

# Fixed Point Theorems in Intuitionistics Fuzzy Metric Spaces Using Implicit Relations

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## Abstract

**In this paper, we proved some fixed point theorems in intuitionistic fuzzy metric spaces applying the properties of weakly compatible mapping and satisfying the concept of implicit relations for  $t$  norms and  $t$  connorms.**

## Keywords

**Intuitionistic Fuzzy Metric Spaces, Weakly Compatible Mapping Implicit Relations for  $t$  Norms and  $t$  Connorms**

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## 1. Introduction

The concept of fuzzy sets is introduced by Zadeh [1]. In 1975, Kramosil and Michlek [2] introduced the concept of Fuzzy sets, Fuzzy metric spaces. George and Veeramani [3] gave the modified version of fuzzy metric spaces using continuous  $t$  norms. In 2005, Park, Kwun and Park [4] proved some point theorems “intuitionistic fuzzy metrics spaces”. In 1986, Jungck [5] introduced concept of compatible mappings for self mappings. Lots of the theorems were proved for the existence of common fixed points in classical and fuzzy metric spaces. Aamri and Moutawakil [6] introduced the concept of non-compatibility using E. A. property and proved several fixed point theorems under contractive conditions. Atanassove [7] introduced the concept of intuitionistic fuzzy sets which is a generalization of fuzzy sets.

In 2004, Park [8] defined intuitionistic fuzzy metric spaces using  $t$ -norms and  $t$  connorms as a generalization of fuzzy metric spaces. Turkoglu [9] generalized Jungck common fixed point theorem to intuitionistic fuzzy metric spaces. In this paper, we used E. A. property in intuitionistic fuzzy metric spaces to prove fixed point theorems for a pair of selfmaps. Kumar, Bhatia and Manro [10] proved common fixed point theorems for weakly maps sa-

tisfying E. A. property in “intuitionistic fuzzy metrics spaces” using implicit relation.

In this paper, we proved fixed point theorems for weakly compatible mappings satisfying E. A. property in “intuitionistic fuzzy metrics spaces” using implicit relation.

## 2. Preliminaries

**Definition 1.1 (t norms).** A binary operation  $*$ :  $[0,1] \times [0,1] \rightarrow [0,1]$  is a continuous  $t$  norms if  $*$  satisfies the following axioms:

- 1)  $*$  is commutative as well as associative
- 2)  $*$  is continuous
- 3)  $a * 1 = a, \forall a \in [0,1]$
- 4)  $a * b \leq c * d \Rightarrow a \leq c$  and  $b \leq d, \forall a, b, c, d \in [0,1]$

**Definition 1.2 (t conorms).** A binary operation  $\diamond$ :  $[0,1] \times [0,1] \rightarrow [0,1]$  is a continuous  $t$  conorms if  $\diamond$  satisfies the following axioms:

- 1)  $\diamond$  is commutative as well as associative
- 2)  $\diamond$  is continuous
- 3)  $a \diamond 0 = a, \forall a \in [0,1]$
- 4)  $a \diamond b \leq c \diamond d \Rightarrow a \leq c$  and  $b \leq d, \forall a, b, c, d \in [0,1]$

Alaca [11] generalized the Fuzzy metric spaces of Kramosil and Michlek [2] and defined intuitionistic fuzzy metric spaces with the help of continuous  $t$ -norms and  $t$  conorms as:

**Definition 1.3 (intuitionistic fuzzy metric spaces).** A 5-tuple  $(X, M, N, *, \diamond)$  is said to be intuitionistic fuzzy metric spaces if  $X$  is a arbitrary set,  $*$  and  $\diamond$  are  $t$ -norms and  $t$  conorms respectively and  $M$  and  $N$  are fuzzy sets on  $X^2 \times [0, \infty)$  satisfying the following axioms:

- 1)  $M(x, y, t) + N(x, y, t) \leq 1, \forall x, y \in X$  and  $t > 0$
- 2)  $M(x, y, 0) = 0, \forall x, y \in X$
- 3)  $M(x, y, t) = 1, \forall x, y \in X$  and  $t > 0$  iff  $x = y$
- 4)  $M(x, y, t) = N(y, x, t), \forall x, y \in X$  and  $t > 0$
- 5)  $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s), \forall x, y, z \in X$  and  $t, s > 0$
- 6)  $M(x, y, \cdot) \square [0, \infty) \rightarrow [0, 1]$  is left continuous  $\forall x, y \in X$
- 7)  $\lim_{n \rightarrow \infty} M(x, y, t) = 1, \forall x, y \in X$  and  $t > 0$
- 8)  $N(x, y, 0) = 1, \forall x, y \in X$
- 9)  $N(x, y, t) = 1, \forall x, y \in X$  and  $t > 0$  iff  $x = y$
- 10)  $N(x, y, t) = N(y, x, t), \forall x, y \in X$  and  $t > 0$
- 11)  $N(x, y, t) \diamond N(y, z, s) \leq N(x, z, t + s), \forall x, y, z \in X$  and  $t, s > 0$
- 12)  $N(x, y, \cdot) \square [0, \infty) \rightarrow [0, 1]$  is right continuous  $\forall x, y \in X$
- 13)  $\lim_{n \rightarrow \infty} N(x, y, t) = 0, \forall x, y \in X$  and  $t > 0$

Then  $(M, N)$  is called an intuitionistic fuzzy metric spaces on  $x$ . The functions  $M(x, y, t)$  and  $N(x, y, t)$  define the degree of nearness and degree of non-nearness between  $x$  and  $y$  with respect to respectively.

**Proposition 1.4.** Every fuzzy metric space  $(X, M, *)$  is an Intuitionistic fuzzy space of the form  $(X, M, 1 - M, *, \diamond)$  if  $*$  and  $\diamond$  are associate as

$$x \diamond y = 1 - ((1 - x) * (1 - y)), \forall x, y \in X$$

**Proposition 1.4.** In intuitionistic fuzzy metric spaces  $(X, M, N, *, \diamond)$ ,  $M(x, y, *)$  is increasing and  $N(x, y, \diamond)$  is decreasing  $\forall x, y \in X$ .

**Lemma 1.5.** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric spaces. Then

- 1) A sequence  $\{x_n\}$  in  $X$  is convergent to a point  $x \in X$  if, for  $t > 0$

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_n, x, t) = 0$$

- 2) A sequence  $\{x_n\}$  in  $X$  is Cauchy sequence if, for  $t > 0$  and  $p > 0$

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(x_{n+p}, x, t) = 0$$

3) An intuitionistic fuzzy metric spaces  $(X, M, N, *, \diamond)$  is said to be complete if every Cauchy sequence in  $X$  is convergent.

**Example 1.6.** Consider  $X = \left\{ \frac{1}{n} \square_{\mathbb{R}} \in N \cup \{0\} \right\}$ , and continuous  $t$  norm  $*$  and continuous  $t$  conorm  $\diamond$  as  $\forall a, b \in [0, 1], a * b = ab$  and  $a \diamond b = \min\{1, a + b\}$ . If  $\forall x, y \in X$  and  $t > 0$ ,  $(M, N)$  is defined as

$$M(x, y, t) = \begin{cases} \frac{t}{t + |x - y|} & t > 0 \\ 0 & t = 0 \end{cases} \text{ and } N(x, y, t) = \begin{cases} \frac{t}{t + |x - y|} & t > 0 \\ 0 & t = 0 \end{cases}$$

Then  $(X, M, N, *, \diamond)$  is complete intuitionistic fuzzy metric spaces.

**Proposition 1.7.** A pair of self mappings  $(f, g)$  of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is called commuting if  $\forall x \in X$

$$M(fgx, gfx, t) = 1 \text{ and } N(fgx, gfx, t) = 0$$

**Proposition 1.8.** A pair of self mappings  $(f, g)$  of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is called weakly compatible if they commute at coincidence point *i.e.*, for  $v \in X$  we have  $fv = gv$ , then  $fgv = gfv$ .

**Proposition 1.9.** A pair of self mappings  $(f, g)$  of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  is said to satisfy E. A. property if there exist a sequence  $\{x_n\}$  of  $x$  such that

$$\lim_{n \rightarrow \infty} M(fx_n, gx_n, t) = 1 \text{ and } \lim_{n \rightarrow \infty} N(fx_n, gx_n, t) = 0.$$

### 3. Implicit Functions

Popa [12] defined the concept of implicit function in proving of fixed point theorems in hybrid metric spaces. Implicit function can be described as, let  $\emptyset$  be the family of lower semi-continuous functions  $F : R_+^6 \rightarrow R$  satisfying the following conditions:

G<sub>1</sub>:  $F$  is non-increasing in variables  $t_2, t_3, t_4, t_5, t_6$  and non-decreasing in  $t_1$

G<sub>2</sub>:  $\exists h \in (0, 1)$  and  $k > 1$  with  $hk < 1$ , such that  $u \leq kt$  and  $F(t, v, v, u, u + v, o) \leq 0 \Rightarrow t \leq hv$

G<sub>3</sub>:  $f(t, t, t, 0, t, t) > t, t > 0$

Popa [12] defined the following examples of implicit function too,

**Example 2.1.** Let  $f : R_+^6 \rightarrow R$  as

$$f(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - m \max \left\{ t_2, t_3, t_4, \frac{t_5 + t_6}{2} \right\}$$

where  $m \in [0, 1)$ .

**Example 2.2.** Let  $f : R_+^6 \rightarrow R$  as

$$f(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - h \left[ a \max \left\{ t_2, t_3, t_4, \frac{t_5 + t_6}{2} \right\} \right] + (1 - a) \max \{ t_2^2, t_3 t_4, t_3 t_5, t_4 t_6, t_5 t_6 \}^{\frac{1}{2}}$$

where  $h \in [0, 1), a \in [0, 1]$ .

**Example 2.3.** Let  $f : R_+^6 \rightarrow R$  as

$$f(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - m \max \{ t_2^2, t_3 t_4, t_3 t_5, t_4 t_6, t_5 t_6 \}^{\frac{1}{2}}$$

where  $m \in \left[ 0, \frac{1}{\sqrt{2}} \right)$ .

**Example 2.4.** Let  $f : R_+^6 \rightarrow R$  as

$$f(t_1, t_2, t_3, t_4, t_5, t_6) = t_1^3 + t_2^3 + \frac{t_1}{1+t_5 t_6} - [at_2^2 + bt_3^2 + ct_4^2],$$

where  $a + b + c < 1$ .

M. Imdad and Javed Ali [13]-[15] added some implicit functions to prove fixed point theorems for Hybrid contraction. Following are examples are as:

**Example 2.5.** Let  $f : R_+^6 \rightarrow R$  as

$$f(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \alpha \left[ \frac{t_3^2 + t_4^2}{t_3 + t_4} \right] - \beta[t_5 + t_6] - \gamma t_2,$$

where  $0 < 2\alpha + 2\beta + \gamma < 1$ .

**Example 2.6.** Let  $f : R_+^6 \rightarrow R$  as

$$f(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \alpha \left[ \frac{t_5^2 + t_6^2}{t_5 + t_6} \right] - \beta[t_3 + t_4] - \gamma t_2,$$

where  $0 < 2\alpha + 2\beta + \gamma < 1$ .

**Example 2.7.** Let  $f : R_+^6 \rightarrow R$  as

$$f(t_1, t_2, t_3, t_4, t_5, t_6) = t_1^p - \left[ \frac{ct_3 t_4^p + bt_5 t_6^p}{t_3 + t_4} \right] - at_2^p,$$

where  $1 < 2a + c < 2$ ,  $1 < 2a + b < 2$  and  $p \geq 1$ .

If  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space. Continuous  $t$ -norms and  $t$  consnorms are defined as  $a * a \geq a$  and  $(1-a) \diamond (1-a) \leq (1-a)$  respectively, where  $a \in [0, 1]$ .

Then implicit functions can be defined as  $\varphi, \varnothing : [0, 1] \rightarrow [0, 1]$  are mappings and upper semi-continuous, non-decreasing, such that  $\varphi, \varnothing \square [0, 1]^6 \rightarrow [0, 1]$ , then

$$(F_1) \quad \varphi(t_1, t_2, 1, 1, t_5, 1) \geq t, \varnothing(t_1, t_2, 0, 0, t_5, 0) \leq t$$

$$(F_2) \quad \varphi(1, 1, t_3, t_4, 1, 1) \geq t, \varnothing(0, 0, t_3, t_4, 0, 0) \leq t$$

$$(F_3) \quad \varphi(1, t_2, 1, t_4, 1, t_6) \geq t, \varnothing(0, t_2, 0, t_4, 0, t_6) \leq t$$

**Example 2.8.**  $\varphi, \varnothing : [0, 1] \rightarrow [0, 1]$  are mappings and upper semi-continuous, non-decreasing, such that  $\varphi, \varnothing \square [0, 1]^6 \rightarrow [0, 1]$ , then

$$(F_1) \quad \varphi\{1, 1, t_3^2, t_4^2, 1, 1\}^{\frac{1}{2}} \geq t, \varnothing\{0, 0, t_3^2, t_4^2, 0, 0\}^{\frac{1}{2}} \leq t$$

$$(F_2) \quad \varphi\{t_1^2, t_2^2, 1, 1, 1, 1\}^{\frac{1}{2}} \leq t, \varnothing\{t_1^2, t_2^2, 0, 0, 0, 0\}^{\frac{1}{2}} \leq t$$

$$(F_3) \quad \varphi\{t_1^2, 1, t_3^2, 1, t_5^2, 1\}^{\frac{1}{2}} \geq t, \varnothing\{t_1^2, 0, t_3^2, 0, t_5^2, 0\}^{\frac{1}{2}} \leq t, \text{ and } t \in [0, 1]$$

**Example 2.9.**  $\varphi, \varnothing : [0, 1] \rightarrow [0, 1]$  are mappings and upper semi-continuous, non-decreasing, such that  $\varphi, \varnothing \square [0, 1]^6 \rightarrow [0, 1]$ , then

$$(F_1) \quad \varphi\{1, 1, t_3^2, t_4^2, 1, 1\}^{\frac{1}{2}} \geq t, \varnothing\{0, 0, t_3^2, t_4^2, 0, 0\}^{\frac{1}{2}} \leq t$$

$$(F_2) \quad \varphi\{t_1^2, 1, 1, t_4^2, 1, 1\}^{\frac{1}{2}} \leq t, \varnothing\{t_1^2, 0, 0, t_4^2, 0, 0\}^{\frac{1}{2}} \leq t$$

$$(F_3) \quad \varphi\{t_1^2, 1, t_3^2, 1, t_5^2, 1\}^{\frac{1}{2}} \geq t, \varnothing\{t_1^2, 0, t_3^2, 0, t_5^2, 0\}^{\frac{1}{2}} \leq t, \text{ and } t \in [0, 1]$$

**Example 3.0.**  $\varphi, \varnothing : [0, 1] \rightarrow [0, 1]$  are mappings and upper semi-continuous, non-decreasing, such that  $\varphi, \varnothing \square [0, 1]^6 \rightarrow [0, 1]$ , then

$$(F_1) \quad \varphi\{1, 1, t_3, t_4, 1, 1\} \geq t, \varnothing\{0, 0, t_3, t_4, 0, 0\} \leq t$$

$$(F_2) \quad \varphi\{t_1, 1, t_3, 1, 1, t_6\} \leq t, \varnothing\{t_1, 0, t_3, 0, 0, t_6\} \leq t$$

$$(F_3) \quad \varphi\{t_1, 1, t_3, 1, t_5, 1\} \geq t, \varnothing\{t_1, 0, t_3, 0, t_5, 0\} \leq t, \text{ and } t \in [0, 1]$$

### 4. Main Result

**Theorem 3.1.** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space. Continuous  $t$  norms and  $t$  conorms are defined as  $a * a \geq a$  and  $(1 - a) \diamond (1 - a) \leq (1 - a)$  respectively, where  $a \in [0, 1]$ . Let  $T$  and  $S$  be two weakly compatible maps of  $X$  satisfying the following conditions:

- (3.1.1)  $T$  and  $S$  satisfying E.A. properties,
- (3.1.2)  $S$  is the closed subspaces of  $X$ ,
- (3.1.3)  $\forall x, y \in X, t > 0, \alpha + \beta < 1$ , there is  $\alpha \in (0, 1)$ , such that

$$M(Tx, Ty, kt) \geq \varphi \{ M(Sx, Sy, T), M(Tx, Sx, t), M(Ty, Sy, t), M(Tx, Sx, \alpha t) \\ * M(Sx, Sy, \beta t), M(Ty, Sy, \alpha t) * M(Sy, Sx, \beta t) \}$$

$$N(Tx, Ty, kt) \leq \theta \{ N(Sx, Sy, T), N(Tx, Sx, t), N(Ty, Sy, t), N(Tx, Sx, \alpha t) \\ * N(Sx, Sy, \beta t), N(Ty, Sy, \alpha t) * N(Sy, Sx, \beta t) \}$$

where  $\varphi, \theta: [0, 1] \rightarrow [0, 1]$  are mappings and upper semi-continuous, non-decreasing, such that

$$\varphi(1, 1, t, 1, t) \geq t, \varphi(t, 1, 1, t, t) \geq t, \theta(1, 1, t, 1, t) \leq t, \theta(t, 1, 1, t, t) \leq t \text{ and } t \in [0, 1]$$

Then  $S$  and  $T$  have a common fixed point.

**Proof.** From (3.1.1), we have a sequence  $\{x_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} Sx_n = u,$$

for some  $u \in X$ . From (3.1.2),  $S(X)$  is the closed subspace of  $X \Rightarrow$  there is  $v \in X$  such that  $u = Sv$ .

Therefore  $\lim_{n \rightarrow \infty} Tx_n = u = Sv = \lim_{n \rightarrow \infty} Sx_n$ . Now our goal is to prove  $Tv = Sv$ .

In (3.1.3), taking  $x = x_n$  and  $y = v$ , we have

$$M(Tx_n, Tv, kt) \geq \varphi \{ M(Sx_n, Sv, t), M(Tx_n, Sv, t), M(Tv, Sv, t), M(Tx_n, Sx_n, \alpha t) \\ * M(Sx_n, Sv, \beta t), M(Tv, Sv, \alpha t) * M(Sv, Sx_n, \beta t) \}$$

$$M(Tx_n, Tv, kt) \geq \varphi \{ M(Sx_n, Sv, t), M(Tx_n, Sv, t), M(Tv, Sv, t), \\ M(Tx_n, Sv, (\alpha + \beta)t), M(Tv, Sx_n, (\alpha + \beta)t) \}$$

Taking  $\lim n \rightarrow \infty$ , we have

$$M(Sv, Tv, kt) \geq \varphi \{ M(Sv, Sv, t), M(Sv, Sv, t), M(Tv, Sv, t), \\ M(Sv, Sv, (\alpha + \beta)t), M(Tv, Sv, (\alpha + \beta)t) \}$$

$$M(Sv, Tv, kt) \geq \varphi \{ 1, M(Sv, Sv, t), M(Tv, Sv, t), M(Sv, Sv, t), M(Tv, Sv, t) \}$$

Since

$$\alpha + \beta < 1 = \varphi \{ 1, 1, M(Tv, Sv, t), 1, M(Tv, Sv, t) \} \geq M(Tv, Sv, t)$$

Similarly

$$N(Tx_n, Tv, kt) \leq \theta \{ N(Sx_n, Sv, t), N(Tx_n, Sv, t), N(Tv, Sv, t), N(Tx_n, Sx_n, \alpha t) \\ * N(Sx_n, Sv, \beta t), N(Tv, Sv, \alpha t) * N(Sv, Sx_n, \beta t) \}$$

$$N(Tx_n, Tv, kt) \leq \theta \{ N(Sx_n, Sv, t), N(Tx_n, Sv, t), N(Tv, Sv, t), \\ N(Tx_n, Sv, (\alpha + \beta)t), N(Tv, Sx_n, (\alpha + \beta)t) \}$$

Taking  $\lim n \rightarrow \infty$ , we have

$$N(Sv, Tv, kt) \leq \theta \{ N(Sv, Sv, t), N(Sv, Sv, t), N(Tv, Sv, t), N(Sv, Sv, (\alpha + \beta)t), N(Tv, Sv, (\alpha + \beta)t) \}$$

$$N(Sv, Tv, kt) \leq \theta \{1, N(Sv, Sv, t), N(Tv, Sv, t), N(Sv, Sv, t), N(Tv, Sv, t)\}$$

Since

$$\alpha + \beta < 1 = \theta \{1, 1, N(Tv, Sv, t), 1, N(Tv, Sv, t)\} \leq N(Tv, Sv, t)$$

Hence  $Tv = Sv = z$  (say)  $\Rightarrow v$  is a coincident point of  $T$  and  $S$ .

Again  $T$  and  $S$  are compatible mappings, therefore  $v = STv = z \Rightarrow Tz = Sz$ .

Now we are to show that  $v$  is common point of  $T$  and  $S$ . Therefore replacing  $x$  and  $y$  by  $z$  and  $v$  in (3.1.3), we have

$$\begin{aligned} M(Tz, z, t) &= M(Tz, Sz, t) \geq \varphi \{M(Sz, Sv, t), M(Tz, Sz, t), M(Tv, Sv, t), \\ &\quad M(Tz, Sv, (\alpha + \beta)t), M(Tv, Sz, (\alpha + \beta)t)\} \\ M(Tz, z, t) &\geq \varphi \{M(Tz, z, t), M(Tz, Tz, t), M(z, z, t), M(Tz, z, t), M(z, Tz, t)\} \end{aligned}$$

Since  $\alpha + \beta < 1$

$$M(Tz, z, t) \geq \varphi \{M(Tz, z, t), 1, 1, M(Tz, z, t), M(z, Tz, t)\} \geq M(Tz, z, t)$$

Similarly

$$\begin{aligned} N(Tz, z, t) &= N(Tz, Sz, t) \leq \theta \{N(Sz, Sv, t), N(Tz, Sz, t), N(Tv, Sv, t), \\ &\quad N(Tz, Sv, (\alpha + \beta)t), N(Tv, Sz, (\alpha + \beta)t)\} \\ N(Tz, z, t) &\leq \theta \{N(Tz, z, t), N(Tz, Tz, t), N(z, z, t), N(Tz, z, t), N(z, Tz, t)\} \end{aligned}$$

Since  $\alpha + \beta < 1$

$$N(Tz, z, t) \leq \theta \{N(Tz, z, t), 1, 1, N(Tz, z, t), N(z, Tz, t)\} \leq N(Tz, z, t)$$

$\Rightarrow Tz = Sz = z \Rightarrow z$  is a common fixed point for  $T$  and  $S$ .

Uniqueness of the point will be proved by contradiction. For that suppose  $p$  and  $q$  be two fixed points. Therefore from (3.1.3) we have

$$\begin{aligned} M(Tp, Tq, kt) &\geq \varphi \{M(Sp, Sq, t), M(Tp, Sp, t), M(Tq, Sq, t), M(Tp, Sp, \alpha t) \\ &\quad * M(Sp, Sq, \beta t), M(Tq, Sq, \alpha t) * M(Sq, Sp, \beta t)\} \\ M(p, q, kt) &\geq \varphi \{M(p, q, t), M(p, p, t), M(q, q, t), M(p, q, (\alpha + \beta)t), M(q, p, (\alpha + \beta)t)\} \\ M(p, q, kt) &\geq \varphi \{M(p, q, t), M(p, p, t), M(q, q, t), M(p, q, t), M(q, p, t)\} \end{aligned}$$

Since  $\alpha + \beta < 1$

$$M(p, q, kt) \geq \varphi \{M(p, q, t), 1, 1, M(p, q, t), M(q, p, t)\} \geq M(p, q, t)$$

Similarly

$$\begin{aligned} N(Tp, Tq, kt) &\leq \theta \{N(Sp, Sq, t), N(Tp, Sp, t), N(Tq, Sq, t), N(Tp, Sp, \alpha t) \\ &\quad * N(Sp, Sq, \beta t), N(Tq, Sq, \alpha t) * N(Sq, Sp, \beta t)\} \\ N(p, q, kt) &\leq \theta \{N(p, q, t), N(p, p, t), N(q, q, t), N(p, q, (\alpha + \beta)t), N(q, p, (\alpha + \beta)t)\} \\ N(p, q, kt) &\leq \theta \{N(p, q, t), N(p, p, t), N(q, q, t), N(p, q, t), N(q, p, t)\} \end{aligned}$$

Since  $\alpha + \beta < 1$

$$N(p, q, kt) \leq \theta \{ N(p, q, t), 1, 1, N(p, q, t), N(q, p, t) \} \leq N(p, q, t)$$

$\Rightarrow p = q$

Hence mappings  $T$  and  $S$  have a unique fixed point.

This completes the proof.

**Theorem 3.2.** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space. Continuous  $t$  norms and  $t$  conorms are defined as  $a * a \geq a$  and  $(1-a) \diamond (1-a) \leq (1-a)$  respectively, where  $a \in [0, 1]$ .

Let  $T$  and  $S$  be two weakly compatible maps of  $X$  satisfying the following conditions:

(3.2.1)  $T$  and  $S$  satisfying E.A. properties,

(3.2.2)  $S$  is the closed subspaces of  $X$ ,

(3.2.3)  $\forall x, y \in X, t > 0$ , such that

$$\begin{aligned} & \varphi \{ M(Sx, Sy, T), M(Tx, Sx, t), M(Ty, Sy, t), M(Tx, Sy, t), M(Ty, Sx, t) \} < 0 \\ & \theta \{ N(Sx, Sy, T), N(Tx, Sx, t), N(Ty, Sy, t), N(Tx, Sy, t), N(Ty, Sx, t) \} < 0 \end{aligned}$$

where  $\varphi, \theta: [0, 1] \rightarrow [0, 1]$  are mappings and upper semi-continuous, non-decreasing, such that

(3.2.4)  $\varphi(1, 1, t, t) \geq t, \theta(t, 1, 1, t) \geq t, \theta(1, 1, t, t) \leq t, \theta(t, 1, 1, t) \leq t$  and  $t \in [0, 1]$

Then  $S$  and  $T$  have a common fixed point.

**Proof.** From (3.2.1), we have a sequence  $\{x_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Tx_n = \lim_{n \rightarrow \infty} Sx_n = u,$$

for some  $u \in X$ . From (3.2.2),  $S(X)$  is the closed subspace of  $X \Rightarrow$  there is  $v \in X$  such that  $u = Sv$ .

Therefore  $\lim_{n \rightarrow \infty} Tx_n = u = Sv = \lim_{n \rightarrow \infty} Sx_n$ . Now our goal is to prove  $Tv = Sv$ .

In (3.2.3), taking  $x = x_n$  and  $y = v$ , we have

$$\varphi \{ M(Sx_n, Sv, t), M(Tx_n, Sv, t), M(Tv, Sv, t), M(Tx_n, Sv, t), M(Tv, Sx_n, t) \} < 0$$

Taking  $\lim n \rightarrow \infty$ , we have,

$$\begin{aligned} & \varphi \{ M(Sv, Sv, t), M(Sv, Sv, t), M(Tv, Sv, t), M(Sv, Sv, t), M(Tv, Sv, t) \} < 0 \\ & = \varphi \{ 1, 1, M(Tv, Sv, t), 1, M(Tv, Sv, t) \} = M(Tv, Sv, t) < 0 \end{aligned} \quad (3.2.5)$$

Similarly

$$\theta \{ N(Sx_n, Sv, t), N(Tx_n, Sv, t), N(Tv, Sv, t), N(Tx_n, Sv, t), N(Tv, Sx_n, t) \} > 0$$

Taking  $\lim n \rightarrow \infty$ , we have

$$\begin{aligned} & \theta \{ N(Sv, Sv, t), N(Sv, Sv, t), N(Tv, Sv, t), N(Sv, Sv, t), N(Tv, Sv, t) \} > 0 \\ & = \varphi \{ 1, 1, N(Tv, Sv, t), 1, N(Tv, Sv, t) \} \leq N(Tv, Sv, t) > 0 \end{aligned} \quad (3.2.6)$$

(3.2.5) and (3.2.6) both are the contradiction of (3.2.4).

Hence  $Tv = Sv = z$  (say)  $\Rightarrow v$  is a coincident point of  $T$  and  $S$ . Again  $T$  and  $S$  are compatible mappings, therefore  $v = STv = z \Rightarrow z = Tz = Sz$ .

Now we are to show that  $v$  is common point of  $T$  and  $S$ . Therefore replacing  $x$  and  $y$  by  $z$  and  $v$  in (3.2.3), we have

$$\begin{aligned} & \varphi \{ M(Sz, Sv, t), M(Tz, Sz, t), M(Tv, Sv, t), M(Tz, Sv, t), M(Tv, Sz, t) \} \\ & = \varphi \{ M(Tz, z, t), M(Tz, Tz, t), M(z, z, t), M(Tz, z, t), M(z, Tz, t) \} \\ & = \varphi \{ M(Tz, z, t), 1, 1, M(Tz, z, t), M(z, Tz, t) \} = M(Tz, z, t) > 0 \end{aligned}$$

This is a contradiction. Similarly

$$\begin{aligned} & \theta\{N(Sz, Sv, t), N(Tz, Sz, t), N(Tv, Sv, t), N(Tz, Sv, t), N(Tv, Sz, t)\} \\ &= \theta\{N(Tz, z, t), N(Tz, Tz, t), N(z, z, t), N(Tz, z, t), N(z, Tz, t)\} \\ &= \theta\{N(Tz, z, t), 1, 1, N(Tz, z, t), N(z, Tz, t)\} = N(Tz, z, t) < 0 \end{aligned}$$

This is a contradiction again. Hence  $Tz = Sz = z \Rightarrow z$  is a common fixed point for  $T$  and  $S$ .

Uniqueness of the point will be proved by contradiction. For that suppose  $p$  and  $q$  be two fixed points.

Therefore from (3.2.3), we have

$$\begin{aligned} & \varphi\{M(Sp, Sq, t), M(Tp, Sp, t), M(Tq, Sq, t), M(Tp, Sq, t), M(Tq, Sp, t)\} \\ &= \varphi\{M(p, q, t), M(p, p, t), M(q, q, t), M(p, q, t), M(q, p, t)\} \\ &= \varphi\{M(p, q, t), M(p, p, t), M(q, q, t), M(p, q, t), M(q, p, t)\} \\ &= \varphi\{M(p, q, t), 1, 1, M(p, q, t), M(q, p, t)\} = M(p, q, t) > 0 \end{aligned}$$

Similarly

$$\begin{aligned} & \varphi\{N(Sp, Sq, t), N(Tp, Sp, t), N(Tq, Sq, t), N(Tp, Sq, t), N(Tq, Sp, t)\} \\ &= \varphi\{N(p, q, t), N(p, p, t), N(q, q, t), N(p, q, t), N(q, p, t)\} \\ &= \varphi\{N(p, q, t), N(p, p, t), N(q, q, t), N(p, q, t), N(q, p, t)\} \\ &= \varphi\{N(p, q, t), 1, 1, N(p, q, t), N(q, p, t)\} = N(p, q, t) < 0, \end{aligned}$$

This is the contradiction of (3.2.4).

$\Rightarrow p = q$ . Hence mappings  $T$  and  $S$  have a unique fixed point.

This completes the proof.

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