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Application of Iterative Approaches in Modeling the Efficiency of ARIMA-GARCH Processes in the Presence of Outliers

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Abstract

The study explored both Box and Jenkins, and iterative outlier detection procedures in determining the efficiency of ARIMA-GARCH-type models in the presence of outliers using the daily closing share price returns series of four prominent banks in Nigeria (Skye (Polaris) bank, Sterling bank, Unity bank and Zenith bank) from January 3, 2006 to November 24, 2016. The series consists of 2690 observations for each bank. The data were obtained from the Nigerian Stock Exchange. Unconditional variance and kurtosis coefficient were used as criteria for measuring the efficiency of ARIMA-GARCH-type models and our findings revealed that kurtosis is a better criterion (as it is a true measure of outliers) than the unconditional variance (as it can be depleted or amplified by outliers). Specifically, the strength of this study is in showing the applicability and relevance of iterative methods in time series modeling.

Keywords

Heteroscedasticity, Kurtosis, Model Efficiency, Outliers, Unconditional Variance, Volatility

1. Introduction

The generalized autoregressive conditional heteroscedastic (GARCH-type) models were introduced to account for heteroscedasticity (changing variance), a phenomenon which occurs as a result of violation of assumption of constant variance in time series. The GARCH-type models are further divided into symmetric and asymmetric. The symmetric GARCH models (for example ARCH and

GARCH) rely on modeling the conditional variance as a linear function of squared past residuals. The strength of this specification is in allowing the conditional variance to depend only on the modulus of the past variables (past positive and negative innovations have the same effect on the current conditional variance). The most interesting feature not addressed by GARCH model is the leverage effect which occurs when an unexpected drop in price (bad news) increases predictable volatility more than an unexpected increase in price (good news) of similar magnitude (Engle and Ng [1]; Francq and Zakoian [2]). The asymmetric specifications (for example EGARCH and GJR-GARCH) allow for the signs of the innovations (returns) to have impact on the volatility apart from magnitude.

Originally, the GARCH model was specified based on the normal distribution for the innovations yet could not capture the heavy-tailed characterizations. Similarly, the student-t distribution which is traditionally specified to remedy the weakness of the normal distribution in accommodating the heavy-tailed property, is found wanting in many applications to account for excess kurtosis and thus, the resulting estimates of GARCH models are not efficient (Moffat and Akpan [3]; Feng and Shi [4]).

Furthermore, previous studies have shown that the heavy-tailed property indicates the presence of excess kurtosis which in turn is a measure of outliers (Moffat and Akpan [5]; Cain, Zhang and Yuan [6]; Fiori and Beltrami [7]; Westfall [8]). Therefore, to completely account for excess kurtosis, it is required that outliers (which are the observations that deviate from the overall pattern of the distribution of the data) be adjusted for.

Hence, the aim of this study is to determine the efficiency of GARCH-type models with outliers taken into consideration using kurtosis coefficient which is at least approximately mesokurtic, and in particular seeks to improve on the work of Akpan, Lasisi and Adamu [9] who used the minimum unconditional variance (which is the standard measure of the variance of a variable) as a measure of efficiency of GARCH-type models in the presence of outliers. However, the major drawback to this approach is that, the unconditional variance pertaining to GARCH models fitted to the outlier contaminated series could be smaller than or equal to that of the outlier adjusted series against the expectation that the GARCH models fitted to outlier adjusted series would produce the minimum unconditional variance.

Moreover, the remaining part of this work is organized as follows; Section 2 takes care of materials and method then followed by results and discussion in Section 3, while the conclusion of overall results is handled in Section 4.

2. Materials and Methods

2.1. Return

The return series R_t can be obtained given that P_t is the price of a unit share at time, t and P_{t-1} is the share price at time $t-1$.

$$R_t = \nabla \ln P_t = (1-B)\ln P_t = \ln P_t - \ln P_{t-1} \tag{1}$$

The R_t in Equation (1) is regarded as a transformed series of the share price, P_t meant to attain stationarity, that is, both mean and variance of the series are stable (Akpan and Moffat [9]). The letter B is the backshift operator.

2.2. Autoregressive Integrated Moving Average (ARIMA) Model

Box, Jenkins and Reinsel [10] considered the extension of ARMA model to deal with homogenous non-stationary time series in which X_t, itself is non-stationary but its d^{th} difference is a stationary ARMA model. Denoting the d^{th} difference of X_t by

$$\varphi(B) = \phi(B)\nabla^d X_t = \theta(B)\varepsilon_t \tag{2}$$

where $\varphi(B)$ is the nonstationary autoregressive operator such that d of the roots of $\varphi(B)=0$ are unity and the remainder lie outside the unit circle. $\phi(B)$ is a stationary autoregressive operator.

2.3. Heteroscedastic Models

2.3.1. Autoregressive Conditional Heteroscedastic (ARCH) Model

The first model that provides a systematic framework for modeling heteroscedasticity is the ARCH model of Engle [11]. Specifically, an ARCH (q) model assumes that,

$$R_t = \mu_t + a_t,\ a_t = \sigma_t e_t,$$
$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_q a_{t-q}^2. \tag{3}$$

where $[e_t]$ is a sequence of independent and identically distributed (i.i.d.) random variables with mean zero, that is $\mathrm{E}(e_t)=0$ and variance 1, that is $\mathrm{E}(e_t^2)=1$, $\omega>0$, and $\alpha_1,\cdots,\alpha_q \ge 0$ (Francq and Zakoian [2]). The coefficients α_i, for $i>0$, must satisfy some regularity conditions to ensure that the unconditional variance of a_t is finite.

2.3.2. Generalized Autoregressive Conditional Heteroscedastic (GARCH) Model

Although the ARCH model is simple, it often requires many parameters to adequately describe the volatility process of a share price return. Some alternative models must be sought. Bollerslev [12] proposed a useful extension known as the generalized ARCH (GARCH) model. For a return series, R_t, let $a_t = R_t - \mu_t$ be the innovation at time *t*. Then, a_t follows a GARCH (q, p) model if

$$a_t = \sigma_t e_t,$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2. \tag{4}$$

where again e_t is a sequence of i.i.d. random variance with mean, 0, and variance, 1, $\omega>0, \alpha_i \ge 0, \beta_j \ge 0$ and $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$ (Tsay [13]).

Here, it is understood that $\alpha_i = 0$, for $i > p$, and $\beta_i = 0$, for $i > q$. The latter constraint on $\alpha_i + \beta_i$ implies that the unconditional variance of a_t is finite, whereas its conditional variance σ_t^2, evolves over time.

2.3.3. Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH)

Model

The EGARCH model represents a major shift from ARCH and GARCH models (Nelson [14]). Rather than modeling the variance directly, EGARCH models the natural logarithm of the variance, and so no parameter restrictions are required to ensure that the conditional variance is positive. The EGARCH (q, p) is defined as,

$$R_t = \mu_t + a_t,\ \ a_t = \sigma_t e_t,$$

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \left| \frac{a_{t-i}}{\sqrt{\sigma_{t-i}^2}} \right| + \sum_{k=1}^{r} \gamma_k \left(\frac{a_{t-k}}{\sqrt{\sigma_{t-k}^2}} \right) + \sum_{j=1}^{p} \beta_j \ln \sigma_{t-j}^2. \tag{5}$$

where again, e_t is a sequence of i.i.d. random variance with mean, 0, and variance, 1, and γ_k is the asymmetric coefficient.

2.3.4. Glosten, Jagannathan and Runkle (GJR-GARCH) Model

The GJR-GARCH (q, p) model proposed by Glosten, Jagannathan and Runkle [15] is a variant, represented by

$$a_t = \sigma_t e_t$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{i=1}^{p} \gamma_i I_{t-i} a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2. \tag{6}$$

where I_{t-1} is an indicator for negative a_{t-i}, that is,

$$I_{t-1} = \begin{cases} 0 & \text{if } a_{t-i} < 0, \\ 1 & \text{if } a_{t-i} \ge 0, \end{cases}$$

and α_i, γ_i and β_j are nonnegative parameters satisfying conditions similar to those of GARCH models. Also the introduction of indicator parameter of leverage effect, I_{t-1} in the model accommodates the leverage effect, since it is supposed that the effect of a_{t-i}^2 on the conditional variance σ_t^2 is different accordingly to the sign of a_{t-i}.

To successfully fit the processes describe by subsections 2.2 and 2.3, the approach is based on Box and Jenkins three iterative procedures which includes model identification, model estimation and diagnostic checking are summarized as follows.

Identification Stage: employs the plots of estimated acf (autocorrelation function) and pacf (partial autocorrelation function) as guides to selecting one or more ARIMA models that appears suitable. At this stage, models whose theoretical acf and pacf most closely resemble the estimated acf and pacf are selected tentatively.

Estimation Stage: The tentatively entertained model is fitted to data and its parameters estimated using maximum likelihood techniques.

Diagnostic checking Stage: Diagnostic checks are carried to help determine if an estimated model is statistically significant targeted at uncovering possible lack-of-fit. If no lack-of-fit is indicated, the model is ready to use. If any inadequacy is found, the iterative cycle of identification, estimation, and diagnostic checking is repeated until a suitable representation is found. (See Box, Jenkins and Reinsel [10]; Akpan, Lasisi and Adamu [8] for more details on the procedures and its application, respectively.)

2.4. Outliers in Time Series

Generally, a time series might contain several, say k outliers of different types and we have the following general outlier model;

$$Y_t = \sum_{j=1}^{k} \tau_j V_j(B) I_t^{(T)} + X_t, \tag{7}$$

where $X_t = (\theta(B))/(\varphi(B)) a_t$, $V_j(B) = 1$ for an AO, and $V_j(B) = \dfrac{\theta(B)}{\varphi(B)}$ for an IO at $t = T_j$, $V_j(B) = (1-B)^{-1}$ for a LS, $V_j(B) = (1-\delta B)^{-1}$ for an TC, and τ is the size of outlier. For more details on the types of outliers and estimation of the outliers effects (see Moffat and Akpan [16]; Sanchez and Pena [17]; Box, Jenkins and Reinsel [10]; Wei [18]; Chen and Liu [19]; Chang, Tiao and Chen [20]).

Moreover, in financial time series, the residual series, a_t is assumed to be uncorrelated with its own past, so additive, innovative, temporary change and level shift outliers coincide, and where both the mean and variance equations evolves together, we have for example GARCH(1,1) model:

$$R_t - \mu_t = \tilde{a}_t + \tau I_t^{(T)}. \tag{8}$$

$$\tilde{a}_t = \sigma_t e_t. \tag{9}$$

$$\sigma_t^2 = \omega + \alpha_1 \tilde{a}_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{10}$$

where $\tilde{a}_t$ is the outliers contaminated residuals.

2.5. Methods of Outliers Detection in Heteroscedasticity

One approach for correcting the series for outliers is using standard criteria and then estimates the conditional variance. This approach involves detecting and correcting of outliers before estimating the conditional variance (Carnero, Pena and Ruiz [21]). This very method is based on the iterative framework of Chen and Liu [19] summarized in the following steps;

Step I: Given an ARIMA model fitted to the data, all the potential outliers are detected based on preliminary model parameter estimates.

Step II: Joint estimates of the model parameters and outliers effects are obtained using the accommodated outlier information of step I.

Step III: Outliers are identified and their effects estimated again based on the adjusted estimates of model parameters obtained in Step II.

2.6. Efficiency of Heteroscedastic Models

Efficiency is a measure of quality of an estimator of a model. It is often expressed using variance or mean square error. For the purpose of this study which looks at a unified effect of outliers, unconditional variance and coefficient of kurtosis are considered as the measures of efficiency of estimator of heteroscedastic model. The application of coefficient of kurtosis in this case is to ensure that the existence of heavy-tailed is taken care of.

For ARCH(q) model which is equivalent to GARCH(q, 0) model, the unconditional variance is given as follows:

$$\sigma^2 = \frac{\omega}{1-\sum_{i=1}^{q}\alpha_i}. \tag{11}$$

For GARCH(q, p) model, the unconditional variance is expressed thus:

$$\sigma^2 = \frac{\omega}{1-\sum_{i=1}^{q}\alpha_i-\sum_{j=1}^{p}\beta_i}. \tag{12}$$

For EGARCH(q, p) model, the unconditional variance is expressed as follows:

$$\sigma^2 = \exp\left(\frac{\omega}{1-\sum_{j=1}^{p}\beta_i}\right), \tag{13}$$

where exp represent natural exponential function.

For GJR-GARCH(q, p) model

$$\sigma^2 = \frac{\omega}{1-\sum_{i=1}^{q}\alpha_i-\sum_{i=1}^{q}\frac{\gamma_i}{2}-\sum_{j=1}^{p}\beta_i}. \tag{14}$$

2.7. Kurtosis

Kurtosis coefficient for a centered (that is, zero-mean) distribution is defined as the ratio of the fourth-order moment, which is assumed to exist, to the squared second-order moment (Francq and Zakoian [2]). This coefficient is equal to 3 for a normal distribution (mesokurtosis). For heavy-tailed distribution, the coefficient is greater than 3 (leptokurtosis). The excess kurtosis of GARCH(1,1) model can be obtained as follows:

$$a_t = \sigma_t e_t, \tag{15}$$

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{16}$$

Note that, $\mathrm{E}(e_t)=0$, $\mathrm{Var}(e_t)=1$, and $\mathrm{E}(e_t^4)=K_e+3$, where K_e is the excess kurtosis of the innovation, e_t.

Also,

$$\mathrm{Var}(a_t)=\mathrm{E}(\sigma_t^2)=\frac{\omega}{\left[1-(\alpha_1+\beta_1)\right]}. \tag{17}$$

$\mathrm{E}(a_t^4)=(K_e+3)\mathrm{E}(\sigma_t^4)$ provided that $\mathrm{E}(\sigma_t^4)$ exists.

But,

$$\mathrm{E}\left(\sigma_t^4\right)=\frac{\omega^2\left(1+\alpha_1+\beta_1\right)}{\left[1-\left(\alpha_1+\beta_1\right)\right]\left[1-\alpha_1^2\left(K_e+2\right)-\left(\alpha_1+\beta_1\right)^2\right]}, \tag{18}$$

provided that $1>\alpha_1+\beta_1\geq 0$ and $1-\alpha_1^2\left(K_e+2\right)-\left(\alpha_1+\beta_1\right)^2>0$. the excess kurtosis of a_t, if it exists, is then

$$K_a=\frac{\mathrm{E}\left(a_t^4\right)}{\left[\mathrm{E}\left(a_t^2\right)\right]^2}-3=\frac{\left(K_e+3\right)\left[1-\left(\alpha_1+\beta_1\right)^2\right]}{1-2\alpha_1^2-\left(\alpha_1+\beta_1\right)^2-K_e\alpha_1^2}-3. \tag{19}$$

This excess kurtosis can be written in an informative expression. Considering the case where e_t follows a normal distribution, $K_e=0$,

$$K_a^{(g)}=\frac{6\alpha_1^2}{1-2\alpha_1^2-\left(\alpha_1+\beta_1\right)^2}, \tag{20}$$

where the superscript, *g*, is used to denote the Gaussian distribution. The same idea applies to other GARCH-type models (Tsay [13]).

3. Results and Discussion

3.1. Data

Data collection is based on secondary source as documented in the records of Nigerian Stock Exchange. The documented data on the daily closing share prices of the sampled banks (Skye bank, Sterling bank, Unity bank and Zenith bank) from January 3, 2006 to November 24, 2016 were purchased from the Nigerian Stock Exchange and delivered through contactcentre@nigerianstockexchange.com. Since the data were obtained from a credible and secured source therefore reliable.

3.2. Interpretation of Time Plot

The share prices of the four prominent Nigerian banks considered are found to be nonstationary given the random fluctuations away from the common mean (see Figures 1-4).

To achieve stationarity, Equation (1) was applied to the share price series and these transformed series were found to cluster round the common mean and thus indicated the presence of heteroscedasticity (see Figures 5-8).

3.3. Modeling Joint ARIMA-GARCH-Type Processes of Return Series of Nigerian Banks

Based on Box and Jenkins procedures, out of the several models identified tentatively, the following joint ARIMA-GARCH-type models with respect to both normal (norm) and student-t (std) distributions in (Table 1) were considered and selected on the grounds of smallest information criteria and model adequacy (see Table 2).

3.4. Identification of Outliers in the Residual Series of ARIMA Models Fitted to the Return Series of Nigerian Bank

Here, we examined the residuals series of the fitted ARIMA models for detection

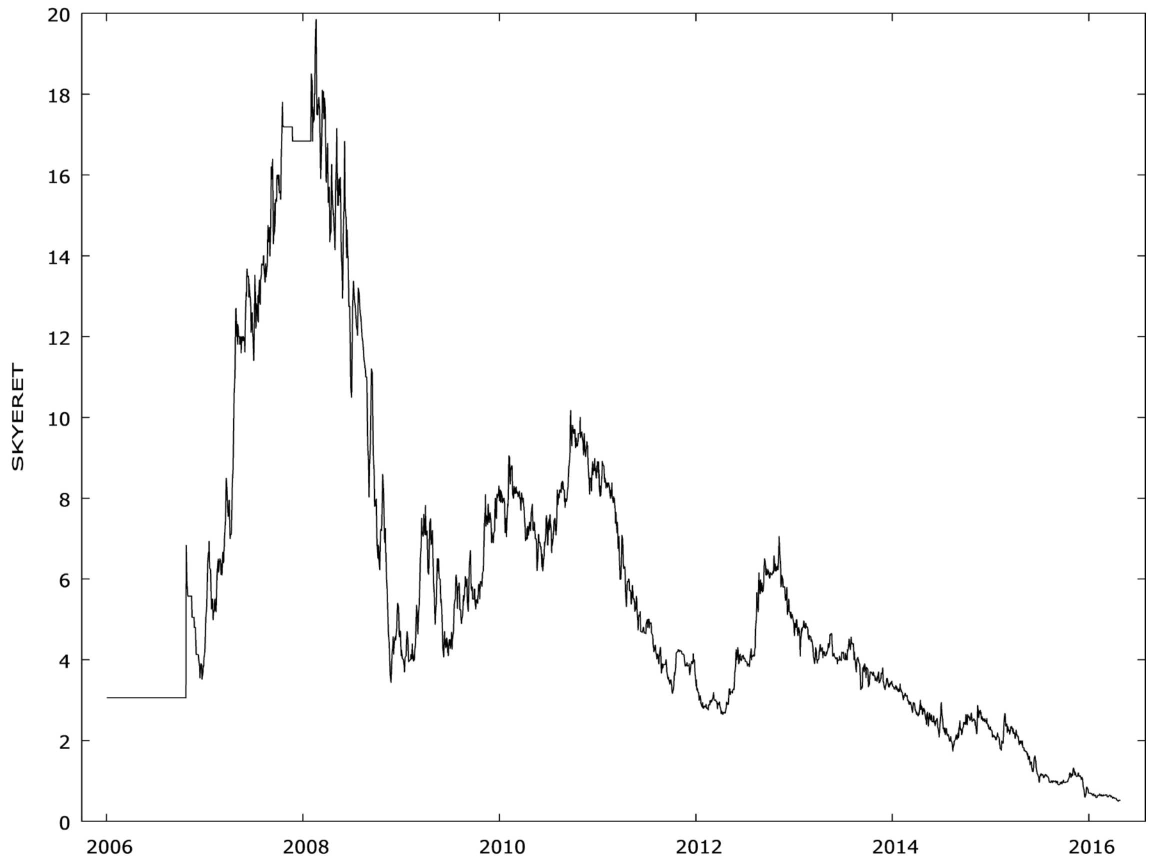

Figure 1. Share price series of Skye Bank.

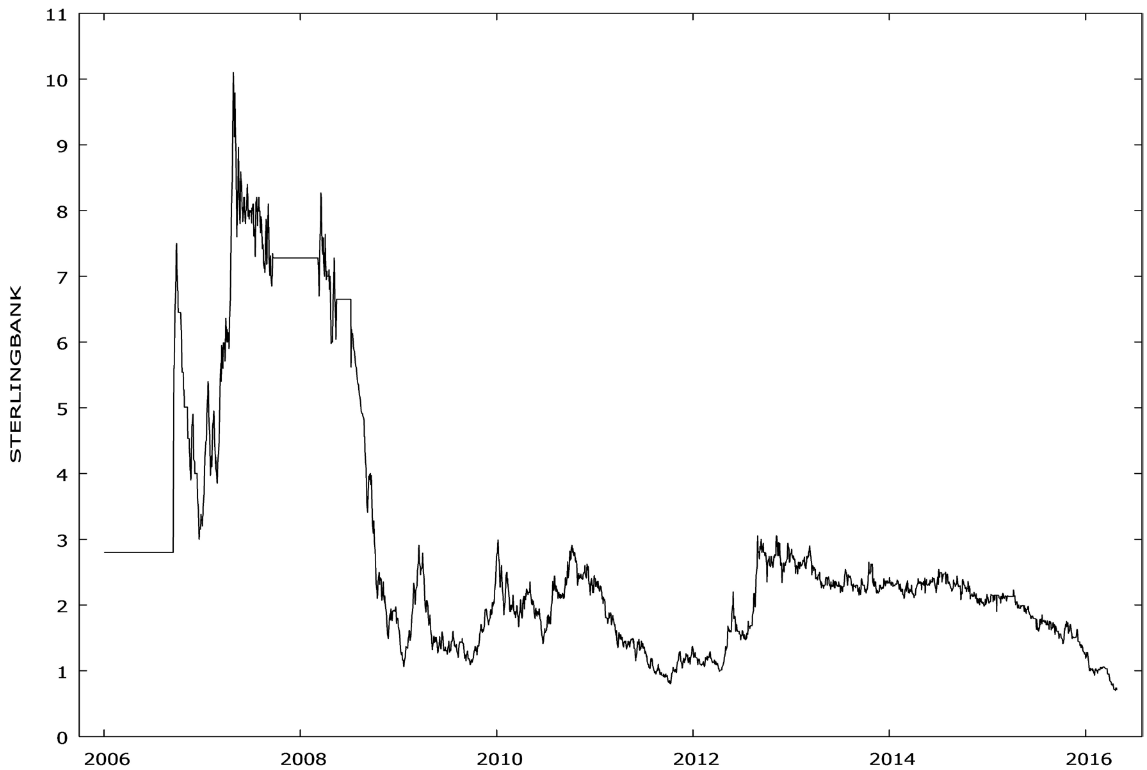

Figure 2. Share price series of Sterling Bank.

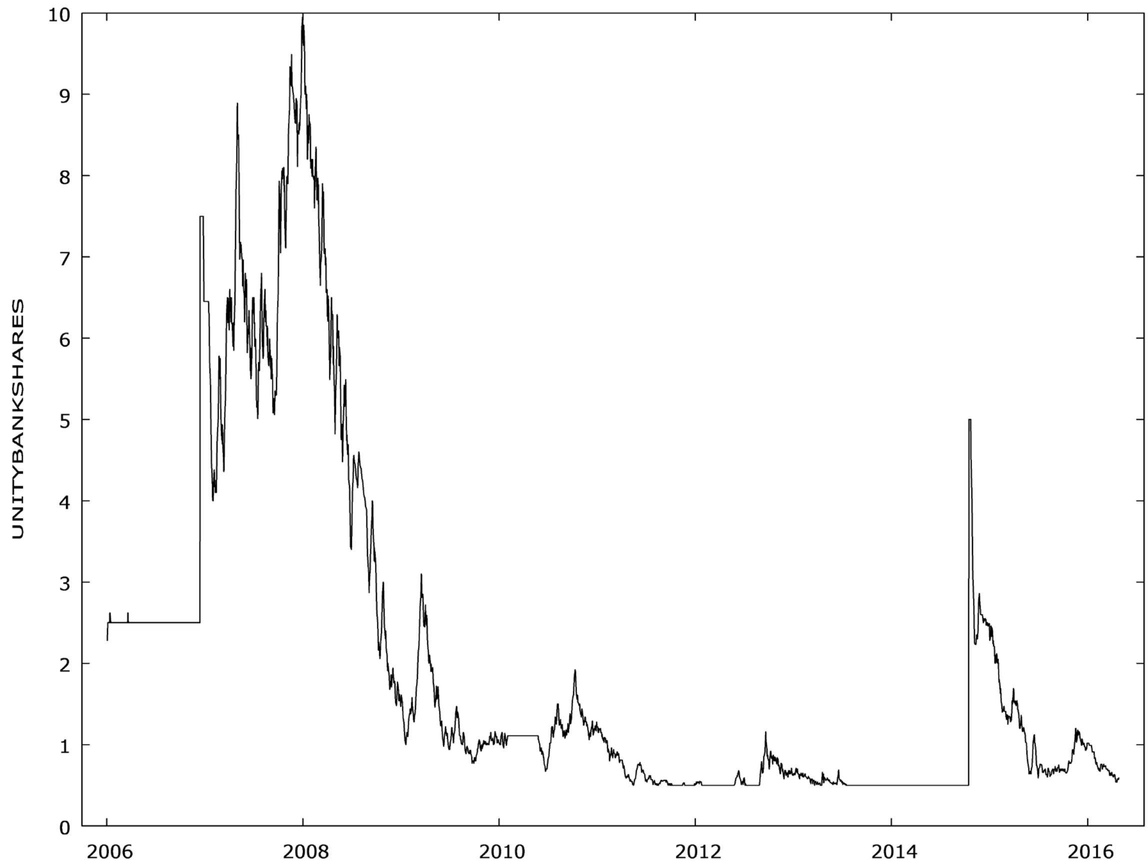

Figure 3. Share price series of Unity Bank.

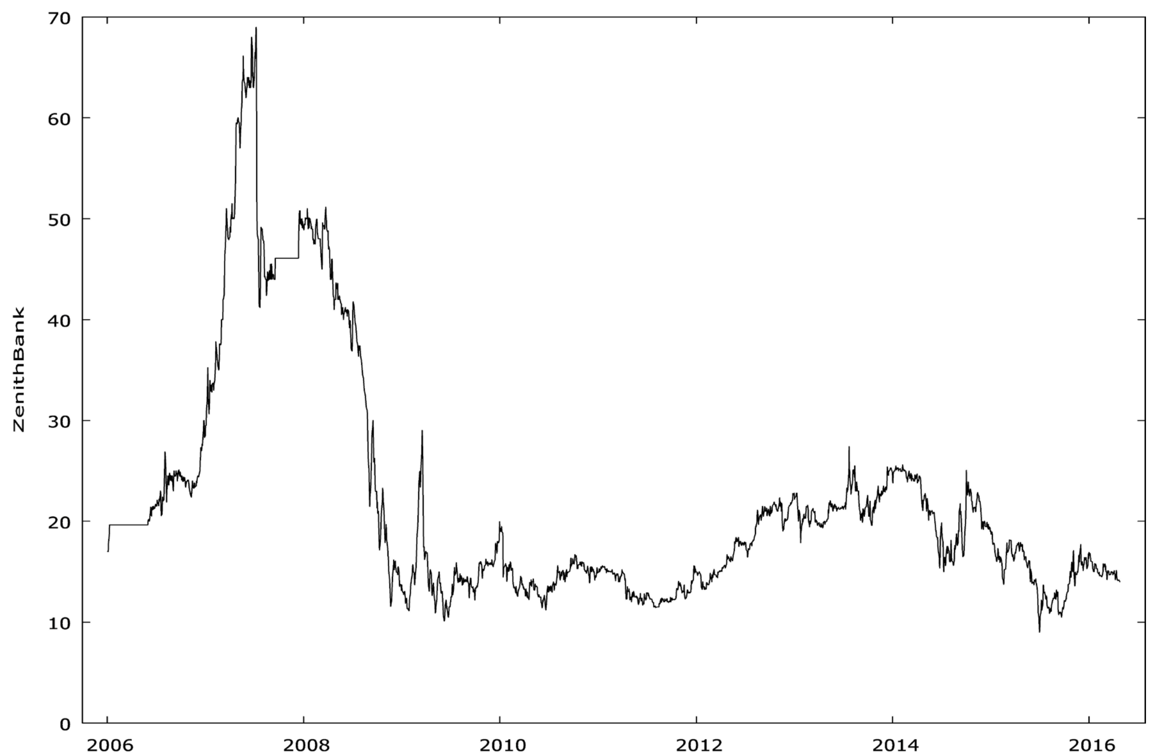

Figure 4. Share price series of Zenith Bank.

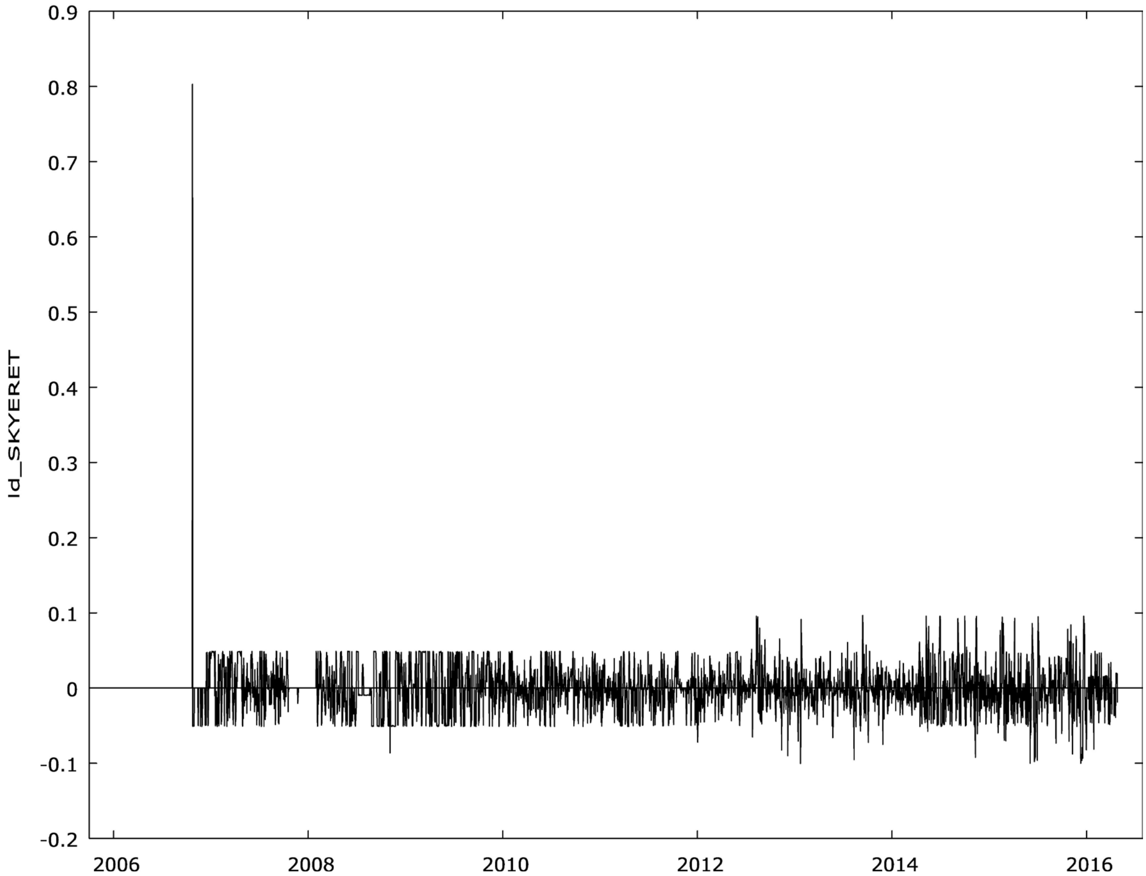

Figure 5. Return series of Skye Bank.

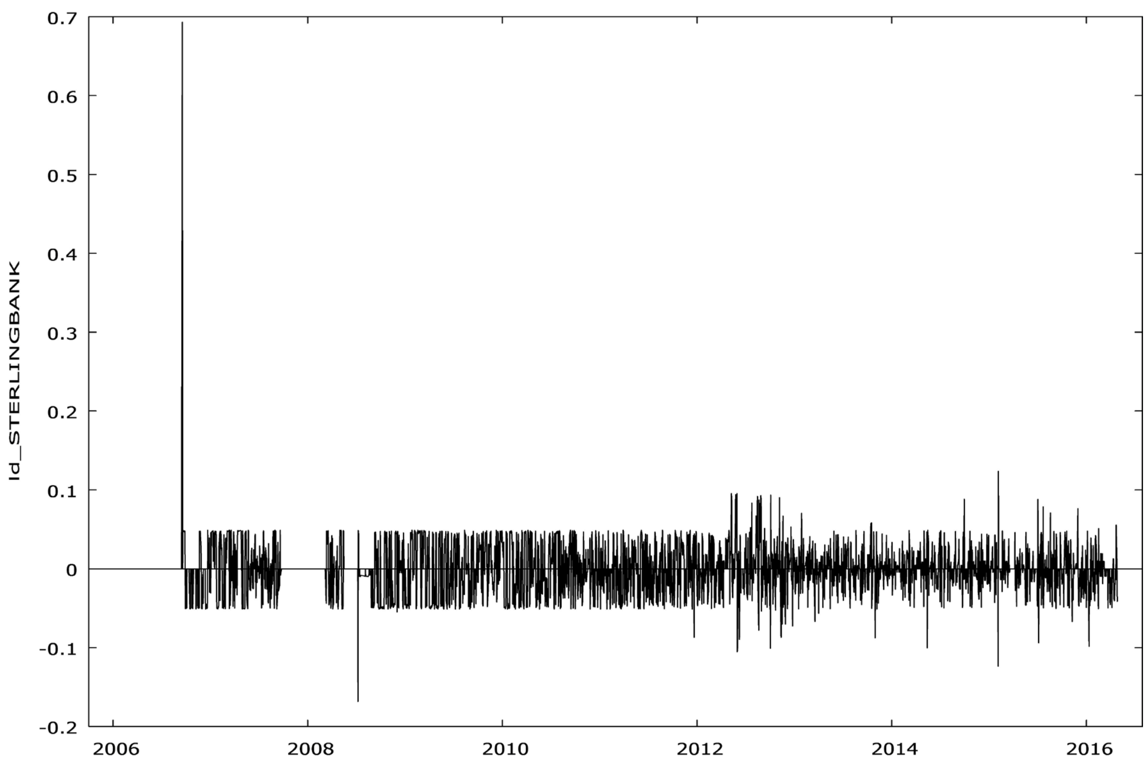

Figure 6. Return series of Sterling Bank.

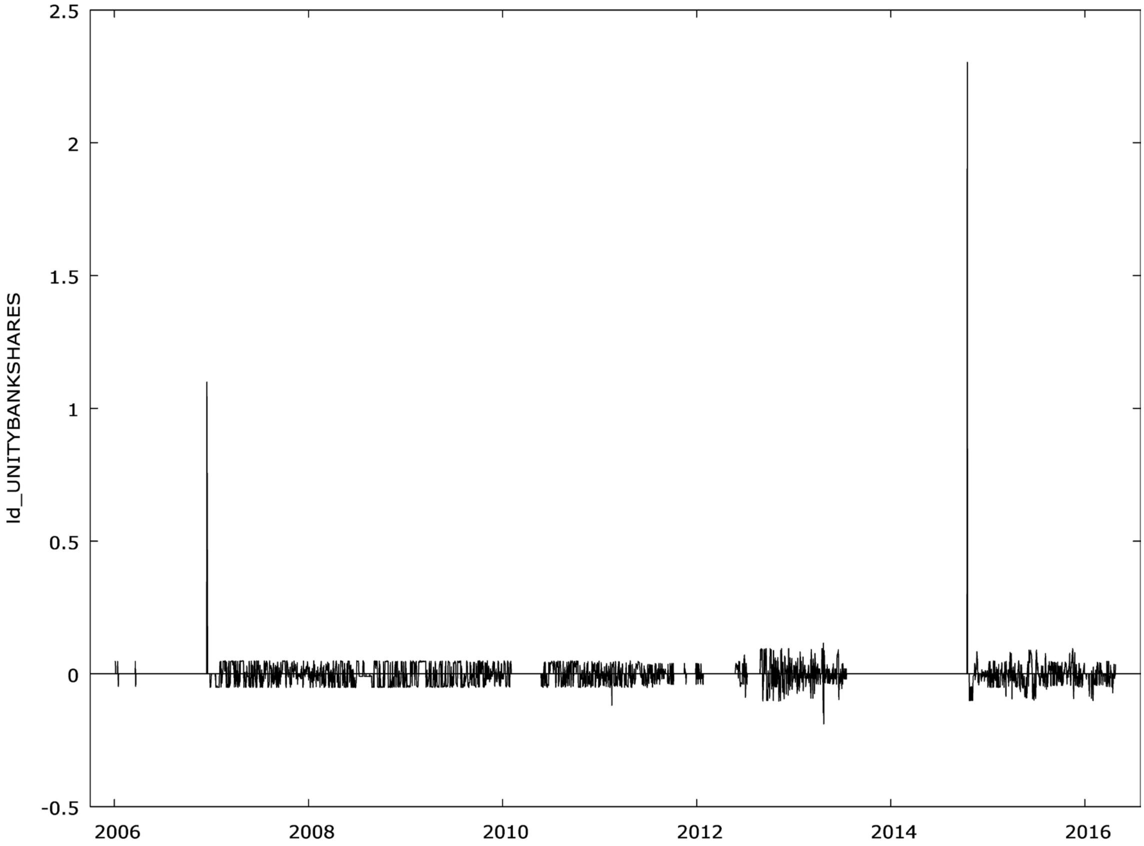

Figure 7. Return series of Unity Bank.

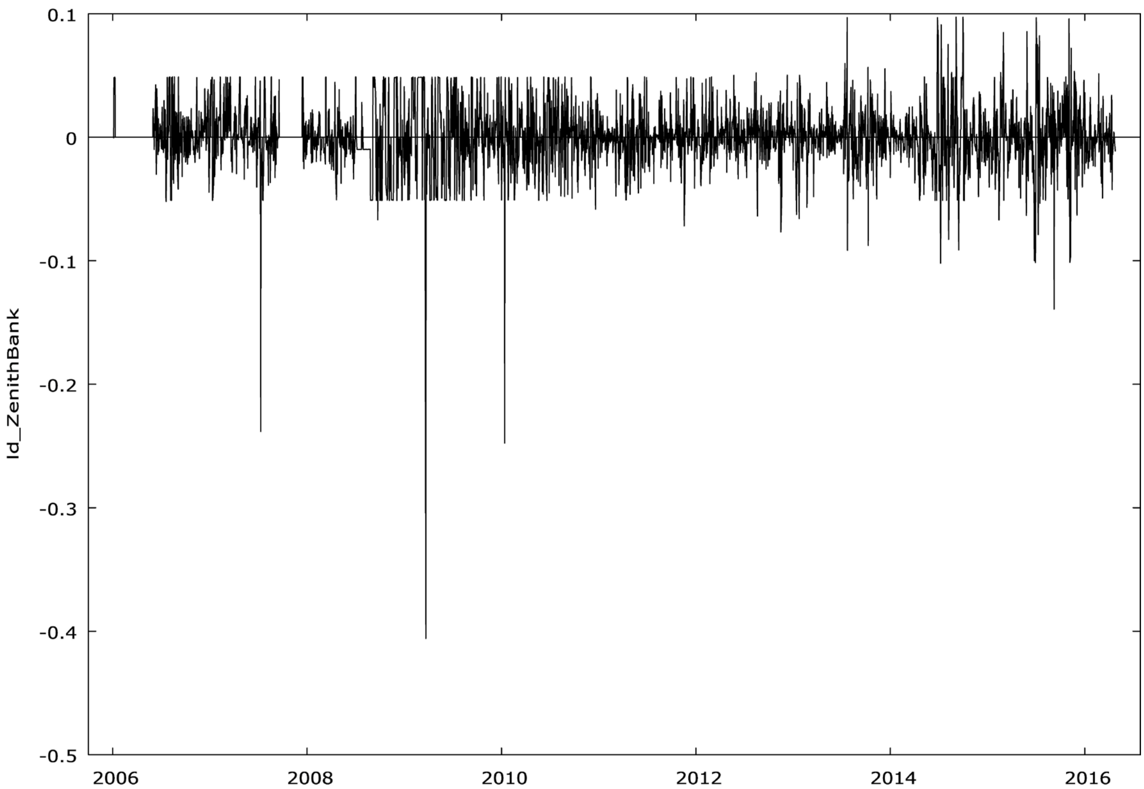

Figure 8. Return series of Zenith Bank.

Table 1. Output of ARIMA-GARCH-type models of returns series of Nigerian Banks.

Bank	Model	Parameter	Estimate	s.e	t-ratio	p-value	Information Criteria		
							AIC	BIC	HQIC
SKYE	ARIMA(1,1,0)-GARCH(1,1)-norm	μ	$-2.6e^{-5}$	$1.1e^{-5}$	−2.3973	0.0165			
		φ_1	$3.87e^{-4}$	0.0214	0.0181	0.9855			
		ω	$2.0e^{-6}$	0.0000	73.3223	0.0000	−4.5293	−4.5183	−4.5253
		α_1	0.1305	$7.996e^{-3}$	16.3261	0.0000			
		β_1	0.8685	$6.671e^{-3}$	130.1908	0.0000			
STERLING	ARIMA(2,1,0)-EGARCH(1,1)-norm	μ	$-2.89e^{-4}$	$1.57e^{-4}$	−1.8335	0.06673			
		φ_1	−0.0190	0.0102	−1.8642	0.0623			
		φ_2	0.0298	$5.205e^{-3}$	5.7258	0.0000			
		ω	−0.0943	$1.607e^{-3}$	−58.7069	0.0000	−4.3888	−4.3734	−4.3832
		α_1	−0.0734	$4.024e^{-3}$	−18.2286	0.0000			
		β_1	0.9859	$1.7e^{-5}$	5863.5496	0.0000			
		γ_1	0.0975	$3.588e^{-3}$	27.1638	0.0000			
UNITY	ARIMA(0,1,1)-GARCH (1,1)-norm	μ	$1.38e^{-4}$	$1.9e^{-5}$	7.1606	0.0000			
		θ_1	0.1009	0.0236	4.2767	$1.9e^{-5}$			
		ω	$4.0e^{-6}$	0.0000	115.7800	0.0000	−4.8711	−4.8601	−4.8671
		α_1	0.2368	0.0117	20.2352	0.0000			
		β_1	0.7622	$7.927e^{-3}$	96.1509	0.0000			
ZENITH	ARIMA(2,1,1)-EGARCH(1,1)-std	μ	0.0000	0.0000	1.2065	0.2276			
		φ_1	−0.3086	0.0106	−29.0801	0.0000			
		φ_2	0.0495	0.0171	2.8972	0.0038			
		θ_1	0.2912	0.0105	27.7875	0.0000	−7.0189	−6.9992	−7.0118
		ω	−0.0291	$8.62e^{-4}$	−33.7888	0.0000			
		α_1	−0.6979	$9.8e^{-5}$	−7150.4179	0.0000			
		β_1	0.9996	$6.3e^{-5}$	15825.7924	0.0000			
		γ_1	0.6983	$9.8e^{-5}$	7147.6333	0.0000			

of possible potential outliers in the returns series of the banks under study. The iterative procedure of Chen and Liu [19] was applied and those statistics that are in absolute value higher than a threshold (critical value, C) identify the time point of a potential outlier. In this study, C = 4 is chosen on the condition that the number of observations, T ≥ 450 and where C = 4 is not sufficient, C = 5 is used.

Table 2. Diagnostic Checking for ARIMA-GARCH-type models of returns series of Nigerian Banks

Bank	Model	Standardized Residuals			Standardized Squared Residuals					
		Lag	Weighted LB	p-value	Lag	Weighted LB	p-value	Lag	Weighted ARCH –LM	p-value
SKYE	ARIMA(1,1,0)-GARCH(1,1)-norm	1	0.0132	0.9085	1	0.0004	0.9833	3	0.0005	0.9829
		2	0.015	1.0000	5	0.0014	1.000	5	0.0011	1.0000
		5	0.0237	1.0000	9	0.0023	1.000	7	0.0016	1.0000
STERLING	ARIMA(2,1,0)-EGARCH(1,1)-norm	1	0.0944	0.7587	1	0.0003	0.9851	3	0.0004	0.9837
		5	0.1192	1.0000	5	0.0012	1.0000	5	0.0010	1.0000
		9	0.1588	1.0000	9	0.0021	1.0000	7	0.0015	1.0000
UNITY	ARIMA(0,1,1)-GARCH (1,1)-norm	1	0.0031	0.9557	1	0.0008	0.9776	3	0.0008	0.9776
		2	0.0031	1.0000	5	0.0024	1.0000	5	0.0019	1.0000
		5	0.0035	1.0000	9	0.0039	1.0000	7	0.0028	1.0000
ZENITH	ARIMA(2,1,1)-EGARCH(1,1)-std	1	0.0014	0.9697	1	0.0014	0.9704	3	0.0014	0.9704
		8	0.0066	1.0000	5	0.0041	1.0000	5	0.0033	0.9999
		14	0.0111	1.0000	9	0.0069	1.0000	7	0.0049	1.0000

LB = Ljung-Box, LM = Lagrange Multiplier.

3.4.1. Identification of Outliers in the Residual Series of ARIMA (1, 1, 0) Model Fitted to the Return Series of Skye Bank

About twenty six (26) different outliers were identified to have contaminated the residuals series of ARIMA(1,1,0) model using the critical value, C = 4; six (6) innovation outliers (IO), six (6) additive outliers (AO) and fourteen (14) temporary change (TC) as indicated in (Table 3).

3.4.2. Identification of Outliers in the Residual Series of ARIMA (2, 1, 0) Model Fitted to the Return Series of Sterling Bank

About seven (7) different outliers were identified to have contaminated the residual series of ARIMA(2,1,0) model using the critical value, C = 5 one (1) innovation outlier (IO), four (4) additive outliers (AO) and two (2) temporary change (TC) as shown in (Table 4).

3.4.3. Identification of Outliers in the Residual Series of ARIMA (1, 1,0) Model Fitted to the Return Series of Unity Bank

About thirty three (33) different outliers were identified to have contaminated the residual series of ARIMA(1,1,0) model using the critical value, C = 5; two (2) innovation outliers (IO), six (6) additive outliers, fifteen (15) temporary change (TC) and ten (10) level shift (LS) as indicated in (Table 5).

3.4.4. Identification of Outliers in the Residual Series of ARIMA (2, 1, 1) Model Fitted to the Return Series of Zenith Bank

About forty two (42) different outliers were identified to have contaminated the residual series of ARIMA(2,1,1) model using the critical value, C = 5; thirteen (13) innovation outliers (IO), nine (9) additive outliers and twenty (20) temporary change (TC) as shown in (Table 6).

Table 3. Outliers identified in the residual series of ARIMA(1, 1, 0) model fitted to return series of Skye Bank.

Type	Observation index	Location	Estimate	T-statistic
IO	211	13/11/2006	−0.20150630	−8.487698
IO	1841	21/06/2013	−0.10241849	−4.313995
IO	1843	25/06/2013	0.09870872	4.157735
IO	2178	31/10/2014	0.10295915	4.336768
IO	2263	05/05/2015	0.09758512	4.110407
IO	210	10/11/2006	0.81215294	34.804236
AO	1726	04/01/2013	0.09492679	4.068020
AO	1984	21/01/2014	−0.10371058	−4.444443
AO	2281	31/05/2015	0.09978000	4.276001
AO	2414	15/10/2015	0.10169110	4.357900
AO	2456	14/12/2015	−0.09871728	−4.230459
AO	209	09/11/2006	0.20948475	10.861708
TC	740	09/01/2009	−0.09161349	−4.750126
TC	742	12/01/2009	−0.07866550	−4.078778
TC	827	18/05/2009	0.07862559	4.076708
TC	1723	13/12/2012	0.08946532	4.638744
TC	2311	15/05/2015	0.09068887	4.702185
TC	2381	26/08/2015	0.09747559	5.054074
TC	2468	05/01/2016	−0.10240036	−5.309421
TC	2590	29/06/2016	−0.08395679	−4.353129
TC	2592	01/07/2016	−0.12692535	−6.581033
TC	2599	15/07/2016	0.10544854	5.467469
TC	2314	20/05/2015	−0.10163346	−4.363007
TC	212	14/11/2006	−0.10846069	−5.731470
TC	741	12/01/2009	0.07648229	4.225631
TC	2589	28/06/2016	0.07256176	4.009023

Table 4. Outliers identified in the residual series of ARIMA(2, 1, 0) model fitted to return series of Sterling Bank.

Type	Observation index	Location	Estimate	T-statistic
AO	184	03/10/2006	0.6913146	27.668748
AO	655	02/09/2008	−0.1764822	−7.063413
AO	2371	12/08/2015	−0.1398721	−5.598155
TC	183	29/09/2006	0.2415834	11.950355
TC	1672	22/05/2012	−0.1075351	−5.319415
IO	185	04/10/2006	−0.1964258	−7.900298
AO	2372	12/08/2015	0.1407112	5.678009

Table 5. Outliers identified in the residual series of ARIMA(1, 1, 0) model fitted to return series of Unity Bank.

Type	Observation index	Location	Estimate	T-statistic
IO	2293	20/04/2015	−0.180979695	−7.444781
AO	248	10/01/2007	1.098612289	45.331893
AO	1906	24/09/2013	−0.200532990	−8.274566
AO	2292	17/04/2015	2.302585093	95.011264
TC	247	09/01/2007	0.365004553	19.903211
TC	1736	18/01/2013	0.107790532	5.877674
TC	1745	01/02/2013	0.112419182	6.130068
TC	1753	13/02/2013	−0.118923561	−6.484743
TC	1762	26/02/2013	0.091895380	5.010932
TC	2291	16/04/2015	0.758297010	41.348923
TC	2298	27/04/2015	−0.142415961	−7.765752
TC	2304	06/04/2015	−0.098876918	−5.391626
TC	2446	30/11/2015	−0.093629262	−5.105479
TC	2458	16/12/2015	0.112419118	6.130064
TC	2460	18/12/2015	0.104980801	5.724463
TC	2467	04/01/2016	−0.106045605	−5.782525
TC	2469	06/01/2016	−0.119002493	−6.489047
IO	1905	23/09/2013	0.127354627	5.132279
AO	1904	20/09/2013	−0.163097022	−6.592937
LS	243	29/12/2006	−0.003141767	−5.772181
LS	251	15/01/2007	−0.002771753	−5.084049
LS	347	11/06/2007	−0.002837928	−5.102009
LS	520	19/06/2008	−0.003114010	−5.387808
LS	598	13/06/2008	−0.003027395	−5.143002
LS	613	04/07/2008	−0.003035068	−5.137530
LS	631	30/07/2008	−0.002988789	−5.037234
LS	635	05/08/2008	−0.003001055	−5.052994
LS	2286	09/04/2015	−0.008202488	−6.130741
TC	1901	17/09/2013	0.097493034	5.208012
TC	2477	18/01/2016	0.096324642	5.145597
LS	607	26/06/2008	0.022239276	28.623733
AO	1336	09/06/2011	−0.128187865	−5.110079
AO	1872	05/08/2013	−0.144642972	−5.766045

Table 6. Outliers identified in the residual series of ARIMA(2, 1, 1) model fitted to return series of Zenith Bank.

Type	Observation index	Location	Estimate	T-statistic
IO	396	17/08/2007	−0.09816377	−13.221339
IO	840	05/06/2009	0.04253167	5.728444
IO	2221	06/01/2015	−0.03927918	−5.29037
IO	2263	05/03/2015	0.04378397	5.897112
IO	2281	18/03/2013	0.03787680	5.101495
IO	2473	12/01/2016	−0.03936074	−5.301362
IO	2475	14/01/2016	−0.04230834	−5.698364
IO	2525	24/03/2016	−0.06416968	−8.642792
IO	2565	24/05/2016	0.04357221	5.868590
IO	2568	27/05/2016	−0.03892676	−5.242911
AO	839	04/06/2009	−0.17023181	−23.610345
AO	1051	13/04/2010	−0.10685231	−14.819909
AO	1971	31/12/2013	−0.04930715	−6.838668
AO	2027	21/03/2014	−0.04348949	−6.031786
AO	2223	08/01/2015	0.04365213	6.054343
AO	2389	07/09/2015	0.03931499	5.452803
AO	2453	09/12/2015	0.04052320	5.620376
AO	2483	26/01/2016	−0.03680867	−5.105188
TC	395	16/08/2007	−0.04944665	−8.279793
TC	691	27/10/2008	−0.03018987	−5.055264
TC	710	21/11/2008	−0.03064893	−5.132133
TC	747	20/01/2009	−0.03068035	−5.137395
TC	802	08/04/2009	0.02900192	5.029855
TC	838	03/06/2009	−0.05601975	−9.380451
TC	2477	18/01/2016	0.03332783	5.580712
IO	1970	30/12/2013	0.03696756	5.089711
IO	2269	13/03/2015	−0.03667538	−5.049484
TC	698	05/11/2008	0.03138467	5.372151
TC	754	29/01/2009	0.03144461	5.382412
TC	802	08/04/2009	0.02900192	5.029855
IO	2569	31/05/2016	−0.04117313	−5.684147
TC	394	15/08/2007	0.03079012	5.941387
TC	833	26/05/2009	0.02933330	5.660273
TC	850	19/06/2009	−0.02610763	−5.037836
TC	2212	18/12/2014	0.02755604	5.317326

Continued

AO	2282	25/01/2016	0.03601640	5.029504
TC	2484	27/01/2016	0.02586343	5.048003
TC	824	13/05/2009	0.02437288	5.000302
TC	890	14/08/2009	−0.02533852	−5.198411
TC	2217	29/12/2014	−0.02467081	−5.061424
TC	2450	04/12/2015	−0.02471591	−5.070677
TC	919	28/09/2009	0.02369936	5.067229

3.5. Modeling Joint ARIMA-GARCH-Type Processes of Outlier Adjusted Return Series of Nigerian Banks

However, with the identified outliers being adjusted for, we obtained a new series (outlier adjusted return series). Again, Box and Jenkins iterative procedures were applied and those models in (**Table 7**) were selected based on smallest information criteria and model adequacy (**Table 8**).

3.6. Determination of Efficiency of ARIMA-GARCH-Type Models of Returns Series of Nigerian Banks

The ARIMA(1, 1, 0)-GARCH(1, 1)-norm model fitted to both outlier adjusted return series and outlier contaminated series with the same unconditional variance, 0.0016. Again, the value of kurtosis, 2.9465 captured by GARCH(1, 1)-norm model on outlier adjusted return series is nearly the value accommodated by the normal distribution while GARCH(1, 1)-norm model on outlier contaminated series seems inferior with excess kurtosis of 132.8707 (see **Table 9**).

From (**Table 10**), evidence shows that ARIMA(2, 1, 2)-EGARCH(1, 1)-std model fitted to outlier adjusted return series appeared to be more efficient given that the unconditional variance and kurtosis coefficient are smaller than that of the ARIMA(2, 1, 0)-EGARCH(1, 1)-norm model fitted to the outlier contaminated series.

From (**Table 11**), evidence shows that ARIMA(1, 1, 0)-GJR-GARCH(1, 0)-norm model fitted to outlier adjusted return series appeared to be more efficient given that the unconditional variance and kurtosis value are smaller than that of the ARIMA(0,1,1)-GARCH(1,1)-norm model fitted to the outlier contaminated series.

The ARIMA(2, 1, 1)-EGARCH(1, 1)-std model fitted to both outlier adjusted return series and outlier contaminated series converges to respective unconditional variances of $5.118125e^{-5}$ and $5.887684e^{-36}$ with corresponding kurtosis values of 3.5746 and 26.3794. Though, the unconditional variance of ARIMA(2, 1, 1)-EGARCH(1, 1)-std model fitted to the outlier contaminated series is smaller than that of the outlier adjusted return series, the model for the outlier adjusted return series is more efficient given that the kurtosis value is near three, the value occupied by normal distribution (**Table 12**).

Table 7. Output of ARIMA-GARCH-type models of outlier adjusted returns series of Nigerian Banks.

Bank	Model	Parameter	Estimate	s.e	t-ratio	p-value	Information Criteria		
							AIC	BIC	HQIC
SKYE	ARIMA(1,1,0)-GARCH(1,1)-norm	μ	$-1.36e^{-4}$	$5.0e^{-6}$	−25.9283	0.0000	−4.6518	−4.6409	−4.6479
		φ_1	−0.0466	0.0211	−2.2101	0.0271			
		ω	$2.0e^{-6}$	0.0000	67.0558	0.0000			
		α_1	0.1228	$7.675e^{-3}$	15.9933	0.0000			
		β_1	0.8762	$6.665e^{-3}$	131.4656	0.0000			
STERLING	ARIMA (2,1,2)-EGARCH(1,1)-std	μ	0.0000	0.0000	−1.51	0.1310	−5.7621	−5.7401	−5.7541
		φ_1	$6.085e^{-3}$	$1.0e^{-6}$	5673.10	0.0000			
		φ_2	0.9386	$1.58e^{-4}$	5935.97	0.0000			
		θ_1	0.2025	$3.2e^{-5}$	6283.61	0.0000			
		θ_2	−0.7371	$1.2e^{-4}$	−6163.17	0.0000			
		ω	−0.0929	$2.74e^{-4}$	−338.38	0.0000			
		α_1	0.5597	$6.7e^{-5}$	8413.54	0.0000			
		β_1	0.9925	$1.42e^{-4}$	6970.63	0.0000			
		γ_1	0.5599	$6.5e^{-5}$	8667.79	0.0000			
UNITY	ARIMA(1,1,0)-GJR-GARCH (1,0)-norm	μ	$-2.284e^{-3}$	$1.0e^{-5}$	−220.0970	0.0000	−4.2296	−4.2406	−4.2336
		φ_1	0.0559	$2.66e^{-4}$	209.9780	0.0000			
		ω	$1.0e^{-6}$	0.0000	3.4940	0.0005			
		α_1	0.8566	$3.888e^{-3}$	220.3020	0.0000			
		γ_1	0.2175	0.0114	19.0470	0.0000			
ZENITH	ARIMA(2,1,1)-EGARCH(1,1)-std	μ	0.0000	0.0000	0.3698	0.7116	−7.0635	−7.0438	−7.0564
		φ_1	0.6094	0.0092	66.5553	0.0000			
		φ_2	0.0852	0.0045	18.7897	0.0000			
		θ_1	−0.6012	0.0038	−159.8213	0.0000			
		ω	−0.0960	$2.3e^{-4}$	−418.1551	0.0000			
		α_1	−0.8581	$1.05e^{-4}$	−8158.1158	0.0000			
		β_1	0.9903	$8.6e^{-5}$	11523.0226	0.0000			
		γ_1	0.8591	$1.05e^{-4}$	8183.2788	0.0000			

4. Conclusion

Our study has shown that the use of minimum unconditional variance as a measure of efficiency of heteroscedastic models in the presence of outliers is not heuristic as outliers are capable of inflating or reducing the unconditional variance. To this end, the use of kurtosis coefficient as a measure of heteroscedastic

Table 8. Diagnostic checking for ARIMA-GARCH-type models of outlier adjusted returns series of Nigerian Banks.

Bank	Model	Standardized Residuals			Standardized Squared Residuals					
		Lag	Weighted LB	p-value	Lag	Weighted LB	p-value	Lag	Weighted ARCH-LM	p-value
SKYE	ARIMA(1,1,0) -GARCH(1,1)-norm	1	1.323	0.2500	1	2.227	0.1356	3	1.113	0.2914
		2	2.710	0.0617	5	3.072	0.3941	5	1.375	0.6257
		5	4.284	0.1838	9	3.533	0.6687	7	1.472	0.8269
STERLING	ARIMA(2,1,2) -EGARCH(1,1)-std	1	0.0014	0.9707	1	0.0013	0.9716	3	0.0013	0.9716
		11	0.0087	1.0000	5	0.0038	1.0000	5	0.0030	0.9999
		19	0.0145	1.0000	9	0.0063	1.0000	7	0.0045	1.0000
UNITY	ARIMA(1,1,0) -GJR-GARCH(1,0) -norm	1	0.0081	0.9283	1	0.3091	0.5782	2	0.0990	0.7530
		2	0.0158	1.0000	2	0.3587	0.7639	4	2.5450	0.3332
		5	4.3656	0.1730	5	2.3489	0.5386	6	2.8354	0.5386
ZENITH	ARIMA(2,1,1) -EGARCH(1,1)-std	1	0.0010	0.9752	1	0.0009	0.9756	3	0.0009	0.9756
		8	0.0058	1.0000	5	0.0028	1.0000	5	0.0022	0.9999
		14	0.0100	1.0000	9	0.0047	1.0000	7	0.0033	1.0000

LB = Ljung-Box, LM = Lagrange Multiplier.

Table 9. Efficiency of ARIMA-GARCH-type model of Skye Bank.

Efficiency Measurement Criteria	ARIMA(1,1,0)-GARCH(1,1)–norm Model fitted to Returns Series of Skye Bank	ARIMA(1,1,0)-GARCH(1,1)-norm Model fitted to Outlier Adjusted Return Series of Skye Bank
Unconditional Variance	0.0016	0.0016
Kurtosis Coefficient	132.8707	2.9465

Table 10. Efficiency of ARIMA-GARCH-type model of Sterling Bank.

Efficiency Measurement Criteria	ARIMA(2,1,0)-EGARCH(1,1)-norm Model fitted to Returns Series of Sterling Bank	ARIMA(2,1,2)-EGARCH(1,1)-std Model fitted to Outlier Adjusted Return Series of Sterling Bank
Unconditional Variance	0.0012288	$4.234715e^{-6}$
Kurtosis Coefficient	80.0303	3.6829

Table 11. Efficiency of ARIMA-GARCH-type model of Unity Bank.

Efficiency Measurement Criteria	ARIMA(0,1,1)-GARCH (1,1)-norm Model fitted to Returns Series of Unity Bank	ARIMA(1,1,0) -GJR-GARCH(1,0)-norm Model fitted to Outlier Adjusted Return Series of Unity Bank
Unconditional Variance	0.0037	$2.104825e^{-5}$
Kurtosis Coefficient	888.5032	3.2678

Table 12. Efficiency of ARIMA-GARCH-type model of Zenith Bank.

Efficiency Measurement Criteria	ARIMA(2,1,1)-EGARCH(1,1)-std Model fitted to Returns Series of Zenith Bank	ARIMA(2,1,1)-EGARCH(1,1)-std Model fitted to Outlier Adjusted Return Series of Zenith Bank
Unconditional Variance	$5.887684e^{-36}$	$5.118125e^{-5}$
Kurtosis Coefficient	26.3794	3.5746

models in the presence of outliers becomes more tractable irrespective of the choice of distribution of the innovations. In addition, this study highlights that the applicability of iterative methods in time series modeling and in gauging model efficiency yet failed to consider the application of iterative methods in forecasting. It is recommended that further studies be extended to focus mainly on the application of iterative methods in maximum likelihood estimation of GARCH parameters.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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