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Assessments of Some Simultaneous Equation Estimation Techniques with Normally and Uniformly Distributed Exogenous Variables

O. O. Alabi[1], B. A. Oyejola[2]

[1]Department of Statistics, Federal University of Technology, Akure, Nigeria
[2]Department of Statistics, University of Ilorin, Ilorin, Nigeria
Email: a.olatayoolusegunvictor@gmail.com, alabioo@futa.edu.ng, boyejola@unilorin.edu.ng

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Abstract

In each equation of simultaneous Equation model, the exogenous variables need to satisfy all the basic assumptions of linear regression model and be non-negative especially in econometric studies. This study examines the performances of the Ordinary Least Square (OLS), Two Stage Least Square (2SLS), Three Stage Least Square (3SLS) and Full Information Maximum Likelihood (FIML) Estimators of simultaneous equation model with both normally and uniformly distributed exogenous variables under different identification status of simultaneous equation model when there is no correlation of any form in the model. Four structural equation models were formed such that the first and third are exact identified while the second and fourth are over identified equations. Monte Carlo experiments conducted 5000 times at different levels of sample size (n = 10, 20, 30, 50, 100, 250 and 500) were used as criteria to compare the estimators. Result shows that OLS estimator is best in the exact identified equation except with normally distributed exogenous variables when $n \geq 100$. At these instances, 2SLS estimator is best. In over identified equations, the 2SLS estimator is best except with normally distributed exogenous variables when the sample size is small and large, $n = 10$ and $n \geq 250$; and with uniformly distributed exogenous variables when n is very large, $n = 500$, the best estimator is either OLS or FIML or 3SLS.

Keywords

Normally Distributed Exogenous Variables, Uniformly Distributed Exogenous Variables, Identification Status, Estimators, Exact Identified Equation, Over Identified Equation

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1. Introduction

A simultaneous equation system is a regression equation system where two types of variables (the endogenous, the predetermined or exogenous variable) appear with disturbance terms. [1] defined simultaneous equation as the process of modeling more than one equation at a time; a multi-equation modeling. It is proposed to solve the problem of correlation.

In a multi-equation model, the dependent variable Y appears as endogenous variable in one equation and as explanatory variable in another equation of the model. The X variable appears as the explanatory variable in the equations. This creates problems of equation identification, multicollinearity and choice of estimation techniques among others.

Identification problem which creates estimation problems has the same features with multicollinearity [2]. Correlation between the pairs of exogenous or independent variables is an important problem in econometrics especially in single equation estimation. The impact of multicollinearity is less serious when attention is focused on predicting or forecasting values of the endogenous variables than when the analyst is interested in estimating the parameters [3].

Econometric variables are often non-negative and can exhibit violation against the normality assumption of classical models which inevitably influences the performances of the estimation techniques. This study therefore examines the performances of four (4) common estimation techniques namely; Ordinary Least squares (OLS), Two-stage Least squares (2SLS), Three-stage Least squares (3SLS) and Full Information Maximum Likelihood Estimators under both normally and uniformly distributed exogenous variables for different equation identification status. Here it is assumed that there is no form of correlation in the simultaneous equation model.

Most economic data are often positive [4] and correlated [5]. However, various works done in the recent time especially on correlation studies on simultaneous equation model have being with normally distributed exogenous variables, exhibiting both positive and negative values [6] [7].

2. Methodology

The methodology followed in this research work is as follows:

The Model and Its Description

Consider the simultaneous Equation model of the for

$$
\begin{aligned}
y_{1t} &= \beta_{12}y_{2t} + \beta_{13}y_{3t} + \beta_{14}y_{4t} + \gamma_{14}x_{4t} + u_{1t} && \text{(i)} \\
y_{2t} &= \beta_{23}y_{3t} + \gamma_{21}x_{1t} + \gamma_{23}x_{3t} + u_{2t} && \text{(ii)} \\
y_{3t} &= \beta_{31}y_{1t} + \beta_{34}y_{4t} + \gamma_{31}x_{1t} + \gamma_{32}x_{2t} + u_{3t} && \text{(iii)} \\
y_{4t} &= \beta_{41}y_{1t} + \beta_{42}y_{42} + \gamma_{42}x_{2t} + u_{4t} && \text{(iv)}
\end{aligned}
\tag{1}
$$

where y_{it} is an endogenous variable, $i = 1,2,3,4$; x_{1t}, x_{2t}, x_{3t} and x_{4t} are the exogenous variables

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} \sim N(0, \Sigma)$$

where $t = 1,2,3,\cdots,n$ and $\beta_{12}, \beta_{13}, \beta_{14}, \beta_{23}, \beta_{31}, \beta_{34}, \beta_{41}, \beta_{42}, \gamma_{14}, \gamma_{21}, \gamma_{23}, \gamma_{31}, \gamma_{32}$ and γ_{42} are the structural parameters of the model. Equations (i) and (iii) are exactly identified while Equations (ii) and (iv) were over identified by both order and rank condition.

For the simulation study, Equation (1) was expressed as follows:

$$
\begin{aligned}
& y_{1t} - \beta_{12}y_{2t} - \beta_{13}y_{3t} - \beta_{14}y_{4t} + 0x_{1t} + 0x_{2t} + 0x_{3t} - \gamma_{14}x_{4t} = u_{1t} && \text{(i)} \\
& -\beta_{31}y_{1t} + 0y_{2t} + y_{3t} - \beta_{34}y_{4t} - \gamma_{31}x_{1t} - \gamma_{32}x_{2t} + 0x_{3t} + 0x_{4t} = u_{3t} && \text{(iii)} \\
& -\beta_{41}y_{1t} - \beta_{42}y_{2t} + 0y_{3t} + y_{4t} + 0x_{1t} - \gamma_{42}x_{2t} + 0x_{3t} + 0x_{4t} = u_{4t} && \text{(iv)}
\end{aligned}
\tag{4}
$$

This can be written in matrix form as:

$$\beta y_t + \Gamma x_t = u_t \tag{5}$$

where

$$\beta = \begin{bmatrix} 1 & -\beta_{12} & -\beta_{13} & -\beta_{14} \\ 0 & 1 & -\beta_{23} & 0 \\ -\beta_{31} & 0 & 1 & -\beta_{34} \\ -\beta_{41} & -\beta_{42} & 0 & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0 & 0 & 0 & -\gamma_{14} \\ -\gamma_{21} & 0 & -\gamma_{23} & 0 \\ -\gamma_{31} & -\gamma_{32} & 0 & 0 \\ 0 & -\gamma_{42} & 0 & 0 \end{bmatrix},$$

$$y_t = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}, \quad x_t = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \text{ and } u_t = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

Now, from (5),

$$\beta^{-1}\beta y_t = \beta^{-1}\Gamma x_t = \beta^{-1}u_t$$

$$y_t = \beta^{-1}\Gamma x_t + \beta^{-1}u_t \tag{6}$$

Equation (6) was used to generate the endogenous variables by taking the true value of the parameters as:

$$\beta = \begin{bmatrix} 1 & -\beta_{12} & -\beta_{13} & -\beta_{14} \\ 0 & 1 & -\beta_{23} & 0 \\ -\beta_{31} & 0 & 1 & -\beta_{34} \\ -\beta_{41} & -\beta_{42} & 0 & 1 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 0 & 0 & 0 & -\gamma_{14} \\ -\gamma_{21} & 0 & -\gamma_{23} & 0 \\ -\gamma_{31} & -\gamma_{32} & 0 & 0 \\ 0 & -\gamma_{42} & 0 & 0 \end{bmatrix}$$

Monte Carlo experiments were performed 5000 times for seven sample sizes (n = 10, 20, 30, 50, 100, 250 and 500) when there is no correlation of any form among exogenous variables and the error terms. The two forms of exogenous variables are generated to follow normal and uniform distribution.

3. Data Generation of Exogenous Variables

The generation was done as follows:

1) Normally Distributed Exogenous Variables

The exogenous were generated to be normal with mean zero and variance unity *i.e.* $x_i \sim N(0,1)$, $i = 1,2,3,4$ and $|\rho_{ij}| < 1$ is the value of correlation between the two variables *i* and *j*, [8]. In this study, $\rho_{12} = \rho_{13} = \rho_{14} = \rho_{23} = \rho_{24} = \rho_{34} = \rho = 0$, was adopted as a situation with no forms of any correlation between the exogenous variables.

2) Correlated Uniformly Distributed Variables

Using the generated normally distributed exogenous variables above, $X_i \sim N(0,1)$, I = 1, 2, 3, 4; we further utilize the properties of random variables that cumulative distribution function of normal distribution produces $U(0, 1)$ without affecting the correlation among the variables to generate correlated uniformly distributed exogenous variables, $X_i \sim U(0,1)$, I = 1, 2, 3, 4; [9].

3) Generation of Correlated Error Terms

Equation provided by [8] was modified when the mean of the error terms are zero and variance is unit (1). Also, $|\lambda_{ij}| < 1$ is the value of correlation between the two error terms *i* and *j*. In this study, $\lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{23} = \lambda_{24} = \lambda_{34} = \lambda = 0$, was adopted as no form of correlation between the error terms.

4) Method of Generating the Data of Endogenous Variables

Equation (6) was used to generate the endogenous variables by taking the true value of the parameters as:

$$\beta=\begin{bmatrix}1 & -\beta_{12} & -\beta_{13} & -\beta_{14}\\ 0 & 1 & -\beta_{23} & 0\\ -\beta_{31} & 0 & 1 & -\beta_{34}\\ -\beta_{41} & -\beta_{42} & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -3.2 & -0.2 & -1.2\\ 0 & 1 & -3.8 & 0\\ -1.6 & 0 & 1 & -1.0\\ -2.8 & -2.2 & 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & -0.8\\ -3.0 & 0 & -1.3 & 0\\ -1.5 & -0.6 & 0 & 0\\ 0 & -2.6 & 0 & 0\end{bmatrix}$$

5) Criteria Used for Assessment of the Estimators

To assess the performance of the estimators, the finite properties were used (Bias, Absolute bias, Variance and Mean Square Error):

Mathematically, MSE is the addition of the Bias squared to the variance. For any estimator $\hat{\theta}_{ij}$ of θ_{ij} in model (1),

$$\hat{\theta}_{ij}=\frac{1}{R}\sum_{l=1}^{R}\hat{\theta}_{ijl}\ ,\quad l=1,2,\cdots,R$$

where R = 5000

$$\text{BIAS}\left(\hat{\theta}_{ij}\right)=\hat{\theta}_{ij}-\theta_{ij}\quad \text{ABSBIAS}\left(\hat{\theta}_{ij}\right)=\frac{1}{R}\sum_{l=1}^{R}\left|\hat{\theta}_{ijl}-\theta_{ij}\right|,$$

as a measure of dispersion.

$$\text{MSE}\left(\hat{\theta}_{ij}\right)=\frac{1}{R}\sum_{l=1}^{R}\left(\hat{\theta}_{ijl}-\theta_{ij}\right)^2$$

$$\text{Var}\left(\hat{\theta}_{ij}\right)=\text{MSE}\left(\hat{\theta}_{ij}\right)-\left[\text{Bias}\left(\hat{\theta}_{ij}\right)\right]^2$$

In this paper, $\hat{\theta}$ is used to represent any of the parameters in (1).

4. Results and Discussion

4.1. Performance of the Estimator Based on the Bias Criterion

The performances of the estimators was examined, ranked and summed over all the parameters in each equation, having examined and summed the rank of the bias of each parameter over all the parameters in each equation the outcome is given in **Table 1**.

The preferred estimators for different identification status differ. The preferred estimators are also slightly affected by the two exogenous variables.

In the exact identified equation with normally distributed exogenous variables, OLS or 2SLS or both are generally preferred except for very large samples sizes $(n \geq 250)$. At this instance, 3SLS is preferred. Also, with uniformly distributed exogenous variables, OLS or 2SLS estimators are generally preferred.

In the over identified equation with normally distributed exogenous variable, 2SLS estimator is generally preferred except when $n \leq 20$ and when $n = 500$. At these instances, FIML estimator is preferred.

Also, with uniformly distributed exogenous variable 2SLS estimator is generally preferred except at very large sample sizes when $n = 50$ and when $n = 250$. At these instances, the FIML estimator is preferred.

4.2. Performances of the Estimators Based on Absolute Bias Criterion

The performances of the estimators was examined, ranked and summed over all the parameters in each equation, having examined and summed the rank of absolute bias of each parameter over all the parameters in each equation the outcome is given in **Table 3**.

From **Table 3**, at each level of sample size the sum rank were further added over the equations and preferred estimator under both exact and over identification model with the two exogenous variables were bolded. **Table 4** gives the summary of the findings as follows:

The preferred estimators differ in term of identification status. The preferred estimators are slightly affected

Table 1. Summary of the total ranks of the parameter based on bias criterion when there is no correlation.

N	Estimator	Normal Exogenous Variables						Uniform Exogenous Variables					
		Exact			Over			Exact			Over		
		Equation (1)	Equation (3)	Tot	Equation (2)	Equation (4)	Tot	Equation (1)	Equation (3)	Tot	Equation (2)	Equation (4)	Tot
10	OLS	6	6	**12**	9	12	21	5	4	**9**	9	10	19
	2SLS	6	6	**12**	6	9	15	7	8	15	5	4	**9**
	3SLS	13	13	26	12	6	18	12	15	27	4	9	13
	FIML	15	15	30	3	3	**6**	16	13	29	12	7	19
20	OLS	8	8	16	12	12	24	4	5	**9**	9	9	18
	2SLS	5	4	**9**	6	9	15	8	7	15	6	3	**9**
	3SLS	11	13	24	9	3	12	12	14	26	3	6	**9**
	FIML	16	15	31	3	6	**9**	16	14	30	12	12	24
30	OLS	5	4	**9**	12	12	24	6	4	**10**	12	11	23
	2SLS	7	8	15	6	3	**9**	6	8	14	3	3	**6**
	3SLS	16	13	29	9	6	15	13	12	25	7	6	13
	FIML	12	15	27	3	9	12	15	16	31	8	10	18
50	OLS	8	7	15	12	12	24	6	9	15	12	12	24
	2SLS	4	5	**9**	6	5	**11**	6	4	**10**	9	3	12
	3SLS	12	12	24	9	4	13	12	13	25	6	9	15
	FIML	16	16	32	3	9	12	16	14	30	3	6	**9**
100	OLS	16	16	32	12	12	24	7	7	14	12	12	24
	2SLS	7	4	**11**	6	3	**9**	5	5	**10**	5	4	**9**
	3SLS	5	10	15	3	6	**9**	13	13	26	8	9	17
	FIML	12	10	22	9	9	18	15	15	30	5	5	10
250	OLS	16	16	32	12	12	24	4	6	**10**	12	12	24
	2SLS	5	12	17	6	3	**9**	8	6	14	9	7	16
	3SLS	7	4	**11**	3	6	**9**	12	15	27	6	8	14
	FIML	12	8	20	9	9	18	16	13	29	3	3	**6**
500	OLS	16	16	32	12	12	24	16	8	24	12	12	24
	2SLS	5	12	17	6	5	11	12	4	**16**	9	5	14
	3SLS	7	5	**12**	9	9	18	7	15	22	5	6	**11**
	FIML	12	7	19	3	4	**7**	5	13	18	4	7	**11**

Source: Computed from simulated results of bias criterion.

by the two exogenous variables. In the exact identified equation with normally distributed exogenous variables under the absolute bias criterion the OLS estimators is preferred when $n \le 50$ and the 2SLS estimator when $n \succ 50$.

With uniformly distributed exogenous variables the OLS estimator is generally preferred.

In the over identified equation with normally distributed exogenous variables under the absolute bias criterion, the OLS estimator is preferred when the sample size is small $n = 10$; 2SLS estimator when $20 \le n \le 50$; and FIML when $n \succ 50$.

With uniformly distributed exogenous variables the 2 SLS estimator is preferred over all the sample sizes. Thus, the preferred estimators are more stable in performance with uniformly distributed exogenous variables than the normally distributed exogenous variables.

4.3. Performances of the Estimators Based on Variance Criterion

The performances of the estimators was examined, ranked and summed over all the parameters in each equation, having examined and summed the rank of the variance of each parameter over all the parameters in each equation the outcome is given in **Table 5**.

From **Table 5**, at each level of sample size the sum rank were further added over the equations, and preferred estimator under both exact and over identification model with the two exogenous variables were bolded. **Table 6** gives the summary of the findings as follows:

From **Table 6**, the following are observed about the preferred estimators under the variance criterion.

The preferred estimators differ in term of identification status. The preferred estimators are slightly affected by the two exogenous variables.

In the exact identified equation, the OLS estimator is generally preferred in all the sample sizes for both exogenous variables. In the over identified equation with normally distributed exogenous variables, OLS or 2SLS estimator are preferred when $n \le 50$; but for $n \succ 50$, the OLS or FIML estimator are preferred. With uniformly distributed exogenous variables the OLS or 2SLS estimators are generally preferred. Moreover, the 3SLS estimator replaces the 2SLS when the sample sizes is very large, $n = 500$.

4.4. Performances of the Estimators Based on Mean Squared Error Criterion

The performances of the estimators was examined, ranked and summed over all the parameters in each equation, having examined and summed the rank of the mean squared error of each parameter over all the parameters in each equation the outcome is given in **Table 8**.

From **Table 8** the following are observed about the preferred estimators under the mean squared error criterion.

The preferred estimators differ in term of identification status. The preferred estimators are slightly affected by the two exogenous variables. In the exact identified equation with normally distributed exogenous variables, the OLS estimator is generally preferred except when the sample size is large $n \ge 250$. At these instances 2SLS is generally preferred. With uniformly distributed exogenous variables, the OLS estimator is generally preferred. In the over identified equation with normally distributed exogenous variables, the OLS estimators is preferred when $n \le 20$; 2SLS when $30 \le n \le 100$, and FIML when $n \ge 100$. With uniformly distributed exogenous variables the OLS estimators is preferred except when the sample size is large, $n \ge 250$. At these instances, the 2SLS estimator is generally preferred. The performance of the preferred estimators is more stable with uniformly distributed exogenous variables than the normally distributed exogenous variables.

4.5. Performances of the Estimators Based on All the Criteria

From **Table 10**, the following are observed about the preferred estimators under the overall criteria.

The preferred estimators differ in term of identification status. The preferred estimators are slightly affected by the two exogenous variables.

In the exact identified equation with normally distributed exogenous variables the OLS estimator is generally preferred except when the sample size is large when $n \ge 100$. At these instances, 2SLS is preferred. With uniformly distributed exogenous variable, the OLS estimator is preferred for all sample sizes.

In the over identified equation with normally exogenous variables the OLS estimators is preferred when the samples sizes is small $n = 10$; 2SLS when $20 \le n \le 30$, and $n = 100$; 3SLS when $n = 50$ and FIML estimator with $n \ge 250$.

With uniformly distributed exogenous variable, the 2SLS estimator is preferred except when the sample sizes is large $n = 500$. At this instance the 3SLS estimator is preferred.

From **Table 1**, at each level of sample size the sum rank were further added over the equations, and preferred estimator under both exact and over identification model with the two exogenous variables were bolded. **Table 2** gives the summary of the preferred estimators.

From **Table 3**, at each level of sample size, the sum rank were further added over the equations, and preferred estimator under both exact and over identification model with the two exogenous variables were bolded. **Table 4** gives the summary of the preferred estimators.

From **Table 5**, at each level of sample size the sum rank were further added over the equations. The preferred

Table 2. Summary of the preferred estimators based on bias criterion.

Sample sizes	Normal Exogenous Variables		Uniform Exogenous Variables	
	Exact	**Over**	**Exact**	**Over**
10	OLS\2SLS	FIML	OLS	2SLS
20	2SLS	FIML	OLS	2SLS\3SLS
30	OLS	2SLS	OLS	2SLS
50	2SLS	2SLS	2SLS	FIML
100	2SLS	2SLS\3SLS	2SLS	2SLS
250	3SLS	2SLS\3SLS	OLS	FIML
500	3SLS	FIML	2SLS	2SLS\FIML

Source: Table 1.

Table 3. Performances of the estimator based on absolute bias criterion.

N	**Estimator**	**Normal Exogenous Variables**						**Uniform Exogenous Variables**					
		Exact			**Over**			**Exact**			**Over**		
		Equation (1)	**Equation (3)**	**Tot**	**Equation (2)**	**Equation (4)**	**Tot**	**Equation (1)**	**Equation (3)**	**Tot**	**Equation (2)**	**Equation (4)**	**Tot**
10	OLS	4	4	**8**	3	3	**6**	4	4	**8**	4	3	**7**
	2SLS	8	8	16	6	6	12	8	8	16	5	6	11
	3SLS	12	13	25	9	9	18	12	14	26	9	9	18
	FIML	16	15	31	12	12	24	16	14	30	12	12	24
20	OLS	4	4	**8**	10	12	22	4	4	**8**	5	6	**11**
	2SLS	8	8	16	3	3	**6**	8	8	16	4	4	**8**
	3SLS	12	12	24	6	6	12	12	12	24	9	8	17
	FIML	16	16	32	11	9	20	16	16	32	12	12	24
30	OLS	4	4	**8**	12	12	24	4	4	**8**	9	5	14
	2SLS	8	8	16	3	3	**6**	8	8	16	3	4	**7**
	3SLS	12	13	25	9	6	15	12	12	24	6	9	15
	FIML	16	15	31	6	9	15	16	16	32	12	12	24
50	OLS	5	4	**9**	12	12	24	4	4	**8**	6	9	15
	2SLS	7	8	15	6	3	**9**	8	8	16	3	4	**7**
	3SLS	12	12	24	3	6	**9**	12	13	25	9	7	16
	FIML	16	16	32	9	9	18	16	15	31	12	10	22
100	OLS	16	10	26	12	12	24	4	4	8	10	11	21
	2SLS	4	4	**8**	9	3	12	8	8	16	5	3	8
	3SLS	8	12	20	6	9	15	12	13	25	4	7	11
	FIML	12	14	26	3	6	**9**	16	15	31	11	9	20
250	OLS	16	16	32	12	12	24	4	4	8	12	12	24
	2SLS	4	6	**10**	9	5	14	8	8	16	3	3	6
	3SLS	8	8	16	6	9	15	12	13	25	6	8	14
	FIML	12	10	22	3	4	**7**	16	15	31	9	7	16
500	OLS	16	16	32	12	12	24	10	5	15	12	12	24
	2SLS	4	6	**10**	9	5.5	14.5	6	7	13	9	3	12
	3SLS	8	8.5	16.5	6	8	14	10	14	24	5	8	13
	FIML	12	9.5	21.5	3	4.5	**7.5**	14	14	28	4	7	11

Source: Computed from simulated results of Absolute bias criterion.

Table 4. Summary of the preferred estimators based on absolute bias criterion.

Sample sizes	Normal Exogenous Variables		Uniform Exogenous Variables	
	Exact	**Over**	**Exact**	**Over**
10	OLS	OLS	OLS	2SLS
20	OLS	2SLS	OLS	2SLS
30	OLS	2SLS	OLS	2SLS
50	OLS	2SLS\3SLS	OLS	2SLS
100	2SLS	FIML	OLS	2SLS
250	2SLS	FIML	OLS	2SLS
500	2SLS	FIML	2SLS\OLS	FIML\2SLS\3SLS

Source: Table 3.

Table 5. Performances of the estimators based on variance criterion.

n	**Estimator**	**Normal Exogenous Variables**						**Uniform Exogenous Variables**					
		Exact			**Over**			**Exact**			**Over**		
		Equation (1)	**Equation (3)**	**Total**	**Equation (2)**	**Equation (4)**	**Total**	**Equation (1)**	**Equation (3)**	**Total**	**Equation (2)**	**Equation (4)**	**Total**
10	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	8	**11**
	2SLS	8	8	16	6	8	**14**	8	8	16	6	5	**11**
	3SLS	13	15	28	9	7	16	12	15	27	9	10	19
	FIML	15	13	28	12	6	18	16	13	29	12	7	19
20	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	7	**10**
	2SLS	8	8	16	6	8	**14**	8	8	16	6	4	**10**
	3SLS	12	12	24	9	5	**14**	12	12	24	9	7	16
	FIML	16	16	32	12	8	20	16	16	32	12	12	24
30	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	8	**11**
	2SLS	8	8	16	6	4	**10**	8	8	16	6	4	**10**
	3SLS	12	13	25	9	7	16	12	12	24	9	7	16
	FIML	16	15	31	12	10	22	16	16	32	12	11	23
50	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	9	**12**
	2SLS	8	8	16	9	5	**14**	8	8	16	6	4	**10**
	3SLS	13	12	25	6	6	**12**	12	13	25	9	9	18
	FIML	15	16	31	12	10	22	16	15	31	12	8	20
100	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	9	**12**
	2SLS	14	8	22	12	4	16	8	8	16	8	4	**12**
	3SLS	8	13	21	9	8	17	12	13	25	7	9	16
	FIML	14	15	29	6	9	**15**	16	15	31	12	8	20
250	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	9	**12**
	2SLS	8	12	20	12	5	17	8	8	16	6	7	**13**
	3SLS	12	10.5	22.5	9	8	17	12	14	26	9	8	17
	FIML	16	13.5	29.5	6	8	**14**	16	14	30	12	6	18
500	OLS	4	4	**8**	3	9	**12**	4	4	**8**	3	9	**12**
	2SLS	8	10	18	12	5	17	8	8	16	12	5	17
	3SLS	12	12	24	9	10	19	13	13.5	26.5	6	6	**12**
	FIML	16	14	30	6	6	**12**	15	14.5	29.5	9	10	19

Source: Computed from simulated results of variance criterion.

estimators under both exact and over identification model with the two exogenous variables were made bold. **Table 6** gives the summary of the preferred estimators.

From **Table 7**, at each level of sample size the sum rank were further added over the equations and preferred

Table 6. Summary of the preferred estimators under variance criterion.

Sample sizes	Normal Exogenous Variables		Uniform Exogenous Variables	
	Exact	**Over**	**Exact**	**Over**
10	OLS	OLS\2SLS	OLS	OLS\2SLS
20	OLS	OLS\2SLS\3SLS	OLS	OLS\2SLS
30	OLS	2SLS\OLS	OLS	2SLS\OLS
50	OLS	OLS\2SLS\3SLS	OLS	2SLS\OLS
100	OLS	OLS\FIML	OLS	OLS\2SLS
250	OLS	OLS\FIML	OLS	OLS\2SLS
500	OLS	OLS\FIML	OLS	OLS\3SLS

Source: **Table 5**.

Table 7. Performances of the estimators based on mean squared error criterion.

n	**Estimator**	**Normal Exogenous Variables**						**Uniform Exogenous Variables**					
		Exact			**Over**			**Exact**			**Over**		
		Equation (1)	**Equation (3)**	**Tot**	**Equation (2)**	**Equation (4)**	**Tot**	**Equation (1)**	**Equation (3)**	**Tot**	**Equation (2)**	**Equation (4)**	**Tot**
10	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
	2SLS	8	8	16	6	6	12	8	8	16	6	6	12
	3SLS	13	15	28	9	9	18	12	15	27	9	9	18
	FIML	15	13	28	12	12	24	16	13	29	12	12	24
20	OLS	4	4	**8**	3	3	**6**	4	4	**8**	3	3	**6**
	2SLS	8	8	16	6	6	12	8	8	16	6	6	12
	3SLS	12	12	24	9	9	18	12	12	24	9	9	18
	FIML	16	16	32	12	12	24	16	16	32	12	12	24
30	OLS	4	4	**8**	3	12	15	4	4	**8**	3	3	**6**
	2SLS	8	8	16	6	3	**9**	8	8	16	6	6	12
	3SLS	12	13	25	9	6	15	12	12	24	9	9	18
	FIML	16	15	31	12	9	21	16	16	32	12	12	24
50	OLS	4	4	**8**	6	12	18	4	4	**8**	3	3	**6**
	2SLS	8	8	16	9	3	**12**	8	8	16	6	6	12
	3SLS	13	12	25	3	6	**9**	12	13	25	9	10	19
	FIML	15	16	31	12	9	21	16	15	31	12	11	23
100	OLS	13	4	**17**	12	12	24	4	4	**8**	3	4	**7**
	2SLS	11	8	**19**	9	3	**12**	8	8	16	8	5	13
	3SLS	5	13	**18**	6	9	15	12	13	25	7	9	16
	FIML	11	15	26	3	6	**9**	16	15	31	12	12	24
250	OLS	16	16	32	12	12	24	4	4	**8**	3	12	15
	2SLS	4	8	**12**	9	4.5	13.5	8	8	16	6	3	**9**
	3SLS	8	6.5	**14.5**	6	9	15	12	14	26	9	7	16
	FIML	12	9.5	21.5	3	4.5	**7.5**	16	14	30	12	8	20
500	OLS	16	16	32	12	12	24	4	4	**8**	12	12	24
	2SLS	4	6	**10**	9	3	**12**	8	8	16	9	3	**12**
	3SLS	8	7.5	15.5	6	9	15	13	14.5	27.5	3	7	**10**
	FIML	12	10.5	22.5	3	6	**9**	15	13.5	28.5	6	8	14

Source: Computed from simulated results of Mean squared error criterion.

estimator under both exact and over identification model with the two exogenous variables were bolded. Table 8 gives the summary of the preferred estimators.

4.6. Overall Examination of the Model Parameters

The overall summary of the performances of the estimators having examined and obtained by adding the total ranks over all the criteria is given in Table 9.

From Table 9, at each level of sample size the sum of the rank were further added over all the criteria and preferred estimator under both exact and over identification model with the two exogenous variables were bolded. Table 10 gives the summary of the preferred estimators.

Table 8. Summary of the preferred estimators when there is no correlation under mean squared error criterion.

Sample sizes	Normal Exogenous Variables		Uniform Exogenous Variables	
	Exact	**Over**	**Exact**	**Over**
10	OLS	OLS	OLS	OLS
20	OLS	OLS	OLS	OLS
30	OLS	2SLS	OLS	OLS
50	OLS	3SLS\2SLS	OLS	OLS
100	OLS\2SLS\3SLS	2SLS\FIML	OLS	OLS
250	2SLS\3SLS	FIML	OLS	2SLS
500	2SLS	FIML\2SLS	OLS	3SLS\2SLS

Source: Table 7.

Table 9. Overall performances of the estimators based on all criteria.

N	**Estimator**	**Normal**		**Uniform**	
		EXACT	**OVER**	**EXACT**	**OVER**
10	OLS	**36**	**45**	**33**	**43**
	2SLS	60	53	63	**43**
	3SLS	107	70	107	68
	FIML	117	72	117	86
20	OLS	**40**	64	**33**	45
	2SLS	57	**47**	63	**39**
	3SLS	96	56	98	60
	FIML	127	73	126	96
30	OLS	**33**	75	**34**	54
	2SLS	63	**34**	62	**35**
	3SLS	104	61	97	62
	FIML	120	70	127	89
50	OLS	**40**	78	**39**	57
	2SLS	56	46	58	**41**
	3SLS	98	**43**	100	68
	FIML	126	73	123	74
100	OLS	83	84	**38**	64
	2SLS	**60**	**49**	58	**42**
	3SLS	74	56	101	60
	FIML	103	51	123	74
250	OLS	104	84	**34**	75
	2SLS	**59**	53.5	62	**44**
	3SLS	64	56	104	61
	FIML	93	**46.5**	120	60
500	OLS	104	84	**55**	84
	2SLS	**55**	54.5	61	55
	3SLS	68	66	100	**46**
	FIML	93	**35.5**	104	55

Source: Computed from Table 1, Table 3, Table 5 and Table 7.

Table 10. Overall summary of the best estimators on the basis of all criteria.

Sample sizes	Normal Exogenous Variables		Uniform Exogenous Variables	
	Exact	**Over**	**Exact**	**Over**
10	OLS	OLS	OLS	2SLS\OLS
20	OLS	2SLS	OLS	2SLS
30	OLS	2SLS	OLS	2SLS
50	OLS	3SLS	OLS	2SLS
100	2SLS	2SLS	OLS	2SLS
250	2SLS	FIML	OLS	2SLS
500	2SLS	FIML	OLS	3SLS

Source: Table 4 and Table 9.

5. Conclusions

The performances of the estimators are affected by the distribution of the exogenous variables. The best estimators are more stable over the levels of sample size with uniformly distributed exogenous variables than the normally distributed exogenous variables.

In exact identified equation with normally distributed exogenous variables the following were observed.

For low sample sizes, OLS is the best performed estimator. In medium sample sizes, OLS or 2SLS is the best performed estimator and 2SLS estimator is the best performed in the large sample sizes. Whereas with uniformly distributed exogenous variables, OLS is the best performed estimator in the entire sample sizes category.

In over identified equation with normally distributed exogenous variables, the following are observed.

For low sample sizes, OLS or 2SLS is the best performed estimator, in medium sample sizes 2SLS\3SLS is the best performed estimator and FIML estimator performed best in the large sample sizes. Whereas with uniformly distributed exogenous variables, 2SLS is the best in low and medium sample sizes category, but 2SLS/3SLS estimator is the best in large sample sizes category.

Hence, when there is no correlation of any form in the model the performances of the estimators are affected by the distribution of the exogenous variables in simultaneous equation models.

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