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Fuzzy δ^*-Continuity and Fuzzy δ^{**}-Continuity on Fuzzy Topology on Fuzzy Sets

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ABSTRACT

The concept of a fuzzy topology on a fuzzy set has been introduced in [1]. The aim of this work is to introduce fuzzy δ^*-continuity and fuzzy δ^{**}-continuity in this in new situation and to show the relationships between fuzzy continuous functions where we confine our study to some of their types such as, fuzzy δ-continuity, fuzzy continuity, after presenting the definition of a fuzzy topology on a fuzzy set and giving some properties related to it.

Keywords: Fuzzy δ^*-Continuity; Fuzzy δ^{**}-Continuity; Quasi-Neighbourhood; Fuzzy δ-Open; Quasi-Coincident

1. Introduction

The concept of a fuzzy topology on a fuzzy set has been introduced by Chakrabarty and Ahsanullah [1]. Neighbourhood systems, quasi-neighbourhood system, subspaces of such fuzzy topology space and quasi-coincidence in this new situation have also been discussed by them. Also, the concepts of fuzzy continuity, Hausdorffness, regularity, normality, compactness, and connectedness have been introduced by Chaudhuri and Das [2]. The concepts of fuzzy δ-closed sets, fuzzy δ-open sets fuzzy regular open, fuzzy regular closed, fuzzy δ-continuity and the relation between fuzzy continuity and fuzzy δ-continuity in this new situation was introduced by Zahran [3]. These functions have been characterized and investigated mainly in light of the notions of quasi-neighborhood, quasi-coincidence. In our rummage we confined ourselves to the study of some kinds of these functions, the fuzzy continuous function, fuzzy δ-continuity and some types of fuzzy regular. In this paper, we introduce the concepts of a fuzzy δ^*-continuity, fuzzy δ^{**}-continuity and to show the relationships between types of fuzzy continuous functions in this situation and we examine the validity of the standard results.

2. Preliminaries

Let X and Y be sets and $\tilde{A}$ and $\tilde{E}$ be two subsets of X, Y respectively. Let I denote the closed unit interval $[0,1]$. Let $X=\{x1,x2,\cdots,xn\}$ and $ai \in I$ for $i=1,2,\cdots,n$ By $(a1,a2,\cdots,an)$ we shall mean the fuzzy subset $\tilde{A}$ of X and the value of a fuzzy set $\tilde{A}$ at some $x\in X$ will be denoted by $\mu\tilde{A}(x)$ such that $\mu\tilde{A}(xi)=ai$ for $i=1,2,\cdots,n$, and the support of a fuzzy set $\tilde{A}$ in X will be denoted by $S(\tilde{A})$ such that $\mu\tilde{A}(x)>0$ for all x in X. If $\tilde{A}$ and $\tilde{E}$ are fuzzy sets and $\mu\tilde{E}(x)\le\mu\tilde{A}(x)$ for all x in X, then $\tilde{E}$ is said to be a fuzzy subset of $\tilde{A}$ and denoted by $\tilde{E}\subseteq\tilde{A}$. The set of all fuzzy subsets of a nonempty set X is denoted by I^X.

Definition 2.1. [2] Let $x\in X$, $r\in I$. A fuzzy set $\tilde{A}$ of the form

$$\mu\tilde{A}(y)=\begin{cases}0 & \text{if } y\ne x\\ r & \text{if } y=x\end{cases}$$

is called a fuzzy point with support x and value r. $\tilde{A}$ is often denoted by χr.

For a fuzzy point χr

1) $\chi r\in 1\tilde{A}\Rightarrow r<\mu\tilde{A}(x)$.
2) $\chi r\in\tilde{A}\Rightarrow r\le\mu\tilde{A}(x)$.

Definition 2.2. [1] If $\tilde{E}\subseteq\tilde{A}$, the complement of $\tilde{E}'$ referred to $\tilde{A}$, denoted by $\tilde{E}'_{\tilde{A}}$ is defined by $\tilde{E}'_{\tilde{A}}(x)=\mu\tilde{A}(x)-\mu\tilde{E}(x)$, for each $x\in X$.

Definition 2.3. [2] $\tilde{U}$, $\tilde{V}\subseteq\tilde{A}$ are said to be quasi-coincident (q-coincident, for short) referred to $\tilde{A}$ written as $\tilde{U}q\tilde{V}[\tilde{A}]$ if there exists $x\in X$ such that $\mu\tilde{U}(x)+\mu\tilde{V}(x)>\mu\tilde{A}(x)$. If $\tilde{U}$ and $\tilde{V}$ is not quasi-coincident referred to $\tilde{A}$, we denoted for this by $\tilde{U}q\tilde{V}[\tilde{A}]$.

3. Basic Definitions and Properties

In [4,5] fuzzy function have been introduced in a different way considering them as fuzzy relations with special properties. A special kind of fuzzy functions had been called fuzzy proper functions or proper functions that would be the morphisms in the proposed category FUZZY TOP.

Definition 3.1. [1] A fuzzy subset $\tilde{\boldsymbol{F}}$ of $X \times Y$ is said to be a proper function from $\tilde{A}$ to $\tilde{E}$ if

1) $\tilde{\boldsymbol{F}}(x,y) \le \min\{\mu\tilde{A}(x), \mu\tilde{E}(y)\}$, for each $(x,y) \in X \times Y$.

2) For each $x \in X$, there exists a unique $y_0 \in Y$ such that $\tilde{F}(x,y_0) = \mu\tilde{A}(x)$ and $\tilde{F}(x,y) = 0$ if $y \ne y_0$.

Let $\tilde{\boldsymbol{F}}$ be a proper function from $\tilde{A}$ to $\tilde{E}$.

Definition 3.2. [1] If $\tilde{V} \subseteq \tilde{E}$, then $\tilde{\boldsymbol{F}}^{-1}(\tilde{V}) : \tilde{A} \to I$ is defined by

$$\left(\tilde{\boldsymbol{F}}^{-1}(\tilde{V})\right)(x) = \sup\left\{\min\left\{\tilde{\boldsymbol{F}}(x,y), \mu\tilde{V}(y)\right\}; y \in Y\right\}$$

for each $x \in X$.

Definition 3.3. [2] If $\tilde{U} \subseteq \tilde{A}$, then $\tilde{\boldsymbol{F}}(\tilde{U}) : \tilde{E} \to I$ is defined by

$$\left(\tilde{\boldsymbol{F}}(\tilde{U})\right)(y) = \sup\left\{\min\left\{\tilde{\boldsymbol{F}}(x,y), \mu\tilde{U}(x)\right\}; x \in X\right\}$$

for each $y \in Y$.

Proposition 3.4. [2] For a proper function $\tilde{\boldsymbol{F}}$

1) $\tilde{\boldsymbol{F}}\left(\tilde{\boldsymbol{F}}^{-1}(\tilde{O})\right) \subseteq \tilde{O}$, for each $\tilde{O} \subseteq \tilde{E}$.

2) $\tilde{\boldsymbol{F}}^{-1}\left(\tilde{\boldsymbol{F}}(\tilde{N})\right) \supseteq \tilde{N}$, for each $\tilde{N} \subseteq \tilde{A}$.

3) $\tilde{\boldsymbol{F}}^{-1}(\tilde{N} \cup \tilde{O}) = \tilde{\boldsymbol{F}}^{-1}(\tilde{N}) \cup \tilde{\boldsymbol{F}}^{-1}(\tilde{O})$ and $\tilde{\boldsymbol{F}}^{-1}(\tilde{N} \cap \tilde{O}) = \tilde{\boldsymbol{F}}^{-1}(\tilde{N}) \cap \tilde{\boldsymbol{F}}^{-1}(\tilde{O})$.

Definition 3.5. [2] $\tilde{E} \subseteq \tilde{A}$ is said to maximal if for each $\mu\tilde{E}(x) \ne o \Rightarrow \mu\tilde{E}(x) = \mu\tilde{A}(x)$.

Proposition 3.6. [2] If $\tilde{V}$ is a maximal fuzzy subset of $\tilde{\boldsymbol{F}}^{-1}(\tilde{V}^c\tilde{E}) = \left[\tilde{\boldsymbol{F}}^{-1}(\tilde{V})\right]^c \tilde{A}$.

Definition 3.7. [2] Let $\tilde{E} \subseteq \tilde{A}$. Then $\tilde{\boldsymbol{F}}/\tilde{E}$ defined by

$$\left(\tilde{\boldsymbol{F}}/\tilde{E}\right)(x,y) = \min\left\{\tilde{\boldsymbol{F}}(x,y), \mu\tilde{E}(x)\right\},$$

for each $(x,y) \in X \times Y$, is said to be the restriction of $\tilde{\boldsymbol{F}}$ to $\tilde{E}$.

Proposition 3.8. [2] If $\tilde{V} \subseteq \tilde{A}$, then for each $\tilde{U} \subseteq \tilde{E}$, $\left(\tilde{\boldsymbol{F}}/\tilde{V}\right)^{-1}(\tilde{U}) = \tilde{V} \cap \tilde{\boldsymbol{F}}^{-1}(\tilde{U})$.

Definition 3.9. [1] A collection $\tilde{T}$ of fuzzy subsets of a fuzzy set $\tilde{A}$ is said to be a fuzzy topology on $\tilde{A}$ if

1) $\}', \tilde{A} \in \tilde{T}$.

2) $\tilde{U}_1, \tilde{U}_2 \in \tilde{T}$, then $\tilde{U}_1 \cap \tilde{U}_2 \in \tilde{T}$.

3) $\tilde{U}_i \in \tilde{T}$ for each $i \in I$, then $\bigcup\{\tilde{U}i : i \in I\} \in \tilde{T}$.

$(\tilde{A},\tilde{T})$ is said to be a fuzzy topological space (fts, for short). The members of $\tilde{T}$ are called fuzzy open sets in $\tilde{A}$. The complement of the members of $\tilde{T}$ referred to $\tilde{A}$ are called the fuzzy closed sets in $\tilde{A}$. The family of all fuzzy closed sets in $\tilde{A}$ will be denoted by $C(\tilde{T})$.

Definition 3.10. [1] If $\tilde{E} \subseteq \tilde{A}$, $\tilde{T}_{\tilde{E}} = \{\tilde{E} \cap \tilde{V} : \tilde{V} \in \tilde{T}\}$ is a fuzzy topology on $\tilde{E}$, $(\tilde{E},\tilde{T}_{\tilde{E}})$ is called a subspace of $(\tilde{A},\tilde{T})$.

Definition 3.11. [1] Let $(\tilde{A},\tilde{T})$ be a fts and $\tilde{E} \subseteq \tilde{A}$ then the closure of $\tilde{E}$ denoted by $Cl(\tilde{E})$ is defined by $Cl(\tilde{E}) = \bigcap\{\tilde{U} : \tilde{U} \in \tilde{T}, \tilde{E} \subseteq \tilde{U}\}$. *i.e.* $Cl(\tilde{E})$ is the intersection of all closed fuzzy subsets of $\tilde{A}$ containing $\tilde{E}$.

Definition 3.12. [3] Let $(\tilde{A},\tilde{T})$ be a fts and $\tilde{E} \subseteq \tilde{A}$ then the interior of $\tilde{E}$ denoted by $\operatorname{int}(\tilde{E}) = \bigcup\{\tilde{U} : \tilde{U} \in \tilde{T}, \tilde{U} \subseteq \tilde{E}\}$. *i.e.* $\operatorname{int}(\tilde{E})$ is the union of all open fuzzy subsets of $\tilde{A}$ which contained in $\tilde{E}$.

Definition 3.13. [1] Let $(\tilde{A},\tilde{T})$ be a fts, a fuzzy subset $\tilde{E}$ of $\tilde{A}$ is called

1) Neighbourhood (nbd, for short) of the fuzzy point $\chi r \in \tilde{A}$ if there exists $\tilde{U} \in \tilde{T}$ such that $\chi r \in \tilde{U} \subseteq \tilde{E}$;

2) Quasi-neighbourhood (q-nbd, for short) of the fuzzy point $\chi r \in \tilde{A}$ if there exists $\tilde{U} \in \tilde{T}$ such that $\chi r q \tilde{U}[\tilde{A}]$, $\tilde{U} \subseteq \tilde{E}$.

The set $U\chi r$ of all q-neighbourhood of χr is called the system of q-nbd of χr.

Proposition 3.14. [2] If $(\tilde{E},\tilde{T}_{\tilde{E}})$ is a maximal subspace of $(\tilde{A},\tilde{T})$, then $Cl_{\tilde{E}}(\tilde{U}) = \tilde{E} \cap Cl_{\tilde{A}}(\tilde{U})$, where $\tilde{U} \subseteq \tilde{E}$.

Definition 3.15. [3]

1) $\tilde{U} \subseteq \tilde{A}$ is said to be a fuzzy regular open set in a fts $(\tilde{A},\tilde{T})$ if $\operatorname{int}(Cl(\tilde{U})) = \tilde{U}$.

2) $\tilde{U} \subseteq \tilde{A}$ is said to be a fuzzy regular closed set in a fts $(\tilde{A},\tilde{T})$ if $\tilde{U}^c_{\tilde{A}}$ is fuzzy regular open.

Definition 3.16. [3] A fuzzy point $\chi r \in \tilde{A}$ is said to be a fuzzy δ-cluster (resp. θ-cluster) point of a fuzzy subset $\tilde{V}$ of $\tilde{A}$ if for each fuzzy regularly open (resp. fuzzy open) q-nbd of $\chi r, \tilde{U} q \tilde{V}[\tilde{A}]$ $\left(resp.\, Cl(\tilde{U}) q \tilde{V}[\tilde{A}]\right)$. The set of all fuzzy δ-cluster (resp. fuzzy θ-cluster) points of $\tilde{V}$ is called fuzzy δ-cluster (resp. fuzzy θ-closure) and is denoted by $\delta - cl(\tilde{V})$ $\left(resp. \theta - cl(\tilde{V})\right)$. A fuzzy subset $\tilde{V} \subseteq \tilde{A}$ is called a fuzzy δ-closed (resp. θ-closed) if $\tilde{V} = \delta - cl(\tilde{V})$ (resp. $\tilde{V} = \theta - cl(\tilde{V})$) and the complement of a fuzzy δ-closed (resp. θ-closed) set is called fuzzy δ-open (resp. θ-open).

Remark 3.17. [3] It is clear that fuzzy regular open (fuzzy regular closed) implies fuzzy δ-open (fuzzy δ-closed) implies fuzzy open (fuzzy closed) but the converses are not true in general.

In this paper, the family of all fuzzy regular open (resp. fuzzy regular closed, fuzzy δ-open, fuzzy δ-closed, fuzzy open, fuzzy closed) sets in $\tilde{A}$ will be denoted by $\left[FRO(\tilde{A})\right]$, $\left(resp, \left[FRC(\tilde{A})\right], \left[F\delta O(\tilde{A})\right], \left[F\delta C(\tilde{A})\right], \left[FO(\tilde{A})\right], \left[FC(\tilde{A})\right]\right)$.

4. Fuzzy δ^*-Continuity

Unless otherwise mentioned $\tilde{T}, \tilde{T}'$ are two fuzzy topologies on $\tilde{A}$, $\tilde{E}$ respectively, and $\tilde{\boldsymbol{F}}$ a proper function from $\tilde{A}$ to $\tilde{E}$.

Definition 4.1. A proper function $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ is called fuzzy δ^*-continuous if $\tilde{\boldsymbol{F}}^{-1}(\tilde{V})\in[F\delta O(\tilde{A})]$ for each $\tilde{V}\in[FO(\tilde{E})]$.

Example 4.2. Let

$$X=\{x,y\}, Y=\{a,b\},\quad \tilde{A}=\{(x,0.7),(y,0.6)\},$$

$$\tilde{W}=\{(x,0.4),(y,0.3)\},\quad \tilde{U}=\{(x,0.3),(y,0.3)\}\in I^X$$

and

$$\tilde{E}=\{(a,0.8),(b,0.6)\},\ \tilde{V}=\{(a,0.3),(b,0.3)\}\in I^Y.$$

Consider the fuzzy topologies on $\tilde{A}$, $\tilde{E}$ resp. $\tilde{T}=\{\}',\tilde{A},\tilde{U},\tilde{W}\}$ and $\tilde{T}'=\{\}',\tilde{E},\tilde{V}\}$. Let the proper function $\tilde{F}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ defined by $\tilde{F}(x,a)=0.7$, $\tilde{F}(x,b)=0$, $\tilde{F}(y,a)=0$, $\tilde{F}(y,b)=0.6$, one may notice that the only fuzzy open sets in $(\tilde{E},\tilde{T}')$ are $\tilde{V}$, $\}$ and $\tilde{E}$ but $\tilde{\boldsymbol{F}}^{-1}(\}')=\}'$, $\tilde{\boldsymbol{F}}^{-1}(\tilde{E})=\tilde{A}$, $\tilde{\boldsymbol{F}}^{-1}(\tilde{V})=\tilde{U}$ and $\}'$, $\tilde{A}$, $\tilde{U}\in[F\delta O(\tilde{A})]$. Hence $\tilde{\boldsymbol{F}}$ is fuzzy δ^*-continuous.

Theorem 4.3. If $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ be fuzzy δ^*-continuous and $\tilde{E}\subseteq\tilde{A}$, then

$$\tilde{\boldsymbol{F}}/\tilde{E}:(\tilde{E},\tilde{T}_{\tilde{E}})\to\left(\tilde{\boldsymbol{F}}(\tilde{E}),\tilde{T}'_{\tilde{\boldsymbol{F}}(\tilde{E})}\right)$$

is fuzzy δ^*-continuous .

Proof: Let $\tilde{V}\in\tilde{T}'_{\tilde{F}(\tilde{E})}$ such that $\tilde{V}\in\left[FO\left(\tilde{\boldsymbol{F}}(\tilde{E})\right)\right]$. Then there exists fuzzy open $\tilde{U}\in\tilde{T}'$ such that $\tilde{V}=\tilde{\boldsymbol{F}}(\tilde{E})\cap\tilde{U}$.

Now

$$\begin{aligned}&(\tilde{\boldsymbol{F}}/\tilde{E})^{-1}(\tilde{V})\\&=\tilde{E}\cap\tilde{\boldsymbol{F}}^{-1}(\tilde{V})[\text{by Prop.3.8.}]\\&=\tilde{E}\cap\tilde{\boldsymbol{F}}^{-1}\left(\tilde{\boldsymbol{F}}(\tilde{E})\cap\tilde{U}\right)\\&=\tilde{E}\cap\tilde{\boldsymbol{F}}^{-1}\left(\tilde{\boldsymbol{F}}(\tilde{E})\cap\tilde{\boldsymbol{F}}^{-1}\tilde{U}\right)[\text{by Prop.3.4. 3)}]\\&=\tilde{E}\cap\tilde{E}\cap\tilde{\boldsymbol{F}}^{-1}(\tilde{U})\\&=\tilde{E}\cap\tilde{\boldsymbol{F}}^{-1}(\tilde{U}),\end{aligned}$$

but $\tilde{\boldsymbol{F}}$ be fuzzy δ^*-continuous such that $\tilde{\boldsymbol{F}}^{-1}\tilde{U}\in[F\delta O(\tilde{A})]$. Therefore

$$(\tilde{\boldsymbol{F}}/\tilde{E})^{-1}(\tilde{V})\in[F\delta O(\tilde{E})].$$

Hence $\tilde{\boldsymbol{F}}/\tilde{E}$ is fuzzy δ^*-continuous.

Definition 4.4. [2] $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ is said to satisfy property (p) if $\tilde{\boldsymbol{F}}^{-1}(\tilde{V})\in\tilde{T}$, for each $\tilde{V}\in\tilde{T}'$.

Henceforth such functions will be called fuzzy continuous proper function.

Theorem 4.5. If a proper function $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ is fuzzy δ^*-continuous then, it is fuzzy continuous.

Proof: Let $\hat{W}\in[FO(\tilde{E})]$, but $\tilde{\boldsymbol{F}}$ is fuzzy δ^*-continuous. Hence $\tilde{\boldsymbol{F}}^{-1}(\hat{W})\in[F\delta O(\tilde{A})]$ and by (Remark (3.17)) every fuzzy δ-open implies fuzzy open. (*i.e.* $\tilde{\boldsymbol{F}}^{-1}(\hat{W})\in[FO(\tilde{A})]$). Hence $\tilde{\boldsymbol{F}}$ is fuzzy continuous.

We can see from Example (4.2) such that $\tilde{\boldsymbol{F}}^{-1}(\}')=\}'$, $\tilde{\boldsymbol{F}}^{-1}(\tilde{E})=\tilde{A}$, $\tilde{\boldsymbol{F}}^{-1}(\tilde{V})=\tilde{U}$ and $\}'$, $\tilde{A}$, $\tilde{U}\in[FO(\tilde{A})]$ but $\}'$, $\tilde{E}$, $\tilde{V}\in[FO(\tilde{E})]$.

Definition 4.6. [3] A proper function $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ is called fuzzy δ-continuous if $\tilde{\boldsymbol{F}}^{-1}(\tilde{V})\in[F\delta O(\tilde{A})]$ for each $\tilde{V}\in[FRO(\tilde{E})]$.

Remark 4.7. [3] The concepts of fuzzy δ-continuous and fuzzy continuous are independent to each other .

Theorem 4.8. If $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ be fuzzy δ-continuous and $\tilde{E}\subseteq\tilde{A}$, then $\tilde{F}/\tilde{E}:(\tilde{E},\tilde{T}_{\tilde{E}})\to\left(\tilde{\boldsymbol{F}}(\tilde{E}),\tilde{T}'_{\tilde{\boldsymbol{F}}(\tilde{E})}\right)$ is fuzzy δ-continuous.

Proof: Let $\tilde{V}\in\tilde{T}'_{\tilde{F}(\tilde{E})}$ such that $\tilde{V}\in\left[FRO\left(\tilde{\boldsymbol{F}}(\tilde{E})\right)\right]$. $(\tilde{\boldsymbol{F}}/\tilde{E})^{-1}(\tilde{V})=\tilde{E}\cap\tilde{\boldsymbol{F}}^{-1}(\tilde{V})$ [by Prop. 3.8]. But $\tilde{\boldsymbol{F}}$ is fuzzy δ-continuous such that $\tilde{\boldsymbol{F}}^{-1}(\tilde{V})\in[F\delta O(\tilde{A})]$. Therefore $(\tilde{\boldsymbol{F}}/\tilde{E})^{-1}(\tilde{V})\in[F\delta O(\tilde{E})]$. Hence $\tilde{\boldsymbol{F}}/\tilde{E}$ is fuzzy δ-continuous.

Theorem 4.9. If a proper function $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ is fuzzy δ^*-continuous, then it is fuzzy δ-continuous.

Proof: Let $\hat{W}\in[FRO(\tilde{E})]$. And by Remark 3.17 every fuzzy regular open implies fuzzy δ-open implies fuzzy open. (*i.e.* $\hat{W}\in[F\delta O(\tilde{E})]$ and $\hat{W}\in[FO(\tilde{E})]$ but $\tilde{\boldsymbol{F}}$ is fuzzy δ^*-continuous). Hence $\tilde{\boldsymbol{F}}^{-1}(\hat{W})\in[F\delta O(\tilde{E})]$. Therefore $\tilde{\boldsymbol{F}}$ is fuzzy δ-continuous.

5. Fuzzy δ^{**}-Continuity

Definition 5.1. A proper function $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ is called fuzzy δ^{**}-continuous if $\tilde{\boldsymbol{F}}^{-1}(\tilde{V})\in[FO(\tilde{A})]$ for each $\tilde{V}\in[F\delta O(\tilde{E})]$.

Example 5.2. Let

$$X=\{x,y\},\ Y=\{a,b\},\ \tilde{A}=\{(x,0.7),(y,0.6)\}$$

$$\tilde{V}=\{(x,0.3),(y,0.3)\},\hat{W}=\{(x,0.5),(y,0.4)\}\in I^X$$

and

$$\tilde{E}=\{(a,0.7),(b,0.6)\}, \tilde{U}=\{(a,0.3),(b,0.3)\}\in I^Y.$$

Consider the fuzzy topologies on $\tilde{A}$ and $\tilde{E}$ resp. $\tilde{T}=\{\}',\tilde{A},\tilde{V},\hat{W}\}$ and $\tilde{T}'=\{\}',\tilde{E},\tilde{U}\}$. Let the proper function $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ defined by $\tilde{\boldsymbol{F}}(x,a)=0.7$, $\tilde{\boldsymbol{F}}(x,b)=0$, $\tilde{\boldsymbol{F}}(y,a)=0$, $\tilde{\boldsymbol{F}}(y,b)=0.6$ One may notice that the only fuzzy δ-open sets in $(\tilde{E},\tilde{T}')$ are $\tilde{U}$, $\}'$ and $\tilde{E}$ and

$$\tilde{\boldsymbol{F}}^{-1}(\}')=\}'\in[FO(\tilde{A})],$$

$$\tilde{\boldsymbol{F}}^{-1}(\tilde{E})=\tilde{A}\in[FO(\tilde{A})],$$

$$\tilde{\boldsymbol{F}}^{-1}(\tilde{U})=\tilde{V}\in[FO(\tilde{A})].$$

Hence $\tilde{\boldsymbol{F}}$ is fuzzy δ^{**}-continuous.

Theorem 5.3. If $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ be fuzzy δ^{**}-continuous and $\tilde{E}\subseteq\tilde{A}$, then

$\tilde{\boldsymbol{F}}/\tilde{E}:(\tilde{E},\tilde{T}_{\tilde{E}})\to(\tilde{\boldsymbol{F}}(\tilde{E}),\tilde{T}'_{\tilde{\boldsymbol{F}}(\tilde{E})})$ is fuzzy δ^{**}-continuous.

Proof: Let $\tilde{V}\in\tilde{T}'_{\tilde{\boldsymbol{F}}(\tilde{E})}$ such that $\tilde{V}\in[F\delta O(\tilde{\boldsymbol{F}}(\tilde{E}))]$. $(\tilde{\boldsymbol{F}}/\tilde{E})^{-1}(\tilde{V})=\tilde{E}\cap\tilde{\boldsymbol{F}}^{-1}(\tilde{V})$ [by Prop. 3.8]. But $\tilde{F}$ is fuzzy δ^{**}-continuous such that $\tilde{\boldsymbol{F}}^{-1}(\tilde{V})\in[FO(\tilde{A})]$. Therefore $(\tilde{\boldsymbol{F}}/\tilde{E})^{-1}(\tilde{V})\in[FO(\tilde{E})]$. Hence $\tilde{\boldsymbol{F}}/\tilde{E}$ is fuzzy δ^{**}-continuous.

Theorem 5.4. If a proper function $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ is fuzzy δ-continuous, then it is fuzzy δ^{**}-continuous.

Proof: Let $\hat{W}\in[FRO(\tilde{E})]$, and (by Remark 3.17 every fuzzy regular open implies fuzzy δ-open), *i.e.*

$\hat{W}\in[F\delta O(\tilde{E})]$. But $\tilde{\boldsymbol{F}}$ is fuzzy δ-continuous. Hence $\tilde{\boldsymbol{F}}^{-1}(\hat{W})\in[F\delta O(\tilde{A})]$, and (by Remark 3.17 every fuzzy δ-open implies fuzzy open). Therefore,

$\tilde{\boldsymbol{F}}^{-1}(\hat{W})\in[FO(\tilde{A})]$. (*i.e.* $\tilde{\boldsymbol{F}}$ is fuzzy δ^{**}-continuous).

Theorem 5.5. If a proper function $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ is fuzzy continuous, then it is fuzzy δ^{**}-continuous.

Proof: Let $\hat{W}\in[F\delta O(\tilde{E})]$, and (by Remark 3.17 every fuzzy δ-open implies fuzzy open), *i.e.*

$\hat{W}\in[FO(\tilde{E})]$. But $\tilde{\boldsymbol{F}}$ is fuzzy continuous. Hence $\tilde{\boldsymbol{F}}^{-1}(\hat{W})\in[FO(\tilde{A})]$. Therefore $\tilde{\boldsymbol{F}}$ is fuzzy δ^{**}-continuous.

We can see from Example (5.2.).

Remark 5.6. It is clear that not every fuzzy δ^{**}-continuous may be fuzzy δ^{*}-continuous and we can see from example.

Example 5.7. Let

$$X=\{x,y\}, Y=\{a,b\}, \tilde{A}=\{(x,0.7),(y,0.6)\}$$

$$\tilde{V}=\{(x,0.3),(y,0.3)\}, \hat{W}=\{(x,0.5),(y,0.4)\}\in I^X$$

and

$$\tilde{E}=\{(a,0.7),(b,0.6)\}, \tilde{U}=\{(a,0.3),(b,0.3)\}\in I^Y.$$

Consider the fuzzy topologies on $\tilde{A}$ and $\tilde{E}$ resp. $\tilde{T}=\{\}',\tilde{A},\tilde{V},\hat{W}\}$ and $\tilde{T}'=\{\}',\tilde{E},\tilde{U}\}$. Let the proper function $\tilde{\boldsymbol{F}}:(\tilde{A},\tilde{T})\to(\tilde{E},\tilde{T}')$ defined by $\tilde{\boldsymbol{F}}(x,a)=0.7$, $\tilde{\boldsymbol{F}}(x,b)=0$, $\tilde{\boldsymbol{F}}(y,a)=0$, $\tilde{\boldsymbol{F}}(y,b)=0.6$. $\tilde{\boldsymbol{F}}$ is fuzzy δ^{**}-continuous but not fuzzy δ^{*}-continuous such that the only fuzzy δ-open sets in $(\tilde{E},\tilde{T}')$ are $\}'$, $\tilde{E}$ and $\tilde{U}$ but $\tilde{\boldsymbol{F}}^{-1}(\tilde{U})=\tilde{V}\notin[F\delta O(\tilde{A})]$.

From what we have deduced so far, we now obtain:
Fuzzy continuous → Fuzzy δ^{**}-continuous;
Fuzzy δ-continuous → Fuzzy δ^{**}-continuous;
Fuzzy δ^{*}-continuous → Fuzzy continuous;
Fuzzy δ^{*}-continuous → Fuzzy δ-continuous.

6. Conclusion

The main purpose of this paper introduces a new concept in fuzzy set theory, namely that of a fuzzy δ^{*}-continuity and fuzzy δ^{**}-continuity. On the other hand, fuzzy topology on a fuzzy set is a kind of abstract theory of mathematics. First, we present and study fuzzy δ^{*}-continuity and fuzzy δ^{**}-continuity from a fuzzy topological space on a fuzzy set into another. Then, we present the relationships between types of fuzzy continuous functions.

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