $(L_1(M_1),\eta_1)$ *and* $(L_2(M_2),\eta_2)$ *be GTMLs,* $f:L_1\to L_2$ *be a GOH, then f is continuous GOH if and only if for every ideal I of* L_1, $f(limI)\le limf(I)$.

Proof. $(\Rightarrow)$ Let *I* be an ideal of $L_1, a\in M_1$ such that $I\to a$, hence $a\le limI$. We need to show that $f(a)\le limf(I)$. Since *f* is a continuous GOH, then *f* is continuous at $a\in M_1$ and $f(a)\in M_2$. Hence, by **Theorem 4**, we get $f(I)\to f(a)$, *i.e*; $f(a)\le limf(I)$.

Since *f* is a GOH, then *f* preserves arbitrary joins and hence $f(limI)\le limf(I)$.

$(\Leftarrow)$ We want to prove that *f* is continuous at every $a\in M_1$, *i.e*; $\forall F\in\eta_2^*(f(a))$, we have $(f^{\vdash}(F))^-\in\eta_1^*(a)$.

Assume that $a\le(f^{\vdash}(F))^-$. Hence, there exists an ideal *I* such that $I\to a$ and $f^{\vdash}(F)\notin I$. Then $a\le limI$ which implies that $f(a)\le f(limI)\le limf(I)$. Thus, $f(I)\to f(a)$.

So, $\forall F\in\eta_2^*(f(a))$, we have $F\in f(I)$. By the definition of $f(I)$, there exists $A\in I$ such that $\forall a\in M_1, a\not\le A\Rightarrow f(a)\not\le F$ equivalently that $a\not\le A\Rightarrow a\not\le f^{\vdash}(F)$. Hence, $f^{\vdash}(F)\le A$ but $A\in I$, so $f^{\vdash}(F)\in I$. Contradiction.

Then, $a\not\le(f^{\vdash}(F))^-$ and hence $(f^{\vdash}(F))^-\in\eta_1^*(a)$.

Therefore, *f* is continuous GOH. □

4. Relations between Convergence of Molecular Nets and Convergence of Ideals in GTMLs

In [3] and [5], the authors introduced a comparison between convergence of molecular nets and convergence of ideals in TMLs. In similar way, we discuss the relations between them in GTMs.

For a generalized topological molecular lattice $(L(M),\eta)$, let *I* be an ideal in *L*, then the set

$$D(I)=\{(a,A):a\in M, A\in I, a\not\le A\}$$

is a directed set with respect to the relation "$\le$" defined as

$$\forall(a,A),(b,B)\in D(I),(a,A)\le(b,B)\Leftrightarrow A\le B$$

Set $S(a,A)=a$, then the set

$$S(I)=\{S(a,A):(a,A)\in D(I)\}$$

is a molecular net in $L(M)$ called the molecular net generated by the ideal *I*.

Now, let $S=\{S(n):n\in D\}$ be a molecular net in *L*, then the set

$$I(S)=\{A\in L:S(n)\not\leq A \text{ eventually}\}$$

is an ideal in *L* called the ideal generated by *S*.

Theorem 6 *Let $(L(M),\eta)$ be a GTML, $x\in M, I$ be an ideal in L and S be a molecular net in L, then we have*

1) $S(I)\to x\Leftrightarrow I\to x$, (resp. $S(I)\infty x\Leftrightarrow I\infty x$).
2) $I(S)\to x\Leftrightarrow S\to x$.
3) $I=I(S(I))$.

Proof. 1) Case I: Let $S(I)\to x$, then $\forall F\in\eta^*(x), S(I)\not\leq F$ eventually, *i.e*; there exists $(a,A)\in D(I)$ such that $\forall (b,B)\in D(I),(b,B)\geq(a,A)$, we have $S(I)(b,B)=b\not\leq F$. Hence, $\forall b\not\leq A$ we get $b\not\leq F$, so $F\leq A$ but $A\in I$ which implies that $F\in I$.

Therefore, $\eta^*(x)\subseteq I$ and $I\to x$.

Conversely, let $I\to x$, then $\forall F\in\eta^*(x), F\in I$. Since $x\not\leq F$, then $(x,F)\in D(I)$ and $\forall (a,A)\in D(I)$ such that $(a,A)\geq(x,F)$, we have $S(I)(a,A)=a\not\leq A$, but $A\geq F$, hence $S(I)(a,A)=a\not\leq F$. Thus, $S(I)\to x$.

Case II: Let $S(I)\infty x$, $F\in\eta^*(x)$ and $A\in I$, then there exists $a\in M$ with $a\not\leq A$. Thus, $(a,A)\in D(I)$, since $S(I)\infty x$, there exists $(b,B)\in D(I)$ such that $(b,B)\geq(a,A)$ and $S(I)(b,B)=b\not\leq F$. Since $b\in M, b\not\leq B$, then $b\not\leq F\vee B$ but $A\leq B$, so $b\not\leq F\vee A$.

Thus, $\forall F\in\eta^*(x)$ and $A\in I$, $F\vee A\neq\top$. Therefore, $I\infty x$.

Conversely, we need to show that $\forall F\in\eta^*(x), S(I)\not\leq F$ eventually. Let $I\infty x$, then $\forall F\in\eta^*(x)$ and $A\in I, F\vee A\neq \top$.

Now, $\forall (a,A)\in D(I)$, we have $F\vee A\neq\top$, therefore, $\exists b\in M$ such that $b\not\leq F$ and $b\not\leq A$. So, $(b,A)\in D(I)$ and $(b,A)\geq(a,A)$, $S(I)(b,A)=b\not\leq F$.

Therefore, $S(I)\not\leq F$ frequently and $S(I)\infty x$.

2) Let $I(S)\to x$, then $\forall F\in\eta^*(x), F\in I(S)$. By the definition of $I(S)$, we have $S\not\leq F$ eventually which means that $S\to x$.

Conversely, let $S\to x$, then $\forall F\in\eta^*(x), S\not\leq F$ eventually. So, $F\in I(S)$, *i.e*, $\eta^*(x)\subseteq I(S)$ which means $I(S)\to x$.

3) Let $A\in I$, then there exists $a\in M$ such that $(a,A)\in D(I)$ and $\forall (b,B)\in D(I)$ with $(b,B)\geq(a,A)$, we have $S(I)(b,B)=b\not\leq B$. But $B\geq A$, hence $S(I)(b,B)=b\not\leq A$, *i.e*; $S(I)\not\leq A$ eventually.

Thus, $A\in I(S(I))$ and $I\subseteq I(S(I))$.

Now, let $A\in I(S(I))$, then $S(I)\not\leq A$ eventually, *i.e*; there exists $(b,B)\in D(I)$ such that $\forall (e,E)\in D(I),(e,E)\geq(b,B)$, we have $S(I)(e,E)=e\not\leq A$. Since $e\not\leq E$ and $E\geq B$, then $e\not\leq B\Rightarrow e\not\leq A$. Hence, $A\leq B$ and $B\in I$, then $A\in I$ and $I(S(I))\subseteq I$.

Therefore, $I=I(S(I))$. □

According to **Theorem 6**, one can get directly the following result:

Corollary 2 *Let $(L(M),\eta)$ be a GTML, I be an ideal in L and S be a molecular net in L, then the following statements hold*:

1) $limI=limS(I)$.
2) $cluI=cluS(I)$.
3) $limS=limI(S)$.

Theorem 7 *Let $(L(M),\eta)$ be a GTML, I be an ideal in L and S be a molecular net in L, then we have*

$$cluS\leq cluI(S).$$

Proof. Let $x\leq cluS$, then $S\infty x$. So, we need to show that $S\infty x\Rightarrow I(S)\infty x$.

Now, $\forall F\in\eta^*(x), S\not\leq F$ frequently. Also, $\forall A\in I(S), S\not\leq A$ eventually and hence, $S\not\leq F\vee A$ frequently. So, $\forall F\in\eta^*(x)$ and $\forall A\in I(S)$, we get $F\vee A\neq\top$.

Therefore, $I(S)\infty x$ and hence, $x\leq cluI(S)$. □

In 1986, Yang [9] introduced the concepts of maximal ideals and universal nets.

Definition 4.1 [9] *An ideal I in a complete lattice L is called a maximal ideal , if for each ideal J in L such that* $I \subset J$ *, we have* $I = J$.

Definition 4.2 [9] *A molecular net S in a complete lattice L is called a universal net , if there exists a maximal ideal in L such that S is a subnet of* $S(I)$.

Proposition 8 *Let* $(L(M),\eta)$ *be a GTML and I be a maximal ideal in L, then*

$$limI = cluI .$$

Proof. It is clear that $limI \le cluI$. Now, we prove that $cluI \le limI$.

Let $x \le cluI$, then $I \infty x$. Put $J = \{B \in L : \exists A \in I, F \in \eta^*(x), B \le A \vee F\}$

Then *J* is an ideal in *L* and clearly that $I \subseteq J$ and $\eta^*(x) \subseteq J$.

Since *I* is a maximal ideal in *L*, we get $I = J$, hence $\eta^*(x) \subseteq I$.

So, $I \to x$ and $x \le limI$. Therefore, $limI = cluI$. □

Theorem 9 *Let* $(L(M),\eta)$ *be a GTML, then the following conditions are equivalent*:

(i) For every ideal *I*, $\exists x \in M$ such that $I \infty x$.

(ii) For every maximal ideal *I*, $\exists x \in M$ such that $I \to x$.

(iii) For every universal net *S*, $\exists x \in M$ such that $S \to x$.

Proof. (i) $\Rightarrow$ (ii) Let *I* be a maximal ideal, by (i), $\exists x \in M$ such that $I \infty x$. Since, *I* is a maximal, then by **Proposition 8**, we have $I \to x$.

(ii) $\Rightarrow$ (i) Let *I* be an ideal, then there exists a maximal ideal *J* with $I \subseteq J$ and $\exists x \in M$ such that $J \to x$. Hence, $\eta^*(x) \subseteq J$.

So, $\forall A \in I$ and $\forall F \in \eta^*(x), A \vee F \ne \top$. Thus, $I \infty x$.

(ii) $\Rightarrow$ (iii) Let *S* be a universal net and $x \in M$, then by the definition, there exists a maximal ideal *I* such that *S* is a subnet of $S(I)$. By (ii), we have $I \to x$ and hence $S(I) \to x$. Therefore, $S \to x$.

(iii) $\Rightarrow$ (ii) Let *I* be a maximal ideal, then $S(I)$ is a universal net, by (iii), $\exists x \in M$ such that $S(I) \to x$. Then, we get $I \to x$. □

Lastly, we conclude this section by studying the relation between the ideal convergence and the GT_2 separation axiom in GTMLs.

Theorem 10 *Let* $(L(M),\eta)$ *be a GTML, then it is* GT_2*, if and only if for every ideal I in L,* $limI$ *contains no disjoint molecules.*

Proof. $(\Rightarrow)$ Let $(L(M),\eta)$ be GT_2, *I* be an ideal in *L*. Assume that $a,b \le limI$ with $a \wedge b = \perp$. Then there exists $H \in \eta^*(a)$ and $F \in \eta^*(b)$ such that $H \vee F = \top$. Since $I \to a$ and $I \to b$, we have that $\eta^*(a) \subseteq I$ and $\eta^*(b) \subseteq I$. Hence, $F, H \in I$ which implies that $\top = H \vee F \in I$. Contradiction with the definition of *I*.

Therefore, $limI$ contains no disjoint molecules.

$(\Leftarrow)$ Assume that $(L(M),\eta)$ is not GT_2, then $\exists a,b \in M$ with $a \wedge b = \perp$ and $\forall F \in \eta^*(a), H \in \eta^*(b)$, we have $F \vee H \ne \top$. Put

$$I = \{A \in L : A \le F \vee H, F \in \eta^*(a), H \in \eta^*(b)\}$$

Then *I* is an ideal in *L* with $a \le limI$ and $b \le limI$. Hence, $limI$ contains two disjoint molecules $a,b \in M$ which contradicts the assumption. Therefore, $(L(M),\eta)$ is GT_2. □

Corollary 3 *Let* $(L(M),\eta)$ *be a GTML, then the following statements are equivalents*:

a) $(L(M),\eta)$ is a GT_2.

b) For every molecular net *S* and every $a,b \in \beta^*(limS)$, we have $a \wedge b \ne \perp$.

c) For every ideal *I* in *L* and every $a,b \in \beta^*(limI)$, we have $a \wedge b \ne \perp$.

5. Conclusion

In this paper, we introduced a convergence theory of ideals in generalized topological molecular lattices by using the concept of generalized remote neighborhoods and studied some of its characterization and properties. Then, we investigated the relations between the ideal convergence and the continuity of GOH in GTMLs. Finally, we discussed the relations among the convergence theories of both ideals and molecular nets and also the GT_2 separation axiom.

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