Applied Mathematics, 2017, 8, 1343-1350
http://www.scirp.org/journal/am
ISSN Online: 2152-7393
ISSN Print: 2152-7385

Discrete Heat Equation Model with Shift Values

G. Britto Antony Xavier, S. John Borg, M. Meganathan

Department of Mathematics, Sacred Heart College, Tirupattur, India
Email: brittoshc@gmail.com, sjborg@gmail.com, meganathanmath@gmail.com

How to cite this paper: Xavier, G.B.A., Borg, S.J. and Meganathan, M. (2017) Discrete Heat Equation Model with Shift Values. *Applied Mathematics*, **8**, 1343-1350.
https://doi.org/10.4236/am.2017.89099

Received: August 31, 2017
Accepted: September 24, 2017
Published: September 27, 2017

Abstract

We investigate the generalized partial difference operator and propose a model of it in discrete heat equation in this paper. The diffusion of heat is studied by the application of Newton's law of cooling in dimensions up to three and several solutions are postulated for the same. Through numerical simulations using MATLAB, solutions are validated and applications are derived.

Keywords

Generalized Partial Difference Equation, Partial Difference Operator and Discrete Heat Equation

1. Introduction

In 1984, Jerzy Popenda [1] introduced the difference operator $\underset{\alpha}{\Delta}$ defined on $u(k)$ as $\underset{\alpha}{\Delta}u(k)=u(k+1)-\alpha u(k)$. In 1989, Miller and Rose [2] introduced the discrete analogue of the Riemann-Liouville fractional derivative and proved some properties of the inverse fractional difference operator Δ_{ℓ}^{-1} ([3] [4]). Several formula on higher order partial sums on arithmetic, geometric progressions and products of n-consecutive terms of arithmetic progression have been derived in [5].

In 2011, M. Maria Susai Manuel, *et al.* [6] [7], extended the definition of Δ_{α} to $\underset{\alpha(\ell)}{\Delta}$ defined as $\underset{\alpha(\ell)}{\Delta}v(k)=v(k+\ell)-\alpha v(k)$ for the real valued function v(k), $\ell>0$. In 2014, the authors in [6], have applied q-difference operator defined as $\Delta_{q}v(k)=v(qk)-v(k)$ and obtained finite series formula for logarithmic function. The difference operator $\underset{k(l)}{\Delta}$ with variable coefficients defined as equation $\underset{k(\ell)}{\Delta}v(k)=v(k+\ell)-kv(k)$ equation is established in [6]. Here, we extend the operator Δ_{ℓ} to a partial difference operator.

Partial difference and differential equations [8] play a vital role in heat equations. The generalized difference operator with *n*-shift values

$l=(\ell_1,\ell_2,\ell_3,\cdots,\ell_n)\neq 0$ on a real valued function $v(k):\mathbb{R}^n\to\mathbb{R}$ is defined as,

$$\underset{(\ell)}{\Delta} v(k)=v(k_1+\ell_1,k_2+\ell_2,\cdots,k_n+\ell_n)-v(k_1,k_2,\cdots,k_n) \quad (1)$$

This operator $\underset{(\ell)}{\Delta}$ becomes generalized partial difference operator if some $\ell_i=0$. The equation involving $\underset{(\ell)}{\Delta}$ with at least one $\ell_i=0$ is called generalized partial difference equation. A linear generalized partial difference equation is of the form $\underset{(\ell)}{\Delta} v(k)=u(k)$, then the inverse of generalized partial difference equation is

$$v(k)=\underset{(\ell)}{\Delta^{-1}} u(k) \quad (2)$$

where $\underset{(\ell)}{\Delta}$ is as given in (1), $\ell_i=0$ for some i and $u(k):\mathbb{R}^n\to\mathbb{R}$ is given function.

A function $v(k):\mathbb{R}^n\to\mathbb{R}$ satisfying (2) is called a solution of Equation (2). Equation (2) has a numerical solution of the form,

$$v(k)-v(k-m\ell)=\sum_{r=1}^{m} u(k-r\ell), \quad (3)$$

where $k-r\ell=(k_1-r\ell_1,k_2-r\ell_2,\cdots,k_n-r\ell_n)$, m is any positive integer. Relation (3) is the basic inverse principle with respect to $\underset{(\ell)}{\Delta}$ [6]. Here we form partial difference equation for the heat flow transmission in rod, plate and system and obtain its solution.

2. Solution of Heat Equation of Rod

Consider temperature distribution of a very long rod. Assume that the rod is so long that it can be laid on top of the set $\Re$ of real numbers. Let $v(k_1,k_2)$ be the temperature at the real position k_1 and real time k_2 of the rod. Assume that diffusion rate γ is constant throughout the rod shift value $\ell>0$. By Fourier law of Cooling, the discrete heat equation of the rod is,

$$\underset{(0,\ell_2)}{\Delta} v(k_1,k_2)=\gamma \underset{(\pm\ell_1,0)}{\Delta} v(k_1,k_2), \quad (4)$$

where $\underset{(\pm\ell_1,0)}{\Delta}=\underset{(\ell_1,0)}{\Delta}+\underset{(-\ell_1,0)}{\Delta}$. Here, we derive the temperature formula for $v(k_1,k_2)$ at the general position (k_1,k_2).

Theorem 2.1. *Assume that there exists a positive integer m, and a real number* $\ell_2>0$ *such that* $v(k_1,k_2-m\ell_2)$ *and* $\underset{(\pm\ell_1,0)}{\Delta} v(k_1,k_2)=\underset{\pm\ell_1}{u}(k_1,k_2)$ *are known then the heat Equation* (4) *has a solution* $v(k_1,k_2)$ *of the form*

$$v(k_1,k_2)=v(k_1,k_2-m\ell_2)+\gamma\sum_{r=1}^{m} u_{\pm\ell_1}(k_1,k_2-r\ell_2). \quad (5)$$

Proof. Taking $\underset{(\pm\ell_1,0)}{\Delta} v(k_1,k_2)=u_{\pm\ell_1}(k_1,k_2)$ in (4) gives

$$v(k_1,k_2)=\gamma \underset{(0,\ell_2)}{\Delta^{-1}} u_{\pm\ell_1}(k_1,k_2). \quad (6)$$

The proof of (5) follows by applying the inverse principle (3) in (6). □

Example 2.2. *From* (2) *we get,* $\overset{-1}{\underset{(0,\ell_2)}{\Delta}} e^{i(k_1+k_2)} = \frac{e^{i(k_1+k_2-\ell_2)} - e^{i(k_1+k_2)}}{2(1-\cos\ell_2)}$,

whose imaginary parts yield

$$\overset{-1}{\underset{(0,\ell_2)}{\Delta}} \sin(k_1+k_2) = \frac{\sin(k_1+k_2-\ell_2) - \sin(k_1+k_2)}{2(1-\cos\ell_2)}. \tag{7}$$

Taking $u_{\pm\ell_1}(k_1,k_2) = \sin(k_1+k_2+\ell_1) - \sin(k_1+k_2-\ell_1)$ in (6), using (7) and (5),

$$\begin{aligned} &\frac{u_{\pm\ell_1}(k_1,k_2-\ell_2) - u_{\pm\ell_1}(k_1,k_2)}{2(1-\cos\ell_2)} \\ &= \frac{u_{\pm\ell_1}(k_1,k_2-(m+1)\ell_2) - u_{\pm\ell_1}(k_1,k_2-m\ell_2)}{2(1-\cos\ell_2)} \\ &+ \sum_{r=1}^{m} u_{\pm\ell_1}(k_1,k_2-r\ell_2) \end{aligned} \tag{8}$$

The matlab coding for verification of (8) for $m=50$, $k_1=2$, $\ell_1=3$, $k_2=4$, $\ell_2=5$ as follows,

$$\begin{aligned} &(\sin(4)-\sin(9)+\sin(-2)-\sin(3))/(2\times(1-\cos(5))) \\ &= (\sin(-246)-\sin(-241)+\sin(-252)-\sin(-247)) \\ &\quad /(2*(1-\cos(5))) + \text{symsum}(\sin(9-5*r)+\sin(3-5*r),r,1,50) \end{aligned}.$$

Theorem 2.3. *Consider* (4) *and denote* $v(k_1\pm\ell_1,*) = v(k_1+\ell_1,*) + v(k_1-\ell_1,*)$ *and* $v(*,k_2\pm\ell_2) = v(*,k_2+\ell_2) + v(*,k_2-\ell_2)$. *Then, the following four types solutions of the Equation* (4) *are equivalent*:

(a)
$$\begin{aligned} v(k_1,k_2) &= (1-2\gamma)^m v(k_1,k_2-m\ell_2) \\ &+ \sum_{r=0}^{m-1} \gamma(1-2\gamma)^r \left[v(k_1\pm\ell_1,k_2-(r+1)\ell_2)\right] \end{aligned} \tag{9}$$

(b)
$$\begin{aligned} v(k_1,k_2) &= \frac{1}{(1-2\gamma)^m} v(k_1,k_2+m\ell_2) \\ &- \sum_{r=1}^{m} \frac{\gamma}{(1-2\gamma)^r} v(k_1\pm\ell_1,k_2+(r-1)\ell_2) \end{aligned} \tag{10}$$

(c)
$$\begin{aligned} v(k_1,k_2) &= \frac{1}{\gamma^m} v(k_1-m\ell_1,k_2+m\ell_2) - \sum_{r=1}^{m} \frac{1-2\gamma}{\gamma^r} v(k_1-r\ell_1,k_2+(r-1)\ell_2) \\ &- \sum_{s=0}^{m-1} \frac{1}{\gamma^s} v(k_1-(s+2)\ell_1,k_2+s\ell_2) \end{aligned} \tag{11}$$

(d)
$$\begin{aligned} v(k_1,k_2) &= \frac{1}{\gamma^m} v(k_1+m\ell_1,k_2+m\ell_2) - \sum_{r=1}^{m} \frac{1-2\gamma}{\gamma^r} v(k_1+r\ell_1,k_2+(r-1)\ell_2) \\ &- \sum_{s=0}^{m-1} \frac{1}{\gamma^s} v(k_1+(s+2)\ell_1,k_2+s\ell_2) \end{aligned} \tag{12}$$

Proof. (a). From (4), we arrive the relation

$$v(k_1,k_2)=(1-2\gamma)v(k_1,k_2-\ell_2)+\gamma v(k_1\pm\ell_1,k_2-\ell_2). \quad (13)$$

By replacing k_2 by $k_2-r\ell_2$ in (13) gives expressions for $v(k_1,k_2-r\ell_2)$ and $v(k_1\pm\ell_1,k_2-r\ell_2)$. Now proof of (a) follows by applying all these values in (13).

(b). The heat Equation (4) directly derives the relation

$$v(k_1,k_2)=\frac{1}{(1-2\gamma)}v(k_1,k_2+\ell_2) -\frac{\gamma}{(1-2\gamma)}v(k_1\pm\ell_1,k_2). \quad (14)$$

Replacing k_2 by $k_2+r\ell_2$ and substituting corresponding *v*-values in (14) yields (b).

(c). The proof of (c) follows by replacing k_1 by $k_1-r\ell_1$ and k_2 by $k_2+r\ell_2$ and

$$v(k_1,k_2)=\frac{1}{\gamma}v(k_1-\ell_1,k_2+\ell_2)-\frac{1-2\gamma}{\gamma}v(k_1-\ell_1,k_2)-v(k_1-2\ell_1,k_2).$$

(d). The proof of (d) follows by replacing k_1 by $k_1+r\ell_1$ and k_2 by $k_2+r\ell_2$ and

$$v(k_1,k_2)=\frac{1}{\gamma}v(k_1+\ell_1,k_2+\ell_2)-\frac{1-2\gamma}{\gamma}v(k_1+\ell_1,k_2)-v(k_1+2\ell_1,k_2). \square$$

Example 2.4. *The following example shows that the diffusion rate of rod can be identified if the solution* $v(k_1,k_2)$ *of* (4) *is known and vice versa. Suppose that* $v(k_1,k_2)=a^{k_1+k_2}$ *is a closed form solution of* (4), *then we have the relation* $\underset{(0,\ell_2)}{\Delta} a^{k_1+k_2}=\gamma\left[\underset{(\ell_1,0)}{\Delta} a^{k_1+k_2}+\underset{(-\ell_1,0)}{\Delta} a^{k_1+k_2}\right]$, which yields $a^{k_1+k_2+\ell_2}-a^{k_1+k_2}=\gamma\left[a^{k_1+k_2+\ell_1}+a^{k_1+k_2-\ell_1}-2a^{k_1+k_2}\right]$. Cancelling $a^{k_1+k_2}$ on both sides derives $\gamma=\dfrac{a^{\ell_2}-1}{a^{\ell_1}+a^{-\ell_1}-2}$.

Theorem 2.5. *Assume that the heat difference* $\underset{(-\ell_1,0)}{\Delta} v(k_1,k_2)$ *is proportional to* $\underset{(\ell_1,0)}{\Delta} v(k_1,k_2)$ *i.e.,* $\underset{(-\ell_1,0)}{\Delta} v(k_1,k_2)=\delta\underset{(\ell_1,0)}{\Delta} u(k_1,k_2)$. *In this case the heat Equation* (4) *has a solution* $\cos(k_1+k_2)$ *if and only if either* $\cos(k_1+k_2)=0$ *or* $\sin\ell_1=0$.

Proof. From the heat Equation (4), and the given condition, we derive

$$\underset{(0,\ell_2)}{\Delta} v(k_1,k_2)=\gamma(1+\delta)\underset{(\ell_1,0)}{\Delta} v(k_1,k_2). \quad (15)$$

If, $\cos(k_1+k_2)=\dfrac{e^{i(k_1+k_2)}+e^{-i(k_1+k_2)}}{2}=v(k_1,k_2)$, then (15) becomes, $\underset{(0,\ell_2)}{\Delta}\left[e^{i(k_1+k_2)}+e^{-i(k_1+k_2)}\right]=\gamma(1+\delta)\underset{(\ell_1,0)}{\Delta}\left[e^{i(k_1+k_2)}+e^{-i(k_1+k_2)}\right]$ which yields,

$$e^{i(k_1+k_2+l_2)}+e^{-i(k_1+k_2+l_2)}-e^{i(k_1+k_2)}-e^{-i(k_1+k_2)}$$
$$=\gamma(1+\delta)e^{i(k_1+k_2+l_1)}+e^{-i(k_1+k_2+l_1)}-e^{i(k_1+k_2)}-e^{-i(k_1+k_2)}$$

By rearranging the terms, we get

$$e^{i(k_1+k_2)}\left[e^{il_2}-1-\gamma(1+\delta)e^{i\ell_1}-1\right]=e^{-i(k_1+k_2)}\left[e^{i\ell_2}-1-\gamma(1+\delta)e^{-i\ell_1}-1\right]$$

which yields either $e^{i(k_1+k_2)}+e^{-i(k_1+k_2)}=0$ or $e^{i\ell_1}=e^{-i\ell_2}$ and hence $\cos(k_1+k_2)=0$ or $\sin l_1=0$. Retracing the steps gives converse. □

3. Heat Equation for Thin Plate and Medium

In the case of thin plate, let $v(k_1,k_2,k_3)$ be the temperature of the plate at position $v(k_1,k_2)$ and time k_3. The heat equation for the plate is

$$\underset{(0,0,\ell_3)}{\Delta} v(k)=\gamma \underset{\pm\ell_{(1,2)}}{\Delta} v(k), \tag{16}$$

where $\underset{(\pm\ell_{(1,2)})}{\Delta} = \underset{(\ell_1,0,0)}{\Delta} + \underset{(-\ell_1,0,0)}{\Delta} + \underset{(0,\ell_2,0)}{\Delta} + \underset{(0,-\ell_2,0)}{\Delta}$

Theorem 3.1. *Consider the heat Equation* (16). *Assume that there exists a positive integer m, and a real number* $\ell_3>0$ *such that* $v(k_1,k_2,k_3-ml_3)$ *and the partial differences* $\underset{(\pm\ell_{(1,2)})}{\Delta} v(k_1,k_2,k_3)=\underset{(\pm\ell_{(1,2)})}{u}(k_1,k_2,k_3)$ *are known functions then the heat Equation* (16) *has a solution* $v(k_1,k_2,k_3)$ *as,*

$$v(k)=v(k_1,k_2,k_3-m\ell_3)+\gamma\sum_{r=1}^{m}\underset{\pm\ell_{(1,2)}}{u}(k_1,k_2,k_3-r\ell_3). \tag{17}$$

Proof. Taking $\underset{\pm\ell_{(1,2)}}{\Delta} v(k)=\underset{\pm\ell_{(1,2)}}{u}(k)$ in (16), we arrive

$$v(k)=\gamma\underset{0,0,\ell_3}{\Delta^{-1}}\underset{\pm\ell_{(1,2)}}{u}(k). \tag{18}$$

The proof follows by applying inverse principle of $\underset{\ell_3}{\overset{-1}{\Delta}}$ in (18).

Consider the notations in the following theorem:

$v\left(k_{(1,2)}\pm l_{(1,2)},*\right)=v(k_1\pm\ell_1,k_2,*)+v(k_1,k_2\pm\ell_2,*)$ also

$v\left(k_{(2,3)}\pm l_{(2,3)},*\right)=v(*,k_2\pm\ell_2,k_3)+v(*,k_2,k_3\pm\ell_3)$.

Theorem 3.2. *Assume that* $v(k_1,k_2,k_3)$ *is a solution of Equation* (16)*,* $v(k_1\pm r\ell_1,k_2\pm r\ell_2)$ *exist and denote*

$v(k_1\pm\ell_1,*,*)=v(k_1+\ell_1,*,*)+v(k_1-\ell_1,*,*)$,

$v(*,*,k_3\pm\ell_3)=v(*,*,k_3+\ell_3)+v(*,*,k_3-\ell_3)$. Then the following are equivalent:

(a)

$$\begin{aligned}v(k_1,k_2,k_3)&=(1-4\gamma)^m v(k_1,k_2,k_3-m\ell_3)+\sum_{r=0}^{m-1}\gamma(1-4\gamma)^r\\&\times\left[v(k_1\pm\ell_1,k_2,k_3-(r+1)\ell_3)+v(k_1,k_2\pm\ell_2,k_3-(r+1)\ell_3)\right]\end{aligned} \tag{19}$$

(b)

$$\begin{aligned}v(k_1,k_2,k_3)&=\frac{1}{(1-4\gamma)^m}v(k_1,k_2,k_3+m\ell_3)\\&-\sum_{r=1}^{m}\frac{\gamma}{(1-4\gamma)^r}\left[v\left(k_{(1,2)}\pm l_{(1,2)},k_3+(r-1)\ell_3\right)\right]\end{aligned} \tag{20}$$

(c)

$$\begin{aligned}v(k)=&\frac{1}{\gamma^m}v(k_1-m\ell_1,k_2,k_3+m\ell_3)\\&-\sum_{r=1}^{m}\frac{1-4\gamma}{\gamma^r}v(k_1-r\ell_1,k_2,k_3+(r-1)\ell_3)\\&-\sum_{r=0}^{m-1}\frac{1}{\gamma^r}v(k_1-(r+2)\ell_1,k_2,k_3+r\ell_3)\\&-\sum_{r=0}^{m-1}\frac{1}{\gamma^r}v(k_1-(r+1)\ell_1,k_2\pm\ell_2,k_3+r\ell_3)\end{aligned}\quad(21)$$

(d)

$$\begin{aligned}v(k)=&\frac{1}{\gamma^m}v(k_1+m\ell_1,k_2,k_3+m\ell_3)-\sum_{r=1}^{m}\frac{1-4\gamma}{\gamma^r}v(k_1+r\ell_1,k_2,k_3+(r-1)\ell_3)\\&-\sum_{r=0}^{m-1}\frac{1}{\gamma^r}v(k_1+(r+2)\ell_1,k_2,k_3+r\ell_3)\\&-\sum_{r=0}^{m-1}\frac{1}{\gamma^r}v(k_1+(r+1)\ell_1,k_2\pm\ell_2,k_3+r\ell_3)\end{aligned}\quad(22)$$

Proof. The proof of this theorem is easy and similar to the proof of the Theorem (2.3). From (16) and (1), we arrive

(i) $$\begin{aligned}v(k)=&(1-4\gamma)v(k_1,k_2,k_3-\ell_3)\\&+\gamma\left[v(k_1\pm\ell_1,k_2,k_3-\ell_3)+v(k_1,k_2\pm\ell_2,k_3-\ell_3)\right]\end{aligned}$$

(ii) $v(k)=\frac{1}{1-4\gamma}v(k_1,k_2,k_3+\ell_3)-\frac{\gamma}{1-4\gamma}\left[v(k_1\pm\ell_1,k_2,k_3)+v(k_1,k_2\pm\ell_2,k_3)\right]$.

(iii) $$\begin{aligned}v(k)=&\frac{1}{\gamma}v(k_1-\ell_1,k_2,k_3+\ell_3)-\frac{1-4\gamma}{\gamma}v(k_1-\ell_1,k_2,k_3)\\&-v(k_1-2\ell_1,k_2,k_3)-v(k_1-\ell_1,k_2\pm\ell_2,k_3)\end{aligned}.$$

(iv) $$\begin{aligned}v(k)=&\frac{1}{\gamma}v(k_1+\ell_1,k_2,k_3+\ell_3)-\frac{1-4\gamma}{\gamma}v(k_1+\ell_1,k_2,k_3)\\&-v(k_1+2\ell_1,k_2,k_3)-v(k_1+\ell_1,k_2\pm\ell_2,k_3)\end{aligned}.$$

Now the proof of (a), (b), (c), (d) follows by replacing

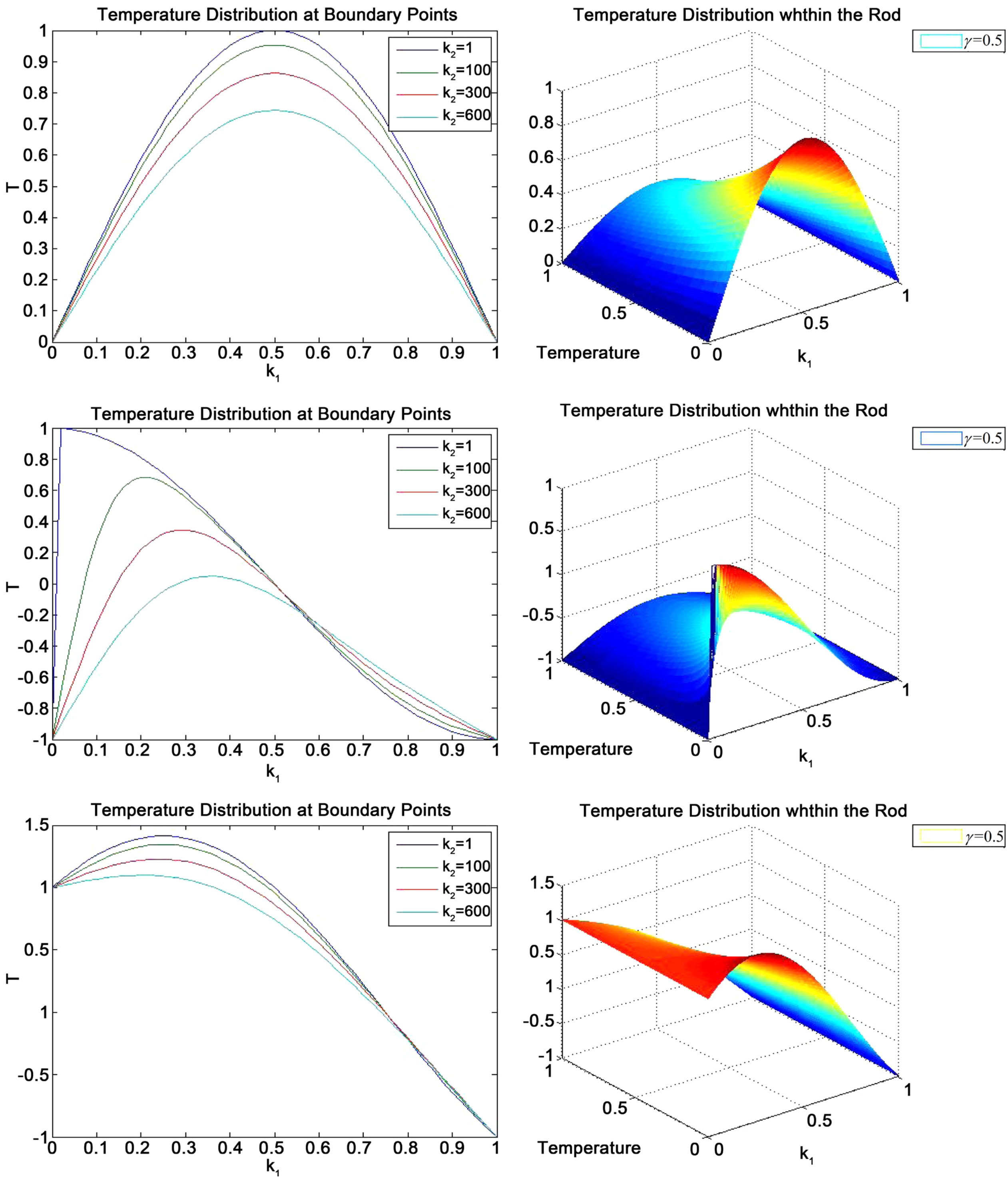
k_3 by $k_3-\ell_3,k_3-2\ell_3,\cdots,k_m-m\ell_3$, k_3 by $k_3+\ell_3,k_3+2\ell_3,\cdots,k_m+m\ell_3$, k_1 and k_3 by $k_1-\ell_1,k_1-2\ell_1,\cdots,k_m-m\ell_1$, $k_3+\ell_3,k_3+2\ell_3,\cdots,k_m+m\ell_3$, k_1 and k_3 by $k_1+\ell_1,k_1+2\ell_1,\cdots,k_m+m\ell_1$, $k_3+\ell_3,k_3+2\ell_3,\cdots,k_m+m\ell_3$ in (i), (ii), (iii), (iv) respectively. □

The following diagrams (generated by MATLAB) are obtained by using 13, $\gamma=0.5$ and taking $\ell_1=\frac{1}{50}$, $\ell_2=\frac{1}{2500}$,

(i) sine function; boundary values(BV) are $v(k_1,1)=\sin\pi\ell_1$, $v(1,k_2)=0$, $v(51,k_2)=0$,

(ii) cosine function; BV are $v(k_1,1)=\cos\pi\ell_1$, $v(1,k_2)=-1$, $v(51,k_2)=-1$,

(iii) sum of sine and cosine function; BV are $v(k_1,1)=\sin\pi\ell_1+\cos\pi\ell_1$, $v(1,k_2)=1$, $v(51,k_2)=-1$ respectively.



From the above diagrams, when the transmission of heat is known at the boundary points then the diffusion within the material under study can be easily determined.

4. Conclusion

The study of partial difference operator has wide applications in discrete fields and heat equation is one such. The core theorems (2.1), (2.3) and (3.2) provide the possibility of predicting the temperature either for the past or the future after getting the know the temperature at few finite points at present time. The above study helps us in making a wise choice of material(γ) for better propagation of heat. In the converse, it also shows the nature of transmission of heat for the material under study. Thus in conclusion, we can say that the above research helps us in reducing any wastage of heat and also enables us in making a optimal choice of material (γ).

References

[1] Popenda, J. and Szmanda, B. (1984) On the Oscillation of Solutions of Certain Difference Equations. *Demonstratio Mathematica*, **XVII**, 153-164.

[2] Miller, K.S and Ross, B. (1989) Fractional Difference Calculus in Univalent Functions. Horwood, Chichester, 139-152.

[3] Bastos, N.R.O., Ferreira, R.A.C. and Torres, D.F.M. (2011) Discrete-Time Fractional Variational Problems. *Signal Processing*, **91**, 513-524. https://doi.org/10.1016/j.sigpro.2010.05.001

[4] Ferreira, R.A.C. and Torres, D.F.M. (2011) Fractional h-Difference Equations Arising from the Calculus of Variations. *Applicable Analysis and Discrete Mathematics*, **5**, 110-121. https://doi.org/10.2298/AADM110131002F

[5] Susai Manuel, M., Britto Antony Xavier, G., Chandrasekar, V. and Pugalarasu, R. (2012) Theory and application of the Generalized Difference Operator of the n^{th} kind (Part I). *Demonstratio Mathematica*, **45**, 95-106. https://doi.org/10.1515/dema-2013-0347

[6] Britto Antony Xavier, G., Gerly, T.G. and Nasira Begum, H. (2014) Finite Series of Polynomials and Polynomial Factorials arising from Generalized q-Difference Operator. *Far East Journal of Mathematical Sciences*, **94**, 47-63.

[7] Maria Susai Manuel, M., Chandrasekar, V. and Britto Antony Xavier, G. (2011) Solutions and Applications of Certain Class of α-Difference Equations. *International Journal of Applied Mathematics*, **24**, 943-954.

[8] Ablowitz, M.J. and Ladik, J.F. (1977) On the Solution of a Class of Nonlinear Partial Difference Equations. *Studies in Applied Mathematics*, **57**, 1-12. https://doi.org/10.1002/sapm19775711