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# The Quasi-Order of Matching Energy of Circum Graph with Chord

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## Abstract

The matching energy of graph $G$ is defined as $ME(G)=\sum_{i=1}^{n}|\lambda_i|$, where $\lambda_1,\lambda_2,\cdots,\lambda_n$ be the roots of matching polynomial of graph $G$. In order to compare the energies of a pair of graphs, Gutman and Wager further put forward the concept of quasi-order relation. In this paper, we determine the quasi-order relation on the matching energy for circum graph with one chord.

## Keywords

Matching Polynomial, Matching Energy, Matching Root

## 1. Introduction

All graphs considered are finite, undirected, loopless and without multiple edges. The terminology and nomenclature of [1] will be used. Throughout this paper, $G$ will denote a graph with vertex set $V(G)=\{u,v,v_1,v_2,\cdots,v_{n-2}\}$ and edge set $E(G)$. By $G-u$ denote the induced subgraph obtained from $G$ by deleting vertex $u$ together with its incident edges and by $G-e$ the edge-induced subgraph obtained from $G$ by deleting edge $e$. As usual, use $P_n$ and $C_n$ to denote the path and cycle on $n$ vertices, respectively.

Let $m(G,k)$ be the number of $k$-matchings in graph $G$. The matching polynomial of a graph $G$ is defined in [2] as

$$\alpha(G,\lambda)=\sum_{k\geq 0}(-1)^k m(G,k)\lambda^{n-2k}. \quad (1)$$

where $m(G,0)=1$ and $m(G,k)\geq 0$ for all $k=1,2,\cdots,\left\lfloor\frac{n}{2}\right\rfloor$.

Gutman and Wager defined the quasi-order "$\succeq$" of two graphs $G$ and $H$ as follows:

If $G$ and $H$ have the matching polynomials in the form (1), then the quasi-order "$\succeq$" is defined by

$$G \succeq H \Leftrightarrow m(G,k) \geq m(H,k) \text{ for all } k = 0,1,\cdots,\lfloor n/2 \rfloor. \tag{2}$$

In particular, if $G \succeq H$ and $m(G,k) > m(H,k)$ for some $k$, then we write $G \succ H$.

We call $G$ and $H$ matching-equivalent if both $G \succeq H$ and $H \succeq G$ hold and denoted by $G \sim H$. Further, Gutman and Wagner introduced the concept of matching energy $ME(G)$ of a graph $G$ in [3] and defined in different expressions as follows:

$$ME(G) = \frac{2}{\pi}\int_0^{\infty} \frac{1}{x^2} \ln\left[\sum_{k\geq 0} m(G,k)x^{2k}\right] \mathrm{d}x. \tag{3}$$

and

$$ME(G) = \sum_{i=1}^{n} |\lambda_i|$$

where $\lambda_1, \lambda_2, \cdots, \lambda_n$ be the roots of matching polynomial of graph $G$.

The matching energy $ME(G)$ of a graph $G$ is an important index, which is widely used in the field of molecular orbital theory. There are many literatures about this parameter. See [4]-[14].

By the above definitions, it is immediately to get

$$G \succeq H \Rightarrow ME(G) \geq ME(H) \text{ and } G \succ H \Rightarrow ME(G) > ME(H). \tag{4}$$

In fact, this property provide an important technique to determine the order relation of the matching energy for graphs. In this paper, we discuss the order of the matching energy for circum graph with chord.

Let $C_n = uvv_1v_2 \cdots v_{n-2}u$ be a cycle with order $n$, the circum graph with chord is obtained by adding one edge $uv_i$ for some $i \in \{1,2,\cdots,n-3\}$ to $C_n$, which is denoted by $G(n;i,n-2-i)$ and simplified as $G_{u\sim v_i}$.

## 2. Preliminaries

**Lemma 1.** [2] *Let* $e = uv$ *be an edge of graph G and* $k \geq 1$. *Then* $m(G,k) = m(G-e,k) + m(G-u-v,k-1)$.

By Lemma 1, it is easy to get

**Lemma 2.** *Let e be an edge of graph G. Then* $ME(G-e) < ME(G)$.

**Lemma 3.** [15] *Let u be a vertex of graph G. Then*
$m(G,k) = m(G-u,k) + \sum_v m(G-u-v,k-1)$, *where the summation goes over all vertices v adjacent to the vertex u.*

**Lemma 4.** [16] *Let* $n = 4k+i, i \in \{0,1,2,3\}$ *for* $k \geq 1$. *Then* $P_n \succ P_2 \cup P_{n-2} \succ P_4 \cup P_{n-4} \succ \cdots \succ P_{2k} \cup P_{n-2k} \succ P_{2k+1} \cup P_{n-2k-1} \succ P_{2k-1} \cup P_{n-2k+1} \succ \cdots \succ P_3 \cup P_{n-3} \succ P_1 \cup P_{n-1}$.

By the definition of graph $G_{u\sim v_i}$, we can immediately get

**Lemma 5.** $G_{u\sim v_i} \sim G_{u\sim v_{n-2-i}}$ *for* $i \in \{1,2,\cdots,n-3\}$.

*Proof.* Since $G_{u\sim v_i} = G(n;i,n-2-i)$ and $G_{u\sim v_{n-2-i}} = G(n;,n-2-i,i)$, so we get $G_{u\sim v_i} \sim G_{u\sim v_{n-2-i}}$.□

## 3. Main Results

**Theorem 6.** Let $G_{u\sim v_i}$ be a circum graph with chord and n is an even. Then $G_{u\sim v_1} \prec G_{u\sim v_3} \prec \cdots \prec G_{u\sim v_{\frac{n-2}{2}-1}} \prec G_{u\sim v_{\frac{n-2}{2}}} \prec G_{u\sim v_{\frac{n-2}{2}-2}} \prec \cdots \prec G_{u\sim v_4} \prec G_{u\sim v_2}$.

*Proof.* First we consider the graph $G_{u\sim v_s}$ and $G_{u\sim v_{s-2}}$. Since $n$ is even, by Lemma 5, we only consider $1 \le s \le \frac{n-2}{2}$. By Lemma 3, we obtain that

$$\begin{aligned} m\left(G_{u\sim v_s},k\right) &= m\left[G\left(n;s,n-2-s\right),k\right] \\ &= m\left(P_{s+(n-2-s)+1},k\right) + m\left(P_{(s-1)+(n-2-s)+1},k-1\right) \\ &\quad + m\left(P_{s+(n-2-s-1)+1},k-1\right) + m\left(P_s \cup P_{n-2-s},k-1\right) \\ &= m\left(P_{n-1},k\right) + 2m\left(P_{n-2},k-1\right) + m\left(P_s \cup P_{n-2-s},k-1\right) \end{aligned} \quad (5)$$

Similarly,

$$\begin{aligned} m\left(G_{u\sim v_{s-2}},k\right) &= m\left[G\left(n;s-2,n-s\right),k\right] \\ &= m\left(P_{(s-2)+(n-s)+1},k\right) + m\left(P_{(s-2-1)+(n-s)+1},k-1\right) \\ &\quad + m\left(P_{(s-2)+(n-s-1)+1},k-1\right) + m\left(P_{s-2} \cup P_{n-s},k-1\right) \\ &= m\left(P_{n-1},k\right) + 2m\left(P_{n-2},k-1\right) + m\left(P_{s-2} \cup P_{n-s},k-1\right) \end{aligned} \quad (6)$$

Based on (5) and (6), we immediately get

$$m\left(G_{u\sim v_s},k\right) - m\left(G_{u\sim v_{s-2}},k\right) = m\left(P_s \cup P_{n-2-s},k-1\right) - m\left(P_{s-2} \cup P_{n-s},k-1\right).$$

**Case 1.** $s$ is even.

By Lemma 4, we can obtain that $P_s \cup P_{n-2-s} \prec P_{s-2} \cup P_{n-s}$. Thus, for some $k$, there be $m\left(G_{u\sim v_{s-2}},k\right) > m\left(G_{u\sim v_s},k\right)$. This means that $G_{u\sim v_2} \succ G_{u\sim v_4} \succ G_{u\sim v_6} \succ \cdots \succ G_{u\sim v_{\frac{n-2}{2}}}$.

**Case 2.** $s$ is odd.

By Lemma 4, using a similar argument as in the previous proof we conclude that $P_s \cup P_{n-2-s} \succ P_{s-2} \cup P_{n-s}$. Thus, for some $k$, there be $m\left(G_{u\sim v_s},k\right) > m\left(G_{u\sim v_{s-2}},k\right)$. This imply that $G_{u\sim v_{\frac{n-2}{2}-1}} \succ G_{u\sim v_{\frac{n-2}{2}-3}} \succ \cdots \succ G_{u\sim v_3} \succ G_{u\sim v_1}$.

For graph $G_{u\sim v_{\frac{n-2}{2}}}$ and $G_{u\sim v_{\frac{n-2}{2}-1}}$, we get that

$$m\left(G_{u\sim v_{\frac{n-2}{2}}},k\right) - m\left(G_{u\sim v_{\frac{n-2}{2}-1}},k\right) = m\left(P_{\frac{n-2}{2}} \cup P_{\frac{n-2}{2}},k-1\right) - m\left(P_{\frac{n-2}{2}-1} \cup P_{\frac{n-2}{2}+1},k-1\right)$$

Repeating the same argument as in the previous proof, combine the fact $n$ is even, we have $P_{\frac{n-2}{2}} \cup P_{\frac{n-2}{2}} \succ P_{\frac{n-2}{2}-1} \cup P_{\frac{n-2}{2}+1}$. Thus $G_{u\sim v_{\frac{n-2}{2}}} \succ G_{u\sim v_{\frac{n-2}{2}-1}}$. Sum up all, we get $G_{u\sim v_1} \prec G_{u\sim v_3} \prec \cdots \prec G_{u\sim v_{\frac{n-2}{2}-1}} \prec G_{u\sim v_{\frac{n-2}{2}}} \prec G_{u\sim v_{\frac{n-2}{2}-2}} \prec G_{u\sim v_2}$. □

**Theorem 7.** *Let n is an odd number. Then*

1) If $\left\lfloor \frac{n-2}{2} \right\rfloor$ is also odd, then $G_{u\sim v_2} \succ G_{u\sim v_4} \succ G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -1}} \succ G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor}} \succ G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -2}} \succ \cdots \succ G_{u\sim v_3} \succ G_{u\sim v_1}$;

2) If $\left\lfloor \frac{n-2}{2} \right\rfloor$ is even, then $G_{u\sim v_2} \succ G_{u\sim v_4} \succ G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor}} \succ G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -1}} \succ G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -3}} \succ \cdots \succ G_{u\sim v_3} \succ G_{u\sim v_1}$.

*Proof.* First consider the graph $G_{u\sim v_s}$ and $G_{u\sim v_{s-2}}$. Since $n$ is odd, similar as Lemma 5, we only consider $1 \le s \le \left\lfloor \frac{n-2}{2} \right\rfloor$. By Lemma 1, we get

$m\left(G_{u\sim v_s}, k\right) - m\left(G_{u\sim v_{s-2}}, k\right) = m\left(P_s \cup P_{n-2-s}, k-1\right) - m\left(P_{s-2} \cup P_{n-s}, k-1\right)$.

**Case 1.** $s$ is even.

By Lemma 4, we have $P_s \cup P_{n-2-s} \prec P_{s-2} \cup P_{n-s}$. Thus, $m\left(G_{u\sim v_{s-2}}, k\right) > m\left(G_{u\sim v_s}, k\right)$.

If $\left\lfloor \frac{n-2}{2} \right\rfloor$ is odd, then $G_{u\sim v_2} \succ G_{u\sim v_4} \succ G_{u\sim v_6} \succ \cdots \succ G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -1}}$.

If $\left\lfloor \frac{n-2}{2} \right\rfloor$ is even, then $G_{u\sim v_2} \succ G_{u\sim v_4} \succ G_{u\sim v_6} \succ \cdots \succ G_{u\sim v_{\frac{n-2}{2}}}$.

**Case 2.** $s$ is odd.

By Lemma 4, we have $P_s \cup P_{n-2-s} \succ P_{s-2} \cup P_{n-s}$. Thus, $m\left(G_{u\sim v_s}, k\right) > m\left(G_{u\sim v_{s-2}}, k\right)$.

If $\left\lfloor \frac{n-2}{2} \right\rfloor$ is odd, then $G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor}} \succ G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -2}} \succ \cdots \succ G_{u\sim v_3} \succ G_{u\sim v_1}$.

If $\left\lfloor \frac{n-2}{2} \right\rfloor$ is even, then $G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -1}} \succ G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -3}} \succ \cdots \succ G_{u\sim v_3} \succ G_{u\sim v_1}$.

Based on the above analysis, if $\left\lfloor \frac{n-2}{2} \right\rfloor$ is odd,

$$\begin{aligned}
& m\left(G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -1}}, k\right) - m\left(G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor}}, k\right) \\
&= m\left[G\left(n; \left\lfloor \frac{n-2}{2} \right\rfloor - 1, \left\lceil \frac{n-2}{2} \right\rceil + 1\right), k\right] - m\left[G\left(n; \left\lfloor \frac{n-2}{2} \right\rfloor, \left\lceil \frac{n-2}{2} \right\rceil + 1\right), k\right] \\
&= m\left(P_{\left\lfloor \frac{n-2}{2} \right\rfloor -1} \cup P_{\left\lceil \frac{n-2}{2} \right\rceil +1}, k-1\right) - m\left(P_{\left\lfloor \frac{n-2}{2} \right\rfloor} \cup P_{\left\lceil \frac{n-2}{2} \right\rceil}, k-1\right)
\end{aligned}$$

By lemma 4, $P_{\left\lfloor \frac{n-2}{2} \right\rfloor -1} \cup P_{\left\lceil \frac{n-2}{2} \right\rceil +1} \succ P_{\left\lfloor \frac{n-2}{2} \right\rfloor} \cup P_{\left\lceil \frac{n-2}{2} \right\rceil}$. Thus for some $k$, we have $m\left(G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -1}}, k\right) > m\left(G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor}}, k\right)$. This means $G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -1}} \succ G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor}}$.

If $\left\lfloor \frac{n-2}{2} \right\rfloor$ is even, then

$$\begin{aligned}
& m\left(G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor}}, k\right) - m\left(G_{u\sim v_{\left\lfloor \frac{n-2}{2} \right\rfloor -1}}, k\right) \\
&= m\left[G\left(n; \left\lfloor \frac{n-2}{2} \right\rfloor, \left\lceil \frac{n-2}{2} \right\rceil\right), k\right] - m\left[G\left(n; \left\lfloor \frac{n-2}{2} \right\rfloor - 1, \left\lceil \frac{n-2}{2} \right\rceil + 1\right), k\right] \\
&= m\left(P_{\left\lfloor \frac{n-2}{2} \right\rfloor} \cup P_{\left\lceil \frac{n-2}{2} \right\rceil}, k-1\right) - m\left(P_{\left\lfloor \frac{n-2}{2} \right\rfloor -1} \cup P_{\left\lceil \frac{n-2}{2} \right\rceil +1}, k-1\right)
\end{aligned}$$

By Lemma 4, $P_{\left\lfloor\frac{n-2}{2}\right\rfloor} \cup P_{\left\lceil\frac{n-2}{2}\right\rceil} \succ P_{\left\lfloor\frac{n-2}{2}\right\rfloor-1} \cup P_{\left\lceil\frac{n-2}{2}\right\rceil+1}$. Thus for some $k$, we have $m\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor}}, k\right) > m\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-1}}, k\right)$. This means that $G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor}} \succ G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-1}}$. Sum up all, for $n$ is an odd, if $\left\lfloor\frac{n-2}{2}\right\rfloor$ is also odd, then $G_{u\sim v_2} \succ G_{u\sim v_4} \succ G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-1}} \succ G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor}} \succ G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-2}} \succ \cdots \succ G_{u\sim v_3} \succ G_{u\sim v_1}$;

If $\left\lfloor\frac{n-2}{2}\right\rfloor$ is an even, then $G_{u\sim v_2} \succ G_{u\sim v_4} \succ G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor}} \succ G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-1}} \succ G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-3}} \succ \cdots \succ G_{u\sim v_3} \succ G_{u\sim v_1}$.

By Theorems 6 and 7, we immediately get our main result as follow.

**Theorem 8.** *Let* $G_{u\sim v_i}\ (i=1,2,\cdots,n-3)$ *be a circum graphs with chord.*

1) If $n$ is an even, then

$$\begin{aligned} &ME\left(G_{u\sim v_1}\right) < ME\left(G_{u\sim v_3}\right) < \cdots < ME\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor}}\right) < ME\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-1}}\right) \\ &< ME\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-3}}\right) < \cdots < ME\left(G_{u\sim v_2}\right) < ME\left(G_{u\sim v_2}\right) \end{aligned} \tag{7}$$

2) If $n$ and $\left\lfloor\frac{n-2}{2}\right\rfloor$ are both odd, then

$$\begin{aligned} &ME\left(G_{u\sim v_1}\right) < ME\left(G_{u\sim v_3}\right) < \cdots < ME\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-2}}\right) < ME\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor}}\right) \\ &< ME\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-1}}\right) < \cdots < ME\left(G_{u\sim v_4}\right) < ME\left(G_{u\sim v_2}\right) \end{aligned} \tag{8}$$

3) If $n$ is an odd and $\left\lfloor\frac{n-2}{2}\right\rfloor$ is an even, then

$$\begin{aligned} &ME\left(G_{u\sim v_1}\right) < ME\left(G_{u\sim v_3}\right) < \cdots < ME\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-3}}\right) < ME\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor-1}}\right) \\ &< ME\left(G_{u\sim v_{\left\lfloor\frac{n-2}{2}\right\rfloor}}\right) < \cdots < ME\left(G_{u\sim v_4}\right) < ME\left(G_{u\sim v_2}\right) \end{aligned} \tag{9}$$

## 4. Conclusions and Suggestions

In this paper, we determine the quasi-order relation on the matching energy for circum graph with one chord. If the chord here can be see $P_2$. Then the general case, determining the quasi-order relation on the matching energy for circum graph with one generalized chord $P_k$ for $2 \le k \le n-3$ is more meaningful.

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