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Real Hypersurfaces in $\mathbb{C}P^2$ and $\mathbb{C}H^2$ Equipped with Structure Jacobi Operator Satisfying $\mathfrak{L}_\xi l = \nabla_\xi l$

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ABSTRACT

Recently in [1], Perez and Santos classified real hypersurfaces in complex projective space $\mathbb{C}P^n$ for $n \geq 3$, whose Lie derivative of structure Jacobi operator in the direction of the structure vector field coincides with the covariant derivative of it in the same direction. The present paper completes the investigation of this problem studying the case *n* = 2 in both complex projective and hyperbolic spaces.

Keywords: Real Hypersurfaces; Complex Projective Space; Complex Hyperbolic Space; Lie Derivative; Structure Jacobi Operator

1. Introduction

A complex n-dimensional Kaehler manifold of constant holomorphic sectional curvature *c* is called a complex space form, which is denoted by $M_n(c)$. A complete and simply connected complex space form is complex analytically isometric to a complex projective space $\mathbb{C}P^n$ a complex Euclidean space $\mathbb{C}^n$ or a complex hyperbolic space $\mathbb{C}H^n$ if $c > 0$, $c = 0$ or $c < 0$ respectively.

The study of real hypersurfaces was initiated by Takagi (see [2]), who classified homogeneous real hypersurfaces in $\mathbb{C}P^n$ and showed that they could be divided into six types, which are said to be of type A_1, A_2, *B*, *C*, *D* and *E*. Berndt (see [3]) classified homogeneous real hypersurfaces in $\mathbb{C}H^n$ with constant principal curvatures. Okumura (see [4]) in $\mathbb{C}P^n$ and Montiel and Romero (see [5]) in $\mathbb{C}H^n$ gave the classification of real hypersurfaces satisfying relation $A\varphi - \varphi A = 0$.

Ki and Liu (see [6]) have given the above classification as follows:

Theorem A *Let M be a real hypersurface of* $M_n(c)$, $c \neq 0$, $(n \geq 2)$. *If it satisfies* $A\varphi - \varphi A = 0$, *then M is locally congruent to one of the following hypersurfaces*:

- *In case* $\mathbb{C}P^n$
  (A_1) *a geodesic hypersphere of radius r, where* $0 < r < \frac{\pi}{2}$,
  (A_2) *a tube of radius r over a totally geodesic* $\mathbb{C}P^k$, $(1 \leq k \leq n-2)$, *where* $0 < r < \frac{\pi}{2}$.
- *In case* $\mathbb{C}H^n$
  (A_0) *a horosphere in* $\mathbb{C}H^n$, *i.e. a Montiel tube*,
  (A_1) *a geodesic hypersphere or a tube over a hyperplane* $\mathbb{C}H^{n-1}$,
  (A_2) *a tube over a totally geodesic* $\mathbb{C}H^k$ $(1 \leq k \leq n-2)$.

Let M be a real hypersurface in $M_n(c)$, $(c \neq 0)$. Then an almost contact metric structure (φ, ξ, η, g) can be defined on *M*. The structure vector field ξ is called principal if $A\xi = \alpha\xi$ holds on *M*, where A is the shape operator of *M* in $M_n(c)$ and α is a smooth function. A real hypersurface is said to be a Hopf hypersurface if ξ is principal.

The Jacobi operator field with respect to *X* on *M* is defined by $R(\cdot, X)X$, where *R* is the Riemmanian curvature of *M*. For $X = \xi$ the Jacobi operator is called structure Jacobi operator and is denoted by $l = R(\cdot, \xi)\xi$. It has a fundamental role in almost contact manifolds. Many differential geometers have studied real hypersurfaces in terms of the structure Jacobi operator.

The Lie derivative of the structure Jacobi operator with respect to ξ was investigated by Perez, Santos, Suh (see [7]). More precisely, they classified real hypersurfaces in $\mathbb{C}P^n$ $(n \geq 3)$, whose structure Jacobi operator satisfies the condition: $\mathfrak{L}_\xi l = 0$. Ivey and Ryan (see [8]) classified real hypersurfaces satisfying the same condition in $\mathbb{C}P^2$ and $\mathbb{C}H^2$.

The study of real hypersurfaces whose structure Jacobi operator is parallel is a problem of great importance. In [9] the nonexistence of real hypersurfaces in nonflat complex space form with parallel structure Jacobi operator ($\nabla l = 0$) was proved. In [10] a weaker condition ($\mathbb{D}$-parallelness), $\nabla_X l = 0$ for any vector field X orthogonal to ξ, was studied and it was proved the non-existence of such hypersurfaces in case of $\mathbb{C}P^n$ ($n \geq 3$). The parallelness of structure Jacobi operator in combination with other conditions was another problem that was studied by many others such as Ki, Kim, Perez, Santos, Suh (see [11,12]).

Recently Perez-Santos (see [1]) studied real hypersurfaces in $\mathbb{C}P^n$ for $n \geq 3$, whose structure Jacobi operator satisfies the relation:

$$\mathfrak{L}_\xi l = \nabla_\xi l. \tag{1}$$

In the present paper we go on studying the same problem for $\mathbb{C}P^2$ and $\mathbb{C}H^2$. We prove the following theorem:

Main Theorem *Let M be a real hypersurface in* $\mathbb{C}P^2$ *or* $\mathbb{C}H^2$, *whose structure Jacobi operator satisfies relation* (1). *Then M is locally congruent to*: *a geodesic sphere of radius r*, *where* $0 < r < \frac{\pi}{2}$ *with* $r \neq \frac{\pi}{4}$, *or to a tube of radius* $r = \frac{\pi}{4}$ *over a holomorphic curve in* $\mathbb{C}P^2$ *and to a horosphere*, *a geodesic sphere or a tube over* $\mathbb{C}H^1$ *in* $\mathbb{C}H^2 i$ *or to a Hopf hypersurface in* $\mathbb{C}H^2$ *with* $A\xi = 0$.

2. Preliminaries

Let M be a connected real hypersurface immersed in a nonflat complex space form $(M_n(c), G)$, ($c \neq 0$), with almost complex structure J of constant holomorphic sectional curvature c. Let N be a unit normal vector field on M and $\xi = -JN$. For a vector field X tangent to M we can write $JX = \varphi(X) + \eta(X)N$, where φX and $\eta(X)N$ are the tangential and the normal component of JX respectively. The Riemannian connection $\bar{\nabla}$ in $M_n(c)$ and ∇ in M are related for any vector fields X, Y on M:

$$\bar{\nabla}_Y X = \nabla_Y X + g(AY, X)N$$
$$\bar{\nabla}_X N = -AX$$

where g is the Riemannian metric on M induced from G of $M_n(c)$ and A is the shape operator of M in $M_n(c)$. M has an almost contact metric structure (φ, ξ, η) induced from J on $M_n(c)$ where φ is a (1,1) tensor field and η is a 1-form on M such that

$$g(\varphi X, Y) = G(JX, Y),$$
$$\eta(X) = g(X, \xi) = G(JX, N)$$

(see [13]). Then we have

$$\begin{aligned} &\varphi^2 X = -X + \eta(X)\xi, \quad \eta(\varphi X) = 0, \\ &\varphi\xi = 0, \quad \eta(\xi) = 1 \end{aligned} \tag{2}$$

$$\begin{aligned} &g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \\ &g(X, \varphi Y) = -g(\varphi X, Y) \end{aligned} \tag{3}$$

$$\begin{aligned} &\nabla_X \xi = \varphi A X, \\ &(\nabla_X \varphi)Y = \eta(Y)AX - g(AX, Y)\xi \end{aligned} \tag{4}$$

Since the ambient space is of constant holomorphic sectional curvature c, the equations of Gauss and Codazzi for any vector fields X, Y, Z on M are respectively given by

$$\begin{aligned} R(X,Y)Z = \frac{c}{4}\big[&g(Y,Z)X - g(X,Z)Y + g(\varphi Y, Z)\varphi X \\ &- g(\varphi X, Z)\varphi Y - 2g(\varphi X, Y)\varphi Z\big] \\ &+ g(AY, Z)AX - g(AX, Z)AY \end{aligned} \tag{5}$$

$$\begin{aligned} (\nabla_X A)Y - (\nabla_Y A)X = \frac{c}{4}\big[&\eta(X)\varphi Y - \eta(Y)\varphi X \\ &- 2g(\varphi X, Y)\xi\big] \end{aligned} \tag{6}$$

where R denotes the Riemannian curvature tensor on M.

For every point $P \in M$, the tangent space $T_P M$ can be decomposed as following:

$$T_P M = span\{\xi\} \oplus ker\eta$$

where $ker(\eta) = \{X \in T_P M : \eta(X) = 0\}$. Due to the above decomposition, the vector field $A\xi$ is decomposed as follows:

$$A\xi = \alpha\xi + \beta U \tag{7}$$

where $\beta = |\varphi\nabla_\xi \xi|$ and $U = -\frac{1}{\beta}\varphi\nabla_\xi \xi \in ker(\eta)$, provided that $\beta \neq 0$.

All manifolds are assumed connected and all manifolds, vector fields etc are assumed smooth (C^∞).

3. Auxiliary Relations

Suppose now that the ambient space is $\mathbb{C}P^2$ or $\mathbb{C}H^2$, (*i.e.* $M_2(c)$, $c \neq 0$), then we consider V be the open subset of points $P \in M$, such that there exists a neighborhood of every P, where $\alpha = 0$ and Ω the open subset of points Q of M such that there exists a neighborhood of every Q, where $a \neq 0$. Since, α is a smooth function on M, then $V \cup \Omega$ is an open and dense subset of M.

Proposition 3.1 *Let M be a real hypersurface in* $M_2(c)$, *equipped with structure Jacobi operator satisfying* (1). *Then*, ξ *is principal on V*.

Proof: The relation (7) on V, takes the form $A\xi = \beta U$. From (5) for $X = \varphi U$, $Y = Z = \xi$ we obtain:

$$l\varphi U = \frac{c}{4}\varphi U. \tag{8}$$

Due to the definition of Lie derivative, the relation (1) for $X=\xi$ yields: $l\nabla_{\xi}\xi=0$. The latter, because of the first relation of (4) and (8) implies $\beta=0$, hence $A\xi=0$. Therefore, ξ is principal on *V*. □

On Ω if $\beta=0$, then ξ is principal. In what follows we work on W ($W\subset\Omega$), which is the open subset of points $Q\in\Omega$ such that $\alpha\beta\neq 0$.

Lemma 3.2 *Let M be a real hypersurface in* $M_2(c)$. *Then the following relations hold on W*:

$$AU=\left(\frac{\beta^2}{\alpha}-\frac{c}{4\alpha}\right)U+\beta\xi,\quad A\varphi U=-\frac{c}{4\alpha}\varphi U \tag{9}$$

$$\nabla_U\xi=\left(\frac{\beta^2}{\alpha}-\frac{c}{4\alpha}\right)\varphi U,\ \nabla_{\varphi U}\xi=\frac{c}{4\alpha}U,\ \nabla_{\xi}\xi=\beta\varphi U \tag{10}$$

$$\nabla_U U=\kappa_1\varphi U,\ \nabla_{\varphi U}U=\kappa_2\varphi U-\frac{c}{4\alpha}\xi,\ \nabla_{\xi}U=\kappa_3\varphi U \tag{11}$$

$$\begin{aligned}&\nabla_U\varphi U=-\kappa_1 U+\left(\frac{c}{4\alpha}-\frac{\beta^2}{\alpha}\right)\xi,\\&\nabla_{\varphi U}\varphi U=-\kappa_2 U,\quad \nabla_{\xi}\varphi U=-\kappa_3 U-\beta\xi\end{aligned} \tag{12}$$

where $\kappa_1,\kappa_2,\kappa_3$ are smooth functions on *M*.

Proof: If $\{U,\varphi U,\xi\}$ is an orthonormal basis, then because of (7) we have:

$$AU=\gamma U+\delta\varphi U+\beta\xi,\quad A\varphi U=\delta U+\rho\varphi U, \tag{13}$$

where γ, δ and ρ are smooth functions on *M*.

The first relation of (4), because of (13) yields:

$$\begin{aligned}&\nabla_U\xi=-\delta U+\gamma\varphi U,\quad \nabla_{\varphi U}\xi=-\rho U+\delta\varphi U,\\&\nabla_{\xi}\xi=\beta\varphi U.\end{aligned} \tag{14}$$

The relation (5), using (13) can be written:

$$\begin{aligned}&lU=\left(\frac{c}{4}+\alpha\gamma-\beta^2\right)U+\alpha\delta\varphi U,\\&l\varphi U=\alpha\delta U+\left(\alpha\rho+\frac{c}{4}\right)\varphi U.\end{aligned} \tag{15}$$

By the definition of Lie derivative, the relation (1) takes the form:

$$\nabla_{lX}\xi=l\nabla_X\xi.$$

The latter for $X\in\{\xi,\varphi U\}$ implies $lU=0,\ l\varphi U=0$ and then from (15) we obtain:

$$\delta=0,\quad \rho=-\frac{c}{4\alpha},\quad \gamma=\frac{\beta^2}{\alpha}-\frac{c}{4\alpha}. \tag{16}$$

The relations (13) and (14), because of (16) imply (9) and (10) respectively.

From the well known relation:

$$Xg(Y,Z)=g(\nabla_X Y,Z)+g(Y,\nabla_X Z)$$

for $X,Y,Z\in\{\xi,U,\varphi U\}$, using (16) we obtain (11) and (12), where κ_1, κ_2 and κ_3 are smooth functions on *M*. □

The Codazzi equation for *X*, *Y* $\in\{\xi,U,\varphi U\}$, because of Lemma 3.2 yields:

$$U\cdot\beta=\beta\kappa_2\left(\frac{4\beta^2}{c}+1\right), \tag{17}$$

$$\frac{\beta^2\kappa_3}{\alpha}=\beta\kappa_1+\frac{c}{4\alpha}\left(\frac{\beta^2}{\alpha}-\frac{c}{4\alpha}\right), \tag{18}$$

$$U\cdot\alpha=\xi\cdot\beta=\frac{4\alpha\beta^2\kappa_2}{c}, \tag{19}$$

$$\xi\cdot\alpha=\frac{4\alpha^2\beta\kappa_2}{c}, \tag{20}$$

$$(\varphi U)\cdot\alpha=\beta\left(\alpha+\kappa_3+\frac{3c}{4\alpha}\right), \tag{21}$$

$$(\varphi U)\cdot\beta=\beta^2+\beta\kappa_1+\frac{c}{2\alpha}\left(\frac{\beta^2}{\alpha}-\frac{c}{4\alpha}\right), \tag{22}$$

$$(\varphi U)\cdot\left(\frac{\beta^2}{\alpha}-\frac{c}{4\alpha}\right)=\beta\left(\frac{\beta^2}{\alpha}+\frac{\beta\kappa_1}{\alpha}-\frac{3c}{4\alpha}\right). \tag{23}$$

The Riemannian curvature on *M* satisfies (5) and on the other hand is given by the relation $R(X,Y)Z=\nabla_X\nabla_Y Z-\nabla_Y\nabla_X Z-\nabla_{[X,Y]}Z$. From these two relations and because of (16) for *X*, *Y* $\in\{\xi,U,\varphi U\}$ we obtain:

$$\begin{aligned}U\cdot\kappa_2-(\varphi U)\cdot\kappa_1=&\frac{c}{2\alpha}\left(\frac{\beta^2}{\alpha}-\frac{c}{4\alpha}\right)+\frac{c\kappa_3}{2\alpha}\\&-\frac{\beta^2\kappa_3}{\alpha}-\kappa_1^2-\kappa_2^2-c\end{aligned} \tag{24}$$

$$U\cdot\kappa_3-\xi\cdot\kappa_1=\kappa_2\left(\frac{\beta^2}{\alpha}-\frac{c}{4\alpha}-\kappa_3\right) \tag{25}$$

$$(\varphi U)\cdot\kappa_3-\xi\cdot\kappa_2=\kappa_1\left(\kappa_3+\frac{c}{4\alpha}\right)+\beta\left(\kappa_3-\frac{c}{2\alpha}\right) \tag{26}$$

Relation (23), because of (18), (21) and (22), yields:

$$\kappa_3=-4\alpha \tag{27}$$

and so relation (18) becomes:

$$\beta\kappa_1=\frac{c}{4\alpha}\left(\frac{c}{4\alpha}-\frac{\beta^2}{\alpha}\right)-4\beta^2. \tag{28}$$

Differentiating the relations (27) and (28) with respect to U and ξ respectively and substituting in (25) and due to (19), (20) and (27) we obtain:

$$\kappa_2\left(c-2\beta^2-4\alpha^2\right)=0. \tag{29}$$

Owing to (29), we consider W_1, ($W_1 \subset W$) the open subset of points $Q \in W$, where $\kappa_2 \neq 0$ in a neighborhood of every Q.

Lemma 3.3 *Let M be a real hypersurface in* $M_2(c)$, *equipped with Jacobi operator satisfying* (1). *Then* W_1 *is empty.*

Proof: Due to (29) we obtain: $2\beta^2 + 4\alpha^2 = c$ on W_1. Differentiation of the last relation along ξ and taking into account (19), (20) and $2\beta^2 + 4\alpha^2 = c$ yields: $c = 0$, which is a contradiction. Therefore, W_1 is empty. □

On W, because of Lemma 3.3, we have $\kappa_2 = 0$, hence the relations (17), (19) and (20) become:

$$U \cdot \alpha = U \cdot \beta = \xi \cdot \alpha = \xi \cdot \beta = 0.$$

Using the last relations and Lemma 3.2 we obtain:

$$[U,\xi]\alpha = U \cdot \xi \cdot \alpha - \xi \cdot U \cdot \alpha = 0,$$

$$[U,\xi]\alpha = \left(\nabla_U \xi - \nabla_\xi U\right)\alpha = \frac{1}{4\alpha}\left(4\beta^2 + 16\alpha^2 - c\right)(\varphi U)\cdot\alpha.$$

Combining the last two relations we have:

$$\left(4\beta^2 + 16\alpha^2 - c\right)(\varphi U)\cdot\alpha = 0. \quad (30)$$

Let W_2, ($W_2 \subset W$) be the set of points $Q \in W$, for which there exists a neighborhood of every Q such that $(\varphi U)\cdot\alpha \neq 0$. So from (30) we have: $16\alpha^2 + 4\beta^2 = c$. Differentiating the last relation with respect to φU and taking into account (21), (22), (27) and (28), we have: $\alpha^2 = 0$, which is impossible. So W_2 is empty. Hence, on W we have $(\varphi U)\cdot\alpha = 0$. Then, relations (21) and (28), because of (27) imply respectively: $c = 4\alpha^2$ and $\beta\kappa_1 = \alpha^2 - 5\beta^2$. Relation (26), because of $(\varphi U)\cdot\alpha = 0$, $c = 4\alpha^2$ and (27) yields: $\kappa_1 = -2\beta$. Substitution of κ_1 in $\beta\kappa_1 = \alpha^2 - 5\beta^2$ yields: $3\beta^2 = \alpha^2$. Differentiation of the last one along φU and taking into account (22) leads to: $\beta = 0$, which is a contradiction. So we obtain the following proposition:

Proposition 3.4 *Let M be a real hypersurface in* $M_2(c)$, *equipped with Jacobi operator satisfying* (1). *Then M is a Hopf hypersurface.*

4. Proof of Main Theorem

Since M is a Hopf hypersurface, we can write $AZ = \lambda Z$ and $A\varphi Z = \mu\varphi Z$ with $\{\xi, Z, \varphi Z\}$, being a local orthonormal basis and the following relation holds: $\lambda\mu = \frac{\alpha}{2}(\lambda + \mu) + \frac{c}{4}$ (Corollary 2.3 [14]). Furthermore, due to Theorem 2.1 [14], we have that α is constant.

From (5), due to $AZ = \lambda Z$ and $A\varphi Z = \mu\varphi Z$ we get:

$$lZ = \left(\alpha\lambda + \frac{c}{4}\right)Z, \quad l\varphi Z = \left(\alpha\mu + \frac{c}{4}\right)\varphi Z. \quad (31)$$

The relation (1), because of (31) implies: $\alpha\lambda(\lambda - \mu) = 0$, for $X = Z$ and $\alpha\mu(\lambda - \mu) = 0$, for $X = \varphi Z$. Combining the last two relations leads to: $\alpha(\lambda - \mu)^2 = 0$. If $\alpha = 0$, in case of $\mathbb{C}P^2$, M is locally congruent to a tube of radius $\frac{\pi}{4}$ over a holomorphic curve in $\mathbb{C}P^2$ due to [15] and to a Hopf hypersurface with $A\xi = 0$ in $\mathbb{C}H^2$. If $\alpha \neq 0$, the last relation implies: $\lambda = \mu$ and we obtain:

$$(\varphi A - A\varphi)X = 0, \quad \forall X \in TM,$$

which because of Theorem A completes the proof of Main Theorem.

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