

Chaos Behavior and Estimation of the Unknown Parameters of Stochastic Lattice Gas for Prey-Predator Model with Pair-Approximation

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Abstract

In this paper, the problem of chaos, stability and estimation of unknown parameters of the stochastic lattice gas for prey-predator model with pair-approximation is studied. The result shows that this dynamical system exhibits an oscillatory behavior of the population densities of prey and predator. Using Liapunov stability technique, the estimators of the unknown probabilities are derived, and also the updating rules for stability around its steady states are derived. Furthermore the feedback control law has been as non-linear functions of the population densities. Numerical simulation study is presented graphically.

Keywords

Stochastic Lattice Gas Model, Prey-Predator, Updating Rules, Estimation, System State

1. Introduction

A lattice model usually represents the motion of a network of particles, where the motion is produced by forces acting between the neighboring particles. Lattice models also are used to simulate the structure of polymers and can exhibit its dynamic behaviors.

The interaction between particles of the systems, which is the subject of this study, is continuous-time Markov process on certain spaces of configurations of particles. These systems began as a branch of probability theory in the 1960's. Most of the inputs came from the work of Spitzer in United States [1] and of Dobrushin in the Soviet Union [2]. From a mathematical point of view, interacting particles systems represent a natural departure from the established theory of Markov processes. The lattice can be in one, two, three or complex dimensions. The one-dimensional lattices consider a linear arrangement between the sequences of particles, and each particle connected to

the next one by a spring depends upon a condition. There are many classifications for the two-dimension lattice, such as a centered rectangular lattice, a hexagonal lattice, square lattice and so on (see **Figure 1**).

Many of models of interacting particles system have shown a chaos behavior. The problems of estimating and controlling stochastic systems are far from solved, and a considerable amount of research is under way. Estimation of the internal states of a stochastic dynamical system is a topic with important applications in different fields such as physics, biology and medicine [3]-[5].

A spatial stochastic model to discuss strategies to control the epidemic was introduced by Schinazi [6]. El-Gohary has proposed a stochastic model to study the problem of optimal controlling the epidemic [3] [4]. El-Gohary and Al-Ruziza have suggested a non-linear stochastic model to investigate the optimal control of a non-homogenous prey-predator model. They have derived the feedback control law as non-linear functions of the population densities [7] [8]. El-Gohary has studied the problems of chaos and optimal control cancer model with complete unknown parameters [9]. Al-Mahdi and Khirallah have studied stability and bifurcation analysis of a model of cancer [10] [11]. Alwan and El-Gohary have studied the chaos, estimation and optimal control of habitat destruction model and genital herpes epidemic models with uncertain parameters [12] [13].

A stochastic lattice gas model is proposed to describe the dynamics of two animals populations, one being a prey and the other a predator [14]. El-Gohary and Alwan have discussed the problem of chaos and control of a stochastic lattice gas model for prey-predator when one-site approximation is used. They have studied stability of the system and derived the optimal control inputs. They have also derived the estimators of the unknown parameters and probabilities [15]. This paper is considered as an extension of the paper [15] where it will discuss the stability and estimation of the unknown parameters of the stochastic lattice gas model for prey-predator when two-site approximation is used.

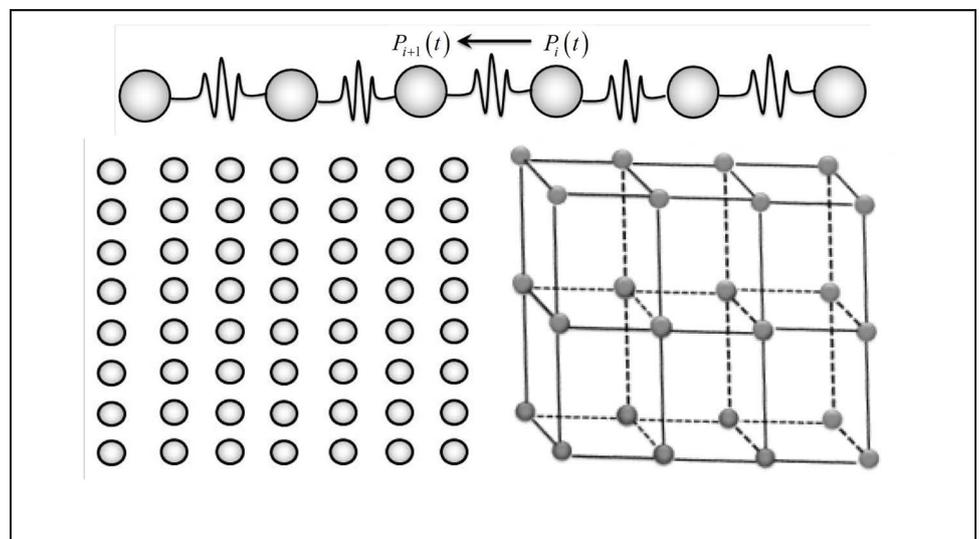


Figure 1. Linear, square and simple 3D lattices.

This paper has the following structure. In section 2, the stochastic probability model and its proposed rules will be discussed, and also the pair approximation mathematical model and its analytical solution will be presented. In section 3, stability analysis of the system will be studied and presented graphically. Estimation of the unknown parameters and the updating rules are derived in section 4. In section 5, numerical solutions are derived and presented graphically. Finally, conclusions are provided in Section 6.

2. Stochastic Probability Model

In this section, we will describe the stochastic rules of the proposed stochastic lattice gas model for prey-predator.

The lattice gas models describing special chemical reaction. Let us consider a lattice of N sites, every site can be either empty (O) or occupied by a prey (1) or occupied by a predator (2). At any time step a site is randomly chosen. For that site, we suppose that n_1 and n_2 are the numbers of the nearest neighbors of that site occupied by prey or predator, respectively and S is the total number of nearest neighbors of this site. The transition matrix of the model is given in **Table 1** or graphically in **Figure 2**.

This Markov process contains three probability parameters p_1 , p_2 and p_3 , which are associated to the process: birth of prey process, death of prey and simultaneous birth of predator, and spontaneous death of the predator [14]-[17]. The parameters p_1 , p_2 and p_3 satisfy the condition:

Table 1. The transition matrix of the model.

	0	1	2
0	$1 - p_1 n_1 / S$	$p_1 n_1 / S$	0
1	0	$1 - p_2 n_2 / S$	$p_2 n_2 / S$
2	p_3	0	$1 - p_3$

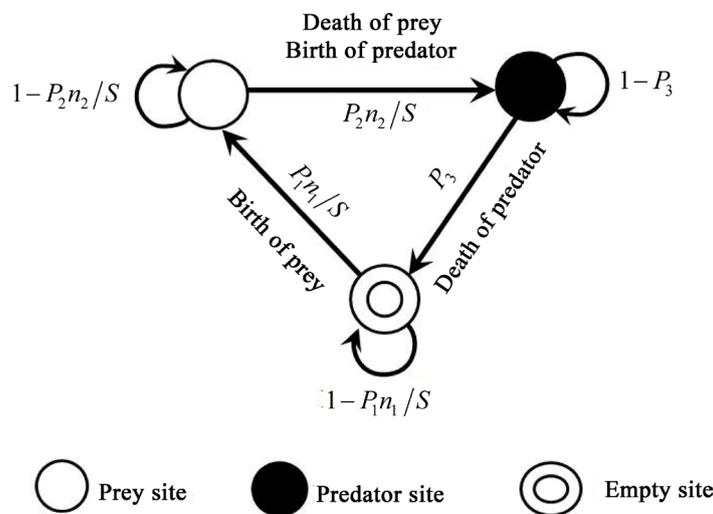


Figure 2. The transitions of the model.

$$p_1 + p_2 + p_3 = 1. \tag{1}$$

Let us consider the system state as $\mu = (\mu_1, \mu_2, \dots, \mu_N)$ where $\mu_i, i = 0, 1$ or 2 according whether the state i empty, occupied by prey or occupied by predator respectively. Let $P(\mu, t)$ be the probability of the state μ at time t . The time evolution of the probability of the state μ $P(\mu, t)$ and the mathematical system is given in details in [14]. The final mathematical expression of the model is a hierarchical system of equations, where the one-site correlations $P_i(\theta_1)$ involve the two-site correlations $P_{ij}(\theta_1\theta_2)$ and the two-site correlations involve three-site correlation $P_{ijk}(\theta_1\theta_2\theta_3)$ and so on, where θ_1, θ_2 and θ_3 take one values of 0, 1, or 2 (see Equation (6) in [14]). The mean field approximation theory is used to derive a closed set of equations. The one-site approximation of this hierarchical system of Equation (6) in [14] is accomplished by writing the two-site correlation $P_{ij}(\theta_1\theta_2)$ as the product $P_i(\theta_1)P_j(\theta_2)$ in the hierarchical system. The two-site approximation model which is the subject of this paper will be discussed in the next section.

Pair Approximation Mathematical Model

In this section, the two-site approximation will be presented and the final mathematical model will be written.

In the hierarchical system Equation (6) in [14], the mean field theory is suggest an approximation to derive a closed set of equations as: The three-site correlations $P_{ijk}(\theta_1\theta_2\theta_3)$ can be expressed in terms of two-site and one-site correlations as follows

$$P_{ij}(\theta_1\theta_2) \approx P_i(\theta)P_j(\theta) \tag{2}$$

$$P_{ijk}(\theta_1\theta_2\theta_3) \approx \frac{P_{ij}(\theta_1\theta_2)P_{jk}(\theta_2\theta_3)}{P_j(\theta_2)} \tag{3}$$

where sites i and k are the nearest neighbors of site j . We also seek for spatially homogeneous and isotropic solutions of Equation (6) in [14]. In this case we may drop the indexes in $P_i(\theta_1)$ and $P_{ij}(\theta_1\theta_2)$. We then have three one-site correlations

$P_i(\theta), \theta = 0, 1$ or 2 , and nine two-site correlations $P_{ij}(\theta\varphi), \theta, \varphi = 0, 1$ or 2 , which are presented in the transition matrix in **Table 1**. However, only five of them are independent. We choose them to be $P(1) = x$ (the prey density), $P(2) = y$ (the predator density), $P(3) = P(01) = u$, $P(12) = P(4) = v$, and $P(02) = P(5) = w$. Let us consider also $P(0) = z = 1 - x - y$ (the vacuum site density). The equations for these variables and the final mathematical system according this pair approximation are given by:

$$\frac{d}{dt}P(1) = p_1P(3) - p_2P(4), \tag{4a}$$

$$\frac{d}{dt}P(2) = p_2P(4) - p_3P(2), \tag{4b}$$

$$\frac{d}{dt}P(3) = \frac{S-1}{S} \left(p_1 \frac{qP(3) - P^2(3)}{z} - p_2 \frac{P(3)P(4)}{P(1)} \right) + p_3P(4) - p_1 \frac{P(3)}{S}, \tag{4c}$$

$$\frac{d}{dt}P(4) = \frac{S-1}{S} \left(p_1 \frac{P(3)P(5)}{z} + p_2 \frac{rP(4) - P^2(4)}{P(1)} \right) - p_2 \frac{P(4)}{S} - p_3 P(4), \quad (4d)$$

$$\frac{d}{dt}P(5) = \frac{S-1}{S} \left(p_2 \frac{P(3)P(4)}{P(1)} - p_1 \frac{P(3)P(5)}{z} \right) + p_3 (l - P(5)), \quad (4e)$$

where

$$\left. \begin{aligned} z &= P(0) = 1 - P(1) - P(2), \\ q &= P(00) = z - P(3) - P(5), \\ r &= P(11) = P(1) - P(3) - P(4), \\ l &= P(22) = P(2) - P(4) - P(5) \end{aligned} \right\} \quad (5)$$

and S is the total number of nearest neighbors of the site. This is a nonlinear system of differential equations. The analytical solution for this system is given by solving the system of equation:

$$\frac{d}{dt}P(i) = 0, \quad i = 1, 2, \dots, 5. \quad (6)$$

By Maple program and [14], when $S = 4$, the system in Equation (6) has the following general solution

$$\bar{P}(1) = \left(1 + u_1 p_1 / p_3 + 2u_1 / (\alpha_1 + \alpha_2 - (\beta_1 + 1)u_1) \right)^{-1}, \quad (7a)$$

$$\bar{P}(2) = p_1 \bar{P}(1) u_1 / p_3, \quad (7b)$$

$$\bar{P}(3) = \bar{P}(1) u_1, \quad (7c)$$

$$\bar{P}(4) = \bar{P}(1) u_1 p_1 / p_2, \quad (7d)$$

$$\bar{P}(5) = (\beta_1 u_1 - \alpha_2) (1 - \bar{P}(1) - \bar{P}(2)), \quad (7e)$$

where,

$$\alpha_1 = (p_3 S + p_2 (S - 2)) / p_2 (S - 1), \quad \alpha_2 = (p_2 (S - 2) - p_3 S) / p_2 (S - 1),$$

$$\beta_1 = 1 + 2 p_1 / p_2, \quad \beta_2 = 2 S p_3 / p_1 (S - 1),$$

$$u_1 = \left(B - (B^2 - 4AC)^{1/2} \right) / 2A, \quad A = (\beta_1 - 1)(\beta_1 + 1),$$

$$B = (\gamma + \alpha_2)(\beta_1 + 1) + (\beta_1 - 1)(\alpha_1 + \alpha_2) + 2\beta_1 \beta_2,$$

$$C = (\gamma + \alpha_2)(\alpha_1 + \alpha_2) + 2\beta_2 \alpha_2 \quad \text{and} \quad \gamma = s(1 - p_3 / p_2) / (S - 1)$$

(for more details, see [14]), and the following two trivial fixed stationary states as special solutions: $D_1 = (z, x, y, u, v, w) = (1, 0, 0, 0, 0, 0)$ corresponds to the vacuum-absorbing state, and $D_2 = (z, x, y, u, v, w) = (0, 1, 0, 0, 0, 0)$ corresponds to the prey-absorbing state.

3. Stability Analysis

Study of the stability and the chaos of the system will be discussed in this section, also

some of the equilibrium point will be presented.

For the case of $S = 2$, the system has only special stationary solution which is prey-absorbing state $(o, x, y) = (0, 1, 0)$ [14]. This solution is unstable when $p_2 > p_3$, and the general stationary solutions: $((p_2 - p_3)/(p_1 + p_3))$, p_3/p_2 ,

$p_1(p_2 - p_3)/[p_2(p_1 + p_3)]$ is unstable also, when $p_2 < p_3$ as in [15]. For the case $S = 4$, the system (3) has two trivial fixed stationary states, that are given by:

$D_1 = (z, x, y, u, v, w) = (1, 0, 0, 0, 0, 0)$, that correspond to the vacuum-absorbing state, and $D_2 = (z, x, y, u, v, w) = (0, 1, 0, 0, 0, 0)$, that correspond to the prey-absorbing state.

The Jacobian matrix of the system (4) is given by

$$J = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \tag{8}$$

where

$$a_{11} = a_{12} = 0, a_{13} = p_1, a_{14} = -p_2, a_{15} = a_{21} = 0, a_{22} = -p_3, a_{23} = 0,$$

$$a_{24} = p_2, a_{25} = 0, a_{31} = \frac{S-1}{S} \left(p_1 \frac{-2P^2(3) - P(3)P(5)}{1 - P(1) - P(2)} + \frac{p_2 P(3)P(4)}{P^2(1)} \right),$$

$$a_{32} = \frac{S-1}{S} \left(p_1 \frac{-2P^2(3) - P(3)P(5)}{1 - P(1) - P(2)} \right),$$

$$a_{33} = \frac{S-1}{S} \left(p_1 \frac{1 - P(1) - P(2) - 4P(3) - P(5)}{1 - P(1) - P(2)} - \frac{p_2 P(4)}{P(1)} \right) - \frac{p_1}{S},$$

$$a_{34} = \frac{S-1}{S} \left(\frac{-p_2 P(3)}{P(1)} \right) + p_3, \quad a_{35} = \frac{S-1}{S} \left(\frac{-p_1 P(3)}{1 - P(1) - P(2)} \right) + p_3,$$

$$a_{41} = \frac{S-1}{S} \left(\frac{p_1 P(3)P(5)}{(1 - P(1) - P(2))^2} + \frac{p_2 P(3)P(4)}{P^2(1)} \right),$$

$$a_{42} = \frac{S-1}{S} \left(\frac{p_1 P(3)P(5)}{(1 - P(1) - P(2))^2} \right), \quad a_{43} = \frac{S-1}{S} \left(\frac{p_1 P(5)}{1 - P(1) - P(2)} - \frac{p_2 P(4)}{P(1)} \right),$$

$$a_{44} = \frac{S-1}{S} \left(p_2 \frac{P(1) - P(3) - 4P(4)}{P(1)} \right) - \frac{p_2}{S} - p_3, \quad a_{45} = \frac{S-1}{S} \left(\frac{p_1 P(3)}{1 - P(1) - P(2)} \right),$$

$$a_{51} = \frac{S-1}{S} \left(\frac{-p_2 P(3)P(4)}{P^2(1)} - \frac{p_1 P(3)P(5)}{(1 - P(1) - P(2))^2} \right),$$

$$a_{52} = \frac{S-1}{S} \left(-\frac{p_1 P(3)P(5)}{(1 - P(1) - P(2))^2} \right) + p_3,$$

$$a_{53} = \frac{S-1}{S} \left(\frac{-p_2 P(4)}{P(1)} - \frac{p_1 P(5)}{1 - P(1) - P(2)} \right)$$

$$a_{54} = \frac{S-1}{S} \left(\frac{p_2 P(3)}{P(1)} \right) - p_3, \quad a_{55} = \frac{S-1}{S} \left(\frac{-p_1 P(3)}{1-P(1)-P(2)} \right) - 2p_3.$$

The Jacobian matrix in Equation (8) of the system in Equation (4) evaluated at the vacuum-absorbing state D_1 converges to

$$JD_1 \approx \begin{pmatrix} 0 & 0 & p_1 & -p_2 & 0 \\ 0 & -p_3 & 0 & p_2 & 0 \\ 0 & 0 & p_1/2 & p_3 & 0 \\ 0 & 0 & 0 & p_2/2 - p_3 & 0 \\ 0 & p_3 & 0 & -p_3 & -2p_3 \end{pmatrix} \quad (9)$$

and its eigenvalues just are the elements of the main diameter, which are

$$\lambda_1 = 0, \lambda_2 = -p_3 < 0, \lambda_3 = p_1/2 > 0, \lambda_4 = \frac{p_2}{2} - p_3, \lambda_5 = -2p_3 < 0. \quad (10)$$

Using the linear stability analysis, since λ_3 is a positive eigenvalue at least, hence this stationary solution is absolutely unstable.

Similarly, we get the Jacobian matrix in Equation (8) of the system in Equation (4) evaluated at the prey-absorbing state D_2 converges to $JD_2 = JD_1$. Accordingly JD_2 has the same eigenvalues. Therefore, the stationary state D_2 is also absolutely unstable. The linear stability analysis for these two stationary states indicates sufficiently that, the system of stochastic lattice gas of prey-predator system according to the pair-approximation is absolutely unstable at least in two dimensions. But the stability conditions are requiring more study.

Such behavior for the system in prey, predator and vacuum densities in **Figures 3(a)-(c)** respectively, represents an oscillatory behavior. For limit-cycle that appear in **Figures 3(d)-(f)** where all the neighboring trajectories tend to a limit-cycle at time tends to infinity, causing the so-called a stable limit-cycle, which indicates that the system stochastic lattice gas of prey-predator system according to the pair-approximation has an oscillatory behavior.

4. Estimations of the Unknown Probabilities

In this section we will derive the dynamic estimators of the unknown probabilities p_1, p_2 and p_3 from the conditions of the asymptotic stability of the system in Equation (4) about its stationary states assistance of some feedback variables.

At the beginning, let us assume the modified model with unknown probabilities in Equation (4) to become as follows

$$\frac{d}{dt} P(1) = \hat{p}_1(t) P(3) - \hat{p}_2(t) P(4) + E_1 \quad (11a)$$

$$\frac{d}{dt} P(2) = \hat{p}_2(t) P(4) - \hat{p}_3(t) P(2) + E_2 \quad (11b)$$

$$\frac{d}{dt} P(3) = \frac{S-1}{S} \left(\hat{p}_1(t) \frac{qP(3) - P^2(3)}{z} - \hat{p}_2(t) \frac{P(3)P(4)}{P(1)} \right) + \hat{p}_3(t) P(4) - \hat{p}_1(t) \frac{P(3)}{S} + E_3 \quad (11c)$$

$$\frac{d}{dt}P(4) = \frac{S-1}{S} \left(\hat{p}_1(t) \frac{P(3)P(5)}{z} + \hat{p}_2(t) \frac{rP(4) - P^2(4)}{P(1)} \right) - \hat{p}_2(t) \frac{P(4)}{S} - \hat{p}_3(t)P(4) + E_4 \tag{11d}$$

$$\frac{d}{dt}P(5) = \frac{S-1}{S} \left(\hat{p}_2(t) \frac{P(3)P(4)}{P(1)} - \hat{p}_1(t) \frac{P(3)P(5)}{z} \right) + \hat{p}_3(t)(l - P(5)) + E_5 \tag{11e}$$

where $\hat{p}_s(t)$ are the estimators of the unknown probabilities $p_s, (s=1,2,3)$ and $E_j, j=1, \dots, 5$ are the control inputs that will be derived to make the trajectory of the system specified by the steady-states D_1, D_2 and the general solution in Equation (7) to any of these states. If $E_j = 0, j=1, \dots, 5$, then the system in Equation (11) has an unstable special solution:

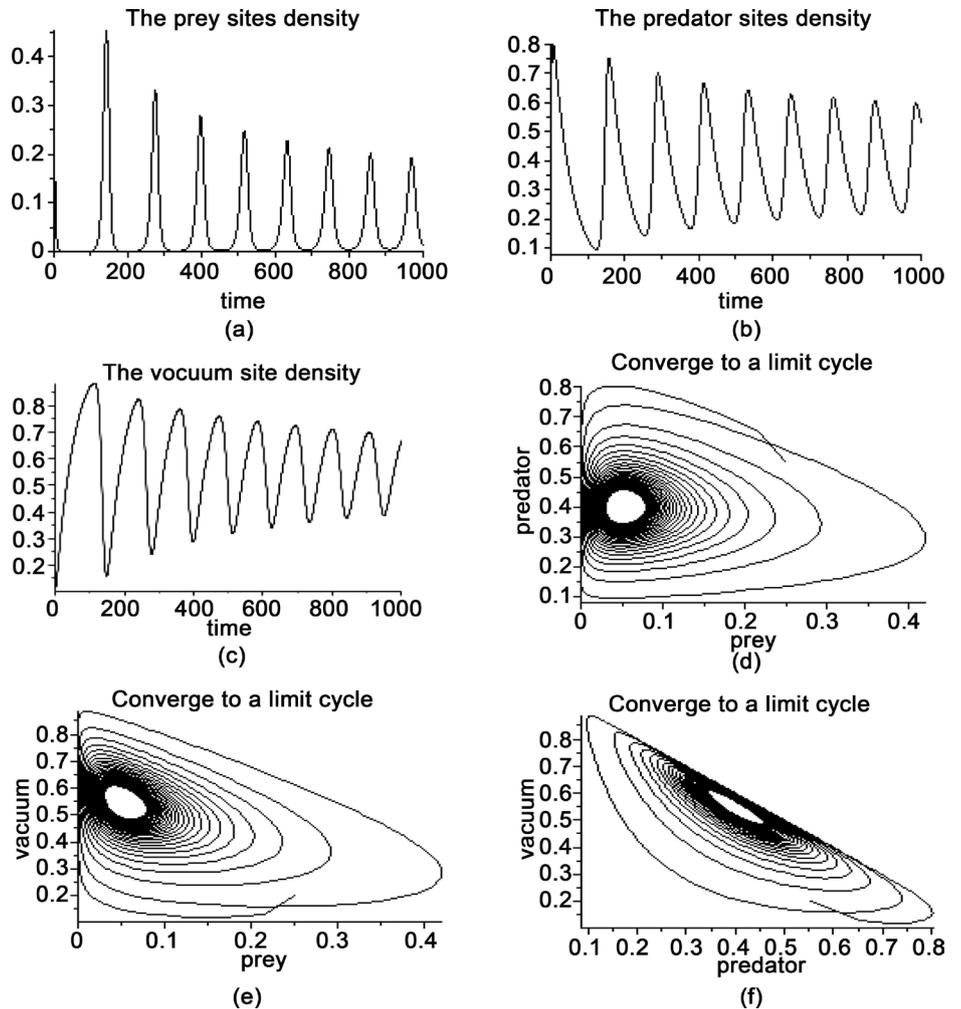


Figure 3. The densities of the prey, predator and vacuum sites and their limit cycle, respectively for $p_1 = 0.49, p_2 = 0.49$ and $p_3 = 0.02$, at the initial densities $x|_0 = 0.25, y|_0 = 0.5, u|_0 = 0.305, v|_0 = 0.29$ and $w|_0 = 0.2$.

$$P(j) = \bar{P}(j), j = 1, \dots, 5 \text{ and } \hat{p}_s(t) = P_s, (s = 1, 2, 3) \quad (12)$$

where $\bar{P}(j), j = 1, \dots, 5$ are the steady-states of the uncontrolled system in Equation (4) that should be stabilized by finding the controllers E_j that causes the system in Equation (11) to follow a stable trajectory. Therefore, the problem is now equivalent to stabilize the steady-states in Equation (12) and determine the update rules of the estimators $\hat{p}_s(t)$ of the system in Equation (11) with the help of the controllers $E_j, j = 1, \dots, 5$.

To solve the problem of this stabilization, we will use the Liapunov stability technique. Constructive upon this, let us consider the following positive definite form of Liapunov function

$$2V(P(j), \hat{p}_s; t) = \sum_{j=1}^5 (P(j) - \bar{P}(j))^2 + \sum_{s=1}^3 (\hat{p}_s - p_s)^2 \quad (13)$$

then

$$\dot{V}(P(j), \hat{p}_s; t) = \sum_{j=1}^5 (P(j) - \bar{P}(j)) \dot{P}(j) + \sum_{s=1}^3 (\hat{p}_s - p_s) \dot{\hat{p}}_s(t). \quad (14)$$

By substituting $\dot{P}(j), j = 1, \dots, 5$ from Equations (11) and choosing the following controllers inputs

$$E_1 = p_2 P(4) - p_1 P(3) - k_1 (P(1) - \bar{P}(1)) \quad (15a)$$

$$E_2 = p_3 P(2) - p_2 P(4) - k_2 (P(2) - \bar{P}(2)) \quad (15b)$$

$$E_3 = \frac{S-1}{S} \left(\frac{p_1 (P^2(3) - qP(3))}{z} + \frac{p_2 P(3) P(4)}{P(1)} \right) - p_3 P(4) + \frac{p_1 P(3)}{S} - k_3 (P(3) - \bar{P}(3)) \quad (15c)$$

$$E_4 = \frac{S-1}{S} \left(\frac{p_2 (P^2(4) - rP(4))}{P(1)} - \frac{p_1 P(3) P(5)}{z} \right) + p_2 \frac{P(4)}{S} + p_3 P(4) - k_4 (P(4) - \bar{P}(4)) \quad (15d)$$

$$E_5 = \frac{S-1}{S} \left(p_1 \frac{P(3) P(5)}{z} - p_2 \frac{P(3) P(4)}{P(1)} \right) - p_3 (l - P(5)) - k_5 (P(5) - \bar{P}(5)) \quad (15e)$$

and the following update rules of the unknown probabilities

$$\begin{aligned} \frac{d}{dt} \hat{p}_1(t) = & (P(3) - \bar{P}(3)) \left[\frac{P(3) - (S-1)}{S} - \frac{qP(3) - P^2(3)}{z} \right] \\ & - P(3) (P(1) - \bar{P}(1)) + \frac{S-1}{S} \frac{P(3) P(5)}{z} \\ & \times \left[(P(5) - \bar{P}(5)) - (P(4) - \bar{P}(4)) \right] - m_1 (\hat{p}_1(t) - p_1(t)) \end{aligned} \quad (16a)$$

$$\begin{aligned} \frac{d}{dt} \hat{p}_2(t) &= P(4) \left[(P(1) - \bar{P}(1)) - (P(2) - \bar{P}(2)) \right] + (P(4) - \bar{P}(4)) \\ &\quad \times \left[\frac{P(4)}{S} - \frac{S-1}{S} \frac{rP(4) - P^2(4)}{P(1)} \right] + \frac{S-1}{S} \frac{P(3)P(4)}{P(1)} \\ &\quad \times \left[(P(3) - \bar{P}(3)) - (P(5) - \bar{P}(5)) \right] - m_2 (\hat{p}_2(t) - p_2(t)) \end{aligned} \quad (16b)$$

$$\begin{aligned} \frac{d}{dt} \hat{p}_3(t) &= P(2)(P(2) - \bar{P}(2)) + P(4) \left[(P(4) - \bar{P}(4)) - (P(3) - \bar{P}(3)) \right] \\ &\quad - (P(5) - \bar{P}(5))(l - P(5)) - m_3 (\hat{p}_3(t) - p_3(t)) \end{aligned} \quad (16c)$$

the total time derivative of the Liapunov function in takes the form:

$$\dot{V} = - \sum_{j=1}^5 k_j (P(j) - \bar{P}(j))^2 - \sum_{s=1}^3 m_s (\hat{p}_s - p_s)^2 \quad (17)$$

where $k_j, j=1, \dots, 5$ and $m_s, s=1, 2, 3$ are non-negative control constants. In this case \dot{V} is a negative definite if $k_j > 0$ and $m_s > 0$ and a negative semi-definite if $k_j = 0$ and $m_s = 0$. This implies that, under the action of the controllers in Equation (15) and updating rules in Equation (16) of the unknown system probabilities, the solution in Equation (7) of the systems in Equation (4) and Equation (17) is asymptotic stable in the Liapunov sense if $k_j > 0$ and $m_s > 0$ and only stable but not necessarily asymptotic stable if $k_j = 0$ and $m_s = 0$. Since V is radially unbounded, and widen over time, therefore the solution in Equation (12) is globally asymptotically stable which completes the proof.

By substituting Equation (15) in the modified controlled system in Equation (11), in addition the update rules in Equation (16) we get the final system as follows

$$\dot{P}(1) = -k_1 (P(1) - \bar{P}(1)) + (\hat{p}_1(t) - p_1(t))P(3) - (\hat{p}_2(t) - p_2(t))P(4) \quad (18a)$$

$$\dot{P}(2) = -k_2 (P(2) - \bar{P}(2)) + (\hat{p}_2(t) - p_2(t))P(4) - (\hat{p}_3(t) - p_3(t))P(2) \quad (18b)$$

$$\begin{aligned} \dot{P}(3) &= -k_3 (P(3) - \bar{P}(3)) + (\hat{p}_1(t) - p_1(t)) \left(\frac{S-1}{S} \frac{qP(3) - P^2(3)}{z} - \frac{P(3)}{S} \right) \\ &\quad - (\hat{p}_2(t) - p_2(t)) \left(\frac{S-1}{S} \frac{P(3)P(4)}{P(1)} \right) + (\hat{p}_3(t) - p_3(t))P(4) \end{aligned} \quad (18c)$$

$$\begin{aligned} \dot{P}(4) &= -k_4 (P(4) - \bar{P}(4)) + (\hat{p}_1(t) - p_1(t)) \frac{S-1}{S} \frac{P(3)P(4)}{z} \\ &\quad + (\hat{p}_2(t) - p_2(t)) \left(\frac{S-1}{S} \frac{rP(4) - P^2(4)}{P(1)} - \frac{P(4)}{S} \right) \end{aligned} \quad (18d)$$

$$\begin{aligned} \dot{P}(5) &= -k_5 (P(5) - \bar{P}(5)) + (\hat{p}_2(t) - p_2(t)) \left(\frac{S-1}{S} \frac{P(3)P(4)}{P(1)} \right) \\ &\quad - (\hat{p}_1(t) - p_1(t)) \left(\frac{S-1}{S} \frac{P(3)P(5)}{z} \right) \\ &\quad + (\hat{p}_3(t) - p_3(t))(l - P(5)) \end{aligned} \quad (18e)$$

$$\begin{aligned} \dot{\hat{p}}_1(t) = & m_1(\hat{p}_1(t) - p_1(t)) + (P(3) - \bar{P}(3)) \left[\frac{P(3)}{S} - \frac{S-1}{S} \frac{qP(3) - P^2(3)}{z} \right] \\ & - P(3)(P(1) - \bar{P}(1)) + \frac{S-1}{S} \frac{P(3)P(5)}{z} [(P(5) - \bar{P}(5)) - (P(4) - \bar{P}(4))] \end{aligned} \quad (18f)$$

$$\begin{aligned} \dot{\hat{p}}_2(t) = & -m_2(\hat{p}_2(t) - p_2(t)) + P(4) [(P(1) - \bar{P}(1)) - (P(2) - \bar{P}(2))] \\ & + (P(4) - \bar{P}(4)) \left[\frac{P(4)}{S} - \frac{S-1}{S} \frac{rP(4) - P^2(4)}{P(1)} \right] \\ & + \frac{S-1}{S} \frac{P(3)P(4)}{P(1)} [(P(3) - \bar{P}(3)) - (P(5) - \bar{P}(5))] \end{aligned} \quad (18g)$$

$$\begin{aligned} \dot{\hat{p}}_3(t) = & -m_3(\hat{p}_3(t) - p_3(t)) + P(2)(P(2) - \bar{P}(2)) + P(4) \\ & \times [(P(4) - \bar{P}(4)) - (P(3) - \bar{P}(3))] - (P(5) - \bar{P}(5))(l - (P(5))) \end{aligned} \quad (18h)$$

where z, q, r and l as defined in Equation (5). This system of nonlinear differential equations will be solved numerically and displayed some graphical solutions as examples.

5. Numerical Solution

This section presents some numerical solutions of the controlled nonlinear system of the stochastic lattice gas of prey-predator model in Equation (18) and the estimators of the system unknown probabilities to show how the control for this system is possible. Also, numerical examples for controlled stochastic lattice gas of prey-predator model were carried out for various probabilities values and different initial densities. For illustration purpose, we display the numerical solutions of the system graphically. Furthermore, the percentage error in the estimate for real values of the parameters will be calculated. The percentage error $PE_{\hat{p}}$ of the estimator \hat{p} of the parameter p is given by the following rule:

$$PE_{\hat{p}} = \frac{|p - \hat{p}|}{p}. \quad (19)$$

The following figures display two examples of numerical solutions of the non-linear system in Equation (18). The first solution is shown below.

Clearly, the densities $P(2)$ to $P(5)$ in **Figures 4(b)-(e)** respectively, tend to the steady states. While, the density $P(1)$ in **Figure 4(a)** converges to the steady state. Also, the estimator $\hat{p}_1(t)$ in **Figure 4(f)** tends to the real value p_1 . While the estimators $\hat{p}_2(t)$ and $\hat{p}_3(t)$ in **Figure 4(g)** and **Figure 4(h)** converge to the real values p_2 and p_3 respectively. The steady state and the real values are represented by the dotted lines.

The second numerical example of solution is given below.

The densities $P(1), P(4)$ and $P(5)$ in **Figure 5(a), Figure 5(d)** and **Figure 5(e)** respectively, tend to the steady states. While, the densities $P(2)$ and $P(3)$ in **Figure 5(b)** and **Figure 5(c)** respectively, converge to the steady state. Also, the estimators

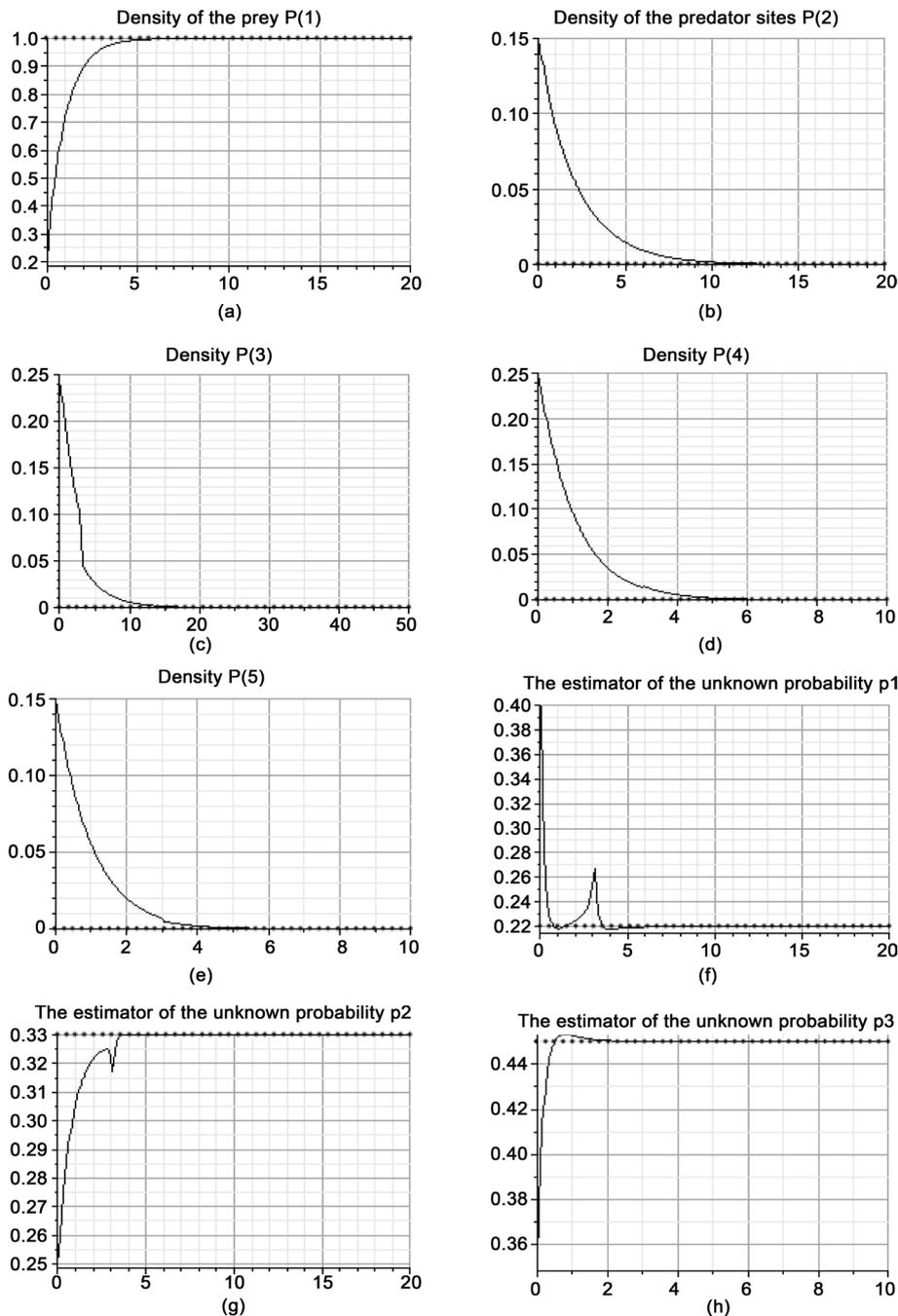


Figure 4. The controlled densities and the estimators of the unknown parameters of the stochastic lattice gas of prey-predator model with pair approximation for some values of parameters and initial densities as follows:

k_1	k_2	k_3	k_4	k_5	m_1	m_2	m_3	$p_1(t)$	$p_2(t)$	$p_3(t)$
1	0.45	0.3	1	1	5	5	5	0.22	0.35	0.45
$P_1 _0$	$P_2 _0$	$P_3 _0$	$P_4 _0$	$P_5 _0$	$\hat{p}_1 _0$	$\hat{p}_2 _0$	$\hat{p}_3 _0$			
0.2	0.15	0.25	0.25	0.15	0.4	0.25	0.35			

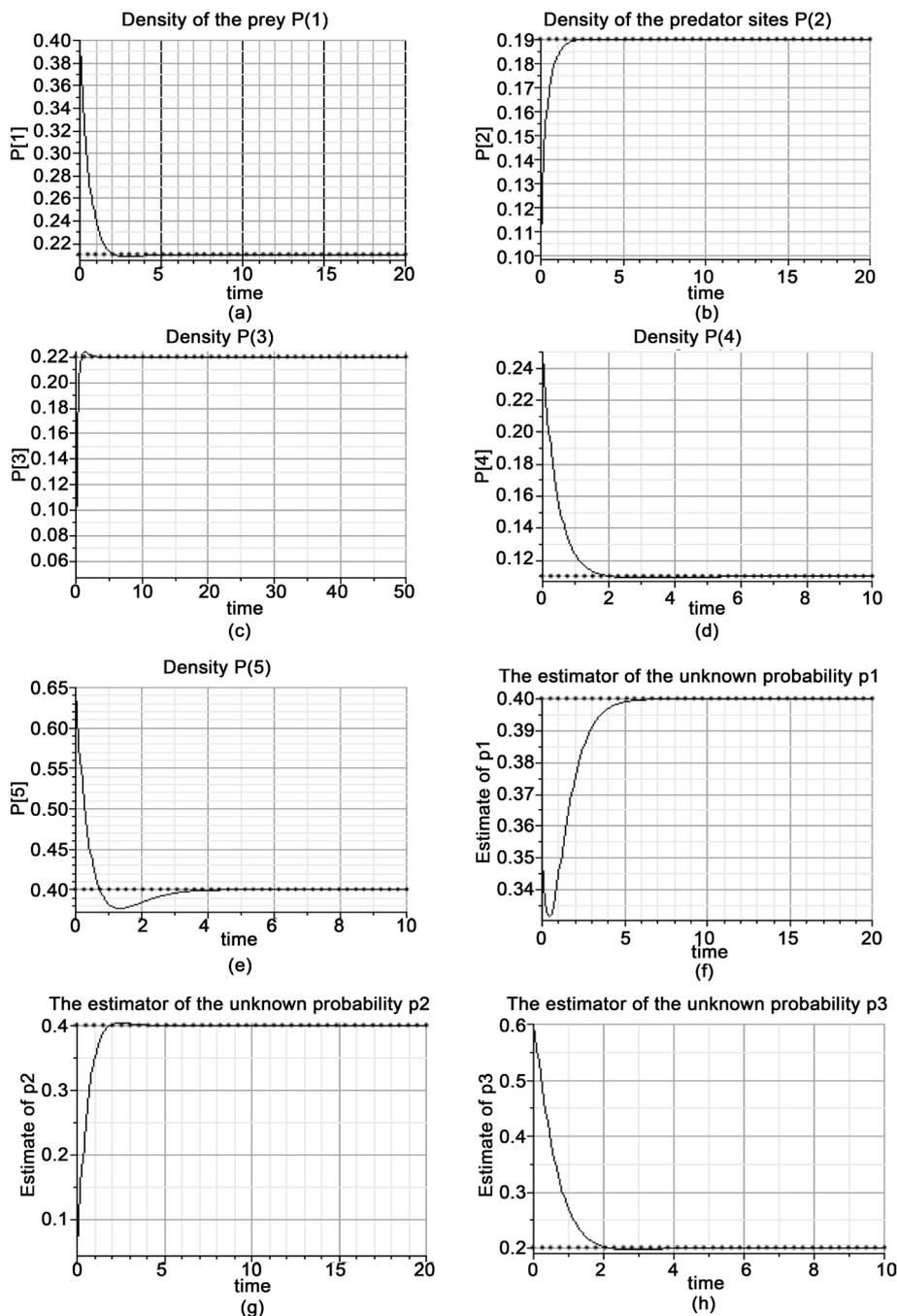


Figure 5. The controlled densities and the estimators of the unknown parameters of the stochastic lattice gas of prey-predator model with pair approximation for some values of parameters and initial densities as follows:

k_1	k_2	k_3	k_4	k_5	m_1	m_2	m_3	$p_1(t)$	$p_2(t)$	$p_3(t)$
2	5	3	2	1	1	2	2	0.4	0.4	0.2
$P_1 _0$	$P_2 _0$	$P_3 _0$	$P_4 _0$	$P_5 _0$	$\hat{p}_1 _0$	$\hat{p}_2 _0$	$\hat{p}_3 _0$			
0.4	0.1	0.05	0.25	0.65	0.35	0.05	0.6			

Table 2. The estimated values of the unknown parameters against the real values, and the percentage errors for some real values.

p_1	\hat{p}_1	PE_{p_1}	p_2	\hat{p}_2	PE_{p_2}	p_3	\hat{p}_3	PE_{p_3}
0.4	4.00000E-01	7.19101E-11	0.4	4.00000E-01	1.64001E-11	0.2	2.00000E-01	1.72100E-11
0.3	3.00000E-01	6.07999E-11	0.3	3.00000E-01	1.38334E-11	0.4	4.00000E-01	5.43746E-12
0.33	3.30000E-01	6.49486E-11	0.33	3.30000E-01	1.47910E-11	0.34	3.40000E-01	7.52927E-12
0.15	1.50000E-01	2.76466E-11	0.65	6.50000E-01	1.51383E-12	0.2	2.00000E-01	2.61000E-12
0.46	4.60000E-01	7.41087E-11	0.22	2.20000E-01	3.52318E-11	0.32	3.20000E-01	1.26968E-11
0.1	1.00000E-01	2.11099E-11	0.3	3.00000E-01	1.62000E-12	0.6	6.00000E-01	4.28361E-13
0.5	5.00000E-01	7.77060E-11	0.4	4.00000E-01	2.21476E-11	0.1	1.00000E-01	4.64799E-11
0.002	2.00000E-03	5.23409E-09	0.661	6.61000E-01	3.54767E-12	0.337	3.37000E-01	3.63507E-12
0.1	1.00000E-01	9.38000E-12	0.8	8.00000E-01	1.93734E-13	0.1	1.00000E-01	7.25114E-13
0.5	5.00000E-01	7.66520E-11	0.25	2.50000E-01	3.48760E-11	0.25	2.50000E-01	1.82880E-11

$\hat{p}_1(t)$ and $\hat{p}_2(t)$ in **Figure 5(f)** and **Figure 5(g)** tends to the real values p_1 and p_2 respectively. While the estimator $\hat{p}_3(t)$ in **Figure 5(h)** converges to the real values p_3 . The steady states and the real values are represented by the dotted lines. The previous figures show the approaching of the trajectories of the system to its steady states and approaching of the estimated values to the real values of the system unknown parameters over time.

In the following, numerical calculation for the percentage error in each estimate for different real values of the system unknown parameters.

Table 2 represents the comparing between the real values of the unknown parameters and its estimated values, in addition to the infinitesimal values of the percentage errors indicate to a strong convergence between the assumed real values and its estimated values so they are almost equally. These so good results are shown a high efficiency of the proposed Liapunov technique in the estimation process.

6. Conclusion

In this paper, we have introduced the mathematical model of stochastic lattice gas of prey-predator model with pair-approximation. The stability for this model has been discussed and it is found that this system has a chaos behavior. The estimators of the unknown parameters and the updating rules are derived according the conditions of the asymptotic stability. Some numerical solutions to show the stable system are presented graphically.

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