

Penrose Transform on Induced $D_{G/H}$ -Modules and Their Moduli Stacks in the Field Theory*

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Received December 14, 2012; revised January 15, 2013; accepted January 28, 2013

ABSTRACT

We consider generalizations of the Radon-Schmid transform on coherent $D_{G/H}$ -Modules, with the intention of obtaining the equivalence between geometric objects (vector bundles) and algebraic objects (D -Modules) characterizing conformal classes in the space-time that determine a space moduli [1] on coherent sheaves for the securing solutions in field theory [2]. In a major context, elements of derived categories like D -branes and heterotic strings are considered, and using the geometric Langlands program, a moduli space is obtained of equivalence between certain geometrical pictures (non-conformal world sheets [3]) and physical stacks (derived sheaves), that establishes equivalence between certain theories of super symmetries of field of a Penrose transform that generalizes the implications given by the Langlands program. With it we obtain extensions of a cohomology of integrals for a major class of field equations to corresponding Hecke category.

Keywords: Penrose Transform; Coherent G -Quasi-Equivariant D -Modules; Hecke Sheaf; Moduli Stacks; Moduli Spaces

1. Introduction

We have formulated in the language of the D -modules and their sheaves, like the given in a resolution [APM] $\mathcal{O}_{\mathbb{P}}^{\circ}(h) \rightarrow \dots \rightarrow \mathcal{O}_{\mathbb{P}}^i(h) \rightarrow \mathcal{O}_{\mathbb{P}}^{i+1}(h) \rightarrow \dots \rightarrow 0$, the correspondence between the space of coherent D -modules and the space of equations of massless fields $\square_h \phi = 0$, which can be established if we can grantee the uniqueness of the Penrose transform. This at least, have stayed guarantied of a partial form to conformal class obtained by the classical Penrose transform [4]

$$H^1(U, \mathcal{O}_{\mathbb{P}}(k)) \cong \ker(U, \square_{h(k)}), \quad (1)$$

that in the $D_{\mathbb{M}}$ -modules categories, and to the functors $\Phi_{\mathcal{X}}^0 F$, and $G\Psi_{\mathcal{X}}^0$, is translated in

$$\text{Mod}_{coh}(D\mathcal{L}) \begin{array}{c} \xrightarrow{\Phi_{\mathcal{X}}^0 F} \\ \xleftarrow{G\Psi_{\mathcal{X}}^0} \end{array} \text{Mod}_{coh}(V, \mathcal{O}_Y), \quad (2)$$

where $\text{Mod}_{coh}(D\mathcal{L})$, is the full subcategory of $\text{Mod}_{coh}(D_X)$, whose open sets are of the $D\mathcal{L}$, type to some bundle of lines L .

Precisely these equivalences shape a classification given of the homogeneous vector bundles of lines [2,5] to differential operator classification [1].

*Postdoctoral research supported by COMECYT (Council of Research of the State of Mexico).

Some important facts that demonstrate by means of the use of the Radon transform and their version of D -Module transform (Radon-Schmid transform), is the necessity of include a result that establishes the regularity in the analytical sense of the Riemannian manifold, which shapes the space-time, and that allows the application of the involutive distribution theorem on integral submanifolds as solutions of the corresponding equations of massless fields on submanifolds isomorphic in the Kählerian model inside of the *Flat model* given on $G_{2,4}(\mathbb{C})$.

Of fact, an analogy in the obtaining of models of space-time (*under the same reasoning*) must be realized between *special Lagrangian submanifolds* and m -folds of Calabi-Yau. But to it, we need define the complex micro-local structure that define all the phenomena of strings and branes in microscopic level, which happen in the 6-dimensional component of the universe (*6-dimensional compact Riemannian manifold*) with ratio of the order 10^{-33} cm (Max Plank longitude of a string) [1,6]. The *Penrose transform* is an integral geometric method that interprets elements of various analytic cohomology groups on open subsets of complex projective 3-space as solutions of linear differential equations on the Grassmannian of 2-planes in the 4-space. The motivation for such transform comes from the interpretation of this Grassmannian as the complexification of the conformal

compactification of the Minkowski space and their differential equations being the massless field equations of various helicities $h(k) = (1 + k/2), \forall k \in \mathbb{Z}$.

Nevertheless: what is the global notion of the equivalence created through the Radon-Schmid transform for the general case of classes not necessary conformal? Which are the classes that are extensions of the space of equivalences of the type (1)? Why are necessary these classes to include more phenomena of the space-time? What version of the Penrose transform will be required?

Part of the goal of our research is centered in the extension of the space of equivalences of the type (1), under a more general context given through the language of the D -modules, that is to say, we want extend our research al context of non-conformal manifolds and with it to find some condition at least of sufficiency for the resolution via the generalized Penrose transform of D_M -Modules, which are D_P -Modules and to induce their application to para-conformal classes through of the use of versions more generalized of Radon transforms that are connected with Penrose transforms to create conformal invariance in geometrical images that are physical stacks at least in the first development and tries to obtain an integral operators cohomology that be a sheaf cohomology group that includes the BRST-cohomology studied in field theory and where be applicable the second scheme of gauge theories (topological gauge theory [7]). The topological gauge theory search use the deformation theory in derived sheaves that not requires use excessive functor calculations. In this sense, the most direct and shortest applications of deformation theory are to the study of moduli spaces (remember that we want to obtain equivalences between categories of objects that are D -modules in different scales inside of field theory from a point of view of bundle of lines, worldsheets from \mathbb{P}^{1s} , and all space time \mathbb{M} (for example D -branes and strings)). If we have a pre-deformation functor Φ , obtained from a point x , of a moduli space M , then as was mentioned in [8,9], the notions of tangent space and formal smoothness for M , and for Φ , coincide. Noting that if a manifold is smooth, their dimension is the dimension of their tangent space, one could prove in this way that the Grassmannian $G_{r,d}(\mathbb{C})$, is smooth of dimension $r(d-r)$, although of course a proof of representability gave inside of the deformation theory a more direct and stronger statement in showing that it is covered by copies of $\mathbb{A}_k^{r(d-r)}$. However, later we will study other fine moduli spaces, such as Hilbert spaces, where in many cases deformation theory is the only way of obtaining such statements. Along these lines, one of the archetypical examples is the moduli space M_g , of curves of genus g , which is proposed inside of moduli spaces that appear in the Fukaya category Fuk

$(M_{\text{curve}}(X))$ [14]. This parametrizes flat families of smooth, proper, geometrically connected curves of genus g . Although it is not representable, it is nonetheless rather well-behaved for $g \geq 2$; technically, it is a ‘‘Deligne-Mumford stack’’, but from our point of view, it suffices to take for granted that the notions of smoothness and dimension can be defined independently, and in this sense there are many results, proved via deformation theory [8,10].

Until this point, can establish a correspondence of the genus as geometric images with elements of field theory? Can a space-time carpet be created from the strings tacked and D -branes, like D_P -modules to the creation of D_M -modules that cover all \mathbb{M} ? Which must be these D_P -modules? Are there some space of geometric invariant equivalences that that is adapted to the whole standard model of the space-time, doing that derived categories are applicable to the SUSY theory and QFT theory of same form, that is to say, to a transitive class of moduli spaces in the space \mathbb{P}^3 ?

These and many other aspects can be answered if we generalize functors between categories of coherent D -Modules in equivariant D -modules to sub-classes of the homogeneous space G/H , of G -equivariant D -modules, where the functors of the transform of D -modules are the obtained through the Penrose-Radon transform with close range on D -modules that are certain generalized g -modules [13,28], in the context of the infinite Lie theory, and must be Harish-Chandra funtores of derived categories [20], in these modules. This latter functorial application must record in eigen-sheaves de categorías de Hecke.

2. Revisited Cycles and Equivalences (in the Mirror Symmetry)

Now in the context of the generalized D -modules (inside of the derived categories) to the use of the Schmid-Radon transform, and finally obtain Radon-Penrose Transform, the functor [2]

$$\Phi_S + \text{additional geometrical functor} \tag{3}$$

establish a equivalence between the category $M(D_X\text{-modules/flat connection})$, and $M(D_Y\text{-modules/Singularities to along of the involutive manifold } V/\text{flat connection})$ [4,11]. Then our moduli space that we constructed is the categorization of equivalences:

$$M_{d[S/Y]} = \{M(D_Y\text{-modules / Singularities to along of the involutive manifold } V / \text{flat co-nnection}) \rightleftharpoons H^*(U, \mathcal{O})\}, \tag{4}$$

considering the moduli space as base [4],

$$\mathbf{M}_{d[S/X]} = \left\{ M(D_X\text{-modules } V / \text{flat co-nnection}) \rightleftharpoons H^*(U, \square_{h(k)}) \right\}, \quad (5)$$

The moduli space (5) and under certain hypothesis of the twisted (0, 2)-sigma model whose path integral localizes on holomorphic maps to twistor space [23], is the corresponding flat moduli space

$\mathbf{M}_{\text{Flat}}(\mathbb{L}G, C)|_{M_{\text{good}}(D_X)}$, [6], which satisfies under special conditions of compactification that (considering the formula to helicities when there is holes of genus g , in a Kähler manifold \mathbb{M} ,

$$d = g - 1 + \sum_{k=1}^{k=1} \frac{h+1}{2} n_k, \text{ with } n = 0: \quad (6)$$

$$\mathbf{M}_{\text{Flat}}(\mathbb{L}G, C)|_{M_{\text{good}}(D_X)} = \mathbf{M}_{g,0}(\mathbb{P}^3, d) \cong \mathbb{P}^{3^4},$$

then the cohomology on moduli spaces is the cohomology of the space-time with an equivalence like the given in (1), to a more general cohomological group that the given by $H^1(U, \mathcal{O}_{\mathbb{P}}(k))$, and whose dimension of $H^*(U, \mathcal{O})$, can be calculated by the intersection methods, to a complex sheaf \mathcal{O} [23], which is of the type $\mathcal{O}_{\mathbb{P}}^i(k)$, (for example: elements of the cohomology group $H^1(\mathbb{P}T, \mathcal{O}(-2h-2))$,¹ correspond via the Penrose transform to the spacetime fields of helicity h , so that in particular a negative helicity gluon corresponds to a twistor wavefunction of weight 0. The additional geometrical hypothesis in the functor (3), comes established by the geometrical duality of Langlands [5], which says that the derived category of coherent sheaves on a moduli space $\mathbf{M}_{\text{flat}}(\mathbb{L}G, C)$, where C , is the complex given by

$$C: \dots \xrightarrow{d^{j-1}} \mathcal{E}^{j-1} \xrightarrow{d^j} \mathcal{E}^j \xrightarrow{d^{j+1}} \mathcal{E}^{j+1} \xrightarrow{d^{j+2}} \mathcal{E}^{j+2} \xrightarrow{d^{j+3}} \dots \quad (7)$$

which is equivalent to the derived category of D -modules on the moduli space of holomorphic vector G -bundles given by $B_G(C)$ [6]. These equivalences permit to map points of $\mathbf{M}_{\text{flat}}(\mathbb{L}G, C)$, to eigen-sheaf of Hecke given by $B_G(C)$ [23]. The complex C , is an infinite dimensional $\bar{\partial}$ -complex. Then we need to establish a local

¹One example of these moduli spaces is

$\mathbf{M}_{0,0}(\mathbb{P}^3, 1) \cong G_{2,4}(\mathbb{C})$. The equivalences in this case are obtained by the classic Penrose transform

$$\begin{aligned} \mathcal{P}: H^1(\mathbb{P}^3, \mathcal{O}(-2)) \\ \cong \left\{ \phi \in C^2(U) \mid \nabla^2 \phi = 0, \text{ in } U \subset \mathbb{M} \cong S^4 \right\} = \ker(U, \nabla^2 \phi), \end{aligned}$$

where $\mathcal{O}(-2)$, is a homogeneous holomorphic line bundle on \mathbb{P}^3 . In this case the complex C , is a Dolbeault complex that is an elliptic complex on U [30].

cohomology to coherent D -modules \mathcal{E}^j , to every reduce analytic subspace of \mathbb{M} . In other words, to find the local cohomology that is intrinsic part of the cohomology of (7). We will do the following extension of $X \subset \mathbb{M}$.

Considering the filtration $M = X_0 \supset X_1 \supset X_2 \supset \dots$, of closed subspaces of a Riemannian complex manifold \mathbb{M} , which defines the topology of \mathbb{M} . Exactly like an open covering of \mathbb{M} , this filtration associates to every sheaf \mathcal{D}_X , their Čech complex, where the filtration persists associating to every sheaf a Cousin complex C^* . The required cohomology is on C^* . This cohomology is on sheaves of $D_{\mathbb{M}}$ -modules that are $D_{\mathbb{F}}$ -modules. The $D_{\mathbb{F}}$ -modules there must be the given in the category of (4), with the due meromorphic extensions of one Cousin-Dolbeault-complex to the inclusion of involutive manifolds with singularities [16].

We consider the following fundamental results to the securing of the Hecke sheaves using local cohomology in the Cousin-Dolbeault-complex.

Theorem 2.1. 1) *The local algebraic cohomology is a complex with D -holonomic cohomology;* 2) *We have the canonical morphism in $D(C_X)$*

$$R\text{Hom}_{\mathcal{D}}(R\Gamma_{[Y]}(\mathcal{O}_X); \mathcal{O}_X) \cong C_Y, \quad (8)$$

Proof. [24].

By the kashiwara theorem, we have that the equivalence between the categories $D(C_X)$, and $D(C_Y)$, preserves *coherency* and *holonomicity* [3]. Then preserve *conformability* in \mathbb{M} [12,13].

In this case we want to decompose singular regions $V \subset T^*X$, of the space-time \mathbb{M} , in regions of conformal nature that could be valued through contour integrals, whose cohomology of contours could be related to the cohomology of the total Riemannian complex manifold maintaining the conformability of these micro-components. Then for micro-local analysis, we can observe flatness conditions of the space to scales of the *QFT*, and the singularities that shape V , there are zeros of polynomials of algebraic manifold of the Riemannian complex variety.

In case of the Cousin complexes, the idea is to decompose a complex with non-conformal properties in conformal parts where it exists flatness due to the holonomicity and conformability of \mathbb{M} , to these scales. A promissory result in this sense is the due to Kashiwara applying jointly the Poincaré theorem (*Theorem 2.1*) in local cohomology, where for the resolution proposed in [13], we can investigate in the cohomology of \mathbb{P}^3 , through the moduli space $\mathbf{M}_{g,0}(\mathbb{P}^3, d) \cong \mathbb{P}^{3^4}$.

Then from the depth and substantial of the complex (7) the conformability is maintained by the Cousin complexes that shape quotients of complexes to every component of the complex (7). The natural morphism

$$R\Gamma_{[Y]}(\mathcal{O}_X) \rightarrow R\Gamma_Y \mathcal{O}_X, \tag{9}$$

induces a morphism in $D(\mathcal{D}^\infty)$, that is to say,

$$\mathcal{D}^\infty \otimes_D^L \Gamma_{[Y]}(\mathcal{O}_X) = \mathcal{D}^\infty \otimes_D \Gamma_{[Y]}(\mathcal{O}_X) \rightarrow R\Gamma_Y(\mathcal{O}_X), \tag{10}$$

Theorem 2.2. *The morphism (*) is an isomorphism in and the local cohomology sheaves of Y , are \mathcal{D}^∞ -holonomic and admissible modules.*

Proof. [21,24]. \square

To relate the cohomological groups of the sheaves of $D(C_X)$, with the sheaves of $D(C_Y)$, we use an intertwining operator that compound with the Penrose transform, allows determine the solution space of the differential equations of the operators given for the equivalence of the moduli space (4). In the following section we will use like intertwining operator the twistor transform, which establishes an isomorphism of Harish-Chandra modules.

Then our sheaves complex takes the form as a holomorphic bundles complex and is aquanstance as to this can be associated branes. In this case we associate branes to every possible sheaf: we can map branes to sheaves but the reciprocal thing is not true.

3. Methods: Equivalences of Cycles in Quasi- G -Equivariant D -Modules Category and Their Ramifications in Derived Sheaves

To obtain a generalization of the Kashiwara theorem in the context of the G -equivariant D -modules is necessary the geometric Zuckerman functor. This is the localization of the equivariant Zuckerman functor to the derived equivariant D -module categories on generalized flag varieties (this to apply in our study an appropriate D -module transform of the Penrose transform type). To this case, we define our categories of interest and construct the geometric Zuckerman functor from the basic D -module functors.

The category of (left) G -equivariant \mathcal{O}_X -modules is denoted by $M_G(X)$. The following theorem is well known:

Theorem 3.1. If G , acts on X , freely, there is an equivalence of categories $M_G(X) \cong M(X/G)$.

For homogeneous spaces, we can make a stronger statement. Let B^C , be a complex linear group, let $Rep(B^C)$, be the category of algebraic representations of B^C .

Proof. [20]. \square

Theorem 3.2. If $X = G/B^C$, there is an equivalence of categories $M_G(X) \cong Rep(B^C)$.

Proof. [20]. \square

We consider the Penrose transform between flag manifolds on an algebraic complex group G^c , using the

Kashiwara theorem between quasi- G -equivariant $D_{G/H}$ -modules and some kind representation spaces, which are (\mathfrak{g}, H) -modules (complex vector space endowed always with an action of a Lie algebra \mathfrak{g} , associated to G , and an action H , which is compatible in some sense when H , is a closed algebraic subgroup of G).

Let \mathcal{M}_G , be a quasi- G -equivariant $D_{G/H}$ -module belonging to the subcategory of $D_{G/H}$ -modules given by $Mod_{good}(\mathcal{D}_{G/H})$, (category of the G -equivariant $D_{G/H}$ -modules).

The idea is to obtain equivalence between the different classes of $D_{G/H}$ -modules establishing the isomorphism between the categories $M(\mathcal{M}_G) = Mod_{RS(v)}(\mathcal{D}_{G/H})$, and $M_G = Mod_{good}(\mathcal{D}_{G/H})$, where $RS(v)$, must be the functor of Zuckerman restricted to the subcategory of left G -equivariant $D_{G/H}$ -modules to obtain the derived category that is a product of a triangulated subcategory (like the given in [1,6]) for a factor category that determine us the functor of Harish-Chandra between categories. These equivalences in analogy to established in (4) take the form for specific $D_{G/H}$ -modules

$$\begin{aligned} & \mathbf{M}_{G/H}^G \\ &= \{M_G(\mathcal{D}_{G/H})\} \\ &= \{\text{derived category of } D_{G/H}\text{-modules } G\text{-equivariants}\} \\ &\Leftrightarrow M_G(\mathfrak{g}, H) \cong M_G(\mathcal{M}_G), \end{aligned} \tag{11}$$

considering the moduli space as base,

$$\begin{aligned} & \mathbf{M}_{G/H}^{equivG} = \\ & \{M(D_{G/H}\text{-modules } G\text{-equivariants}) \Leftrightarrow \ker(U, \mathcal{Q}_{BRST})\}, \end{aligned} \tag{12}$$

where $\mathcal{Q}_{BRST} = \bar{\partial} + \Phi_1^i \theta_i - \Phi_2^i \theta_i$, such that $\mathcal{Q}_{BRST}^2 \phi = 0$, where Φ_1^i , and Φ_2^i , are Higgs fields on either side of the open string [23]. This condition established in the kernel of equivalences inside the moduli space $\mathbf{M}_{G/H}^{equivG}$, is analogous to the given by the isomorphism of the Penrose transform discussed in foot of page 2, in the Section 2, in this work. The idea is extend the harmonic condition to the functions to differential operators of derived sheaves that are in BRST-cohomology, such that the equivalences in (5) are defined by the certain functors due to the duality between the BGG-resolution and meromorphic version of the Cousin complex² associated to \mathcal{L}_λ (bundle of lines associated to the flag manifold $X = G/H$, then H , is a Borel subgroup).

Let $Mod_{RS(v)}(\mathcal{D}_{G/H})$, be the thick subcategory of

²Cousin Dolbeault theory refers to the complex Dolbeault infinite dimension.

$Mod_{\text{good}}(\mathcal{D}_{G/H})$, whose objects have regular singularities on V , and by $D_{RS(V)}^b(\mathcal{D}_{G/H})$, the full triangulated subcategory of $D_{\text{good}}^b(\mathcal{D}_{G/H})$, whose objects have cohomology groups belonging to $Mod_{RS(V)}(\mathcal{D}_{G/H})$ [6]:

$$Mod_{\text{good}}(\mathcal{D}_{G/H}) \supset Mod_{RS(V)}(\mathcal{D}_{G/H}) \leftarrow D_{\text{good}}^b(\mathcal{D}_{G/H})$$

$$\begin{array}{ccc} \nwarrow & & \nearrow \\ & D_{RS(V)}^b(\mathcal{D}_{G/H}) & \end{array}$$

Let (\mathfrak{g}, H) , be a Harish-Chandrs pair, let X , be a generalized flag variety for \mathfrak{g} , and let \mathcal{D}_λ , be a homogeneous twisted sheaf of differential operators on X . A weak Harish-Chandra sheaf for the pair (\mathcal{D}_λ, H) , is a quasi-coherent \mathcal{D}_λ -module \mathcal{M} , with a H -invariant \mathcal{O}_X -module structure such that the action of \mathcal{D}_λ , is H -equivariant. A weak Harish-Chandra sheaf is a Harish-Chandra sheaf if additionally the differential of the H -action on M , agrees with the action of \hat{t} , induced by \mathcal{D}_λ .

A morphism of weak Harish-Chandra sheaves is a \mathcal{D}_λ -module homomorphism which respects the underlying H -equivariant structure. As with weakly equivariant Harish-Chandra modules, we will use $Mod_\omega(\mathcal{D}_\lambda, H)$, to denote the category of weak Harish-Chandra sheaves and $Mod(\mathcal{D}_\lambda, H)$, for the category of Harish-Chandra sheaves. There is an equivalence of categories for λ , anti-dominant and regular

$$Mod_\omega(\mathcal{D}_\lambda, H) \xrightarrow{\cong} Mod_\omega(\Gamma(X, \mathcal{D}_\lambda), H), \quad (13)$$

We construct the derived equivariant Harish-Chandra sheaf category in the same way as the derived equivariant Harish-Chandra module category. The derived equivariant Harish-Chandra module category is precisely that we want obtain to consider it inside the base moduli space to determine an extension of this one, in the context of a cousin cohomology.

Theorem 3.3. The morphism (sfc) is an isomorphism in and the local cohomology sheaves of Y , are D^∞ -holonomic and admissible modules.

Proof. [24]. \square

Here the local system on the complex manifold is given by the $D_{\mathfrak{p}}$ -modules inside one Dolbeault-Cousin cohomology that shape a complex as the given in the Morphism (9).

Theorem (Deligne) 3.4. The functor DR , gives a categorical equivalence between the category of regular connections on an algebraic manifold X , and that of local systems on the complex manifold \mathbb{X} .

Proof. [15]. \square

4. Results

Theorem 4.1 (F. Bulnes). The derived category of quasi- G -equivariant $D_{G/H}$ -modules can be identify for twisted

Hecke category if \mathcal{H}_G , is a derived version of the category of Harish-Chandra to a certain $\lambda \in \mathfrak{h}_\lambda^*$.

Proof. If $\mathcal{H}_G = M(D_\lambda^{G/H}) \forall \lambda \in \mathfrak{h}_\lambda^*$ then $\mathcal{H}_G = M(B \backslash G/B)$, of the group $G = G^c$. B -equivariant D -module on the flag manifold $X = G/B$, provide integral kernels and thus integral transforms, to know

$$H^0(X, \mathcal{L}_\lambda) \cong \ker(U^-, \mathcal{Q}_{\text{BRST}}),^3 \quad (14)$$

For other side, realizing the following considerations to the extension of the Langlands geometrical program, the category $M(B \backslash G/B)$ of D -modules is the double quotient stack of a complex reductive group G , by a Borel subgroup $B \subset G$, along with familiar related versions involving twisted D -modules. Our aim is to construct a topological field theory from $M(B \backslash G/B)$, and in particular to relate the abelianization and center of $M(B \backslash G/B)$ to each other and in turn to character sheaves. These character sheaves are the Hecke eigen-sheaves required.

The Hecke category $\mathcal{H}_G = \mathcal{H}_{G,0}$, is a fiber of a natural family of Hecke categories $\mathcal{H}_{G,\lambda}$, labeled by infinitesimal character $\lambda \in \mathfrak{h}^V$, in the dual of the Cartan Lie algebra. The twisted Hecke category $\mathcal{H}_{G,\lambda}$, consists of the natural integral transforms acting on categories of λ -twisted D -modules on the flag variety G/B . (For λ regular, such categories are equivalent to categories of \mathfrak{g} -modules with infinitesimal character λ , modulo W , (Harish-Chandra categories)). Concretely, we can realize $\mathcal{H}_{G,\lambda}$, as the monoidal category of D -modules on $N \backslash G/N$ with strict $(-\lambda, \lambda)$ -monodromy with respect to the left and right actions of $H = B/N$. (For λ , regular, such D -modules are equivalent to Harish

³Our demonstration will consist of the fact that each of these subspaces of the complexes Cousin must be Hecke eigen-sheaves [14]. The sum of Hecke eigen-sheaves is the Hecke category in our result, which is twisted, since there are images of isomorphisms of Harish-Chandra modules (that is to say, in a category of Harish-Chandra whose D -modules are $D_{\mathfrak{p}}$ -modules) created via the Penrose transform on twisted modules. Here the property of being twisted is demonstrated by the argument of the Penrose transform that involve to the twistor transform in field theory.

Then the twistor transform followed from the Penrose transform $\mathcal{P}(\mathcal{T})$, will calculate the kernels of the cohomological groups that are isomorphisms in the sense of the equivalence of the Kashiwara theorem. Nevertheless, we need a version of eigensheaves that include connections with regular singularities that could be extended through a pull-back of certain bundle of lines on $\text{Bun}G$ [6,23], to the irregular connections that there need no to be Verma modules, as it is mentioned in the Recillas conjecture [1]. In fact, the implications to that the above mentioned conjecture is valid, is the fact of considering regular connections to be the established for the Deligne theorem mentioned in the previous section, which is not clear when is want to extend them in the diverse ramifications of Langlands geometrical program [27,31].

Chandra bi-modules for \mathfrak{g} , with infinitesimal character $(-\lambda, \lambda)$ -modulo $W \times W$).

Taking the fibers $\mathcal{H}_{G,\lambda}$, of the above family of Hecke categories, we can allow the parameter $\lambda \in \mathfrak{h}^V$, to vary infinitesimally, and obtain the monodromic Hecke categories $\mathcal{H}_{G,\lambda}$, of natural integral transforms acting on categories of generalized λ -monodromic D -modules on the flag variety G/B . (For λ , regular, such categories are equivalent to categories of \mathfrak{g} -modules with generalized infinitesimal character λ , modulo W). Remember that there are basic integral transforms associated to pairs of flags in a fixed relative position indexed by an element of the Weyl group of G . (These are closely related to the Radon and Penrose transforms of algebraic analysis and with analogous in field theory by the twistor transform (this was established in the Gindikin conjecture [6])).

We consider a complex holomorphic bundle $\text{Bun}_G(C)$, on a hypersurface Σ , and that is a derived category of quasi-coherent D -modules $\mathcal{D}(\text{Bun}_G(\Sigma))$ (that is to say, we have applied the direct Penrose transform to the bundle $\text{Bun}_G(\Sigma)$ [6]) which is isomorphic to the category $\mathcal{D}(H_{G,\lambda})$, to some $\lambda \in \mathfrak{h}_e^*$, of fact, $\mathcal{D}(H_{G,\lambda}) \cong H_{G^{\wedge},0}$. But for Langlands geometrical duality, whose equivalence that shape a moduli stack in the character category context, takes the form

$$\chi_G(\Sigma) = \chi_{G^{\wedge}}(\Sigma), \quad (15)$$

But this means that in duality quasi-coherent versions of the affine Hecke category are images of the Penrose transform applied to D -modules on finite flag manifolds (that is to say, in this context our D -modules are $D_{\mathbb{F}}$ -modules). Thus $\mathcal{H}_{G^{\wedge}}$ and $\mathcal{H}_{G^{\wedge},0}$, lead to D -modules on finite flag varieties⁴.

Newly, consider the Hecke category $\mathcal{H}_{G^{\wedge},\lambda}$. Also consider $H_{G,0}(\Sigma)$, with Σ , a surface in \mathbb{M}^* . Said category assign a Riemannian surface S , the ∞ -category of D -modules given by $\mathcal{D}(\text{Bun}_G(\Sigma))$ on moduli stack of G -bundles, while that $H_{G^{\wedge},\lambda}$ assigns the ∞ -categories of quasi-coherent sheaves on the stack $\text{Loc}_{G^{\wedge}}$ of G^{\wedge} -local systems of Σ , denoted also by ${}^L G$. Then by the theorem given in [6], and by relative discussion to the eigen-object that stays to the apply de MO-duality [23], to A-brane on moduli space $\mathbf{M}_H(G,C)$, we have that

⁴This last, can be interpreted through the field theory like the flatness that characterize the background radiation in the space-time producing the energy support of all variations of strings and heterotic strings that happen inside the deep space-time. To obtain interpretations in the frame of the topological gauge theories is necessary use the dual image of orbit space like orbifolds from the Calabi-Yau manifold that establishes relations between bundles of forms of fields on orbit \mathbb{M}^* , via a enclosed transform with Penrose transform. In this sense the \mathcal{E}_2 -resolution is given by the Penrose transform.

the D -module on $\text{Bun}_G(C)$, corresponding to a point on $M_{\text{flat}}({}^L G, C)$, is Hecke eigen-sheaf. Then in this point, we can affirm that $\mathcal{D}(\text{Bun}_G(\Sigma))$, is the category of the twisted Hecke category $\mathcal{H}_{G,\lambda}$, consists of the natural integral transforms acting on categories of λ -twisted D -modules on the flag variety G/B . Indeed, is necessary go from the co-cycles for the Radon-Penrose transform on $\text{Mod}_{\text{good}}(\mathcal{D}_{G/H})$ (e.g. $\mathcal{H}_{G,\infty}$) to the obtained cycles by Penrose transform on $\text{Mod}_{RS(V)}(\mathcal{D}_{G/H})$, and viceversa. The equivalence of both spaces of co-cycles demonstrates that the functors are the given by $\text{Hom}(\text{Mod}_{\text{good}}(\mathcal{D}_{G/H}), \text{Mod}_{RS(V)}(\mathcal{D}_{G/H}))$, of the moduli space $\mathbf{M}_{G/H}G = \{M_G(\mathcal{D}_{G/H}) \rightleftarrows M_G(\mathfrak{g}, H)\}$, which consider as base the moduli space given in (5).

Then these functors are G -invariants and their image under $\text{Fun}_{\mathcal{D}(G/H)}$, is recorded in a moduli space of holomorphic bundles $\mathcal{D}(\text{Bund}_G(C))$, [27]⁵ which is an extension of the transformed cycles by the classic Penrose transform [17] The equivalence under the G -invariance of holomorphic bundles must be demonstrated using a generalized Penrose transform for D -modules that is the composition of an inverse image functor and a direct image functor on the side D -module, which is foreseen by the geometrical duality of Langlands [5]. \square

Theorem 4.2. (F. Bulnes).

$$\mathbf{M}_{\text{quasi-}G\text{-equivariant}}(G/H, C^{\bullet}) \cong \mathbf{M}_{G\text{-equivariant}}(CY, C^{\bullet*}),$$

being $C^{\bullet*}$ a Cousin-Dolbeault complex

$$\begin{aligned} \mathbb{E}^{n,\bullet} \otimes H^p \text{RDO}_{[X_K/X_{K+1}]}(M)[k] \\ \cong \mathbb{E}^{n,\bullet} \otimes H^p(R(\mathcal{D}\mathcal{O}_{X_I}(\lambda))) \end{aligned}$$

Proof. We realize an extension by local cohomology of the complexes,

$C^{\bullet*} = \text{RHom}_{\mathcal{D}}(R\Gamma_{[Y]}(\mathcal{O}_X); \mathcal{O}_X) \cong C_Y$, using inverse and direct images of the Penrose transform on the corresponding Hochschild cohomologies obtained on the domains CY , an G/H [6], using their developments of Dolbeault-Cousin complexes[16] The bijection of the Penrose transform on orbifolds in both cases gives the result. \square

Corollary (F. Bulnes) 4.1. We consider the corresponding equivalence given by the *Theorem 7.1.* [6], and using the large resolution [1].

$$\mathbb{P}^3 \rightarrow \dots \rightarrow \mathbb{P}^{1+4d} \rightarrow \mathbb{P}^{2+4d} \rightarrow \mathbb{P}^{3+4d} \rightarrow \mathbb{P}^{4+4d} \rightarrow 0, \quad (16)$$

we can to give the following scheme of Cousin complexes $C^{\bullet}(k)$, that brings an equivalence between the spaces $\mathcal{L}^{4|6}$, and $\mathcal{P}^{2|6}$, where $\mathcal{P}^{2|6}$, is the supertwistor space $\mathbb{P}\mathbb{M}$, in [26]. In this case, the equivalence

⁵Using Penrose transform, we obtain the geometrical images of the vector bundles given by the category $\mathcal{D}(\text{Bun}_G(C))$.

is given by the Penrose transform on the quasi-coherent D_λ -modules established by the diagram (\cong to the Verma modules diagram in the conjecture Recillas to the group $SO(1, n+1)$ [6])

$$\begin{array}{ccccccc}
 & \mathcal{L}^{46} & & \mathbb{P}^3 & & & \\
 & \uparrow & & \uparrow \mathcal{M}(\mathbb{P}^3, 0) & & & \\
 \mathbb{P}^{1+4d} & \rightarrow & \mathbb{P}^{2+4d} & \rightarrow & \mathbb{P}^{3+4d} & \rightarrow & \mathbb{P}^{4+4d} \rightarrow 0 \quad (17) \\
 & \downarrow & & \downarrow & & & \\
 & \mathcal{P}^{246} & \rightarrow & \mathcal{P}^{246} & & &
 \end{array}$$

where $M(\mathbb{P}^3, d) \cong \mathbb{P}^{3+4d}$.

Proof. We consider the Cousin complex diagram

$$\begin{array}{ccc}
 C^\bullet & \hookrightarrow & A^\bullet[k] \\
 \pi \nearrow & & \nearrow \\
 A^\bullet & \rightarrow & B^\bullet \xrightarrow{\delta} C^{\bullet\sim}
 \end{array}$$

and applying to the exact succession

$$\mathbb{P}^{1+4d} \rightarrow \mathbb{P}^{2+4d} \rightarrow \mathbb{P}^{3+4d} \rightarrow \mathbb{P}^{4+4d} \rightarrow 0,$$

we find that (in the upper part)

$$\begin{array}{ccccccc}
 & \mathcal{L}^{46} & & \mathbb{P}^3 & \hookrightarrow & A^\bullet[k] \cong & \\
 & \uparrow & \nearrow & \uparrow \mathcal{M}(\mathbb{P}^3, 0) \rightarrow \mathcal{M}(\mathbb{P}^3, 1) & & & \\
 \mathbb{P}^{1+4d} & \rightarrow & \mathbb{P}^{2+4d} & \rightarrow & \mathbb{P}^{3+4d} & \rightarrow & \mathbb{P}^{4+4d} \rightarrow 0,
 \end{array}$$

is commutative. For other side, using the Penrose-Ward transform in the down part of the diagram we have

$$\mathbb{P}^{24} = O(2) \oplus \mathbb{C}^4 \otimes \Pi O(2) \rightarrow \mathbb{P}^1, [29] \quad (18)$$

But $\mathbb{P}^1 = \mathbb{P}^{1+4d}$, in the level of moduli space $M(\mathbb{P}^3, 0)$. To obtain the equivalence between the space \mathcal{P}^{24} , and \mathcal{L}^{46} , is necessary to consider the interpretation of the quadric inside the space $\mathcal{P}^{23} \times \mathcal{P}^{23}$, and construct the double fibration in the twistor scheme corresponding to the supertwistor space $\mathbb{P}\mathbb{M}$. This double fibration takes us to

$$0 \rightarrow \mathcal{L}^{46} \rightarrow \mathcal{P}^{23} \times \mathcal{P}^{23} \rightarrow O_{\mathbb{P}^1_\Delta}(2) \rightarrow 0, \quad (19)$$

which is not vector bundles fibration. Then is necessary the degeneration of cycles given inside of CY -manifold signed in the equivalences of moduli spaces in the *Theorem 7.1*. Using action given by path integrals given in [6], we can to obtain an approximation that can legitimize the extension of the Penrose transform in the linear context (integrals become traces of matrices) of the representations in $SO(1, n+1)$, and the use of strings and heterotic strings to obtain the commutability of all diagram. Details of the calculations will happen in a next

paper. \square

5. Conclusions

Spaces of equivalence modulus a relation of congruence were constructed on field solutions to establish a theory of the universe that includes the theory *QFT (Quantum Field theory)*, the *SUSY (Super-symmetry theory)* and *HST (heterotic string theory)* using the sheaves correspondence of differential operators of the field equations and sheaves of coherent D -Modules. The above mentioned correspondence was constructed using a Zuckerman functor that is a factor of the universal functor of derived sheaves of Harish-Chandra to the Langlands geometrical program in mirror symmetry. The obtained development included complexes of D -modules of infinite dimension, generalizing this way, the *BRST*-cohomology in this context. With it the class of the integrable systems was extended in mathematical physics and the possibility of obtaining a general theory of integral transform for the space-time, and with it the measurement of many of their observables. Also has tends a bridge to complete a classification of the differential operators for the different field equations using on the base of *Verma modules* that are classification spaces of $SO(1, n+1)$, where elements of the Lie algebra $\mathfrak{sl}(1, n+1)$, are differential operators, of the equations in mathematical physics.

The cosmological problem that exists is to reduce the number of field equations that are resolvable under the same gauge field (Verma modules) and to extend the gauge solutions to other fields using the topological groups symmetries that define their interactions.

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