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Distribution Free Waves in Viscoelastic Wedge with an Arbitrary Angle Tops

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Abstract

In work questions of distribution of waves in a viscoelastic wedge with any corner of top is considered. The elastic cylinder with a radial crack is a wedge $|\varphi| < 180^\circ$ corner. The regional task for system of the differential equations in private derivatives is decided by means of a method of straight lines that allows using a method of orthogonal prorace.

Keywords

The Cylinder, The Differential Equations, The Orthogonal Prorace, Approximating Formulas, A Wedge of Corner, A Wave Guide, Dispersive Dependence

1. Introduction

The problems of the propagation of viscoelastic waves in extended laminate and layers of variable thickness are considered in [1] [2] [3]. In these papers, the boundaries in the formation of the structure of the wave field, as a spectrum of Eigen frequencies and Eigen modes, are revealed in a series for simple problems (for unchanged boundaries), and the boundary change is consistently accompanied by increasing difficulties. We also consider the occurrence of local singularities in wave fields.

In this paper, in contrast to the above, the propagation of waves along the z axis in an infinite viscoelastic cylinder with a radial crack is considered, which is a wedge with some angle $|\varphi| < 180^\circ$.

2. Statement of the Problem and Methods of Solution

The basic equations of motion of a deformable cylinder (with a radius R) with a longitudinal crack, which at $\varphi = |\varphi_0| < 180^\circ$, Case describes a wedge. They are given with three groups of relations. The system of equations of motion of a wedge in a cylindrical coordinate system (r, φ, z) it takes the form

$$\begin{aligned}
&\rho \frac{\partial u_r}{dt^2} = \frac{\partial \sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{r\varphi}}{r} + \frac{1}{r}\frac{\partial \sigma_{r\varphi}}{\partial \varphi} + \frac{\partial \sigma_{rz}}{\partial z};\\
&\rho \frac{\partial^2 u_\varphi}{\partial t^2} = \frac{1}{r}\frac{\partial \sigma_{\varphi\varphi}}{\partial \varphi} + \frac{2\sigma_{r\varphi}}{r} + \frac{\partial \sigma_{r\varphi}}{\partial r} + \frac{\partial \sigma_{z\varphi}}{\partial z},\\
&\rho \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{rz}}{\partial r} + \frac{\sigma_{zz}}{r} + \frac{1}{r}\frac{\partial \sigma_{z\varphi}}{\partial \varphi}.
\end{aligned} \tag{1}$$

Here

$$\begin{aligned}
&\varepsilon_{rr} = \frac{\partial u_r}{\partial r};\ \varepsilon_{zz} = \frac{\partial u_z}{\partial z};\ \varepsilon_{\varphi\varphi} = \frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{u_r}{r};\\
&\varepsilon_{r\varphi} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right);\ \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right);\\
&\varepsilon_{\varphi z} = \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right);
\end{aligned} \tag{2}$$

$$\begin{aligned}
&\sigma_{rr} = \tilde{\lambda}\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\tilde{\mu}\frac{\partial u_r}{\partial r};\\
&\sigma_{r\phi} = 2\tilde{\mu}\varepsilon_{r\varphi} = \tilde{\mu}\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right);\\
&\sigma_{rz} = 2\tilde{\mu}\varepsilon_{rz} = \tilde{\mu}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right);\\
&\sigma_{\varphi\varphi} = \tilde{\lambda}\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\tilde{\mu}\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r}\right);\\
&\sigma_{\varphi z} = \tilde{\mu}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right);\\
&\sigma_{zz} = \tilde{\lambda}\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}\right) + 2\tilde{\mu}\frac{\partial u_z}{\partial z}.
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
&\tilde{\lambda} f(t) = \lambda_0\left[f(t) - \int_0^t R_\lambda(t-\tau) f(\tau)\,\mathrm{d}\tau\right],\\
&\tilde{\mu} f(t) = \mu_0\left[f(t) - \int_0^t R_\mu(t-\tau) f(\tau)\,\mathrm{d}\tau\right]
\end{aligned} \tag{4}$$

$f(t)$, is some function; ρ, density of material; $R_\mu(t-\tau)$ and $R_\lambda(t-\tau)$, the relaxation nucleus [4]; λ_0, μ_0, Lame's parameters (instantaneous elastic moduli); $\boldsymbol{u}(u_r, u_\varphi, u_z)$, displacement vector; $\sigma_{rr}, \sigma_{r\varphi}, \sigma_{rz}, \sigma_{\varphi\varphi}, \sigma_{\varphi z}, \sigma_{zz}$, the components of the stress tensor; $\varepsilon_{rr}, \varepsilon_{r\varphi}, \varepsilon_{rz}, \varepsilon_{\varphi\varphi}, \varepsilon_{\varphi z}, \varepsilon_{zz}$, respectively, the components of the strain tensor. The integral terms in (4) are assumed to be small [5]. Let the function f have the form $f(t) = \psi(t)\mathrm{e}^{-i\omega_R t}$, where $\psi(t)$ is a slowly varying function of time; ω_R is the real constant; i, imaginary unit. Using the freezing method [6] in place (4), it is possible to obtain approximate relations:

$$\tilde{\lambda} f(t) \approx \bar{\lambda} f(t) = \lambda \left[1 - \Gamma_{\lambda}^{c}(\omega_R) - i\Gamma_{\lambda}^{s}(\omega_R)\right] f(t),$$

$$\tilde{\mu} f(t) \approx \bar{\mu} f(t) = \mu_m \left[1 - \Gamma_{\mu}^{C}(\omega_R) - i\Gamma_{\mu}^{S}(\omega_R)\right] f(t),$$

$$\Gamma_{\lambda}^{C}(\omega_R) = \int_x^{\infty} R_{\lambda}(\tau) \cos \omega_R \tau \mathrm{d}\tau; \ \Gamma_{\lambda}^{S}(\omega_R) = \int_0^{\infty} R_{\lambda}(\tau) \sin \omega_R \tau \mathrm{d}\tau;$$

$$\Gamma_{\mu}^{C}(\omega_R) = \int_0^{\infty} R_{\mu}(\tau) \cos \omega_R \tau \mathrm{d}\tau; \ \Gamma_{\mu}^{S}(\omega_R) = \int_0^{\infty} R_{\mu}(\tau) \sin \omega_R \tau \mathrm{d}\tau,$$

respectively, the cosine and sine Fourier transforms; ω_R, the real part of the complex frequency ($\omega = \omega_R + i\omega_I$); ρ, density; $R_{\lambda}(t)$ и $R_{\mu}(t)$, the relaxation nucleus of the material.

The relations (1), (2), (3) after identical algebraic transformations are reduced to a system of six differential equations with complex coefficients, solved with respect to the first derivative with respect to the radial coordinate:

$$\begin{cases} \dfrac{\partial u_r}{\partial r} = \dfrac{1}{K}\sigma_{rr} - \dfrac{\bar{\lambda}}{K}\left(\dfrac{1}{r}\dfrac{\partial u_{\varphi}}{\partial \varphi} + \dfrac{u_r}{r} + \dfrac{\partial u_z}{\partial z}\right); \\ \dfrac{\partial u_{\varphi}}{\partial r} = \dfrac{1}{\bar{\mu}}\sigma_{r\varphi} - \dfrac{1}{r}\left(\dfrac{\partial u_r}{\partial \varphi} - u_{\varphi}\right); \\ \dfrac{\partial u_z}{\partial r} = \dfrac{1}{\bar{\mu}}\sigma_{rz} - \dfrac{\partial u_r}{\partial z}; \\ \dfrac{\partial \sigma_{rr}}{\partial r} = \rho\dfrac{\partial^2 u_r}{\partial t^2} - \dfrac{\tilde{A}}{r} - \dfrac{1}{r}\dfrac{\partial \sigma_{r\varphi}}{\partial \varphi} - \dfrac{\partial \sigma_{rz}}{\partial z}; \\ \dfrac{\partial \sigma_{r\varphi}}{\partial r} = \rho\dfrac{\partial^2 u_{\varphi}}{\partial t^2} - \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\left[\sigma_{rr} - \tilde{A}\right] - \dfrac{2\sigma_{r\varphi}}{r} - \dfrac{\partial}{\partial z}\tilde{B}; \\ \dfrac{\partial \sigma_{rz}}{\partial r} = \rho\dfrac{\partial^2 u_z}{\partial t^2} - \dfrac{\partial}{\partial z}\left[\sigma_{rr} - 2\bar{\mu}\left(\dfrac{\partial u_r}{\partial r} - \dfrac{\partial u_z}{\partial z}\right)\right] - \dfrac{\sigma_{rz}}{r} - \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\tilde{B}; \end{cases} \tag{5}$$

where the notation

$$\tilde{A} = 2\bar{\mu}\left[\frac{\partial u_r}{\partial r} - \frac{1}{r}\left(\frac{\partial u_{\varphi}}{\partial \varphi} + u_r\right)\right]; \ \tilde{B} = \bar{\mu}\left(\frac{\partial u_{\varphi}}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right).$$

The boundary conditions are given in the form:

$$r = r_0 \to 0 \ \text{ и } \ R: \sigma_{rz} = \sigma_{rr} = \sigma_{r\varphi} = 0$$

$$\varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2}; \ \sigma_{\varphi\varphi} = \sigma_{\varphi r} = \sigma_{\varphi z} = 0 \tag{6}$$

The periodicity conditions allow us to exclude the dependence of the main unknowns on time and the axial coordinate z by the following change of variables:

$$\begin{aligned} &u_r = w(r,\varphi)\mathrm{e}^{i\kappa(z-ct)}; \ u_{\varphi} = v(r,\varphi)\mathrm{e}^{i\kappa(z-ct)}; \\ &u_z = u(r,\varphi)\mathrm{e}^{i\kappa(z-ct)}; \ \sigma_{rr} = \sigma(r,\varphi)\mathrm{e}^{i\kappa(z-ct)}; \\ &\sigma_{r\varphi} = \tau_{\varphi}(r,\varphi)\mathrm{e}^{i\kappa(z-ct)}; \ \sigma_{rz} = \tau_z(r,\varphi)\mathrm{e}^{i\kappa(z-ct)}. \end{aligned} \tag{7}$$

where

$W(r), v(r), u(r), \sigma(r), \tau_{\varphi}(r), \tau_z(r)$, the amplitude of the oscillations, which are a function of the radial coordinate; κ, wave number; $c = C_R + iC_I$, complex phase velocity; $\omega = \omega_R + i\omega_I$, complex frequency.

Under the condition (6), the separation of variables r and φ, is impossible. Taking into account (7), the system of Equation (5) takes the form:

$$\begin{cases} w' = \dfrac{\sigma}{\kappa} - \dfrac{\bar{\lambda}}{\kappa}\left(ku + \dfrac{1}{r}\left(w + \dfrac{\partial v}{\partial \varphi}\right)\right) \\ v' = \dfrac{\tau_\varphi}{\bar{\mu}} + \dfrac{1}{r}\left(v - \dfrac{\partial w}{\partial \varphi}\right) \\ u' = \dfrac{\tau_z}{\bar{\mu}} + kw \\ \sigma' = -\omega^2 \rho w + \dfrac{1}{r}\left(A - \dfrac{\partial \tau_\varphi}{\partial \varphi}\right) - k\tau_z \\ \tau'_\varphi = -\omega^2 \rho v - \dfrac{1}{r}\left(\dfrac{\partial (A+\sigma)}{\partial \varphi} + 2\tau_\varphi\right) - kB \\ \tau'_z = -\omega^2 \rho u - \dfrac{1}{r}\left(\dfrac{\partial B}{\partial \varphi} + \tau_z\right) + k\left(\sigma + 2\mu\left(ku - w'\right)\right) \end{cases} \tag{8}$$

where

$$A = 2\bar{\mu}\left(\frac{1}{2}\left(\frac{\partial v}{\partial \varphi} + w\right) - w'\right), \quad B = \bar{\mu}\left(\frac{1}{r}\frac{\partial u}{\partial \varphi} - kv\right).$$

Here k is the wave number, which is given in the construction of the dispersion relation [7].

Similarly, boundary conditions are transformed (6)

$$r = 0, R : \sigma = \tau_\varphi = \tau_z = 0 \tag{9,a}$$

It is easy to see that the components of the stress tensor $\sigma_{\varphi\varphi}, \sigma_{\varphi z}, \sigma_{zz}$ are expressed in terms of the basic unknowns by the formulas:

$$\begin{aligned} \sigma_{\varphi\varphi} &= \sigma_{rr} + 2\bar{\mu}\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} - \frac{\partial u_r}{\partial r}\right); \\ \sigma_{\varphi z} &= \bar{\mu}\left(\frac{\partial u_z}{\partial \varphi} + \frac{\partial u_\varphi}{\partial z}\right); \\ \sigma_{zz} &= \sigma_{rr} + 2\bar{\mu}\left(\frac{\partial u_z}{\partial z} - \frac{\partial u_r}{\partial r}\right). \end{aligned} \tag{9,b}$$

Then, taking into account the first equation of system (9,b), the boundary conditions (6) take the form:

$$\begin{aligned} &\sigma_\varphi = A + \sigma_r = a\sigma_r + b\frac{1}{r}\left(\frac{\partial v}{\partial \varphi} + w\right) + cku = 0; \\ &\varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2} : \tau_\varphi = 0; \\ &B = \bar{\mu}\left(\frac{\partial u}{r\partial \varphi} - kr\right) = 0. \end{aligned} \tag{9,c}$$

where,

$$a = 1 + \frac{2\overline{\mu}}{k},\ b = 2\overline{\mu}\left(1 + \frac{1}{k}\right),\ c = 2\overline{\mu}\frac{1}{k},$$

Thus, the spectral boundary value problem (8)-(9) is formulated, which describes the propagation of harmonic waves in an infinite viscoelastic wedge with an arbitrary vertex angle.

As examples of a viscoelastic material, we take three parametric nuclei of relaxation $R_\lambda(t) = R_\mu(t) = Ae^{-\beta t}/t^{1-\alpha}$, possessing a weak singularity.

The boundary value problem for the system of partial differential Equation (8) is reduced to the solution of ordinary differential equations by the method of lines, which will allow using the method of orthogonal sweep in the solution [8]. According to the method of straight lines, the rectangular domain of the definition of the function of the principal unknowns is covered by straight lines parallel to the axis *r* and uniformly spaced from each other.

The solution is sought only on these lines, and the derivative in the direction φ, is replaced by approximate finite differences. The approximating formulas of the second order used for the first and second derivatives have the form:

$$\begin{aligned} y'_{i,\varphi} &\cong \frac{y_{i+1} - y_{i-1}}{2\Delta} \cong \frac{-3y_i + 4y_{i+1} - y_{i+2}}{2\Delta} \cong \frac{3y_i - 4y_{i-1} + y_{i-2}}{2\Delta} \\ y''_{i,\varphi} &\cong \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta^2} \end{aligned} \tag{10}$$

where i varies from 0 to $N+1$, y_i, projection of an unknown function y onto the line with the number *i*; Δ, move partition to the coordinate φ.

As a result of the discrimination, the vector of the main unknowns of the general dimension 6N can be written in the form:

$$Y = \left(\{w_i\},\{v_i\},\{u_i\},\{\sigma_{ri}\},\{\tau_{\varphi i}\},\{\tau_{zi}\}\right)^{\mathrm{T}},\ i = \overline{1,N} \tag{11}$$

Central differences (10), (11) are used for internal straight lines ($1 < i < N$), the left and right differences (10), (11) allow us to take into account the boundary conditions for φ. In the first case, the derivative of φ on the right sides of Equation (8) can be expressed by the formulas [4]:

$$\begin{aligned} & 1 < i < N \\ & w_{i,\varphi} = (w_{i+1} - w_{i-1})/2\Delta; \\ & u_{i,\varphi} = (u_{i+1} - u_{i-1})/2\Delta; \\ & v_{i,\varphi} = (v_{i+1} - v_{i-1})/2\Delta; \\ & \tau_{\varphi i,\varphi} = \left(\tau_{\varphi(i+1)} - \tau_{\varphi(i-1)}\right)/2\Delta; \\ & \sigma_{\varphi i,\varphi} = a\left(\sigma_{i+1} - \sigma_{i-1}\right)/2\Delta + \frac{b}{r}\left[\left(v_{i+1} - 2v_i + v_{i-1}\right)/\Delta^2 + w_{i,\varphi}\right] + cku_{i,\varphi}; \\ & B_i = \left(u_{i+1} - 2u_i + u_{i-1}\right)/\Delta^2/k - kv_{i,\varphi}. \end{aligned} \tag{12}$$

Boundary conditions at $\varphi = -\frac{\varphi_0}{2}$ is taken into account in the equations corresponding to the straight lines $i = 1$.

For the main unknowns that do not enter into the boundary conditions, w_i, v_i,

u_i use the right difference (10):

$$\begin{aligned}
&i=1: \\
&w_{i,\varphi}=(-3w_1+4w_2-w_3)/2\Delta; \\
&v_{i,\varphi}=(-3v_1+4v_2-v_3)/2\Delta; \\
&u_{i,\varphi}=(-3u_1+4u_2-u_3)/2\Delta.
\end{aligned} \tag{13}$$

For a variables τ_φ using central difference

$$\tau_{\varphi_i,\varphi} \approx \left(\tau_{\varphi_0}-\tau_{\varphi_2}\right)/2\Delta=-\tau_{\varphi_2}/2\Delta \tag{14}$$

The first and third of the conditions (9,c) are taken into account when approximating the derivatives of the function σ_φ, B by φ

$$\begin{aligned}
\sigma_{\varphi_1,\varphi} &= \left(\sigma_{\varphi_2}-\sigma_{\varphi_0}\right)/2\Delta=\sigma_{\varphi_2}/2\Delta \\
&= \left(a\sigma_{r_2}+\frac{b}{r}\left[(v_3-v_1)/2\Delta+w_2\right]-cku_2\right)\Big/2\Delta
\end{aligned}$$

$$B_{1,\varphi} \approx (B_2-B_0)/2\Delta=B_2/2\Delta=\left[(u_3-u_1)/2\Delta/r-kv_2\right]/2\Delta.$$

Similarly, derivatives for the straight line with the number $i = N$, taking into account the boundary conditions at $\varphi=\frac{\varphi_0}{2}$. The only difference is the replacement of the right finite difference left:

$$\begin{aligned}
&i=N: \\
&w_{i,\varphi}=(3w_N-4w_{N-1}+w_{N-2})/2\Delta; \\
&v_{i,\varphi}=(3v_N-4v_{N-1}+v_{N+1})/2\Delta; \\
&u_{i,\varphi}=(3u_N-4u_{N-1}+u_{N+1})/2\Delta;
\end{aligned} \tag{15}$$

$$\tau_{\varphi i,\varphi}=-\tau_{\varphi(N-1)}/2\Delta;$$

$$\sigma_{i,\varphi}=-\left(a\sigma_{N-1}+\frac{b}{r}\left[(v_N-v_{N-2})/2\Delta+w_{N-1}\right]+cku_{N-1}\right)\Big/2\Delta=-\frac{\sigma_{N-1}}{2\Delta}$$

$$B_{i,\varphi}=-\left[(u_N-u_{N-2})/2\Delta/r-kv_{N-1}\right]\Big/2\Delta=-\frac{B_{N-1}}{2\Delta}.$$

The number of straight lines can be reduced by using the ant symmetry conditions for the transverse vibrations of a plate at

$$\varphi=0: w=v=\sigma_\varphi=0. \tag{16}$$

The corresponding difference relations that take the conditions (16) can be written in the form

$$\begin{aligned}
&i=N: \\
&w_{i,\varphi}=-w_{N-1}/2\Delta;\ u_{i,\varphi}=-u_{N-1}/2\Delta; \\
&v_{i,\varphi}=(3v_N+4v_{N-1}-v_{N-2})/2\Delta.
\end{aligned} \tag{17}$$

$$\tau_{\varphi_i,\varphi}=\left(3\tau_{\varphi N}-4\tau_{\varphi(N-1)}+\tau_{\varphi(N-2)}\right)/2\Delta;$$

$$\sigma_{i,\varphi}=-\left(a\sigma_{N-1}+\frac{b}{r}\left[(v_N-v_{N-2})/2\Delta+w_{N-1}\right]+cku_{N-1}\right)\Big/2\Delta=-\frac{\sigma_{N-1}}{2\Delta}$$

$$B_{i,\varphi} = \left(-2u_N + u_{N-1}\right)/\Delta^2/r - kv_{i,\varphi}.$$

The resolving system of ordinary differential equations according to (8) has the form:

$$\begin{aligned}
&w_i' = \sigma_i/k - a\left(ku_i + \left(w_i + v_{i,\varphi}\right)/R\right);\\
&v_i' = \tau_{\varphi_i} + \left(v_i - w_{i,\varphi}\right)/R;\\
&u_i' = \tau_{zi} + kw_i;\\
&\sigma_i' = -\omega^2 w_i + \left[2\left(\left(w_i + v_{i,\varphi}\right)/R - w_i'\right) - \tau_{\varphi_i,\varphi}\right]/r - k\tau_{zi};\\
&\tau_{\varphi i}' = -\omega^2 v_i + \left(\sigma_{i,\varphi} + 2\tau_{\varphi i}\right)/r - k\left(u_{i,\varphi}/R - kv_i\right);\\
&\tau_{zi}' = -\omega^2 u_i + \left(B_{i,\varphi} + \tau_{zi}\right)/r + k\left(\sigma_i + 2\left(ku_i - w_i'\right)\right).
\end{aligned} \tag{18}$$

In Equation (18), the expressions for the derivatives $w_{i,\varphi}, v_{i,\varphi}, u_{i,\varphi}, \tau_{\varphi i,\varphi}, \sigma_{i,\varphi}, B_{i,\varphi}$ are chosen from the relations (13)-(17), depending on the boundary conditions with respect to the coordinate φ.

The free surface conditions, the equivalent conditions (9,a) and forming the boundary value problem together with the Equation (18), is obtained in the form

$$\tau_{\varphi i} = 0,\ \sigma_{\varphi i} = 0,\ B_i = 0\ \left(i = 1,\cdots,N\right) \tag{19}$$

Thus, the original spectral problem (8), (9) is reduced to the canonical problem (18) and (19) by discrimination of the coordinate φ by the method of lines, for the solution of which we apply the method of orthogonal sweep.

3. Numerical Results

The results of calculations are obtained on dimensionless parameters $\nu = 0.25$, c/c_s, R/λ, $A = 0.048$, $\beta = 0.05$, $\alpha = 0.1$, where $\lambda = 2\pi/k$ is the wavelength of the wave, the density $\rho = \rho/\rho_0$ ($\lambda = \rho\left(c_p^2 - 2c_s^2\right), \mu = c_s^2\rho$) [9] [10]. For numerical realization of the problem a software tool MAPLE 9.5.

Comparisons of the obtained numerical results (without viscosity $R_\lambda(t) = R_\mu(t) = 0$) with the known results [1] [3] are given in **Table 1**.

The limiting values of the phase velocity of the first edge mode, depending on the wedge angle at the vertex, found for a material with a Poisson's ratio $\upsilon = 0.25$ and $r = R = 1$, $R_\lambda(t) = R_\mu(t) = 0$ are given in **Table 1** of Column 2. Also the results obtained in [11] for a wedge based on plates Variable thickness are given in **Table 1** of column 3 (according to the theory of Kirchhoff-Love plates) and column 4 (according to the theory of Timoshenko's plates). Column 5 corresponds to a calculation variant with three internal straight lines (N = 3) and boundary conditions (9,a), column 6 corresponds to the boundary conditions:

$$\varphi = -\frac{\varphi_0}{2} : \sigma_\varphi = \sigma_{\varphi r} = \sigma_{\varphi z} = 0; \varphi = \frac{\varphi_0}{2} : u_r = u_z = \sigma_{\varphi\varphi} = 0 \tag{20}$$

with the same number of straight lines. From the results, the variants of calculating variable-section plates (according to the Kirchhoff-Love and Tymoshenko theory) and the three-dimensional theory (the proposed method) show that they agree within 7% for wedge angles not exceeding 28˚ (wedge angle $\varphi_0 \le 28^\circ$).

In the framework of the work described in this paper (based on the calculation procedure for a three-dimensional wedge), taking into account the viscous properties of the material $R_\lambda(t) = R_\mu(t) = Ae^{-\beta t}/t^{1-\alpha}$, they are shown in Figure 1.

Table 1. Comparisons of the obtained numerical results.

φ_0	According to the proposed Method of calculation Three-dimensional wedge	By the method of Kirchhoff-Love [11]	By Tymoshenko's method [11]	By the method of calculating a three-dimensional wedge (9,a)	By the method of calculating a three-dimensional wedge (20)
11^0	-	0.2680	0.1962	-	-
17^0	0.3083	0.3906	0.2863	0.3086	0.2989
28^0	0.4757	0.5269	0.4426	0.4758	0.4629
30^0	0.6058	0.8992	0.9232	0.6054	0.5927
53^0	0.7411	1.192	1.6918	0.7413	0.7294

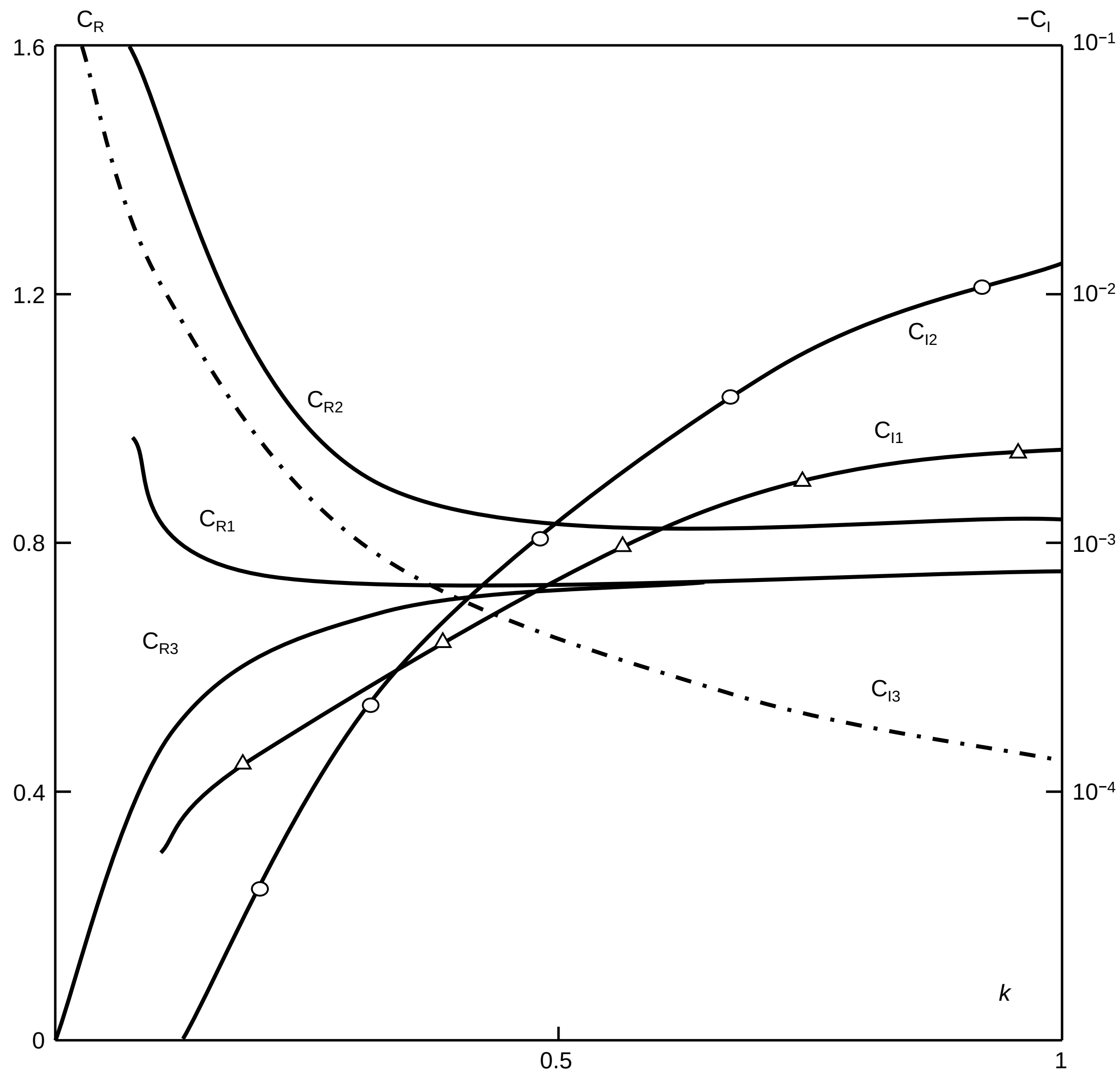

Figure 1. Change in real and imaginary parts of the phase velocity Fluctuations depending onк.

In **Figure 1** shown the real (curves C_{R1}, C_{R2}) and imaginary (curves C_{I1}, C_{I2}) parts of the dispersion curves of the first two modes of oscillations in an infinite viscous elastic wedge with angle ($\varphi_0 = 45^\circ$).

For comparison, the dependence on the wave number of the phase velocity of the first flexural mode of oscillations of a solid cylinder, founded by Pochhomer and Krie, using special functions (curve C_{R3}) [12] [13] and curve C_{I3} [14] is shown in the same figure. We note the characteristic features of the curve C_{R3}: at zero, the phase velocity is zero, and at infinity, tends to the Rayleigh wave velocity for a half-space. In the case of a viscoelastic wedge, the first mode has a cutoff frequency, and the phase velocity tends to infinity. At large wave numbers, the limiting phase velocity of this mode also coincides with the velocity of the Rayleigh wave.

4. Conclusions

Thus, unlike waveguides with a rectangular cross section in wedge-shaped waveguides with a sufficiently small wedge angle, in the analysis of the dispersion dependences of the first mode, it is permissible to use the theory of plates of variable cross section Kirchhoff-Love and Timoshenko. The established fact is explained by the phenomenon of localization of the shape of oscillations near the acute angle of the wedge, described in [3].

Based on the results obtained, the following conclusions are drawn:

- the results of calculating the limiting real velocity part ($c = C_R$) of the propagation of the first mode of a wedge-shaped waveguide in the theory of changing the Kirchhoff-Love plates [10] [11] and according to the dynamic theory of elasticity differ by no more than 6% for corners of the wedge top not exceeding 28˚;
- in the wedge-shaped waveguides, the first mode has a cut-off frequency, and the phase velocity tends to infinity.

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