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Unsteady Flow of a Dusty Visco-Elastic Fluid through an Inclined Channel

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Abstract

The present discussion deals with the study of an unsteady flow of a dusty fluid through an inclined channel under the influence of pulsatile pressure gradient along with the effect of a uniform magnetic field. The analytical solutions of the problem are obtained using variable separable and Fourier transform techniques. The graphs drawn for the velocity fields of both fluid and dust phase under the effect of Reynolds number. The velocity profiles for the liquid and the dust particles decreases at different values of time t increases. As the visco-elastic parameter λ increases the velocity of the liquid and the dust particles deceases. When relaxation time parameter σ increases, the velocity of the liquid and dust particles decreases.

Keywords: Dusty Fluid, Pulsatile Pressure Gradient, Velocities of Dust and Fluid Phase, Inclined Channel, Reynolds Number

1. Introduction

In recent years many authors have studied the flow of immiscible viscous electrically conducting fluids and their different transport phenomena. These fluid also known as non-Newtonian fluids are molten plastics. Plups, emulsion etc., and large variety of industrial product having visco-elastic behavior in their motion. Such fluids are often embedded with spherical non-conducting dust particles in the form of impurities. This fluid also called dusty Rivlin-Ericksen second order fluid. The influence of dust particles on visco-elastic fluid flow has its importance in many applications such as extrusion of plastic in the manufacture of rayon and Nylon, purification of crude oil, pulp oil, pulp, paper industry, textile industry and in different geophysical cases etc. In these cases stratification effect is often observed which are under the action of geomagnetic field.

Saffman *et al.*, (1962) studied the stability of a laminar flow of dusty gas with uniform distribution of dust particles. Michel (1965) considered the Kelvin-Helmholtz instability of the dusty gas. Michael and Miller (1965) discussed the motion of the dusty gas enclosed in the same infinite space above a rigid plane boundary. We have studied the unsteady dusty visco-elastic liquid in a channel bounded by two parallel plates. The change in velocity profiles for dust and liquid particles has been depicted graphically.

2. Theory

Formulation and Solution of the Problem

The X-axis is taken along the plate and the Y-axis normal to it. The basic equations of hydro magnetic flow are

$$\frac{\partial u_1'}{\partial t'}+\left(u_1'\cdot\nabla\right)u_1'=-\frac{1}{\rho}\nabla\cdot P'+\left(\gamma'+\beta\nabla\right)\nabla^2u_1'+\frac{kN_0}{\rho'}\left(u_2'-u_1'\right) \tag{1}$$

$$\frac{\partial u_2'}{\partial t'}+\left(u_2'\cdot\nabla\right)u'_2=\frac{k}{m}\left(u_1'-u_2'\right) \tag{2}$$

$$\mathrm{div}u_1'=0 \tag{3}$$

$$\mathrm{div}u_2'=0 \tag{4}$$

where the u_1', u_2' denotes the velocity vector of fluid and dust particles respectively: p' the pressure: ρ' the density of the fluid: γ' kinematic coefficient of viscosity: t' the time: m, the mass of the dust particles: N_0, the number density of dust particles: *K*, the stokes resistance coefficient which for spherical particles of radius a is $6\pi\mu'$ a: μ, the coefficient of viscosity of fluid particles.

In the present analysis, the following important assumptions are made:

1) The dust particles are spherical in shape are uniformly distributed.

2) Chemical reaction, mass transfer and radiation between the particles and fluid are not considered.

3) The temperature is uniform within a particle.

4) Interaction between particles themselves is not considered.

5) The flow is fully developed.

6) The buoyancy force is neglected.

7) The number density of the dust particles is constant throughout the motion.

8) The displacement current is zero, since the flow velocity is small relative to the speed of light.

9) The Hall effects are negligible.

10) The fluid is electrically neutral, i.e., no surplus electrical charge distribution is present in the fluid.

11) Only the electromagnetic body forces are present.

12) Fluid properties are invariable.

13) Viscous dissipation is neglected.

Maxwell's equations, together with Ohm's law and the law of electromagnetic conservation, are written in the case of zero-displacement and hall currents as:

$$\nabla \times \boldsymbol{B} = \boldsymbol{J} \tag{5}$$

$$\nabla \times \boldsymbol{E} = \frac{-\partial \boldsymbol{B}}{\partial t} \tag{6}$$

$$\boldsymbol{J} = \sigma_1 \left(E + \boldsymbol{V} \times \boldsymbol{B} \right) \tag{7}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{8}$$

$$\nabla \cdot \boldsymbol{E} = 0 \tag{9}$$

The usual Prandtl boundary layer assumptions along with assumptions (5)-(9) leads to the following reduction of the previous equations:

$$\frac{\partial u_1'}{\partial t'} = -\frac{1}{\rho}\frac{\partial P'}{\partial x'} + \left(\gamma' + \beta \frac{\partial}{\partial t'} \right) \frac{\partial^2 u_1'}{\partial y^2} + \frac{KN_0}{\rho}\left(u_2' - u_1' \right) \tag{10}$$

$$\frac{\partial u_2'}{\partial t'} = K\left(u_1' - u_2' \right) \tag{11}$$

which are to be solved subject to the boundary conditions

$$\begin{aligned} & t' = 0, \quad u_1' = u_2' = 0 \\ & t' > 0, \quad -\frac{1}{\rho}\frac{\partial P'}{\partial x'} = C \text{ (constant)} \\ & y' = \pm h, \quad u_1' = 0,\ u_2' = 0 \end{aligned} \tag{12}$$

Changing it into non dimensional form by putting

$$y = \frac{y'}{h},\ x = \frac{x'}{h},\ t = \frac{\gamma' t'}{h^2},\ u = \frac{u_1' h}{\gamma'},\ v = \frac{u_2'}{\gamma'},\ p = \frac{p' h^2}{\rho' \gamma'^2}$$

We have

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \left(1 - \lambda \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial y^2} + \frac{l}{\sigma}\left(v - u \right) \tag{13}$$

$$\sigma \frac{\partial v}{\partial t} = u - v \tag{14}$$

where

$\sigma = \dfrac{m\gamma'}{Kh^2}$ Relaxation time parameter, $l = \dfrac{mN_0}{\rho}$

Mass Concentration $\lambda = \dfrac{-\beta}{h^2}$ Visco-elastic parameter

The boundary conditions are

$$\begin{aligned} & t = 0: \quad u=0,\ v = 0 \\ & t > 0: \quad u=0, \text{ at } y = -1 \\ & \qquad\quad\ \ u=0, \text{ at } y = 1 \end{aligned} \tag{15}$$

take $-\dfrac{\partial p}{\partial x} = C$ (constant) for $t > 0$.

Then the equation (2.2.13) becomes

$$\frac{\partial u}{\partial t} = C + \left(1 - \lambda \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial y^2} + \frac{l}{\sigma}\left(v - u \right) \tag{16}$$

Appling the Laplace Transform, we have from (14) and (16)

$$S\bar{u} = \frac{C}{S} + \left(1 - \lambda S \right) \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{l}{\sigma}\left(\bar{v} - \bar{u} \right) \tag{17}$$

$$\sigma S \bar{v} = \bar{u} - \bar{v} \tag{18}$$

where

$$\bar{u} = \int_0^\infty u \mathrm{e}^{-st} \mathrm{d}t, \quad \bar{v} = \int_0^\infty v \mathrm{e}^{-st} \mathrm{d}t$$

The boundary conditions (15) are transformed to

$$\bar{u} = 0, \bar{v} = 0 \ at\ y = \pm 1 \tag{19}$$

Solving equations (17) and (18) subject to the boundary conditions (19) we have

$$\frac{\mathrm{d}^2 \bar{u}}{\mathrm{d}y^2} - \alpha^2 \bar{u} = \frac{-C}{\left(1 - \lambda S \right) S} \tag{20}$$

where

$$\alpha^2 = \frac{S\left(1 + \sigma S \right) + ls}{\left(1 - \lambda S \right)\left(1 + \sigma S \right)} \tag{21}$$

Finally

$$\bar{u} = \frac{C}{\alpha^2 S \left(1 - \lambda S \right)} \left\{ 1 - \frac{\cosh \alpha y}{\cosh \alpha} \right\} \tag{22}$$

$$\bar{v} = \frac{C}{\alpha^2 S(1-\lambda S)(1+\sigma S)}\left\{1-\frac{\cosh\alpha y}{\cosh\alpha}\right\} \tag{23}$$

Applying Laplace Inversion formula

$$u = \frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}\bar{u}\mathrm{e}^{St}\mathrm{d}t \tag{24}$$

Here δ is greatest then the real part of all the Singularities of $\bar{u}$

$$u = \frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty}\frac{C}{\alpha^2 S(1-\lambda S)}\left\{1-\frac{\cosh\alpha y}{\cosh\alpha}\right\}\mathrm{e}^{St}\mathrm{d}t \tag{25}$$

Taking Inversion Laplace Transform and with the help of calculus of residues the above equations (22) and (23) yields.

$$\frac{2u}{C} = \frac{2}{Q^2}\left\{1-\frac{\cosh Qy}{\cosh Q}\right\} + \frac{8}{\pi}\sum_{r=1}^{\infty}\frac{(-1)^r\cos\left(\frac{2r+1}{2}\right)\pi y(1-\sigma S_1)^2(1+\lambda S_1)\mathrm{e}^{-S_1 t}}{(2r+1)A'_{33}} + \frac{8}{\pi}\sum_{r=1}^{\infty}\frac{(-1)^r\cos\left(\frac{2r+1}{2}\right)\pi y(1-\sigma S_2)^2(1+\lambda S_2)\mathrm{e}^{-S_2 t}}{(2r+1)A''_{33}} \tag{26}$$

and

$$\frac{2v}{C} = \frac{2}{Q^2}\left\{1-\frac{\cosh Qy}{\cosh Q}\right\} + \frac{8}{\pi}\sum_{r=1}^{\infty}\frac{(-1)^r\cos\left(\frac{2r+1}{2}\right)\pi y(1-\sigma S_1)(1+\lambda S_1)\mathrm{e}^{-S_1 t}}{(2r+1)A'_{33}} + \frac{8}{\pi}\sum_{r=1}^{\infty}\frac{(-1)^r\cos\left(\frac{2r+1}{2}\right)\pi y(1-\sigma S_2)(1+\lambda S_2)\mathrm{e}^{-S_2 t}}{(2r+1)A''_{33}} \tag{2.2.25}$$

where

$$Q = \alpha^2 \; at \;\; S \to 0 \qquad A'_{33} = \frac{1+l-2S_1\sigma+\sigma^2 S_1^2+\lambda S_1^2\sigma l}{\left(1-S_1\sigma+\lambda S_1-\lambda S_1^2\sigma\right)}$$

$$A''_{33} = \frac{1+l-2S_2\sigma+\sigma^2 S_2^2+\lambda S_2^2\sigma l}{\left(1-S_2\sigma+\lambda S_2-\lambda S_2^2\sigma\right)}$$

$$S_1 = \frac{-X_1+X}{2\sigma\left(1-\pi^2\lambda\left(\frac{2r+1}{2}\right)^2\right)},\; S_2 = \frac{-X_1-X}{2\sigma\left(1-\pi^2\lambda\left(\frac{2r+1}{2}\right)^2\right)}$$

where

$$X_1 = 1+l+\pi^2\sigma\left(\frac{2r+1}{2}\right)^2-\pi^2\lambda\left(\frac{2r+1}{2}\right)^2$$

$$X = \sqrt{X_1^2-4\sigma\left[1-\pi^2\lambda\left(\frac{2r+1}{2}\right)^2\right]\times\pi^2\left(\frac{2r+1}{2}\right)^2} \tag{27}$$

3. Results and Discussion

The unsteady flow of a dusty visco-elastic fluid through a channel is studied. From **Figures 1** and **2** it shows that the velocity profiles for the liquid and the dust particles decreases at different values of time t increases. As the visco-elastic parameter λ increases the velocity of the liquid and the dust particles deceases as shown in **Figure 3** and **4**. When relaxation time parameter σ increases the velocity of the liquid and dust particles decreases as shown in **Figure 5** and **6**. From **Figure 7** and **8** it can be observed that as mass concentration increases the velocity of the liquid and the dust particles deceases. In case when gravity or inclination angle $\theta \to 0$ and visco-elastic parameter $\lambda \to 0$ and adding the magnetic field term then the present model becomes that of Singh and Ram.

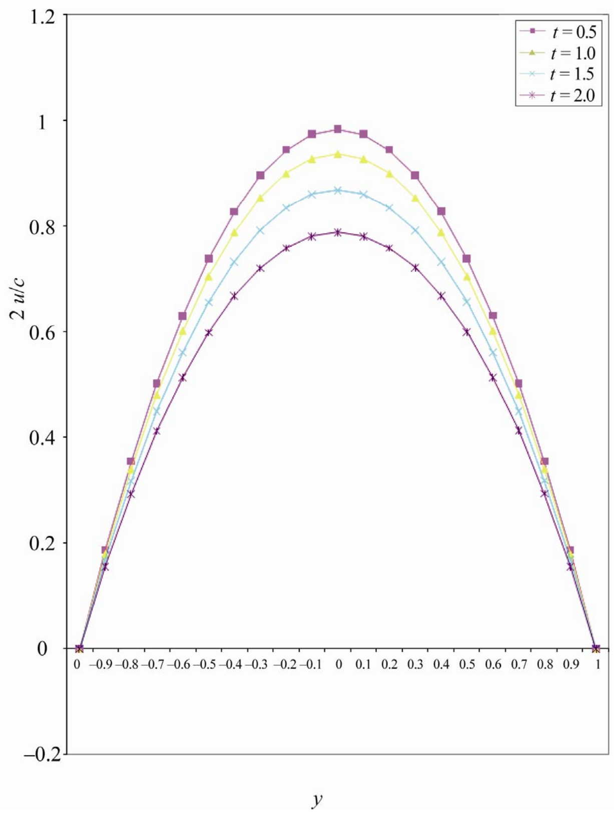

Figure 1. Show the variation of velocity profile of liquid for different value of time at fixed (σ = 0.8, λ = 0.5, I = 0.5).

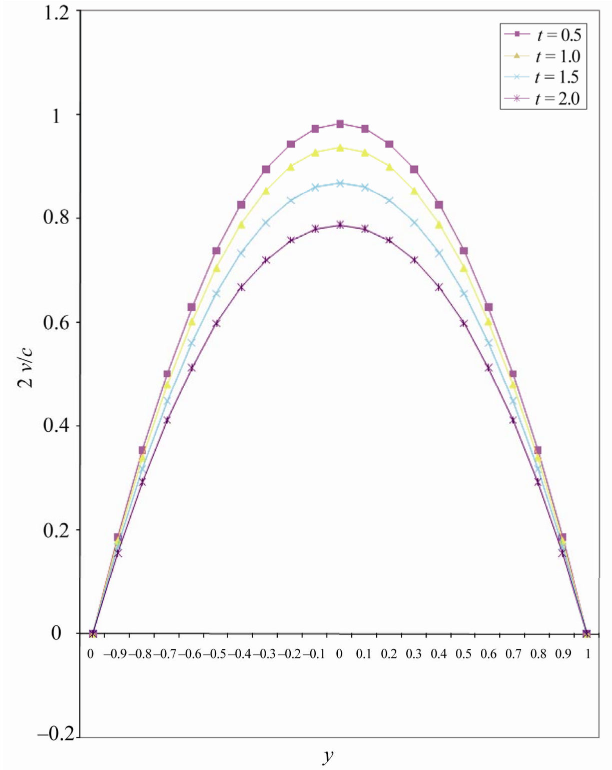

Figure 2. Show the variation of velocity profile of dust for different values of time at fixed (σ = 0.8, λ = 0.5, I = 0.5).

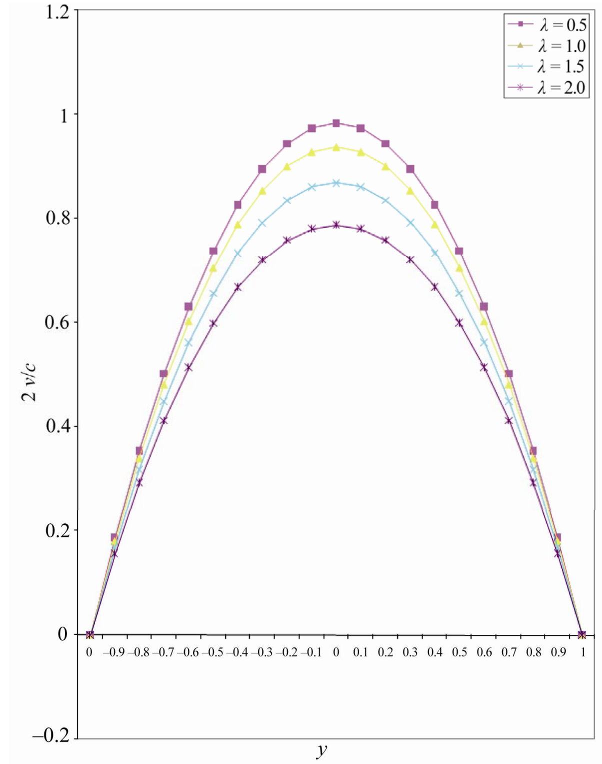

Figure 4. Show the variation of velocity profile of dust for different values of time at fixed (σ = 0.8, λ = 0.5, I = 0.5).

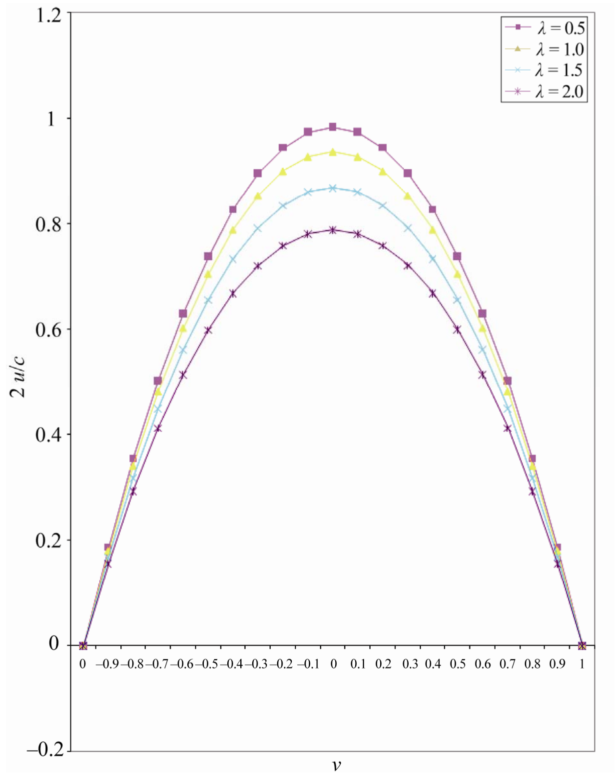

Figure 3. Show the variation of velocity profile of liquid for different values of time at fixed (σ = 0.8, λ = 0.5, I = 0.5).

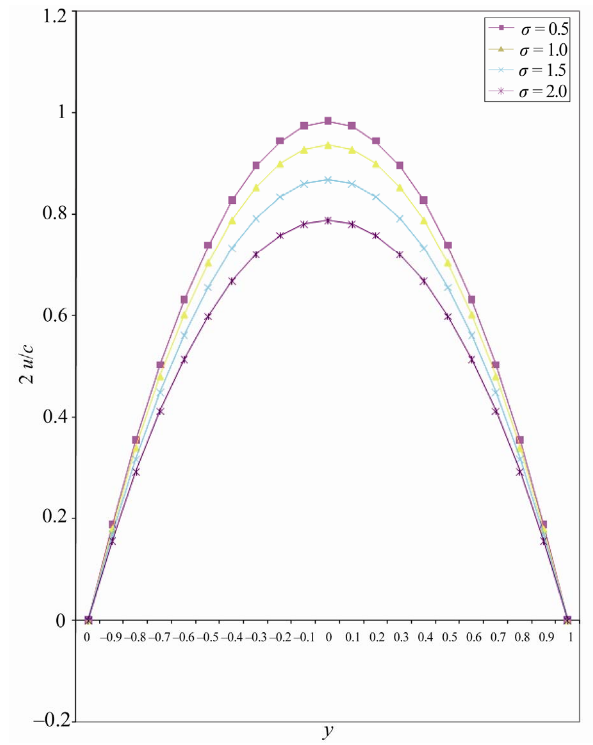

Figure 5. Show the variation of velocity profile of liquid or different values of relaxation and at fixed (λ = 0.8, t = 0.5, I = 0.5).

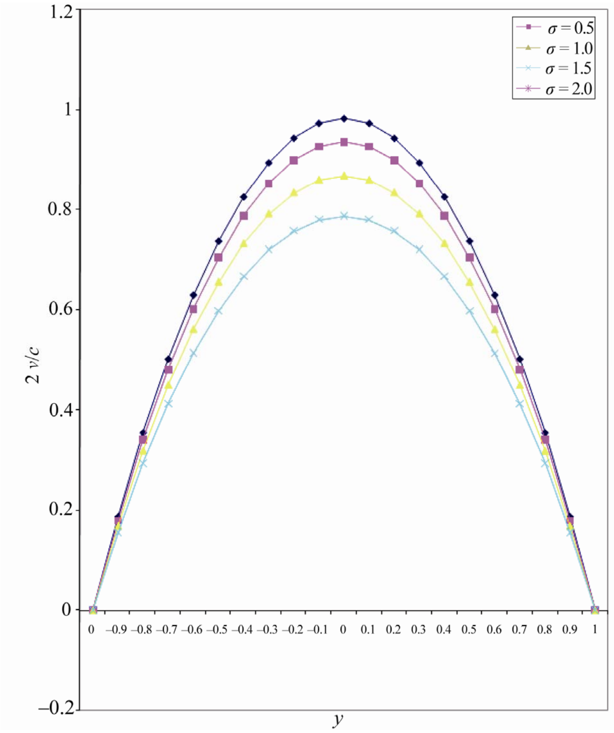

Figure 6. Show the variation of velocity profile of dust or different values of relaxation and at fixed (λ = 0.8, t = 0.5, I = 0.5).

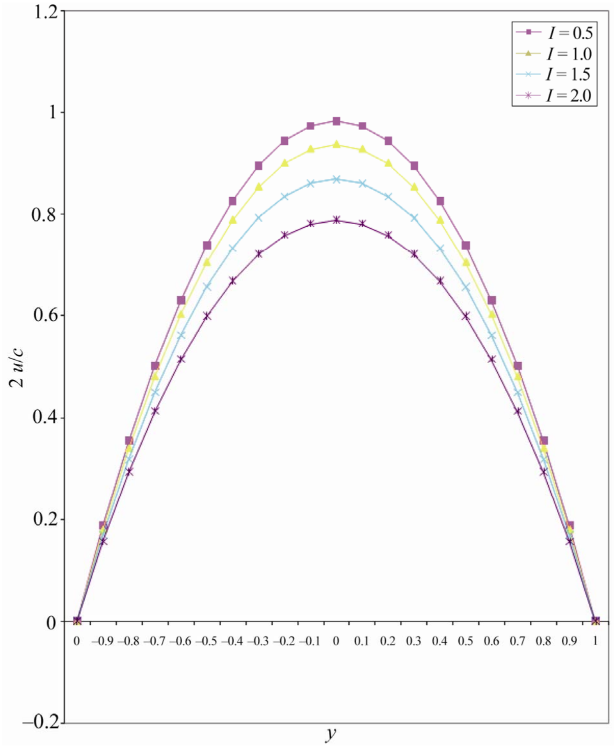

Figure 7. Show the variation of velocity profile of liquid for different values of I mass concentration and at fixed (σ = 0.8, t = 0.5, λ = 0.5).

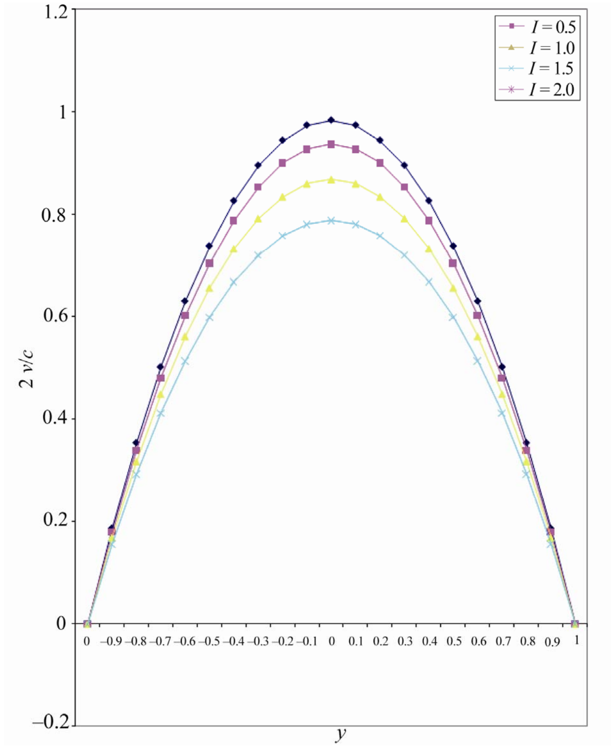

Figure 8. Show the variation of velocity profile of dust for different values of I mass concentration and at fixed (σ = 0.8, t = 0.5, λ = 0.5).

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