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Global Attractor for a Non-Autonomous Beam Equation*

Yonghua Ren[1], Jianwen Zhang[2#]
[1]Department of Mathematic, Taiyuan University of Technology, Taiyuan, China
[2]CSIRO Sustainable Ecosystems, Highett, Australia
Email: renyonghua@tyut.edu.cn, #jianwen.z2008@163.com

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ABSTRACT

This work studies the global attractor for the process generated by a non-autonomous beam equation $u_{tt}+\Delta^2 u+\eta u_t-\left[\beta(t)+M\left(\int_\Omega|\nabla u(x,t)|^2\,\mathrm{d}x\right)\right]\Delta u+g(u,t)=f(x,t)$ Based on a time-uniform priori estimate method, we first in the space $H_0^2(\Omega)\times L^2(\Omega)$ establish a time-uniform priori estimate of the solution u to the equation, and conclude the existence of bounded absorbing set. When the external term $f(x,t)$ is time-periodic, the continuous semigroup of solution is proved to possess a global attractor.

Keywords: Global Attractor; Non-Autonomous Beam Equation; Absorbing Set

1. Introduction

Let Ω be an open bounded connected domain in R^N with smooth boundary $\partial\Omega$. In this work, we are devoted to the investigation of the following problem for a perturbed non-autonomous beam equation

$$u_{tt}+\Delta^2 u+\eta u_t-\left[\beta(t)+M\left(\int_\Omega|\nabla u(x,t)|^2\,\mathrm{d}x\right)\right]\Delta u+g(u,t)=f(x,t),\quad x\in\Omega,\ t>\tau \tag{1}$$

with the following initial and boundary conditions

$$u|_{x\in\partial\Omega}=\Delta u|_{x\in\partial\Omega}=0,\quad t\ge\tau \tag{2}$$

$$u(x,\tau)=u_{0\tau}(x),\ u_t(x,\tau)=u_{1\tau}(x),\ x\in\Omega \tag{3}$$

where $u(x,t)$ denotes a real-valued unknown function, and describes the transversal motion of the non-autonomous beam. $\eta\ge 0$, $g\in C^2(R\times R;R)$, $f\in L^\infty(R;L^2(\Omega))$, $f'(\cdot,t)\in C_b(R;L^2(\Omega))=C(R;L^2(\Omega))\cap L^\infty(R;L^2(\Omega))$, and $\beta(t)$, $g(u,t)$, $f(x,t)$ is the time-periodic external force.

System (1)-(3) is derived from the vibrations of an non-autonomous beam equation, and its dynamical setting is presented by Woinowsky-Krieger [1] as the new idea in fields of J. Appl. Mech. For the non-autonomous wave equations, there are many interesting results were published focusing different respects (see [2,3]). The focus of this work is the study of the long-term properties of the dynamical system generated by global attractor, please refer the reader to [4,5] and references therein.

This paper is organized as follows. In Section 2, we introduce the main assumptions and discuss the existence of a family of solution operators $S(t,\tau)$ to the problem (1)-(3). Section 3 is devoted to the existence of the bounded absorbing set. In Section 4, we show the continuity of the semigroup operator. Finally, in Section 5, the global attractor is obtained.

2. Assumptions and the Existence of the Solution Operators *S*(*t*, *τ*)

Firstly, we assume that the nonlinear functions M, g satisfied the following conditions. Namely, there exist two constants $C_1, C_2>0$, such that

$$(H1)\quad \liminf_{|s|\to+\infty}\frac{G(s,t)}{s^2}\ge 0,\ G(s,t)=\int_0^s g(s,t)\,\mathrm{d}s \tag{4}$$

$$(H2)\quad \liminf_{|s|\to+\infty}\frac{sg(s,t)-C_1G(s,t)}{s^2}\ge 0 \tag{5}$$

$$(H3)\quad G_t'(u,t)\le\gamma G(u,t)+C_2,\ \gamma\le\frac{\varepsilon C_1}{2} \tag{6}$$

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#Corresponding author.

(H4) $M'(s) \geq 0, M(z)z \geq \tilde{M}(z)$ (7)

For simplicity, we define

$$\phi(u,t) = \int_\Omega G(u(x),t)\mathrm{d}x, \tilde{M}(z) = \int_0^z M(s)\mathrm{d}s,$$

and

$$z(t) = |\nabla u(t)|^2$$

with the usual notation, we write $H = L^2(\Omega)$, $V = H_0^2(\Omega)$, $E = H_0^2(\Omega)\times L^2(\Omega)$ and the scalar products and norms on H, V and E, respectively

$$(u,v) = \int_\Omega uv\mathrm{d}x, |u|^2 = (u,u), \forall u,v \in L^2(\Omega)$$

$$((u,v)) = \int_\Omega \Delta u \Delta v \mathrm{d}x, \|u\|^2 = ((u,u)), \forall u,v \in H_0^2(\Omega)$$

$$\forall \varphi_i = (u_i, v_i)^T, i = 1,2$$

$$(\varphi_1, \varphi_2)_E = ((u_1,u_2)) + (v_1,v_2), |\varphi|_E^2 = (\varphi,\varphi)_E,$$

$$\varphi = (u,v)^T \in E$$

For the linear self-adjoint and positive operator A, let us write $A = \Delta^2$, $A: D(A) \to H$.

The space $D(A) = \{v \in V, Av \in H\}$ is dense in H. Next, we define the power A^s of A, $\forall s \in R$, which operate on the spaces $D(A^s)$, and write $V_{2s} = D(A^s)$, which turns out to be a Hilbert space with the inner product and the norm

$$(u,v)_{2s} = (A^s u, A^s v), \|u\|_{2s}^2 = ((u,u))_{2s}, \forall u,v \in D(A^s)$$

and A^γ is an isomorphism from $D(A^s)$ onto $D(A^{s-\gamma})$, $\forall s,\gamma \in R$.

From the Poincaré inequality, there exists the constant $\lambda_1 > 0$, such that:

$$|\Delta u| \geq \lambda_1 |u|, \forall u \in V,$$

where λ_1 is the first-eigenvalue of $A^{\frac{1}{2}}$. For $\eta > 0$, we consider the abstract Cauchy problem on E in the unknown variables $u = u(\cdot,t)$

$$\begin{cases} u_{tt} + Au + \eta u_t + \left[\beta(t) + M\left(\int_\Omega \left|A^{\frac{1}{2}}u(x,t)\right|^2 \mathrm{d}x\right)\right]A^{\frac{1}{2}}u \\ \qquad + g(u,t) = f(x,t) \\ u(x,\tau) = u_{0\tau}(x), \quad u_t(x,\tau) = u_{1\tau}(x) \end{cases} \tag{8}$$

The following well-posedness result holds.

Theorem 1. Suppose that $\eta > 0$ and the conditions (4)-(7) hold. If the nonlinear functions $g(u,t) \in C^2(R\times R;R)$, $f(\cdot,t) \in L^\infty(R;L^2(\Omega))$. Then, for any initial value $(u_{0\tau}, u_{1\tau})$ is given in E, problem (8) admits a unique solution $u = u(x,t)$ in the class

$$(u(t), u_t(t)) \in C(R_\tau;V)\times C(R_\tau;H)$$

where $R_\tau = [\tau, +\infty)$.

Furthermore, calling $\varphi(t)$ the difference of any two solutions corresponding to initial data having norm less than or equal to $\tau \geq 0$, there exists $C = C(\tau) \geq 0$ such that

$$\varphi(t,\varphi_{0\tau}) = e^{C(t-\tau)}\varphi_{0\tau} + \int_\tau^t e^{C(t-s)}F(\varphi(s),s)\mathrm{d}s \tag{9}$$

We omit the proof, based on a standard Feado-Galerkin approximation procedure together with a slight generalization of the usual Gronwall's lemma. Theorem 1 translates into the existence of the solution operators $S(t,\tau): E \to E, \quad t \geq \tau$ acting as

$$\varphi_{0\tau} = \{u_{0\tau}, u_{1\tau}\} \mapsto S(t,\tau)\varphi_{0\tau} = \varphi(t) = (u(t), u_t(t))$$

Remark 1. In the non-autonomous case, namely, when both $\beta(t)$, $g(u,t)$ and $f(x,t)$ are time-dependent, the two-parameter family $S(t,\tau)$ fulfills the semigroup property

$$S(t+s) = S(t) + S(s), \forall t,s \geq 0.$$

$S(\tau,\tau) = I$ is the identity operator, $\tau \geq 0$.

Thus, $S(t,\tau)$ is a continuous semigroup of operators on E.

3. The Absorbing Set

In this section, we prove the existence of an absorbing set for the semigroup $S(t,\tau)$. Combining with (4) and (5), there exist two constants $K_1, K_2 > 0$ such that

$$\varphi(u,t) + \frac{1}{8}\|u\|^2 \geq -K_1 \tag{10}$$

$$\int_\Omega ug(u,t)\mathrm{d}x - C_1\varphi(u,t) + \frac{1}{4}\|u\|^2 \geq -K_2 \tag{11}$$

Lemma 1. Under the hypotheses of Theorem 1, For the ball $B = B(0,M)$, $\forall (u_{0\tau}, u_{1\tau}) \in E$, centered at 0 of radius *M*, is an absorbing set for the semigroup $S(t,\tau)$ in *E*.

Proof. Let us begin with $\eta > 0$ be fixed and ε is chosen such that $0 < \varepsilon < \min\left\{\frac{\eta}{4}, \frac{\lambda_1^2}{2\eta}\right\}$. We set $v = u_t + \varepsilon u$ and rewrite (1) as follows

$$\begin{aligned} &v_t + (\eta - \varepsilon)v + \Delta^2 u - \left[\beta(t) + M\left(\int_\Omega |\nabla u|^2 \mathrm{d}x\right)\right]\Delta u \\ &-\varepsilon(\eta - \varepsilon)u + g(u,t) = f(x,t) \end{aligned} \tag{12}$$

Taking the scalar product in *H* of (12) with v, and we obtain the desired form

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[|v|^2+|\Delta u|^2+\beta(t)|\nabla u|^2+\tilde{M}(z)\right]+(\eta-\varepsilon)|v|^2$$
$$+\varepsilon|\Delta u|^2-\varepsilon(\eta-\varepsilon)(u,v)+\varepsilon\tilde{M}(z)+(g(u,t),v) \quad (13)$$
$$+\left(\varepsilon\beta(t)-\frac{\dot{\beta}(t)}{2}\right)|\nabla u|^2\le(f(x,t),v)$$

Using the Hölder inequality and the Poincaré inequality, we have the estimate

$$\begin{aligned}&(\eta-\varepsilon)|v|^2+\varepsilon|\Delta u|^2-\varepsilon(\eta-\varepsilon)(u,v)\\&\ge\frac{3\eta}{4}|v|^2+\varepsilon|\Delta u|^2-\frac{\varepsilon(\eta-\varepsilon)}{\sqrt{\lambda_1}}|\Delta u||v|\\&\ge\frac{3\eta}{4}|v|^2+\varepsilon|\Delta u|^2-\frac{\varepsilon}{2}|\Delta u|^2-\frac{\varepsilon\eta^2}{2\lambda_1}|v|^2\\&\ge\frac{\varepsilon}{2}|\Delta u|^2+\frac{\eta}{2}|v|^2\end{aligned} \quad (14)$$

Exploiting (11), we lead to

$$\begin{aligned}&(g(u,t),v)\\&=(g(u,t),u_t)+\varepsilon(g(u,t),u)\\&=\frac{\mathrm{d}}{\mathrm{d}t}\phi(u,t)-G_t'(u,t)+\varepsilon(g(u,t),u)\\&\ge\frac{\mathrm{d}}{\mathrm{d}t}\left[G(u,t)\right]+\frac{\varepsilon C_1}{2}G(u,t)-\frac{\varepsilon}{4}|\Delta u|^2-\varepsilon K_2-|\Omega|C_2\end{aligned} \quad (15)$$

and,

$$(f,v)\le\frac{|f|^2}{\eta}+\frac{\eta}{4}|v|^2 \quad (16)$$

where $|f|=\sup_{t\in R}|f(x,t)|$. From (13)-(16), it follows that

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[|v|^2+|\Delta u|^2+\beta(t)|\nabla u|^2+\tilde{M}(z)+2G(u,t)\right]$$
$$+\left(\varepsilon\beta(t)-\frac{\dot{\beta}(t)}{2}\right)|\nabla u|^2+\varepsilon\tilde{M}(z)+\frac{\varepsilon C_1}{2}G(u,t) \quad (17)$$
$$+\frac{\eta}{4}|v|^2+\frac{\varepsilon}{4}|\Delta u|^2\le\varepsilon K_2+|\Omega|C_2+\frac{|f|^2}{\eta}$$

So, in the light of condition (10), we have

$$L(t)=|v|^2+|\Delta u|^2+\beta(t)|\nabla u|^2+\tilde{M}(z)+2G(u,t)+2K_1>0$$

Taking $\theta=\min\left\{\frac{\varepsilon}{2},\frac{\varepsilon C_1}{2},\left(2\varepsilon-\frac{\dot{\beta}(t)}{\beta(t)}\right)\right\}$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}L(t)+\theta L(t)\le C \quad (18)$$

where $C=2\theta K_1+2\varepsilon K_2+2|\Omega|C_2+\frac{2|f|^2}{\eta}$.

Applying the Gronwall's Lemma, we obtain the following absorbing inequality

$$\begin{aligned}|\varphi(t)|_E^2&\le L(t)\\&\le L(\tau)\exp(-\theta(t-\tau))+\frac{C}{\theta}\left(1-\exp(-\theta(t-\tau))\right)\end{aligned}$$

or

$$\limsup_{t\to+\infty}|\varphi(t)|_E^2\le M,\ t\ge\tau \quad (19)$$

Taking

$$M=L(\tau)\exp(-\theta(t-\tau))+\frac{C}{\theta}\left(1-\exp(-\theta(t-\tau))\right).$$

Obviously, $|\varphi(t)|_E^2\le M$. Furthermore, let us denote B be a bounded closed ball of E centered at 0 with radius M

$$\begin{aligned}B&=B_E(0,M)\\&=\left\{(u,v)\in E:|\varphi(t)|_E^2=\|u(t)\|^2+|v(t)|^2\le M\right\}\end{aligned}$$

So, B is a bounded absorbing set of analytic semigroup $\{S(t,\tau)\}$ of (1)-(3). We complete the proof.

4. Continuity of the Semigroup

In this section, we prove the continuity of the semigroup $S(t,\tau)$ in E. For this reason, we assume

$$\limsup_{|s|\to+\infty}\frac{|g'(s,t)|}{|s|^2}=0,\ \forall 0\le\gamma_0<+\infty,\quad s\in R \quad (20)$$

Lemma 2. Under the hypotheses of Theorem 1, the mapping $\{u_{0\tau},u_{1\tau}\}\to\{u(t),u_t(t)\}$, for $\forall(u_{0\tau},u_{1\tau})\in E$, is continuous in E, and the semigroup $S(t,\tau)$ associated with the initial-boundary value problem (1) is a C_0-semigroup in E.

Proof. Assume that $B\subset E$ be a bounded positive invariant set for $S(t,\tau)$, and initial-data $\{u_{0\tau},u_{1\tau}\},\{v_{0\tau},v_{1\tau}\}\in E$. Let u,v be the two corresponding solutions of (1).

Assume $\xi=u-v$. We claim that the proof is similar to the Lemma 1. Then, from (1) we have

$$\begin{aligned}&\xi_{tt}+\Delta^2\xi-\beta(t)\Delta\xi+\eta\xi\\&-M\left(\int_\Omega|\nabla u(x,t)|^2\,\mathrm{d}x\right)\Delta u\\&+M\left(\int_\Omega|\nabla v(x,t)|^2\,\mathrm{d}x\right)\Delta v\\&+(g(u,t)-g(v,t))=f_1(x,t)-f_2(x,t)\end{aligned} \quad (21)$$

By multiplying (21) by $w=\xi_t+\varepsilon\xi$ and integrating over Ω, we find

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[|w|^2+|\Delta\xi|^2\right]+(\eta-\varepsilon)|w|^2+\varepsilon|\Delta\xi|^2$$
$$-\varepsilon(\eta-\varepsilon)(\xi,w)-\beta(t)(\Delta\xi,w)$$
$$\left(-M\left(\int_\Omega|\nabla u(x,t)|^2\,\mathrm{d}x\right)\Delta u+M\left(\int_\Omega|\nabla v(x,t)|^2\,\mathrm{d}x\right)\Delta v,w\right)$$
$$+\left(g(u,t)-g(v,t),w\right)=\left(f_1(x,t)-f_2(x,t),w\right) \tag{22}$$

Next, we are devoted to estimate (22). By the same method that we obtained (14), it follows that

$$\begin{aligned}&(\eta-\varepsilon)|w|^2+\varepsilon|\Delta\xi|^2-\varepsilon(\eta-\varepsilon)(\xi,w)\\&\ge\frac{\eta}{2}|w|^2+\frac{\varepsilon}{2}|\Delta\xi|^2\end{aligned} \tag{23}$$

And, according to (20), for $\forall\varepsilon>0$, there exists a constant $C_\varepsilon>0$, such that

$$|g'(s,t)|\le\varepsilon|s|\gamma_0+C_\varepsilon,\ \forall 0\le\gamma_0<+\infty \tag{24}$$

By (24) and the Sobolev embedding theorem, we can obtain $g(u,t),g'(u,t)$ are uniformly bounded in L^∞, that is, there exists a constant $K_3>0$, such that

$$|g(u,t)|_{L^\infty}\le K_3,\ |g'(u,t)|_{L^\infty}\le K_3 \tag{25}$$

From (24), (25), it follows that

$$\begin{aligned}&\left(g(u,t)-g(v,t),w\right)=\int_\Omega g'(u+\vartheta\xi)\cdot\xi\cdot w\mathrm{d}x,\\&\vartheta=\vartheta(x)\in[0,1]\end{aligned} \tag{26}$$

Thus

$$\begin{aligned}&|(g(u,t)-g(v,t),w)|\\&=\left|\int_\Omega g'(u+\vartheta\xi)\cdot\xi\cdot w\mathrm{d}x\right|\\&\ge-K_3\int_\Omega|\xi|\cdot|w|\,\mathrm{d}x\ge-\frac{K_3}{\lambda_1}\int_\Omega|\Delta\xi|\cdot|w|\,\mathrm{d}x\end{aligned}$$

Meanwhile, we know easily

$$\begin{aligned}&|\beta(t)(\Delta(\xi,w)|\\&\le\beta(t)|\Delta\xi||w|\le\frac{C_3}{2}\left(|\Delta\xi|^2+|w|^2\right)\end{aligned} \tag{27}$$

Due to the continuity of M', we can note the fact that

$$\begin{aligned}&M(x^2)-M(y^2)\\&\le M'\left(\sup\{x^2,y^2\}\right)|x+y|\cdot|x-y|.\end{aligned}$$

We see that easily

$$\begin{aligned}&\left(M\left(\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta u-M\left(\int_\Omega|\nabla v|^2\,\mathrm{d}x\right)\Delta v,w\right)\\&\le C_4|\Delta\xi||w|\end{aligned}.$$

This implies

$$\begin{aligned}&\left|(M\left(\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta u-M\left(\int_\Omega|\nabla v|^2\,\mathrm{d}x\right)\Delta v,w\right|\\&=\left|M\left(\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta u-M\left(\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta v\right.\\&\quad\left.+M\left(\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta v-M\left(\int_\Omega|\nabla v|^2\,\mathrm{d}x\right)\Delta v,w\right|\\&=\left|M\left(\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta\xi\right.\\&\quad\left.+\left(M\left(\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)-M\left(\int_\Omega|\nabla v|^2\,\mathrm{d}x\right)\right)\Delta v,w\right|\\&\le C_5|\Delta\xi|\cdot|w|+C_4|\Delta\xi|\cdot|w|\\&\le\frac{C_6}{2}\left(|\Delta\xi|^2+|w|^2\right)\end{aligned} \tag{28}$$

where $C_6=\max\{C_4,C_5\}$.

Combining with (27) and (28), it is obtained directly

$$\begin{aligned}&|\beta(t)(\Delta\xi,w)|\\&+\left(M\left(\int_\Omega|\nabla u|^2\,\mathrm{d}x\right)\Delta u-M\left(\int_\Omega|\nabla v|^2\,\mathrm{d}x\right)\Delta v,w\right)\\&\le\frac{1}{2}(C_3+C_6)\left(|\Delta\xi|^2+|w|^2\right)\le C_7\left(|\Delta\xi|^2+|w|^2\right)\end{aligned} \tag{29}$$

Thanks to (22)-(29), and the usual Gronwall's Lemma, we have

$$|\Delta\xi(t)|^2+|w(t)|^2\to0,\text{ as }|\Delta\xi(\tau)|^2+|w(\tau)|^2\to0.$$

So we complete the proof.

5. Existence of the Global Attractor

Theorem 2. Under the hypotheses of Theorem 1, the semigroup $S(t,\tau)$ associated with the initial-boundary value problem (1) possesses a global attractor β in E which attracts all bounded subsets of E.

Proof. Let $(u(t),u_t(t))$ be a solution of (12) with initial value, for any $(u_{0\tau},u_{1\tau})\in E$, and, it can be decomposed into $(u,u_t)=(u_1,u_{1t})+(u_2,u_{2t})$, where $v_i(t)=u_{it}(t)+\varepsilon u_i(t)$, $i=1,2$, satisfy, respectively,

$$\begin{cases}v_t+(\eta-\varepsilon)v+\Delta^2u-\varepsilon(\eta-\varepsilon)u=0, & t\ge\tau\\ u_1(\tau)=u_{0\tau}, \quad u_{1t}(\tau)=u_{1\tau}\end{cases} \tag{30}$$

and

$$\begin{cases}v_t+(\eta-\varepsilon)v+\Delta^2u-\varepsilon(\eta-\varepsilon)u=b(t), & t\ge\tau\\ u_2(\tau)=0, \quad u_{2t}(\tau)=0,\\ b(t)=\left[\beta(t)+M\left(\int_\Omega|\nabla u|^2\mathrm{d}x\right)\right]\Delta u-g(u,t)+f(x,t)\end{cases} \tag{31}$$

Applying two lemmas showed below. We also define a new inner product and norm in $E=H_0^2(\Omega)\times L^2(\Omega)$. Obviously, by embedding theorem, (30) are easily concluded through some simple computation $\left|v_1(t)\right|_E^2 \le C_8$. It is sufficient to prove that $S(t,\tau)$ is asymptotically smooth in E.

Taking the inner product in E of (12) with $v_2 = u_{2t} + \varepsilon u_2$, we obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\Big[\left|v_2\right|^2 + \left|\Delta u_2\right|^2 + \beta(t)\left|\nabla u_2\right|^2 + \tilde{M}(z)\Big] + (\eta - \varepsilon)\left|v_2\right|^2$$
$$+\varepsilon\left|\Delta u_2\right|^2 - \varepsilon(\eta - \varepsilon)(u_2, v_2) + \varepsilon\tilde{M}(z) + \big(g(u,t), v_2\big)$$
$$+\left(\varepsilon\beta(t) - \frac{\dot{\beta}(t)}{2}\right)\left|\nabla u_2\right|^2 \le \big(f(x,t), v_2\big)$$

According to the uniform boundedness of u in space V, the Sobolev Embedding theorem and the Gromwall's Lemma, combining with Lemma 1, we have the conclusion similar to below, $\left|v_2(t)\right|_E^2 \le C_9$. Thus, this leads to $S(t,\tau)$ possesses a global attractor β in E which attracts all bounded subsets of E. So we end the proof.

REFERENCES

[1] S. Woinowsky-Krieger, "The Effect of Axial Force on the Vibration of Hinged Bars," *Journal of Applied Mechanics*, Vol. 17, 1950, pp. 35-36.

[2] O. F. Ma, S. H. Wang and C. K. Zhong, "Necessary and Sufficient Conditions for the Existence of Global Attractor for Semigroup and Application," *Indiana University Mathematics Journal*, Vol. 51, 2002, pp. 529-551. doi.org/10.1512/iumj.2002.51.2255

[3] R. Temam, "Infinite Dimensional Dynamical System in Mechanics and Physical," 2nd Edition, Spring-Verlag, Nork York, 1997.

[4] Q. Z. Ma and C. K. Zhong, "Existence of Strong Global Attractors for Hyperbolic Equation with Linear Memory," *Applied Mathematics and Computation*, Vol. 157, No. 1, 2004, pp. 745-758. doi:10.1016/j.amc.2003.08.080

[5] Q. Z. Ma and C. K. Zhong, "Global Attractors of Strong Solutions for Nonclassical Diffusion Equation," *Journal of Lanzhou University*, Vol. 40, 2004, pp. 7-9.

