

From Highly Structured E-Infinity Rings and Transfinite Maximally Symmetric Manifolds to the Dark Energy Density of the Cosmos

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Abstract

Starting from well established results in pure mathematics, mainly transfinite set theory, E-infinity algebra over operads, fuzzy manifolds and fuzzy Lie symmetry groups, we construct an exact Weyl scaling for the highly structured E-infinity rings corresponding to E-infinity theory of high energy physics. The final result is an exact expression for the energy density of the cosmos which agrees with previous analysis as well as accurate cosmological measurements and observations, such as COBE, WMAP and Planck. The paper is partially intended as a vivid demonstration of the power of pure mathematics in physics and cosmology.

Keywords

Highly Structured Rings, E-Infinity, Loop Spaces, High Energy Physics, Dark Energy, Einstein Relativity, Fractal-Cantorian Spacetime, Nonlinear Dynamics, Quantum Chaos

1. Introduction

There are a few scientists, the present author included, who feel quite strongly that at its deepest level physics in general and high energy particle physics in particular is governed by the laws of pure mathematics [1]-[5], that is to say the laws of thought. Thus pure mathematics could be seen as the foundation of the very roots of real physics to the extent that any true mathematical theorem, no matter how pure it is, will very likely have an application in the so called “real world” domain [5]-[25]. The present author believes that the work of L. Euler, F. Gauss, J. von Neumann, A. Connes as well as E-infinity theory are accurate examples for this scientific philos-

ophy [1]-[62].

The main purpose of the present paper is twofold. It is intended first to make the mainstream applied physicists more aware of the treasures of pure mathematics waiting to be discovered by them for application [53]-[62] and second to make pure mathematicians more aware of the indispensable role played by their pure science in applied sciences, particularly high energy physics and quantum cosmology [60]-[62]. The present author thinks, and so do others, that his main contribution was to help bring E-infinity theory and related areas such as transfinite numbers and Cantorian fractals to quantum physics and cosmology [24]-[42]. The Klein-Penrose fractal universe [40] which is used here directly and indirectly, is a clear cut example of noncommutative geometry as well as being a prototype of K-theory and Cantorian-E-infinity theory [12] [40]. Noting the well known intimate relationship of K-theory, n-categories and E-infinity algebra over operads [42] [43] and the associated loop spaces and E-infinity spectra, the reader can sense the deep relations between E-infinity Cantorian spacetime of high energy physics on the one side and the afore mentioned enormously rich and versatile fields of pure mathematics. In fact, as mentioned earlier, from the viewpoint of the present work, the basis of the foundation of theoretical quantum physics and cosmology, is without a doubt, pure mathematics [9] [10]. The author seriously believes that any mathematically consistent structure in pure mathematics corresponds to something out there in the real world so that the division between pure mathematics and physics is a largely an artificial one.

The present paper will use transfinite set theoretical results [7] and results from K-theory and E-infinity theory and demonstrate that dark energy and ordinary energy apart from quantum wave collapse and measurement can be fully understood [60]-[62].

2. A Transfinite Cellular Automata as a Golden Turing

The main pillar upon which our highly structured E-infinity golden ring [42] [43] rests is the golden mean “binary” system. Before going into that we recall first a few facts concerning E-infinity in general. First E_∞ ring is a commutative monad in the $(\infty,1)$ -category of E-infinity spectra. E-infinity algebra on the other hand is an algebra over an operad for E-infinity operad [42] [43]. In chain complex this is similar, in fact equivalent to those in Abelian symplectic group which brings us nearer to our treatment of the compactified symplectic modular space of Klein and Penrose [38] [40]. For our present E-infinity “physical” version of P. May E-infinity, the golden mean ring introduces incredible simplification to the entire problem [11] so that the E-infinity algebra is de facto replaced by an almost elementary E-infinity arithmetic manipulating $\phi = (\sqrt{5} - 1)/2$ in all its forms of union and intersections in a way analogous to a Suslin operation. For instance the vital $\langle n \rangle = 4 + \phi^3$ is found from [11] [35] [37]

$$\langle n \rangle = \sum_0^\infty n(d_{(0)})^n = (1 + d_c^{(0)}) / (1 - d^{(0)}) \tag{1}$$

to be nothing but

$$\begin{aligned} \langle n \rangle &= \sum_0^\infty n^2 \phi^n / \sum_0^\infty n \phi^n = \phi + 2\phi^2 + 3\phi^3 + 4\phi^4 + \dots \\ &= (1 + \phi) / (1 - \phi) = 1/\phi^3 = 4 + \phi^3 \\ &= 4.236067977. \end{aligned} \tag{2}$$

Similarly one finds that the same value is obtained for the average Hausdorff dimension [11] [35] [37]

$$\langle d_c \rangle = \frac{1}{d_c^{(0)}(1 - d_c^{(0)})} = \frac{1}{\phi(1 - \phi)} = 1/\phi^3 = 4 + \phi^3. \tag{3}$$

The above constitutes the required minimum to understand the rest of the paper which is devoted to a major cosmological problem considered by many as the greatest mystery of our time [63].

3. Analysis of the Energy Density of the Cosmos

We start from the obvious fact that Einstein’s space is 4 dimensional, *i.e.* 3 spatial dimensions fused to 1 time dimension and that the classical photon of the electromagnetic field moves in these 4 dimensions [56] [57] [59]. At least formally, Einstein’s density could be given within this picture as the ratio of the spacetime background

dimension $D = 4$ to itself as the dimension of the photon “space” so that we obtain the trivial result

$$\gamma = (D = 4)/(D = 4) = 1 \quad (4)$$

and the corresponding Einstein special relativity formula is again formally given by the trivial tautology

$$E = \gamma mc^2 = mc^2. \quad (5)$$

Now we move to the superstring picture [29] which after decades long debate is commonly perceived as a substantially correct quantum gravity theory. The spacetime of this theory is, as is well known, 10 dimensional and 6 of these dimensions are considered compactified in various ways [28] [29]. Concurrently we can in the meantime view the photon and in fact for the same purpose, the electron to be moving in a space with 137 dimensions equal to the inverse of the electromagnetic fine structure constant which can be shown to be a scale down of the E8E8 dimensions of superstring, *i.e.* the 496 massless gauge bosons [34] [35]. Seen that way we must scale γ , *i.e.* we must scale $E = mc^2$ with the intuitively obvious scaling, namely

$$\lambda = \frac{D(\text{string}) - D(\text{Einstein})}{\bar{\alpha}_o - D(\text{Einstein})} = \frac{10 - 4}{137 - 4} = 6/133 = 1/22.166. \quad (6)$$

i.e. it is approximately 1/22. This is a remarkable result because it means

$$E(\text{ordinary}) = (\gamma)(\lambda)mc^2 \cong mc^2/22 \quad (7)$$

which is complete agreement with all known cosmological measurements [30] of the ordinary measureable energy density of the universe. This implies a 95.5 percent dark energy density which being effectively an empty set quantum wave cannot be measured in any ordinary way due to quantum wave collapse [60]. The corresponding formula is

$$E(D) = \left(1 - \frac{1}{22}\right)mc^2 = (21/22)mc^2 \quad (8)$$

The preceding result and conclusion needs a great deal of further elaboration and refinement. To start with one could find it strange to set $\bar{\alpha}_o = 137$ and $D = 4$ on the same footing. However this is perfectly correct when we take Noether’s theorem seriously and differentiate between external dimensions $D = 4$ and internal dimensions such as $D = 10 - 4 = 6$ of superstring theory and $\bar{\alpha}_o = 137$ which may be regarded as internal dimensions of the electromagnetic manifold or even particle-like states contributing to the 496 massless gauge boson as is obvious from the fundamental equation [11] [37]

$$B = H + \bar{E} + G + |\text{SU}(2)| \quad (9)$$

In the above equation B is the 496 bulk of E8E8, H is the holographic boundary, *i.e.* the particle physics given by $|\text{SL}(2,7)| = 336$ of Klein’s modular space, G is the 20 degrees of freedom of pure gravity in eight dimensions of E8 and $|\text{SU}(2)|$ are the three massive photons of the weak force, namely w^+ , w^- and Z^0 [48]. Finally, and that is the main point, $\bar{E} = 137$ which leads to [37]

$$|\text{E8E8}| = 336 + (\bar{\alpha}_o = 137) + 20 + 3 = 496 \quad (10)$$

exactly as should be. Having established that 137 is a dimension or particle-like state we should use then the exact E-infinity value which is $\bar{\alpha}_o = 137 + k_o$ where $k_o = \phi^5(1 - \phi^5)$ and ϕ^5 is Hardy’s quantum entanglement [63]. Inserting in λ one finds

$$\lambda = 6/(137.08203932) = \phi^5/2 = 1/(22 + k) \quad (11)$$

where $\phi = (\sqrt{5} - 1)/2$ and $k = 2\phi^5$ is ‘tHooft’s renormalon [53]. This is indeed the exact theoretical result obtained in earlier publications using other methods. The scaling factor $\lambda = 1/(22 + k)$ makes intuitively a great deal of sense. It is the ratio of the number of spacetime extended dimensions to the total not accounted for internal dimensions. However there are even more interesting interpretations of this result which we discuss further next.

It is clear that the compactification of the 6 dimensions is best dealt with using a Calabi-Yau manifold [16]. This manifold has 6 real dimensions so that 6 can be considered the dimension of this geometrically and topo-

logically remarkable manifold. On the other hand the $133+k_o$ dimensions remaining from $\bar{\alpha}_o = 137+k_o$ could be viewed as a transfinite dimension of E7, namely $|E7|+k_o = 133+0.082039325$ where E7 is the next largest exceptional Lie group in the E-line after E8 and is an important step into the symmetry breaking from E8 to the standard model [25]. On the other hand if we are not interested in the exact solution and would be happy to find the integer solution, then 133 of $|E7|$ could be replaced by $\dim M(6,22) = 132$ modular manifold so that one finds [16]

$$\lambda = \frac{6(\text{Calabi-Yau})}{132(M6,22)} = 6/132 = 1/22 \tag{12}$$

As an aside we mention that at the beginning and due to inaccurate measurements $\bar{\alpha}_o$ was believed by Sir A. Stanley Eddington to be $\bar{\alpha}_o = 136$. Using this wrong value which we could call the naked, undressed value of $\bar{\alpha}_o$, we find $\lambda = 1/22$ as above. It is now relatively easy to reason that the dark energy density is given by the exact expression

$$\gamma(\text{Dark}) = \frac{\bar{\alpha}_o - (D^{(10)} - (D^4 - k))}{137 + k_o} = \frac{\bar{\alpha}_o - (6 + k)}{137 + k} = \frac{21 + k}{22 + k} = 5\phi^2/2. \tag{13}$$

The difficulty in the above is that we are involving 'tHooft's k which leads to 'tHooft-Veltman-Wilson $4-k$ fractal regularization space [53] rather than $D^{(4)}$ of Einstein. The reason is subtle and is best understood from the next analysis of the same problem within the framework of E-infinity applied to dimensional regularization.

We start this time from the bulk E8E8 and note that $k^2/2$ is the transfinite correction leading to 247.98373 [57]. We now have to subtract the $11+\phi^5$ fractal M-theory dimension [41]. Our λ must therefore account for 'tHooft's k and we have thus [53]

$$\begin{aligned} \gamma(\text{Dark}) &= \frac{(496 - k^2) - (2)(11 + \phi^5) - k}{496 - k^2} \\ &= (21 + k)/(22 + k). \end{aligned} \tag{14}$$

However if we work without super symmetry with Einstein and Kaluza-Klein spacetime [63], then we have

$$\lambda^+ \lambda^- = \lambda^2 = \frac{496 - k^2 - (5) - \bar{\alpha}_g}{(496 - k^2) - (D^{(4)} = 4)} \tag{15}$$

where $\bar{\alpha}_g = 42 + 2k$ is the non-super symmetric grand unification coupling constant [28] [29] [37]. Consequently one finds

$$\begin{aligned} \lambda &= \sqrt{\frac{(496 - k^2) - (47 + 2k)}{496 - k^2 - 4}} = \sqrt{449 - k^2 - 2k} / \sqrt{492 - k^2} \\ &= 5\phi^2/2 \cong 21/22. \end{aligned} \tag{16}$$

where we have taken the positive root only as a start. That way we see that $\bar{\alpha}_o$ and $(4-k)$ are related by the scaling $(\bar{\alpha}_o/2)(\phi)^6 = 4-k$ while by contrast $496-k^2$ is not from the same series and the coupling term enters into the calculation via the inverse non-super symmetric "GUT" quantum gravity coupling $\bar{\alpha}_g = 42 + 2k$ [37] [63]. Next we look at the derivation a last time from the viewpoint of the standard model and the model of Witten's 5 Brane model in eleven dimensions.

We recall that the number of states in Witten's model are given by [28] [29]

$$N = \binom{11}{1} + \binom{11}{2} + \binom{11}{5} = 11 + 55 + 462 = 66 + 462 = 528. \tag{17}$$

Furthermore this model represents a maximally symmetric manifold [28] [29] [44] because

$$K_N^{(32)} = (32)(32+1)/2 = 528 = N. \tag{18}$$

Similarly we know from the Heterotic superstring theory that the number corresponding to 66 in Witten's model is 63 and that $(2)(33) = 126$ is exactly the total number of the particle-like states of the standard model. The corresponding number for Witten's model would consequently be $(2)(66) = 132$. This is thus the dimension of the modular manifold $M_{6,22}$ discussed earlier. From the above we see that the scaling factor λ for dark energy λ (Dark) must be simply the ratio of the standard model to the maximally symmetric Witten model provided we trust both models to represent a substantial part of what nature is. Consequently we have

$$\gamma(\text{Dark}) = \frac{126}{132} = 63/66 = 0.954545 = 21/22. \quad (19)$$

which is the exact integer approximation of cosmic dark energy density and as mentioned repeatedly, is very close to all cosmological measurements and observations while the ordinary cosmic energy density is clearly the ratio of $66 - 63 = 3$ to the 66 total which means

$$\gamma(0) = \frac{3}{66} = 1/22 \quad (20)$$

as should be [53]-[60]. This last expression is amiable to a simple physical interpretation when Yang-Mills theory and Witten's model are combined. The number of one and two dimensional Branes making the Witten model is clearly our 66 but there are 4 photons within this expression, namely the classical photon plus the massive photon W^+ , W^- and Z^0 . Einstein's equation $E = mc^2$ has taken care of the ordinary photon so that 3 photons are not accounted for. On the other hand our 66 are part of the 528 massless gauge bosons which in Witten's theory play the role of $|E8E8| = 496$ of superstrings. Thus at the energy level of cosmological measurements, the 3 missing "photons" relate to the 66 via an ordinary energy scaling [56]

$$\lambda(0) = \frac{3}{66} = 1/22 \quad (21)$$

as reasoned earlier. Writing $\lambda(0)$ in a super symmetric form in eight dimensional super space nothing changes because [56]

$$\lambda(0) = \frac{(8)3}{(8)66} = \frac{24}{528} \quad (22)$$

and

$$\lambda(\text{Dark}) = \frac{528 - 504}{528} = \frac{24}{528} \quad (23)$$

are two results obtained a few years ago [24] [42] [60] within a slightly different scenario. However the surprising wonder which is not a wonder is that ordinary energy is the position or potential energy of the quantum particle while dark energy is kinetic energy of the quantum wave but both are nothing more or less than the zero set and the empty set of pure mathematics respectively [60]. The problem is incredibly simple when seen as pure mathematics [61]-[63].

4. Conclusion

By taking internal and external isometries of spacetime carefully into consideration [16] [48] and comparing the standard model of high energy particles with the maximally symmetric spacetime of Witten's 5-Brane in eleven dimensions, both dark energy and ordinary energy densities of the cosmos were determined [63]. The result agrees completely with previously obtained results as well as actual cosmological measurements and observations [30]. The mathematical basis of the present analysis is rooted in the theoretical structure of E-infinity theory of highly structured rings [12] [13] of which E-infinity theory of high energy physics and loop quantum gravity are in the meantime two well known applications. This was accomplished in part by integrating number theory, the queen of mathematical science and consequently of science in physics [10]. We did this before for mechanics and produced the new and vibrant field of nonlinear dynamics and its associated numerical experiments pioneered by leading researchers such as M. Feigenbaum, M. Berry, the late Russian academician B. Chirikov, the Italian quantum chaos physicist G. Casati and many others [12]. However in the present work we went

even one step further by developing a kind of new computational system based on an imaginary Turing-like golden mean computer operating based on a golden mean “binary” system [12] [37]. In view of what we have said in this rather short paper on a vast field, the great paper of Wigner is essentially about the very “reasonable” effectiveness of pure mathematics [60]-[63].

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