

Fourier Coefficients of a Class of Eta Quotients of Weight 16 with Level 12

Barış Kendirli

Istanbul Kultur University, Istanbul, Turkey
Email: baris.kendirli@gmail.com

Received 30 May 2015; accepted 28 July 2015; published 31 July 2015

Copyright © 2015 by author and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY).
<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

Recently, Williams [1] and then Yao, Xia and Jin [2] discovered explicit formulas for the coefficients of the Fourier series expansions of a class of eta quotients. Williams expressed all coefficients of 126 eta quotients in terms of $\sigma(n), \sigma\left(\frac{n}{2}\right), \sigma\left(\frac{n}{3}\right)$ and $\sigma\left(\frac{n}{6}\right)$ and Yao, Xia and Jin, following the method of proof of Williams, expressed only even coefficients of 104 eta quotients in terms of $\sigma_3(n), \sigma_3\left(\frac{n}{2}\right), \sigma_3\left(\frac{n}{3}\right)$ and $\sigma_3\left(\frac{n}{6}\right)$. Here, by using the method of proof of Williams, we will express the even Fourier coefficients of 360 eta quotients *i.e.*, the Fourier coefficients of the sum, $f(q) + f(-q)$, of 360 eta quotients in terms of $\sigma_{15}(n), \sigma_{15}\left(\frac{n}{2}\right), \sigma_{15}\left(\frac{n}{3}\right), \sigma_{15}\left(\frac{n}{4}\right), \sigma_{15}\left(\frac{n}{6}\right)$ and $\sigma_{15}\left(\frac{n}{12}\right)$.

Keywords

Dedekind Eta Function, Eta Quotients, Fourier Series

The divisor function $\sigma_i(n)$ is defined for a positive integer i by

$$\begin{aligned}\sigma_i(n) &:= \sum_{d \text{ positive integer}, d|n} d^i, \text{ if } n \text{ is a positive integer, and} \\ \sigma_i(n) &:= 0 \text{ if } n \text{ is not a positive integer.}\end{aligned}\tag{1}$$

The Dedekind eta function is defined by

$$\eta(z) := q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \quad (2)$$

where

$$q := e^{2\pi iz}, z \in H = \{x + iy : y > 0\} \quad (3)$$

And an eta quotient of level n is defined by

$$f(z) := \prod_{m|n} \eta(mz)^{a_m}, n \in \mathbb{N}, a_m \in \mathbb{Z}, a_n \neq 0. \quad (4)$$

It is interesting and important to determine explicit formulas of the Fourier coefficients of eta quotients since they are the building blocks of modular forms of level n and weight k . The book of Köhler [3] (Chapter 3, p. 39) describes such expansions by means of Hecke Theta series and develops algorithms for the determination of suitable eta quotients. One can find more information in [4]-[8]. I have determined the Fourier coefficients of the theta series associated to some quadratic forms, see [9]-[14].

Recently, Williams, see [1] discovered explicit formulas for the coefficients of Fourier series expansions of a class of 126 eta quotients in terms of $\sigma(n), \sigma\left(\frac{n}{2}\right), \sigma\left(\frac{n}{3}\right)$ and $\sigma\left(\frac{n}{6}\right)$. One example is as follows:

$$\frac{\eta^2(2z)\eta^4(4z)\eta^6(6z)}{\eta^2(z)\eta^2(3z)\eta^4(12z)}$$

gives the expansion found by Williams.

Then Yao, Xia and Jin [2] expressed the even Fourier coefficients of 104 eta quotients in terms of $\sigma_3(n), \sigma_3\left(\frac{n}{2}\right), \sigma_3\left(\frac{n}{3}\right)$ and $\sigma_3\left(\frac{n}{6}\right)$. One example is as follows:

$$\frac{\eta^{25}(2z)\eta^4(3z)}{\eta^{12}(z)\eta^5(4z)\eta^3(6z)\eta(12z)},$$

where the even coefficients are obtained. Motivated by these two results, we find that we can express the even Fourier coefficients of 360 eta quotients in terms of $\sigma_{15}(n), \sigma_{15}\left(\frac{n}{2}\right), \sigma_{15}\left(\frac{n}{3}\right), \sigma_{15}\left(\frac{n}{4}\right), \sigma_{15}\left(\frac{n}{6}\right)$ and $\sigma_{15}\left(\frac{n}{12}\right)$, see **Table 2**. One example is as follows:

$$\frac{\eta^{12}(4z)\eta^{10}(6z)\eta^4(12z)}{\eta^6(2z)}.$$

We see that the odd Fourier coefficients of 875 eta quotients are zero and even coefficients can be expressed by simple formula. Let

$$f_1 = \frac{\eta^{18}(4z)\eta^{16}(6z)\eta^{10}(12z)}{\eta^{12}(2z)},$$

$$f_2 = \frac{\eta^{20}(4z)\eta^2(6z)\eta^{20}(12z)}{\eta^{10}(2z)},$$

$$f_3 = \frac{\eta^{15}(4z)\eta^7(6z)\eta^{19}(12z)}{\eta^9(2z)},$$

$$f_4 = \frac{\eta^{10}(2z)\eta^{12}(4z)\eta^{18}(6z)}{\eta^8(12z)},$$

$$\begin{aligned}
f_5 &= \frac{\eta^5(4z)\eta^{17}(6z)\eta^{17}(12z)}{\eta^7(2z)}, \\
f_6 &= \frac{\eta^{17}(4z)\eta^{17}(6z)\eta^5(12z)}{\eta^7(2z)}, \\
f_7 &= \frac{\eta^{19}(4z)\eta^{15}(6z)\eta^{15}(12z)}{\eta^{17}(2z)}, \\
f_8 &= \frac{\eta^{17}(2z)\eta^{17}(4z)\eta^5(12z)}{\eta^7(6z)}, \\
f_9 &= \frac{\eta^{20}(2z)\eta^{14}(4z)\eta^{14}(12z)}{\eta^{16}(6z)}, \\
f_{10} &= \frac{\eta^{16}(4z)\eta^{18}(6z)}{\eta^2(2z)}, \\
f_{11} &= \frac{\eta^{20}(2z)\eta^{14}(4z)\eta^8(6z)}{\eta^{10}(12z)}, \\
f_{12} &= \eta^{20}(2z)\eta^2(4z)\eta^8(6z)\eta^2(12z), \\
f_{13} &= \frac{\eta^2(2z)\eta^{14}(6z)\eta^{20}(12z)}{\eta^4(4z)}, \\
f_{14} &= \frac{\eta^{16}(2z)\eta^{10}(4z)\eta^{18}(12z)}{\eta^{12}(6z)}, \\
f_{15} &= \frac{\eta^{19}(4z)\eta^9(6z)\eta^{15}(12z)}{\eta^{11}(2z)}, \\
f_{16} &= \frac{\eta^{14}(4z)\eta^{14}(6z)\eta^{14}(12z)}{\eta^{10}(2z)}, \\
f_{17} &= \frac{\eta^{15}(4z)\eta^{13}(6z)\eta^{19}(12z)}{\eta^{15}(2z)}, \\
f_{18} &= \frac{\eta^9(4z)\eta^{19}(6z)\eta^{13}(12z)}{\eta^9(2z)}, \\
f_{19} &= \frac{\eta^{18}(4z)\eta^{10}(6z)\eta^{10}(12z)}{\eta^6(2z)}, \\
f_{20} &= \frac{\eta^{20}(2z)\eta^8(4z)\eta^{20}(6z)}{\eta^{16}(12z)}, \\
f_{21} &= \frac{\eta^{20}(2z)\eta^{20}(4z)}{\eta^4(6z)\eta^4(12z)}, \\
f_{22} &= \frac{\eta^{20}(4z)\eta^{20}(12z)}{\eta^4(2z)\eta^4(6z)},
\end{aligned}$$

$$f_{23} = \frac{\eta^{20}(4z)\eta^{20}(6z)}{\eta^4(2z)\eta^4(12z)},$$

$$f_{24} = \frac{\eta^{15}(4z)\eta(6z)\eta^{19}(12z)}{\eta^3(2z)},$$

$$f_{25} = \frac{\eta^{20}(2z)\eta^{20}(4z)}{\eta^4(6z)\eta^4(12z)},$$

$$f_{26} = \frac{\eta^{20}(2z)\eta^{20}(6z)\eta^8(12z)}{\eta^4(4z)},$$

$$f_{27} = \frac{\eta^{19}(2z)\eta^{13}(4z)\eta^{15}(6z)}{\eta^{15}(12z)}.$$

Now we can state our main Theorem:

Theorem 1 Let b_1, b_2, \dots, b_5 be non-negative integers satisfying

$$b_1 + b_2 + \dots + b_5 \leq 32. \tag{5}$$

Define the integers $a_1, a_2, a_3, a_4, a_6, a_{12}$ by

$$a_1 := -b_1 + 2b_2 - 2b_3 - 4b_4 - b_5 + 32, \tag{6}$$

$$a_2 := 3b_1 + b_2 + 3b_3 + 10b_4 + b_5 - 80, \tag{7}$$

$$a_3 := 3b_1 + 2b_2 + 6b_3 + 4b_4 + 3b_5 - 96, \tag{8}$$

$$a_4 := -2b_1 - b_2 - b_3 - 4b_4 + 2b_5 + 32, \tag{9}$$

$$a_6 := -9b_1 - 7b_2 - 9b_3 - 10b_4 - 7b_5 + 240, \tag{10}$$

$$a_{12} := 6b_1 + 3b_2 + 3b_3 + 4b_4 + 2b_5 - 96. \tag{11}$$

They are functions of q by (3). Now define integers

$$k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, k_{13}, k_{14}, k_{15}, k_{16}, k_{17}, k_{18}, k_{19},$$

$$k_{20}, k_{21}, k_{22}, k_{23}, k_{24}, k_{25}, k_{26}, k_{27}, k_{28}, k_{29}, k_{30}, k_{31} \text{ and } k_{32}$$

by

$$\frac{1}{2^{b_1+b_5}} x^{b_1} (1-x)^{b_2} (1+x)^{b_3} (1+2x)^{b_4} (2+x)^{b_5} \tag{12}$$

$$= k_0 + k_1x + k_2x^2 + k_3x + k_4x^4 + k_5x^5 + k_6x^6 + k_7x^7 + k_8x^8$$

$$+ k_9x^9 + k_{10}x^{10} + k_{11}x^{11} + k_{12}x^{12} + k_{13}x^{13} + k_{14}x^{14} + k_{15}x^{15} \tag{13}$$

$$+ k_{16}x^{16} + k_{17}x^{17} + k_{18}x^{18} + k_{19}x^{19} + k_{20}x^{20} \tag{14}$$

$$+ k_{21}x^{21} + k_{22}x^{22} + k_{23}x^{23} + k_{24}x^{24} + k_{25}x^{25} + k_{26}x^{26} + k_{27}x^{27} \tag{15}$$

$$+ k_{28}x^{28} + k_{29}x^{29} + k_{30}x^{30} + k_{31}x^{31} + k_{32}x^{32}. \tag{16}$$

Define the rational numbers

$$c_1, c_2, c_3, c_4, c_6, c_{12}, r_1, r_2, \dots, r_{26}$$

and r_{27} as in **Table 1**. Here $\{f_1, \dots, f_{27}\} \setminus \{f_7, f_9, f_{17}, f_{20}, f_{25}, f_{26}, f_{27}\} \in S_{16}(\Gamma_0(12))$, $f_7, f_9, f_{17}, f_{20}, f_{25}, f_{26}, f_{27} \in M_{16}(\Gamma_0(12)) \setminus S_{16}(\Gamma_0(12))$ and

Table 1. Coefficients of eisenstein series and some eta quotients.

$$\begin{aligned}
 c_1 := & \frac{72583}{980659548} k_1 - \frac{120832}{245164887} k_0 - \frac{2735}{326886516} k_2 + \frac{277}{490329774} k_3 \\
 & - \frac{7}{490329774} k_4 + \frac{1}{36320724} k_5 - \frac{13}{245164887} k_6 + \frac{25}{245164887} k_7 \\
 & - \frac{16}{81721629} k_8 + \frac{92}{245164887} k_9 - \frac{16}{22287717} k_{10} + \frac{112}{81721629} k_{11} \\
 & - \frac{640}{245164887} k_{12} + \frac{4352}{245164887} k_{15} - \frac{256}{27240543} k_{14} - \frac{1216}{245164887} k_{13} \\
 & - \frac{8192}{245164887} k_{16} + \frac{5120}{81721629} k_{17} - \frac{28672}{245164887} k_{18} + \frac{16384}{22287717} k_{21} \\
 & - \frac{32768}{81721629} k_{20} - \frac{327680}{245164887} k_{22} + \frac{65536}{27240543} k_{23} \\
 & - \frac{1048576}{245164887} k_{24} + \frac{53248}{245164887} k_{19} + \frac{1835008}{245164887} k_{25} \\
 & - \frac{1048576}{81721629} k_{26} + \frac{5242880}{245164887} k_{27} - \frac{8388608}{245164887} k_{28} \\
 & + \frac{4194304}{81721629} k_{29} - \frac{16777216}{245164887} k_{30} + \frac{16777216}{245164887} k_{31}, \\
 c_2 := & \frac{34734530088732674832514185950084774049248}{2142599748818134613113093495072137244191} k_0 - \frac{62956800237180571066473807796775564347301}{25711196985817615357357121940865646930292} k_1 \\
 & + \frac{560501183955670989597137220160641262555}{1977784383524431950565932456989665148484} k_2 - \frac{97964204168754996402004893769133989949}{4285199497636269226226186990144274488382} k_3 \\
 & + \frac{40897072982474021350305265538542513037}{12855598492908807678678560970432823465146} k_4 - \frac{77910984221605457897100564905533152137}{25711196985817615357357121940865646930292} k_5 \\
 & + \frac{7683007415360393635856788341175110341}{2142599748818134613113093495072137244191} k_6 - \frac{32060487020365187360697936804888726131}{6427799246454403839339280485216411732573} k_7 \\
 & + \frac{50628602500203669592250667935460937744}{6427799246454403839339280485216411732573} k_8 - \frac{29085265762693348400769904586321479036}{2142599748818134613113093495072137244191} k_9 \\
 & + \frac{158250367120128739881846817708074446992}{6427799246454403839339280485216411732573} k_{10} - \frac{22656914712598824036276628186340738864}{494446095881107987641483114247416287121} k_{11} \\
 & + \frac{184848740939247688020554787142399039616}{2142599748818134613113093495072137244191} k_{12} - \frac{1048239944217959494935262133888301514304}{6427799246454403839339280485216411732573} k_{13} \\
 & + \frac{25174768088427596815019432883901899008}{164815365293702662547161038082472095707} k_{14} - \frac{1246723832169732120072934552513970970880}{2142599748818134613113093495072137244191} k_{15} \\
 & + \frac{7038115875495624397185080978321395609600}{6427799246454403839339280485216411732573} k_{16} - \frac{13195297276418057599661347671371361539072}{6427799246454403839339280485216411732573} k_{17} \\
 & + \frac{8210357018229573060477785588795679469568}{2142599748818134613113093495072137244191} k_{18} - \frac{45744265651265841051078471764079279534080}{6427799246454403839339280485216411732573} k_{19} \\
 & + \frac{84452773012339913380407176682315598299136}{6427799246454403839339280485216411732573} k_{20} - \frac{51610948844243996791857065512420047634432}{2142599748818134613113093495072137244191} k_{21} \\
 & + \frac{21655225465002831817250269915926736076800}{494446095881107987641483114247416287121} k_{22} - \frac{506736790874131434657008627249238597238784}{6427799246454403839339280485216411732573} k_{23} \\
 & + \frac{300290233988701749697945831633510766477312}{2142599748818134613113093495072137244191} k_{24} - \frac{1576529708086760827933004700454891591958528}{6427799246454403839339280485216411732573} k_{25} \\
 & + \frac{2702628877067406586383696467255320708907008}{6427799246454403839339280485216411732573} k_{26} - \frac{1501462766976347698189587052601574136348672}{2142599748818134613113093495072137244191} k_{27} \\
 & + \frac{7207028070602459124588069514635005543317504}{6427799246454403839339280485216411732573} k_{28} - \frac{10810548170070847789240155429646823984857088}{6427799246454403839339280485216411732573} k_{29} \\
 & + \frac{4804689379205214665850988696123019241979904}{2142599748818134613113093495072137244191} k_{30} - \frac{14414064906651085511759265915202903948132352}{6427799246454403839339280485216411732573} k_{31} \\
 & - \frac{12614340427608524802511638703898624}{189219877290080612110474090437566009} k_{32},
 \end{aligned}$$

Continued

$$\begin{aligned}
 c_3 := & \frac{32}{2187}k_0 - \frac{62}{2187}k_1 + \frac{40}{729}k_2 - \frac{232}{2187}k_3 + \frac{448}{2187}k_4 \\
 & - \frac{32}{81}k_5 + \frac{1664}{2187}k_6 - \frac{3200}{2187}k_7 + \frac{2048}{729}k_8 - \frac{11776}{2187}k_9 \\
 & + \frac{22528}{2187}k_{10} - \frac{14336}{729}k_{11} + \frac{81920}{2187}k_{12} - \frac{155648}{2187}k_{13} \\
 & + \frac{32768}{243}k_{14} - \frac{557056}{2187}k_{15} + \frac{1048576}{2187}k_{16} - \frac{655360}{729}k_{17} \\
 & + \frac{4194304}{729}k_{20} - \frac{23068672}{2187}k_{21} + \frac{41943040}{2187}k_{22} \\
 & - \frac{8388608}{243}k_{23} + \frac{134217728}{2187}k_{24} + \frac{3670016}{2187}k_{18} \\
 & - \frac{6815744}{2187}k_{19} + \frac{2147483648}{2187}k_{30} + \frac{234881024}{2187}k_{25} \\
 & + \frac{134217728}{729}k_{26} - \frac{671088640}{2187}k_{27} + \frac{1073741824}{2187}k_{28} \\
 & - \frac{536870912}{729}k_{29} - \frac{2147483648}{2187}k_{31},
 \end{aligned}$$

$$\begin{aligned}
 c_4 := & -\frac{3438631669957687943317991998219836912041984}{212117375132995326698196256012141587174909}k_0 + \frac{519391370415717958219513005709135610101760}{212117375132995326698196256012141587174909}k_1 \\
 & - \frac{1541655078827786053038522300113680482304}{5438907054692187864056314256721579158331}k_2 + \frac{4861344345762952626180655745881508651008}{212117375132995326698196256012141587174909}k_3 \\
 & - \frac{686662260194965967174344185570009972736}{212117375132995326698196256012141587174909}k_4 + \frac{24223505589260903505657457023611248640}{7856199078999826914748009481931169895367}k_5 \\
 & - \frac{770912905776039933043040352765543514112}{212117375132995326698196256012141587174909}k_6 + \frac{1066674040873884195145072515431390019584}{212117375132995326698196256012141587174909}k_7 \\
 & - \frac{558909424814416346855153477262757068800}{70705791710998442232732085337380529058303}k_8 + \frac{2880943447310005657271900824199337410560}{212117375132995326698196256012141587174909}k_9 \\
 & - \frac{474212116868164817910723564062331109376}{19283397739363211518017841455649235197719}k_{10} + \frac{248759939095225256330737009613353582592}{5438907054692187864056314256721579158331}k_{11} \\
 & - \frac{18261794573801688749784466114240638353408}{212117375132995326698196256012141587174909}k_{12} + \frac{34521130653913088315205006307078765740032}{212117375132995326698196256012141587174909}k_{13} \\
 & - \frac{7252972970368272989918505205394851364864}{23568597236999480744244028445793509686101}k_{14} + \frac{123219038849645502527071198852093878009856}{212117375132995326698196256012141587174909}k_{15} \\
 & - \frac{231919667081212951123368487557465697681408}{212117375132995326698196256012141587174909}k_{16} + \frac{144967222655613673979840116872910152466432}{70705791710998442232732085337380529058303}k_{17} \\
 & - \frac{811966565987547701423795536752529381916672}{212117375132995326698196256012141587174909}k_{18} + \frac{1508222015438432664091595009713631746654208}{212117375132995326698196256012141587174909}k_{19} \\
 & - \frac{928294070025214438646146335719133007839232}{70705791710998442232732085337380529058303}k_{20} + \frac{464214257594362122128001913465962465394688}{19283397739363211518017841455649235197719}k_{21} \\
 & - \frac{714261708179731033246971232677038694989824}{16316721164076563592168942770164737474993}k_{22} + \frac{1857262903145106340717278218703097268338688}{23568597236999480744244028445793509686101}k_{23} \\
 & - \frac{29718557905407543679170452853863882287480832}{212117375132995326698196256012141587174909}k_{24} + \frac{52010743753839595169447408643587812743643136}{212117375132995326698196256012141587174909}k_{25} \\
 & - \frac{29721880606491798348403071893001785716637696}{70705791710998442232732085337380529058303}k_{26} + \frac{148614907659583800278219830959223630146371584}{212117375132995326698196256012141587174909}k_{27} \\
 & - \frac{237790053425604749045832090773182365227548672}{212117375132995326698196256012141587174909}k_{28} + \frac{118896676835312618271459427865760036870225920}{70705791710998442232732085337380529058303}k_{29} \\
 & - \frac{475583378556974223031298806035732201510404096}{212117375132995326698196256012141587174909}k_{30} + \frac{475544908027362753251165705288941682960629760}{212117375132995326698196256012141587174909}k_{31} \\
 & + \frac{69662864213164369817684884330643456}{70081436195928911559647188534724667}k_{32},
 \end{aligned}$$

Continued

$$\begin{aligned}
 c_6 := & \frac{903140823425764539266175742781129563328}{1892198777290080612110474090437566009} k_0 + \frac{1753599251608810820042292756595554516256}{1892198777290080612110474090437566009} k_1 \\
 & - \frac{261382625829009040606391162572202246708}{145553752099236970162344160802889693} k_2 + \frac{6573369312471681371367564937022266290064}{1892198777290080612110474090437566009} k_3 \\
 & - \frac{12697362031501480563543217533293921843408}{1892198777290080612110474090437566009} k_4 + \frac{24491723680826731742452810508492829778096}{1892198777290080612110474090437566009} k_5 \\
 & - \frac{47173190303187096890076255016849180135904}{1892198777290080612110474090437566009} k_6 + \frac{90721607156469354671767678086756795409024}{1892198777290080612110474090437566009} k_7 \\
 & - \frac{174189406811691020942065915513466611942400}{1892198777290080612110474090437566009} k_8 + \frac{333866937216440680723220058100861470373888}{1892198777290080612110474090437566009} k_9 \\
 & - \frac{638705859516034234832558641988331987268352}{1892198777290080612110474090437566009} k_{10} + \frac{93796263570379824927121966639465976592640}{145553752099236970162344160802889693} k_{11} \\
 & - \frac{2322578004133601510600390747954985709790720}{1892198777290080612110474090437566009} k_{12} + \frac{4412902046734060157362588620252943713253888}{1892198777290080612110474090437566009} k_{13} \\
 & - \frac{8361291905582645393038766948930775507203072}{1892198777290080612110474090437566009} k_{14} \\
 & + \frac{15793555169896537058264823055463258597435392}{1892198777290080612110474090437566009} k_{15} \\
 & - \frac{29729048791079112416901620246445683858493440}{1892198777290080612110474090437566009} k_{16} \\
 & + \frac{55741970217875135338106642659799456846036992}{1892198777290080612110474090437566009} k_{17} \\
 & - \frac{104051681439650108926296625737844180389232640}{1892198777290080612110474090437566009} k_{18} \\
 & + \frac{193238840618886918635723645699723731688488960}{1892198777290080612110474090437566009} k_{19} \\
 & - \frac{356748632448054978156317427837374824256110592}{1892198777290080612110474090437566009} k_{20} \\
 & + \frac{654039163047100217481579906467314595064709120}{1892198777290080612110474090437566009} k_{21} \\
 & - \frac{91474009086609878519169859048974830832975872}{145553752099236970162344160802889693} k_{22} \\
 & + \frac{2140491816044378587791643159133715725272612864}{1892198777290080612110474090437566009} k_{23} \\
 & - \frac{3805318787402182990247254077934953884303753216}{1892198777290080612110474090437566009} k_{24} \\
 & + \frac{6659307881158913920815458330065825499748892672}{1892198777290080612110474090437566009} k_{25} \\
 & - \frac{11415956370753930456327232934139762697522380800}{1892198777290080612110474090437566009} k_{26} \\
 & + \frac{19026593954106378493996218523841248653172670464}{1892198777290080612110474090437566009} k_{27} \\
 & - \frac{30442550329135403612241707744616567638358228992}{1892198777290080612110474090437566009} k_{28} \\
 & + \frac{45663825495841001129663585013837084183763222528}{1892198777290080612110474090437566009} k_{29} \\
 & - \frac{60885100662546560092483434885863179799076012032}{1892198777290080612110474090437566009} k_{30} \\
 & + \frac{60885100662544944606303920653576106827577294848}{1892198777290080612110474090437566009} k_{31} \\
 & + \frac{8561044434669014598204843153342595072}{1892198777290080612110474090437566009} k_{32},
 \end{aligned}$$

Continued

$$\begin{aligned}
c_{12} := & \frac{3252809915859073519777303401431108157440}{1892198777290080612110474090437566009} k_0 - \frac{812779358899696260571931072917630803968}{630732925763360204036824696812522003} k_1 \\
+ & \frac{288828819117788504601941284332077178880}{145553752099236970162344160802889693} k_2 - \frac{6861769718417542232659970832848739131392}{1892198777290080612110474090437566009} k_3 \\
+ & \frac{4319640213871583582252549071887097708544}{630732925763360204036824696812522003} k_4 - \frac{24731010874750872036103772996504498094080}{1892198777290080612110474090437566009} k_5 \\
+ & \frac{47390045096215471516305801919735885168640}{1892198777290080612110474090437566009} k_6 - \frac{10101702184638558192238504007106640936960}{210244308587786734678941565604174001} k_7 \\
+ & \frac{174358743315650983724683435099117789904896}{1892198777290080612110474090437566009} k_8 - \frac{334009569552550003296439137577949085761536}{1892198777290080612110474090437566009} k_9 \\
+ & \frac{212939119199173702549123860084134672859136}{630732925763360204036824696812522003} k_{10} - \frac{93801797963924066131835359863821619101696}{145553752099236970162344160802889693} k_{11} \\
+ & \frac{2322594452836691591464083034791767560945664}{1892198777290080612110474090437566009} k_{12} - \frac{1470944291358380225706902495347555354083328}{630732925763360204036824696812522003} k_{13} \\
+ & \frac{8361080409552557422244787219425411938779136}{1892198777290080612110474090437566009} k_{14} \\
- & \frac{15793095034248826921971842405375934932254720}{1892198777290080612110474090437566009} k_{15} \\
+ & \frac{1101042279957564750138573655175586727854080}{70081436195928911559647188534724667} k_{16} \\
- & \frac{55740247326202845516581891125695235332964352}{1892198777290080612110474090437566009} k_{17} \\
+ & \frac{104048462464856587744272659805885512547172352}{1892198777290080612110474090437566009} k_{18} \\
- & \frac{64410959372568544558588221853600930469445632}{630732925763360204036824696812522003} k_{19} \\
+ & \frac{356737658342027134120802811029731871930712064}{1892198777290080612110474090437566009} k_{20} \\
- & \frac{654019094795311412876984593210670908205891584}{1892198777290080612110474090437566009} k_{21} \\
+ & \frac{30490402509700503156745923519010011984822272}{48517917366412323387448053600963231} k_{22} \\
- & \frac{2140426342563992719357502322247849299974094848}{1892198777290080612110474090437566009} k_{23} \\
+ & \frac{3805202487140833038817416109251596329128820736}{1892198777290080612110474090437566009} k_{24} \\
- & \frac{739900496096751216448457373229853046301196288}{210244308587786734678941565604174001} k_{25} \\
+ & \frac{11415607775646961094449597461988589844054802432}{1892198777290080612110474090437566009} k_{26} \\
- & \frac{19026013085993732806392825204320343923582894080}{1892198777290080612110474090437566009} k_{27} \\
+ & \frac{10147207020811412120523586624388195543064248320}{630732925763360204036824696812522003} k_{28} \\
- & \frac{45662431704961318665681528877733054305885945856}{1892198777290080612110474090437566009} k_{29} \\
+ & \frac{60883242347447663868810793893791741513570582528}{1892198777290080612110474090437566009} k_{30} \\
- & \frac{20294414101188258626560211095358731428079599616}{630732925763360204036824696812522003} k_{31} \\
- & \frac{708292761822193495987179298010326630400}{1892198777290080612110474090437566009} k_{32},
\end{aligned}$$

Continued

$$\begin{aligned}
 r_1 := & \frac{91310053715411127712}{63561267} k_0 - \frac{7641452474572136509}{36320724} k_1 + \frac{1237277739610086155}{84748356} k_2 \\
 & + \frac{301998497851066193}{18160362} k_3 - \frac{4445149892148477245}{127122534} k_4 + \frac{634162625358701723}{9416484} k_5 \\
 & - \frac{8243024078809732703}{63561267} k_6 + \frac{15852441259742139299}{63561267} k_7 - \frac{10145856761562783152}{21187089} k_8 \\
 & + \frac{58339579743384278452}{63561267} k_9 - \frac{10146087995299300784}{5778297} k_{10} + \frac{71022855408522794960}{21187089} k_{11} \\
 & - \frac{405845531481576392576}{63561267} k_{12} + \frac{771107108760819174464}{63561267} k_{13} - \frac{162338403816018689792}{7062363} k_{14} \\
 & + \frac{2759753469532240738048}{63561267} k_{15} - \frac{5194830713883325932544}{63561267} k_{16} + \frac{3246769440142703828992}{21187089} k_{17} \\
 & - \frac{18181909703056191509504}{63561267} k_{18} - \frac{20779326348351079361536}{21187089} k_{20} \\
 & + \frac{33766404705203191651328}{63561267} k_{19} + \frac{10389663292872757535744}{5778297} k_{21} \\
 & - \frac{207793267358083979723776}{63561267} k_{22} + \frac{41558653661276242758656}{7062363} k_{23} \\
 & - \frac{664938460496776958624768}{63561267} k_{24} + \frac{1163642307985908317785088}{63561267} k_{25} \\
 & - \frac{664938462468936095771648}{21187089} k_{26} + \frac{3324692314743380900555776}{63561267} k_{27} \\
 & - \frac{5319507705979530190849024}{63561267} k_{28} + \frac{2659753853705085386064896}{21187089} k_{29} \\
 & - \frac{10639015416250982125540352}{63561267} k_{30} + \frac{10639015416250982125540352}{63561267} k_{31}, \\
 \\
 r_2 := & \frac{12586505097374334976}{190683801} k_0 - \frac{3733982941123692544}{27240543} k_1 + \frac{16889620877821442048}{63561267} k_2 \\
 & - \frac{13972654485209821184}{27240543} k_3 + \frac{188725357515715014656}{190683801} k_4 - \frac{13476065128386856960}{7062363} k_5 \\
 & + \frac{700665691647489974272}{190683801} k_6 - \frac{1347366543765495291904}{190683801} k_7 + \frac{862298199446149267456}{63561267} k_8 \\
 & - \frac{4958180642825882009600}{190683801} k_9 + \frac{862290353928014528512}{17334891} k_{10} - \frac{6036029087927587176448}{63561267} k_{11} \\
 & + \frac{34491594748032331546624}{190683801} k_{12} - \frac{65534039862911696896000}{190683801} k_{13} + \frac{13796642211391528763392}{21187089} k_{14} \\
 & - \frac{234542949175674985250816}{190683801} k_{15} + \frac{441492654941393812717568}{190683801} k_{16} - \frac{275932929400260762533888}{63561267} k_{17} \\
 & + \frac{1765970941189026648424448}{63561267} k_{20} + \frac{1545224483122349019234304}{190683801} k_{18} - \frac{2869702711575934430347264}{190683801} k_{19} \\
 & - \frac{882985484621709431013376}{17334891} k_{21} + \frac{17659709879541226999906304}{190683801} k_{22} \\
 & - \frac{3531942000722552879054848}{21187089} k_{23} + \frac{56511072273638284678660096}{190683801} k_{24} \\
 & - \frac{98894376780489799523368960}{190683801} k_{25} + \frac{56511072558923794231066624}{63561267} k_{26} \\
 & - \frac{282555363162368112461348864}{190683801} k_{27} + \frac{452088581437165067101011968}{190683801} k_{28} \\
 & - \frac{226044290833720748870729728}{63561267} k_{29} + \frac{904177163565159426123366400}{190683801} k_{31},
 \end{aligned}$$

Continued

$$\begin{aligned}
r_3 := & \frac{15566728830133141504}{63561267} k_0 + \frac{3185223803802372608}{9080181} k_1 - \frac{17041426163914032640}{21187089} k_2 \\
& + \frac{14256037267597450240}{9080181} k_3 - \frac{192816996480558164992}{63561267} k_4 + \frac{13774493971286120960}{2354121} k_5 \\
& - \frac{716316582824116373504}{63561267} k_6 + \frac{1377567939458816092160}{63561267} k_7 - \frac{881653709229028573184}{21187089} k_8 \\
& + \frac{5069535705690915561472}{63561267} k_9 - \frac{881660623394833989632}{5778297} k_{10} + \frac{6171632372910594031616}{21187089} k_{11} \\
& - \frac{35266495865392543170560}{63561267} k_{12} + \frac{67006370355143754776576}{63561267} k_{13} - \frac{14106607994499260874752}{7062363} k_{14} \\
& + \frac{239812376836928988774400}{63561267} k_{15} - \frac{451411584006449932926976}{63561267} k_{16} + \frac{282132261439511343923200}{21187089} k_{17} \\
& - \frac{1579940744809878345089024}{63561267} k_{18} - \frac{180564669026706414764032}{21187089} k_{20} + \frac{2934175769689493562982400}{63561267} k_{19} \\
& + \frac{902823348363955189317632}{5778297} k_{21} - \frac{18056467151355893015511040}{63561267} k_{22} + \frac{3611293454652083622379520}{7062363} k_{23} \\
& - \frac{57780695532049934081982464}{63561267} k_{24} + \frac{101116217478105169134092288}{63561267} k_{25} \\
& - \frac{57780695813286527873712128}{21187089} k_{26} + \frac{288903479430757697593016320}{63561267} k_{27} \\
& - \frac{462245567464244031099633664}{63561267} k_{28} + \frac{231122783846772861380329472}{21187089} k_{29} \\
& - \frac{924491135616393137182343168}{63561267} k_{30} + \frac{924491135616393137182343168}{63561267} k_{31},
\end{aligned}$$

$$\begin{aligned}
r_4 := & -\frac{131929657504554345952}{190683801} k_0 - \frac{7798583377191259211}{36320724} k_1 + \frac{497214249938415980821}{762735204} k_2 \\
& - \frac{70199075741890437883}{54481086} k_3 + \frac{950187200906606025247}{381367602} k_4 - \frac{407348862148220695163}{84748356} k_5 \\
& + \frac{588474420114898407047}{63561267} k_6 - \frac{3395249820289863776129}{190683801} k_7 + \frac{6519050633956423121840}{190683801} k_8 \\
& - \frac{12494990286877563773116}{190683801} k_9 + \frac{241450319017769846288}{1926099} k_{10} - \frac{45634217333431409584592}{190683801} k_{11} \\
& + \frac{86922417480145120334464}{190683801} k_{12} - \frac{165152690405816521254592}{190683801} k_{13} + \frac{104306996560416660578048}{63561267} k_{14} \\
& - \frac{197024365259424744775424}{63561267} k_{15} + \frac{1112608315908857084529664}{190683801} k_{16} - \frac{2086140756494604496789504}{190683801} k_{17} \\
& + \frac{3894129613648474658561024}{190683801} k_{18} - \frac{2410651748393159873821696}{63561267} k_{19} \\
& + \frac{13351302296637597451316224}{190683801} k_{20} - \frac{2225217083402369781173248}{17334891} k_{21} \\
& + \frac{44504342121021691370875904}{190683801} k_{22} - \frac{26702605453785736254491648}{63561267} k_{23} \\
& + \frac{15823766266379980287331328}{21187089} k_{24} - \frac{249224319446065870083802112}{190683801} k_{25} \\
& + \frac{427241691334835494525561856}{190683801} k_{26} - \frac{712069486501952645203108864}{190683801} k_{27} \\
& + \frac{379770393128547090469430272}{63561267} k_{28} - \frac{1708966769985901373141107712}{190683801} k_{29} \\
& + \frac{2278622360586161474873924608}{190683801} k_{30} - \frac{2278622360586161474873924608}{190683801} k_{31},
\end{aligned}$$

Continued

$$\begin{aligned}
 r_5 := & -\frac{4078493266508992}{3026727}k_0 + \frac{1757093202559117}{672606}k_1 - \frac{30608970079163071}{6053454}k_2 + \frac{29588812154023063}{3026727}k_3 \\
 & -\frac{57137119040209117}{3026727}k_4 + \frac{24487373790016241}{672606}k_5 - \frac{23580465846292238}{336303}k_6 + \frac{408123936542458630}{3026727}k_7 \\
 & -\frac{783598788593292832}{3026727}k_8 + \frac{1501899078618026600}{3026727}k_9 - \frac{87066684321864992}{91719}k_{10} \\
 & + \frac{5485205009982032608}{3026727}k_{11} - \frac{10448015967673411328}{3026727}k_{12} + \frac{19851240835474624640}{3026727}k_{13} \\
 & -\frac{12537631452645220864}{1008909}k_{14} + \frac{23682201810197115392}{1008909}k_{15} - \frac{133734829754049019904}{3026727}k_{16} \\
 & + \frac{250752873123348457472}{3026727}k_{17} - \frac{468072133707326267392}{3026727}k_{18} + \frac{289758992548733034496}{1008909}k_{19} \\
 & -\frac{1604819271844457676800}{3026727}k_{20} + \frac{267469910195425869824}{275157}k_{21} \\
 & -\frac{5349398704186073153536}{3026727}k_{22} + \frac{3209639457729863090176}{1008909}k_{23} \\
 & -\frac{5706026026397239083008}{1008909}k_{24} + \frac{29956637937551790309376}{3026727}k_{25} \\
 & -\frac{51354238139489320960000}{3026727}k_{26} + \frac{85590398952084000997376}{3026727}k_{27} \\
 & -\frac{15216071182490613579776}{336303}k_{28} + \frac{205416963210483884621824}{3026727}k_{29} \\
 & -\frac{273889285778552247025664}{3026727}k_{30} + \frac{273889285778552247025664}{3026727}k_{31},
 \end{aligned}$$

$$\begin{aligned}
 r_6 := & \frac{17850531125341314656}{190683801}k_0 - \frac{1560234802784637731}{108962172}k_1 + \frac{565694547181365397}{254245068}k_2 \\
 & -\frac{75770652342362225}{54481086}k_3 + \frac{954011446348337117}{381367602}k_4 - \frac{136502477859918587}{28249452}k_5 \\
 & + \frac{1775145743398433855}{190683801}k_6 - \frac{3414079397677526531}{190683801}k_7 + \frac{2185090542419623856}{63561267}k_8 \\
 & -\frac{12564463935821044276}{190683801}k_9 + \frac{2185139107269117872}{17334891}k_{10} - \frac{15296021695296107984}{63561267}k_{11} \\
 & + \frac{87405968241034408832}{190683801}k_{12} - \frac{166071462214432294976}{190683801}k_{13} + \frac{34962426591446694656}{21187089}k_{14} \\
 & -\frac{594361378558416382720}{190683801}k_{15} + \frac{1118798026619340021760}{190683801}k_{16} - \frac{69924881811398697984}{63561267}k_{17} \\
 & + \frac{3915793558221726961664}{190683801}k_{18} - \frac{4475192842589763960832}{63561267}k_{20} \\
 & -\frac{2237596446112370425856}{17334891}k_{21} + \frac{7272188241043282374656}{190683801}k_{19} \\
 & + \frac{44751929234907189379072}{190683801}k_{22} - \frac{8950385886365732618240}{21187089}k_{23} \\
 & + \frac{143206174578570883235840}{190683801}k_{24} - \frac{250610805949435611987968}{190683801}k_{25} \\
 & + \frac{143206174985307203403776}{63561267}k_{26} - \frac{716030875419409985093632}{190683801}k_{27} \\
 & + \frac{1145649401161281843822592}{190683801}k_{28} + \frac{572824700727179062132736}{63561267}k_{29} \\
 & -\frac{2291298803201792528973824}{190683801}k_{30} - \frac{2291298803201792528973824}{190683801}k_{31},
 \end{aligned}$$

Continued

$$\begin{aligned}
r_7 := & \frac{182985211619366502400}{27240543} k_0 - \frac{9114335538551103232}{9080181} k_1 + \frac{3015119186275104512}{27240543} k_2 \\
& - \frac{160577127932952064}{27240543} k_3 - \frac{20805865009788416}{27240543} k_4 + \frac{593806988273408}{3026727} k_5 \\
& + \frac{2355269316932608}{9080181} k_6 - \frac{22249981901335552}{27240543} k_7 + \frac{47041546286743552}{27240543} k_8 \\
& - \frac{92318681559486464}{27240543} k_9 + \frac{1794795498225664}{275157} k_{10} - \frac{339752396451659776}{27240543} k_{11} \\
& + \frac{647414622429839360}{27240543} k_{12} - \frac{1230220929648558080}{27240543} k_{13} + \frac{777003747655745536}{9080181} k_{14} \\
& - \frac{1467684753032151040}{9080181} k_{15} + \frac{8288118568652177408}{27240543} k_{16} - \frac{15540230489066110976}{27240543} k_{17} \\
& + \frac{29008434308301783040}{27240543} k_{18} - \frac{17957602863013756928}{9080181} k_{19} \\
& + \frac{99457493780231684096}{27240543} k_{20} - \frac{16576249008379461632}{2476413} k_{21} \\
& + \frac{331524980412700229632}{27240543} k_{22} - \frac{198914988287781830656}{9080181} k_{23} \\
& + \frac{117875548621602881536}{3026727} k_{24} - \frac{1856539890819019767808}{27240543} k_{25} \\
& + \frac{3182639812846948777984}{27240543} k_{26} - \frac{5304399688084778319872}{27240543} k_{27} \\
& + \frac{2829013166979691249664}{9080181} k_{28} - \frac{16974079001880596381696}{27240543} k_{30} \\
& - \frac{16974079001880596381696}{27240543} k_{31} + \frac{12730559251409835065344}{27240543} k_{29}, \\
r_8 := & \frac{1402046646636032}{572051403} k_0 - \frac{404359709107292}{81721629} k_1 + \frac{1838528360224516}{190683801} k_2 \\
& - \frac{1526510201939944}{81721629} k_3 + \frac{20651859976599112}{572051403} k_4 - \frac{1475682550603964}{21187089} k_5 \\
& + \frac{76747948993198640}{572051403} k_6 - \frac{147602599162168688}{572051403} k_7 + \frac{94468543272975104}{190683801} k_8 \\
& - \frac{543201327526722112}{572051403} k_9 + \frac{94470348795976448}{52004673} k_{10} - \frac{661294196377898240}{190683801} k_{11} \\
& + \frac{3778828733290707968}{572051403} k_{12} - \frac{7179779137236426752}{572051403} k_{13} + \frac{1511532964276548608}{63561267} k_{14} \\
& - \frac{25696065395082001408}{572051403} k_{15} + \frac{48369069929078318080}{572051403} k_{16} - \frac{30230670898864546816}{190683801} k_{17} \\
& + \frac{193476311878459657216}{190683801} k_{20} + \frac{169291764793840182272}{572051403} k_{18} \\
& - \frac{314399000941203786752}{572051403} k_{19} - \frac{96738157089236403200}{52004673} k_{21} \\
& + \frac{1934763156414028899328}{572051403} k_{22} - \frac{386952633138707606528}{63561267} k_{23} \\
& + \frac{6191242149010792604672}{572051403} k_{24} - \frac{10834673781543178923008}{572051403} k_{25} \\
& + \frac{6191242168369447930880}{190683801} k_{26} - \frac{30956210865390042385408}{572051403} k_{27} \\
& + \frac{49529937408072029873152}{572051403} k_{28} - \frac{24764968711051435418624}{190683801} k_{29} \\
& + \frac{99059874858236582638592}{572051403} k_{30} - \frac{99059874858236582638592}{572051403} k_{31},
\end{aligned}$$

Continued

$$\begin{aligned}
 r_9 := & \frac{410440630890496}{245164887}k_0 - \frac{822808296656416}{245164887}k_1 + \frac{532781926617632}{81721629}k_2 \\
 & - \frac{3092465057117888}{245164887}k_3 + \frac{5973557948909120}{245164887}k_4 - \frac{426751782276064}{9080181}k_5 \\
 & + \frac{22192952299424128}{245164887}k_6 - \frac{42680608321127296}{245164887}k_7 + \frac{27316205232019456}{81721629}k_8 \\
 & - \frac{157070021108969984}{245164887}k_9 + \frac{27316692114626560}{22287717}k_{10} - \frac{191217453405644800}{81721629}k_{11} \\
 & + \frac{1092672979433414656}{245164887}k_{12} - \frac{2076080468475166720}{245164887}k_{13} + \frac{437069771975139328}{27240543}k_{14} \\
 & - \frac{7430187908813594624}{245164887}k_{15} + \frac{13986237835758608384}{245164887}k_{16} - \frac{8741399233129766912}{81721629}k_{17} \\
 & + \frac{48951837446132015104}{245164887}k_{18} - \frac{55944958707133865984}{81721629}k_{20} \\
 & - \frac{27972479505717747712}{22287717}k_{21} + \frac{90910556978580103168}{245164887}k_{19} \\
 & + \frac{559449591757583753216}{245164887}k_{22} - \frac{111889918530040373248}{27240543}k_{23} \\
 & + \frac{1790238698042727669760}{245164887}k_{24} - \frac{3132917723081066487808}{245164887}k_{25} \\
 & + \frac{1790238699381654839296}{81721629}k_{26} - \frac{8951193498247201366016}{245164887}k_{27} \\
 & + \frac{14321909598400556638208}{245164887}k_{28} + \frac{7160954799535010111488}{81721629}k_{29} \\
 & + \frac{28643819198809504030720}{245164887}k_{30} - \frac{28643819198809504030720}{245164887}k_{31},
 \end{aligned}$$

$$\begin{aligned}
 r_{10} := & \frac{497136946726582528}{190683801}k_0 - \frac{10122366213302674}{27240543}k_1 + \frac{487378431477950}{63561267}k_2 \\
 & + \frac{1798343394456628}{27240543}k_3 - \frac{25298324783503396}{190683801}k_4 + \frac{1806651219262046}{7062363}k_5 \\
 & - \frac{93955276958445080}{190683801}k_6 + \frac{180694470620880824}{190683801}k_7 - \frac{115647804424121216}{63561267}k_8 \\
 & + \frac{664983511054035232}{190683801}k_9 - \frac{115649988633594752}{17334891}k_{10} + \frac{809552148599316608}{63561267}k_{11} \\
 & - \frac{4626018445010825216}{190683801}k_{12} + \frac{8789440995378177536}{190683801}k_{13} - \frac{1850409300232641536}{21187089}k_{14} \\
 & + \frac{31456964494852645888}{190683801}k_{15} - \frac{59213116671659852800}{190683801}k_{16} + \frac{37008200568585167872}{63561267}k_{17} \\
 & - \frac{207245932298079798272}{190683801}k_{18} - \frac{236852504576418688000}{63561267}k_{20} + \frac{384885313359433100288}{190683801}k_{19} \\
 & + \frac{11842625355655441152}{17334891}k_{21} - \frac{2368525087038476502016}{190683801}k_{22} + \frac{473705019402448833536}{21187089}k_{23} \\
 & - \frac{7579280330445604668416}{190683801}k_{24} + \frac{13263740600225661307904}{190683801}k_{25} \\
 & - \frac{7579280350845647793152}{63561267}k_{26} + \frac{37896401778812429495296}{190683801}k_{27} \\
 & - \frac{60634242870485098267648}{190683801}k_{28} + \frac{30317121442518316565504}{63561267}k_{29} \\
 & - \frac{121268485784624801125376}{190683801}k_{30} + \frac{121268485784624801125376}{190683801}k_{31},
 \end{aligned}$$

Continued

$$\begin{aligned}
r_{11} := & -\frac{7845155552}{245164887}k_0 + \frac{1961246513}{980659548}k_1 + \frac{2735}{326886516}k_2 - \frac{277}{490329774}k_3 + \frac{7}{490329774}k_4 - \frac{1}{36320724}k_5 \\
& + \frac{13}{245164887}k_6 - \frac{25}{245164887}k_7 + \frac{16}{81721629}k_8 - \frac{92}{245164887}k_9 + \frac{16}{22287717}k_{10} - \frac{112}{81721629}k_{11} + \frac{640}{245164887}k_{12} \\
& - \frac{1216}{245164887}k_{13} + \frac{256}{27240543}k_{14} - \frac{4352}{245164887}k_{15} + \frac{8192}{245164887}k_{16} - \frac{5120}{81721629}k_{17} + \frac{28672}{245164887}k_{18} - \frac{32768}{81721629}k_{20} \\
& - \frac{16384}{22287717}k_{21} + \frac{327680}{245164887}k_{22} - \frac{65536}{27240543}k_{23} + \frac{53248}{245164887}k_{19} + \frac{1048576}{245164887}k_{24} - \frac{1835008}{245164887}k_{25} + \frac{1048576}{81721629}k_{26} \\
& - \frac{5242880}{245164887}k_{27} + \frac{8388608}{245164887}k_{28} - \frac{16777216}{245164887}k_{30} - \frac{16777216}{245164887}k_{31} + \frac{4194304}{81721629}k_{29},
\end{aligned}$$

$$\begin{aligned}
r_{12} := & \frac{2317220714287552}{190683801}k_0 + \frac{152166700793161}{54481086}k_1 - \frac{1195931905993439}{127122534}k_2 \\
& + \frac{508464732469555}{27240543}k_3 - \frac{6883955676246967}{190683801}k_4 + \frac{983776951113553}{14124726}k_5 \\
& - \frac{25582341097777850}{190683801}k_6 + \frac{49200272990040146}{190683801}k_7 - \frac{31489134650021408}{63561267}k_8 \\
& + \frac{181064925473328184}{190683801}k_9 - \frac{31489736491021856}{17334891}k_{10} + \frac{220428740372507360}{63561267}k_{11} \\
& - \frac{1259594386791424256}{190683801}k_{12} + \frac{2393230849565066624}{190683801}k_{13} - \frac{503838245036658176}{21187089}k_{14} \\
& + \frac{8565251833083414016}{190683801}k_{15} - \frac{16122828865249315840}{190683801}k_{16} + \frac{10076768771844355072}{63561267}k_{17} \\
& - \frac{56429907709061295104}{190683801}k_{18} + \frac{32245663474191887360}{17334891}k_{21} - \frac{64491326181712724992}{63561267}k_{20} \\
& + \frac{104798403091518792704}{190683801}k_{19} - \frac{644913274360271358976}{190683801}k_{22} + \frac{128982655490688214016}{21187089}k_{23} \\
& - \frac{2063722494114835057664}{190683801}k_{24} + \frac{3611514371625725305856}{190683801}k_{25} \\
& - \frac{2063722500567720166400}{63561267}k_{26} + \frac{10318612510686201742336}{190683801}k_{27} \\
& - \frac{16509780024913910139904}{190683801}k_{28} + \frac{8254890014795428563968}{63561267}k_{29} \\
& - \frac{33019560063858661243904}{190683801}k_{30} + \frac{33019560063858661243904}{190683801}k_{31},
\end{aligned}$$

$$\begin{aligned}
r_{13} := & \frac{1713565605536}{3026727}k_0 - \frac{4885835317153}{4035636}k_1 + \frac{28580433958345}{12106908}k_2 - \frac{27640118395825}{6053454}k_3 + \frac{53375164694971}{6053454}k_4 \\
& - \frac{22875194940935}{1345212}k_5 + \frac{33042120966595}{1008909}k_6 - \frac{190628577556325}{3026727}k_7 + \frac{366008629043312}{3026727}k_8 - \frac{701519765914156}{3026727}k_9 \\
& + \frac{40667991081200}{91719}k_{10} - \frac{2562094151981840}{3026727}k_{11} + \frac{4880198724104320}{3026727}k_{12} - \frac{9272412472391104}{3026727}k_{13} \\
& + \frac{5856281324390144}{1008909}k_{14} - \frac{3687300576400640}{336303}k_{15} + \frac{62467405675601920}{3026727}k_{16} - \frac{117126728485473280}{3026727}k_{17} \\
& + \frac{218637154101997568}{3026727}k_{18} - \frac{15038572140081152}{112101}k_{19} + \frac{749616704535101440}{3026727}k_{20} - \frac{124936378636779520}{275157}k_{21} \\
& + \frac{2498732274592317440}{3026727}k_{22} - \frac{1499241872412311552}{1008909}k_{23} + \frac{2665322785088208896}{1008909}k_{24} - \frac{13992962175311544320}{3026727}k_{25} \\
& + \frac{23987960234246144000}{3026727}k_{26} - \frac{39979967159169187840}{3026727}k_{27} + \frac{21322662525272145984}{1008909}k_{28} \\
& + \frac{95952021488282894336}{3026727}k_{29} - \frac{127936055399384350720}{3026727}k_{30} - \frac{127936055399384350720}{3026727}k_{31},
\end{aligned}$$

Continued

$$\begin{aligned}
 r_{14} := & \frac{8258019327389696}{190683801} k_0 - \frac{2364911193971552}{27240543} k_1 + \frac{10718724919702432}{63561267} k_2 - \frac{8886997089409600}{27240543} k_3 + \frac{120154525341999424}{190683801} k_4 \\
 & - \frac{8583279015924320}{7062363} k_5 + \frac{446349030653909888}{190683801} k_6 - \frac{858379450288459136}{190683801} k_7 + \frac{549367570134431744}{63561267} k_8 - \frac{3158876750178781696}{190683801} k_9 \\
 & + \frac{549371059459782656}{17334891} k_{10} - \frac{3845602001029695488}{63561267} k_{11} + \frac{21974883783699685376}{190683801} k_{12} - \frac{41752296617398059008}{190683801} k_{13} \\
 & + \frac{8789959449390325760}{21187089} k_{14} - \frac{149429334740324589568}{190683801} k_{15} + \frac{281278776244639277056}{190683801} k_{16} - \frac{175799246338520276992}{63561267} k_{17} \\
 & + \frac{1125115266765590093824}{63561267} k_{20} + \frac{984475818726279618560}{190683801} k_{18} - \frac{1828312280237141762048}{190683801} k_{19} - \frac{562557638768933888000}{17334891} k_{21} \\
 & + \frac{11251152841939587506176}{190683801} k_{22} - \frac{2250230576600004141056}{21187089} k_{23} + \frac{36003689306605160013824}{190683801} k_{24} - \frac{63006456373979816468480}{190683801} k_{25} \\
 & + \frac{36003689387387099242496}{63561267} k_{26} - \frac{180018447031999325249536}{190683801} k_{27} + \frac{288029515344343620493312}{190683801} k_{28} \\
 & - \frac{144014757699731394486272}{63561267} k_{29} + \frac{576059030854044746424320}{190683801} k_{30} - \frac{576059030854044746424320}{190683801} k_{31},
 \end{aligned}$$

$$\begin{aligned}
 r_{15} := & \frac{577738379470921373970421838621222390291070976}{145553752099236970162344160802889693} k_0 + \frac{389943426768775182610466056652437246979899136}{145553752099236970162344160802889693} k_1 \\
 & - \frac{284368637537941787517107429110680352807201024}{145553752099236970162344160802889693} k_2 + \frac{233719005131796095379317410248817492183195904}{145553752099236970162344160802889693} k_3 \\
 & - \frac{204266613045128794868970582764082926843958784}{145553752099236970162344160802889693} k_4 + \frac{180982375350361123797457444770419729976069632}{145553752099236970162344160802889693} k_5 \\
 & - \frac{159492484645521478135024402908398403472614400}{145553752099236970162344160802889693} k_6 + \frac{139135726848179027535311521784750735180558336}{145553752099236970162344160802889693} k_7 \\
 & - \frac{120051291230827604989855013281820481416740864}{145553752099236970162344160802889693} k_8 + \frac{102405923175215278676932791313675138348163072}{145553752099236970162344160802889693} k_9 \\
 & - \frac{86338518039207496417640219949443743298473984}{145553752099236970162344160802889693} k_{10} + \frac{72043112925773671620161505898310423323652096}{145553752099236970162344160802889693} k_{11} \\
 & - \frac{59828951030446462921950052347269335522385920}{145553752099236970162344160802889693} k_{12} + \frac{50145468776477793634308374381898890313555968}{145553752099236970162344160802889693} k_{13} \\
 & - \frac{43588273326559350446315416451454928314351616}{145553752099236970162344160802889693} k_{14} + \frac{40897809442440987060284075657392619762843648}{145553752099236970162344160802889693} k_{15} \\
 & - \frac{42955861925222786337915243787918244582195200}{145553752099236970162344160802889693} k_{16} + \frac{50780865984879389682162631031032311798038528}{145553752099236970162344160802889693} k_{17} \\
 & - \frac{65521847445400920309068025486770806824632320}{145553752099236970162344160802889693} k_{18} + \frac{88450912673016470567691648142506948199448576}{145553752099236970162344160802889693} k_{19} \\
 & - \frac{120954357456372037081202364100637280542654464}{145553752099236970162344160802889693} k_{20} + \frac{164522623618043879235129804884879526325649408}{145553752099236970162344160802889693} k_{21} \\
 & - \frac{220739419593938362483559138258027962407583744}{145553752099236970162344160802889693} k_{22} + \frac{291270264487621422272510678730059925807693824}{145553752099236970162344160802889693} k_{23} \\
 & - \frac{377850601647604933036669445427582730199105536}{145553752099236970162344160802889693} k_{24} + \frac{482273773863644551181794880666599556095410176}{145553752099236970162344160802889693} k_{25} \\
 & - \frac{606378721801060506038490889931913288422785024}{145553752099236970162344160802889693} k_{26} + \frac{752025380869379181769745921835934191664234496}{145553752099236970162344160802889693} k_{27} \\
 & - \frac{921115675966539272203317511246287289416417280}{145553752099236970162344160802889693} k_{28} + \frac{1117453456888418164994720727898222628872650752}{145553752099236970162344160802889693} k_{29} \\
 & - \frac{1368286209459734662501787199033322451736985600}{145553752099236970162344160802889693} k_{30} + \frac{1869951714602367657515920141303522097465655296}{145553752099236970162344160802889693} k_{31} \\
 & - \frac{3739903429204735315031840282607044194931310592}{145553752099236970162344160802889693} k_{32},
 \end{aligned}$$

Continued

$$\begin{aligned}
r_{16} := & -\frac{8187517611412321816450317517847180936670908416}{1892198777290080612110474090437566009} k_0 - \frac{159533088530622432903456266215894185920157280}{1892198777290080612110474090437566009} k_1 \\
& + \frac{154394218053931204299148068085218437236530464}{145553752099236970162344160802889693} k_2 - \frac{2057350305190694751206512989996888686524292512}{1892198777290080612110474090437566009} k_3 \\
& + \frac{1732404887835464639120997787916801947574461376}{1892198777290080612110474090437566009} k_4 \\
& - \frac{1405990680809729370498243610916705554858690240}{1892198777290080612110474090437566009} k_5 \\
& + \frac{1144488651965894568749363790437907726384136832}{1892198777290080612110474090437566009} k_6 \\
& - \frac{939844587890153231821926456516923457051665920}{1892198777290080612110474090437566009} k_7 \\
& + \frac{775530716458450573108031044536332857186887680}{1892198777290080612110474090437566009} k_8 \\
& - \frac{641215590606143303960670664662711172566602752}{1892198777290080612110474090437566009} k_9 \\
& + \frac{533598746312304368323837358420614698018730496}{1892198777290080612110474090437566009} k_{10} \\
& - \frac{34978573114728227134846608592592432234501632}{145553752099236970162344160802889693} k_{11} \\
& + \frac{410580247182314008263876827317175039683871744}{1892198777290080612110474090437566009} k_{12} \\
& - \frac{410429903343374999751381395132964252726148096}{1892198777290080612110474090437566009} k_{13} \\
& + \frac{466571397956138819403384731590732091436726272}{1892198777290080612110474090437566009} k_{14} \\
& - \frac{594395101150790885129246689338994483319664640}{1892198777290080612110474090437566009} k_{15} \\
& + \frac{812520028837847841388581651545269952176578560}{1892198777290080612110474090437566009} k_{16} \\
& - \frac{1142939484077410993309324621624796185836421120}{1892198777290080612110474090437566009} k_{17} \\
& + \frac{1611122320404854212801419430219237356852150272}{1892198777290080612110474090437566009} k_{18} \\
& - \frac{2246042283397163284978426120120548188157902848}{1892198777290080612110474090437566009} k_{19} \\
& + \frac{3080125446429399549359394621911824478685888512}{1892198777290080612110474090437566009} k_{20} \\
& - \frac{4149118181721195882382697028838574923427086336}{1892198777290080612110474090437566009} k_{21} \\
& + \frac{422452804013804374185584906133418118056509440}{145553752099236970162344160802889693} k_{22} \\
& - \frac{7150157392461355980247977668273897149643358208}{1892198777290080612110474090437566009} k_{23} \\
& + \frac{9168214513517564713799636815721714863928508416}{1892198777290080612110474090437566009} k_{24} \\
& - \frac{11592569199135583258269488881518337074348949504}{1892198777290080612110474090437566009} k_{25} \\
& + \frac{14471584786483274457333207749537373737259892736}{1892198777290080612110474090437566009} k_{26} \\
& - \frac{17854724727195931687505586900600532885838233600}{1892198777290080612110474090437566009} k_{27} \\
& + \frac{21793652701843709390979659009425718706904236032}{1892198777290080612110474090437566009} k_{28} \\
& - \frac{26391696071566916452141489425448739574098427904}{1892198777290080612110474090437566009} k_{29} \\
& + \frac{32307970231440982228678836455867430533548998656}{1892198777290080612110474090437566009} k_{30} \\
& - \frac{44140518551189113781753530516704812452450140160}{1892198777290080612110474090437566009} k_{31} \\
& + \frac{88281037102378227563507061033409624904900280320}{1892198777290080612110474090437566009} k_{32}
\end{aligned}$$

Continued

$$\begin{aligned}
 r_{17} := & -\frac{643329882977685041186429125699313858510848}{19375569863403072037502678610651} k_0 + \frac{147304312960293970271713745600069976768512}{19375569863403072037502678610651} k_1 \\
 & - \frac{1210058973360542406118624016827790540800}{1490428451031005541346359893127} k_2 + \frac{1466508444348165559897402255268056875008}{19375569863403072037502678610651} k_3 \\
 & - \frac{13791830636937415202317438092493963362304}{19375569863403072037502678610651} k_4 + \frac{25583908148846558533738292877982288412672}{19375569863403072037502678610651} k_5 \\
 & - \frac{31793249747431987822197562069245137846272}{19375569863403072037502678610651} k_6 + \frac{33393590156541009877887421501110672687104}{19375569863403072037502678610651} k_7 \\
 & - \frac{32146613512659643567497135901386798530560}{19375569863403072037502678610651} k_8 \\
 & + \frac{29340222814323358302435599361999339782144}{19375569863403072037502678610651} k_9 \\
 & - \frac{25727794074444541470428892293017424625664}{19375569863403072037502678610651} k_{10} \\
 & + \frac{1669909346001985082043437607054018609152}{1490428451031005541346359893127} k_{11} \\
 & - \frac{17485837707243561661643418134480882237440}{19375569863403072037502678610651} k_{12} \\
 & + \frac{13159684454152305001121263037913470861312}{19375569863403072037502678610651} k_{13} \\
 & - \frac{8780131102450043498347309361207922393088}{19375569863403072037502678610651} k_{14} \\
 & + \frac{4371095296436930207744006869605743329280}{19375569863403072037502678610651} k_{15} \\
 & + \frac{56884464447602314319048873700573052928}{19375569863403072037502678610651} k_{16} \\
 & - \frac{4500563563858544575245007184075160027136}{19375569863403072037502678610651} k_{17} \\
 & + \frac{8961145379968385996218648239160669503488}{19375569863403072037502678610651} k_{18} \\
 & - \frac{13443049983427965289224702813189030805504}{19375569863403072037502678610651} k_{19} \\
 & + \frac{17953498180473373613836035222480375775232}{19375569863403072037502678610651} k_{20} \\
 & - \frac{22502443030732959711290340551179531976704}{19375569863403072037502678610651} k_{21} \\
 & + \frac{2084817310202454900942859214086690045952}{1490428451031005541346359893127} k_{22} \\
 & - \frac{31769596617430646921830140919403345870848}{19375569863403072037502678610651} k_{23} \\
 & + \frac{36521700489210666099545283068371263291392}{19375569863403072037502678610651} k_{24} \\
 & - \frac{41379970885435339270027407481861551161344}{19375569863403072037502678610651} k_{25} \\
 & + \frac{46367880678660444594892532483648538804224}{19375569863403072037502678610651} k_{26} \\
 & - \frac{51510835579955268116552733326400612990976}{19375569863403072037502678610651} k_{27} \\
 & + \frac{56838106977416111639012199120574275387392}{19375569863403072037502678610651} k_{28} \\
 & - \frac{62408237647235578770279308087082529325056}{19375569863403072037502678610651} k_{29} \\
 & + \frac{68464086861772293119161703398259966345216}{19375569863403072037502678610651} k_{30} \\
 & - \frac{80575785290845721816926494020614840385536}{19375569863403072037502678610651} k_{31} \\
 & + \frac{161151570581691443633852988041229680771072}{19375569863403072037502678610651} k_{32},
 \end{aligned}$$

Continued

$$\begin{aligned}
r_{18} := & \frac{7200817595893180445536096846534860064747520}{210244308587786734678941565604174001} k_0 - \frac{6998804769585734064045981619545288517039456}{210244308587786734678941565604174001} k_1 \\
+ & \frac{530440027226536076672531909649765764319776}{16172639122137441129149351200321077} k_2 - \frac{6824456919060819825768983080500742798450336}{210244308587786734678941565604174001} k_3 \\
+ & \frac{6745344487584820041562803126194655389563328}{210244308587786734678941565604174001} k_4 - \frac{6631258703671288380471438380101647464928448}{210244308587786734678941565604174001} k_5 \\
+ & \frac{6455113775897888780618057957975709314213504}{210244308587786734678941565604174001} k_6 \\
- & \frac{6181235253021515552883083212777760046814720}{210244308587786734678941565604174001} k_7 \\
+ & \frac{5758119554980403429819093457588757301356544}{210244308587786734678941565604174001} k_8 \\
- & \frac{5110522418215584299116111920300401752339456}{210244308587786734678941565604174001} k_9 \\
+ & \frac{4129576966405346966954698319541915127431680}{210244308587786734678941565604174001} k_{10} \\
- & \frac{204626082710675433817370075612361538504192}{16172639122137441129149351200321077} k_{11} \\
+ & \frac{484884300610749283881335487138637146285056}{210244308587786734678941565604174001} k_{12} \\
+ & \frac{269498547845193869970685224592271879365632}{210244308587786734678941565604174001} k_{13} \\
- & \frac{7282123882134533727480605253259587403565056}{210244308587786734678941565604174001} k_{14} \\
+ & \frac{13807628039746943061694296140356476279492608}{210244308587786734678941565604174001} k_{15} \\
- & \frac{22956294155949443695821205372887579311153152}{210244308587786734678941565604174001} k_{16} \\
+ & \frac{35590428396275037094309238771169584563453952}{210244308587786734678941565604174001} k_{17} \\
- & \frac{52769844592919353622636487797537224668282880}{210244308587786734678941565604174001} k_{18} \\
+ & \frac{75765966797134442187626316939977444164370432}{210244308587786734678941565604174001} k_{19} \\
- & \frac{106068830099335348925610826044828633042780160}{210244308587786734678941565604174001} k_{20} \\
+ & \frac{145386924761036033230957881256012322449915904}{210244308587786734678941565604174001} k_{21} \\
- & \frac{15049279328586904693634985775596926369529856}{16172639122137441129149351200321077} k_{22} \\
+ & \frac{258950098403709600992922925102622957931855872}{210244308587786734678941565604174001} k_{23} \\
- & \frac{337618625261985621511998958776201715539509248}{210244308587786734678941565604174001} k_{24} \\
+ & \frac{434113669315911983121555959326471394635022336}{210244308587786734678941565604174001} k_{25} \\
- & \frac{551043106399733822502402833747296646861946880}{210244308587786734678941565604174001} k_{26} \\
+ & \frac{691107750712581020350483488820461979164999680}{210244308587786734678941565604174001} k_{27} \\
- & \frac{857194293183352945392011830903591609091227648}{210244308587786734678941565604174001} k_{28} \\
+ & \frac{1055076115669848335079415672711933971734724608}{210244308587786734678941565604174001} k_{29} \\
- & \frac{1316548498187790654058570513970701799812759552}{210244308587786734678941565604174001} k_{30} \\
+ & \frac{1839493263223675292016880196488237455968829440}{210244308587786734678941565604174001} k_{31} \\
- & \frac{3678986526447350584033760392976474911937658880}{210244308587786734678941565604174001} k_{32}
\end{aligned}$$

Continued

$$\begin{aligned}
 r_{10} := & -\frac{846331556576961593459224129208633418441519104}{1892198777290080612110474090437566009} k_0 + \frac{376685845259126425679159929193982114613026400}{1892198777290080612110474090437566009} k_1 \\
 & -\frac{15949332300870052520194531877272118975378720}{145553752099236970162344160802889693} k_2 + \frac{157457424374236669142626570672887235024901536}{1892198777290080612110474090437566009} k_3 \\
 & -\frac{142757330201442573398181736488684165478953920}{1892198777290080612110474090437566009} k_4 + \frac{134052406517472853730290982190011910207067840}{1892198777290080612110474090437566009} k_5 \\
 & -\frac{124259776276963153692996588503959035499995776}{1892198777290080612110474090437566009} k_6 \\
 & + \frac{112921269720863783677292219739652208623232512}{1892198777290080612110474090437566009} k_7 \\
 & -\frac{100743677662559060822473284906275626382305280}{1892198777290080612110474090437566009} k_8 \\
 & + \frac{88244241456936317146220738290478244155806720}{1892198777290080612110474090437566009} k_9 \\
 & -\frac{75651309378062869097629433955447348840395264}{1892198777290080612110474090437566009} k_{10} \\
 & + \frac{4849031507067062092856558657652005348873728}{145553752099236970162344160802889693} k_{11} \\
 & -\frac{50417462117028014651397187179764440190501888}{1892198777290080612110474090437566009} k_{12} \\
 & + \frac{37790993654245889191778157000988234520083456}{1892198777290080612110474090437566009} k_{13} \\
 & -\frac{25154311114530973778498491223713198830811136}{1892198777290080612110474090437566009} k_{14} \\
 & + \frac{12501854301022655068735728536552256371814400}{1892198777290080612110474090437566009} k_{15} \\
 & + \frac{174587763380230960804941100576736416768000}{1892198777290080612110474090437566009} k_{16} \\
 & -\frac{12886880971902362237482557821386000286842880}{1892198777290080612110474090437566009} k_{17} \\
 & + \frac{25651762948709973544400826181308544276037632}{1892198777290080612110474090437566009} k_{18} \\
 & -\frac{38492215473544975648972869943687091588956160}{1892198777290080612110474090437566009} k_{19} \\
 & + \frac{51438870966597824196638913979421144158044160}{1892198777290080612110474090437566009} k_{20} \\
 & -\frac{64531272182658442150283331484088449896808448}{1892198777290080612110474090437566009} k_{21} \\
 & + \frac{5986060837788585057029513883773431389356032}{145553752099236970162344160802889693} k_{22} \\
 & -\frac{91361089805231262070827247863264282107445248}{1892198777290080612110474090437566009} k_{23} \\
 & + \frac{105228168991844362841136407442766957360709632}{1892198777290080612110474090437566009} k_{24} \\
 & -\frac{119500079234285383183585405562083118156349440}{1892198777290080612110474090437566009} k_{25} \\
 & + \frac{134265969294232210675100001389251263902777344}{1892198777290080612110474090437566009} k_{26} \\
 & -\frac{149622180436917923372056747595921941211906048}{1892198777290080612110474090437566009} k_{27} \\
 & + \frac{165679438934685980289733783860969791102648320}{1892198777290080612110474090437566009} k_{28} \\
 & -\frac{182659197332223299458687389542144095612829696}{1892198777290080612110474090437566009} k_{29} \\
 & + \frac{2014839555299143130194134055571309361889280}{1892198777290080612110474090437566009} k_{30} \\
 & -\frac{239133471923450830473207623082425736860008448}{1892198777290080612110474090437566009} k_{31} \\
 & + \frac{478266943846901660946415246164851473720016896}{1892198777290080612110474090437566009} k_{32}
 \end{aligned}$$

Continued

$$\begin{aligned}
r_{20} := & \frac{1078859109831985493644073609782886992384}{1892198777290080612110474090437566009} k_0 + \frac{4499676737645056443877707160369439025260}{1892198777290080612110474090437566009} k_1 \\
& - \frac{406636874382244288798717822301159788324}{145553752099236970162344160802889693} k_2 + \frac{5081939553175629806444613010575260341900}{1892198777290080612110474090437566009} k_3 \\
& - \frac{4709782744231551655955564510815646508184}{1892198777290080612110474090437566009} k_4 + \frac{4317649540934262446189484869616220726928}{1892198777290080612110474090437566009} k_5 \\
& - \frac{3924794600960911189951601678871639244640}{1892198777290080612110474090437566009} k_6 + \frac{3532247300518338232573402434653887273472}{1892198777290080612110474090437566009} k_7 \\
& - \frac{3139765231809650633289399363326082586624}{1892198777290080612110474090437566009} k_8 + \frac{2747293623660562738263020611062084637696}{1892198777290080612110474090437566009} k_9 \\
& - \frac{2354822949354024558118710138817669047808}{1892198777290080612110474090437566009} k_{10} + \frac{150950166734930474975341994482256099840}{145553752099236970162344160802889693} k_{11} \\
& - \frac{1569881213199230909496003432450828372992}{1892198777290080612110474090437566009} k_{12} + \frac{1177409985069016148647654302934124985344}{1892198777290080612110474090437566009} k_{13} \\
& - \frac{784938328109830133555525743770575190016}{1892198777290080612110474090437566009} k_{14} \\
& + \frac{392466008874649421729581783440263753728}{1892198777290080612110474090437566009} k_{15} \\
& + \frac{7317383123706780126502652683919360}{1892198777290080612110474090437566009} k_{16} \\
& - \frac{392482148874754034898390804238201520128}{1892198777290080612110474090437566009} k_{17} \\
& + \frac{784959188374318952231532230179666395136}{1892198777290080612110474090437566009} k_{18} \\
& - \frac{1177439400853839509168059996346859618304}{1892198777290080612110474090437566009} k_{19} \\
& + \frac{1569924072548545823008794206264513134592}{1892198777290080612110474090437566009} k_{20} \\
& - \frac{1962414863846032624142286378132182958080}{1892198777290080612110474090437566009} k_{21} \\
& + \frac{181147219062231418506894629602025209856}{145553752099236970162344160802889693} k_{22} \\
& - \frac{2747423529387971157406848123153438048256}{1892198777290080612110474090437566009} k_{23} \\
& + \frac{3139946846873401163761226327938994274304}{1892198777290080612110474090437566009} k_{24} \\
& - \frac{3532487158686654513314341962121808084992}{1892198777290080612110474090437566009} k_{25} \\
& + \frac{3925048200848256111218475758859131617280}{1892198777290080612110474090437566009} k_{26} \\
& - \frac{4317634001914486088109437105736875737088}{1892198777290080612110474090437566009} k_{27} \\
& + \frac{4710249175513135025590092699198268768256}{1892198777290080612110474090437566009} k_{28} \\
& - \frac{5102902948899784086866175932129767358464}{1892198777290080612110474090437566009} k_{29} \\
& + \frac{5495633921862433395733114444001477066752}{1892198777290080612110474090437566009} k_{30} \\
& - \frac{6281095867787732013466991467744896483328}{1892198777290080612110474090437566009} k_{31} \\
& + \frac{12562191735575464026933982935489792966656}{1892198777290080612110474090437566009} k_{32},
\end{aligned}$$

Continued

$$\begin{aligned}
 r_{21} := & -\frac{7750445741315666895918359690548182249472}{1892198777290080612110474090437566009} k_0 + \frac{2422014377531290030063231597884443416864}{630732925763360204036824696812522003} k_1 \\
 & -\frac{173888214081398885127485081793020111200}{48517917366412323387448053600963231} k_2 + \frac{6297237498249787169137694842496337310880}{1892198777290080612110474090437566009} k_3 \\
 & -\frac{215290171235759979706752511437242707264}{70081436195928911559647188534724667} k_4 + \frac{21927702636736253909769767204008529984}{7786826243992101284405243170524963} k_5 \\
 & -\frac{4844028855842384549488996906287563499136}{1892198777290080612110474090437566009} k_6 + \frac{1453208656801679115994283739232375591424}{630732925763360204036824696812522003} k_7 \\
 & -\frac{1291741028294309961352031286542241658880}{630732925763360204036824696812522003} k_8 + \frac{3390820199402965390086824038297507251200}{1892198777290080612110474090437566009} k_9 \\
 & -\frac{968805771340934266889385663715470591488}{630732925763360204036824696812522003} k_{10} + \frac{62102934072012225679663699479265444352}{48517917366412323387448053600963231} k_{11} \\
 & -\frac{1937611543862008717801004653814447653888}{1892198777290080612110474090437566009} k_{12} + \frac{161467628815955912265240324530668350464}{210244308587786734678941565604174001} k_{13} \\
 & -\frac{107645086166824739290713105589082937344}{210244308587786734678941565604174001} k_{14} \\
 & +\frac{484402892714176251898753051218969374720}{1892198777290080612110474090437566009} k_{15} \\
 & -\frac{3850139043385407228861034405888}{630732925763360204036824696812522003} k_{16} \\
 & -\frac{161467622383797383766962957562346373120}{630732925763360204036824696812522003} k_{17} \\
 & +\frac{968805742178455434765255479159039787008}{1892198777290080612110474090437566009} k_{18} \\
 & -\frac{484402870604201311791880441930598580224}{630732925763360204036824696812522003} k_{19} \\
 & +\frac{645870491224182246878144575975001686016}{630732925763360204036824696812522003} k_{20} \\
 & -\frac{2422014324531738330746397181737959686144}{1892198777290080612110474090437566009} k_{21} \\
 & +\frac{24841172308681606029242723054178009088}{16172639122137441129149351200321077} k_{22} \\
 & -\frac{37675774986982332891457366807146921984}{210244308587786734678941565604174001} k_{23} \\
 & +\frac{3875222761791135228424253796407111057408}{1892198777290080612110474090437566009} k_{24} \\
 & -\frac{1453208504019684655763100085135207825408}{630732925763360204036824696812522003} k_{25} \\
 & +\frac{1614676071637458574404128160319563366400}{630732925763360204036824696812522003} k_{26} \\
 & -\frac{5328430857078332966992982826454813769728}{1892198777290080612110474090437566009} k_{27} \\
 & +\frac{1937611140095688696613755582220613976064}{630732925763360204036824696812522003} k_{28} \\
 & -\frac{2099078622666285291392526383705156485120}{630732925763360204036824696812522003} k_{29} \\
 & +\frac{6781638084716758981489948524062295719936}{1892198777290080612110474090437566009} k_{30} \\
 & -\frac{287053426598243155411655084072442527744}{70081436195928911559647188534724667} k_{31} \\
 & +\frac{574106853196486310823310168144885055488}{70081436195928911559647188534724667} k_{32},
 \end{aligned}$$

Continued

$$\begin{aligned}
 r_{22} := & \frac{7330595268122946758044910723058938685685760}{145553752099236970162344160802889693} k_0 + \frac{9006593137467592173065483152486192627712000}{145553752099236970162344160802889693} k_1 \\
 & - \frac{10379142738069975712231358759430273972699136}{145553752099236970162344160802889693} k_2 + \frac{11447140100956022325813815640931025173479424}{145553752099236970162344160802889693} k_3 \\
 & - \frac{12210136358542666643094706709896919005265920}{145553752099236970162344160802889693} k_4 + \frac{12667964437324526499003420546606850309357568}{145553752099236970162344160802889693} k_5 \\
 & - \frac{12820567619976158014667894839843376314449920}{145553752099236970162344160802889693} k_6 + \frac{12667928623549064239962099890421036386091008}{145553752099236970162344160802889693} k_7 \\
 & - \frac{12210042926973817754320116320247206096404480}{145553752099236970162344160802889693} k_8 + \frac{11446909727181089695980728439132181434990592}{145553752099236970162344160802889693} k_9 \\
 & - \frac{10378529252103120978703064202643082366156800}{145553752099236970162344160802889693} k_{10} + \frac{9004902143711410259623970831102950246449152}{145553752099236970162344160802889693} k_{11} \\
 & - \frac{7326029453415436983893725519258810924924928}{145553752099236970162344160802889693} k_{12} + \frac{5341912832285938079558450269256713313976320}{145553752099236970162344160802889693} k_{13} \\
 & - \frac{3052554837953837399143495350450541960364032}{145553752099236970162344160802889693} k_{14} \\
 & + \frac{457959373624724012185649114651507210321920}{145553752099236970162344160802889693} k_{15} \\
 & + \frac{2441867700628816200519995307346254708604928}{145553752099236970162344160802889693} k_{16} \\
 & - \frac{5646917742386191009346909071518117451005952}{145553752099236970162344160802889693} k_{17} \\
 & + \frac{9157178250108114291442578304123637510176768}{145553752099236970162344160802889693} k_{18} \\
 & - \frac{12972631511676105821867427090785057634254848}{145553752099236970162344160802889693} k_{19} \\
 & + \frac{17093252977057450133634166920944605064069120}{145553752099236970162344160802889693} k_{20} \\
 & - \frac{21519009378845941910931166701457110097461248}{145553752099236970162344160802889693} k_{21} \\
 & + \frac{26249856640493667160119769663408643413901312}{145553752099236970162344160802889693} k_{22} \\
 & - \frac{31285737610373723282181200189371267095199744}{145553752099236970162344160802889693} k_{23} \\
 & + \frac{36626579656463088928460136994542787282075648}{145553752099236970162344160802889693} k_{24} \\
 & - \frac{42272292191224065403399105528669284346101760}{145553752099236970162344160802889693} k_{25} \\
 & + \frac{48222764056882861161254508819533198092402688}{145553752099236970162344160802889693} k_{26} \\
 & - \frac{54477858295986760799528689159929498165051392}{145553752099236970162344160802889693} k_{27} \\
 & + \frac{61037386351725201202215867372678213886017536}{145553752099236970162344160802889693} k_{28} \\
 & - \frac{67900971110477056137304485103419403899240448}{145553752099236970162344160802889693} k_{29} \\
 & + \frac{75372669275255740137195981870145542496976896}{145553752099236970162344160802889693} k_{30} \\
 & - \frac{90316065604813108136978975403597819692449792}{145553752099236970162344160802889693} k_{31} \\
 & + \frac{180632131209626216273957950807195639384899584}{145553752099236970162344160802889693} k_{32}
 \end{aligned}$$

Continued

$$\begin{aligned}
 r_{23} := & \frac{4904993794976991099823904992355941897166848}{1892198777290080612110474090437566009} k_0 - \frac{821130356296232050443487392140562335570368}{1892198777290080612110474090437566009} k_1 \\
 & - \frac{8279856796378227846817000556675613572800}{145553752099236970162344160802889693} k_2 + \frac{269327618836431288290571063581891599820224}{1892198777290080612110474090437566009} k_3 \\
 & - \frac{271729813216736475220990849337008590235264}{1892198777290080612110474090437566009} k_4 + \frac{24936726721318030261116340318864818363520}{1892198777290080612110474090437566009} k_5 \\
 & - \frac{225649489483342663184680707166035600733952}{1892198777290080612110474090437566009} k_6 + \frac{202714277096662969776878653545620790993920}{1892198777290080612110474090437566009} k_7 \\
 & - \frac{180119193510954322575954518104304140251136}{1892198777290080612110474090437566009} k_8 + \frac{157598066119643530302179040552462253287424}{1892198777290080612110474090437566009} k_9 \\
 & - \frac{135083529689461825490979043654947111156736}{1892198777290080612110474090437566009} k_{10} + \frac{8659093361664868199144013300859307058176}{145553752099236970162344160802889693} k_{11} \\
 & - \frac{90051646126494768276572685726275511744512}{1892198777290080612110474090437566009} k_{12} + \frac{67533092422513980993802478813703822522368}{1892198777290080612110474090437566009} k_{13} \\
 & - \frac{45011427064271071866336070367548073046016}{1892198777290080612110474090437566009} k_{14} \\
 & + \frac{22484954875052346128059614752663873609728}{1892198777290080612110474090437566009} k_{15} \\
 & + \frac{48828618001527661737477211041575813120}{1892198777290080612110474090437566009} k_{16} \\
 & - \frac{22593544739321788598541316502269149773824}{1892198777290080612110474090437566009} k_{17} \\
 & + \frac{45154305249628187288757857213057840840704}{1892198777290080612110474090437566009} k_{18} \\
 & - \frac{67738135284274891919957102500686651719680}{1892198777290080612110474090437566009} k_{19} \\
 & + \frac{90354409548604405009285724469652339294208}{1892198777290080612110474090437566009} k_{20} \\
 & - \frac{113015247716454551201736620703781406375936}{1892198777290080612110474090437566009} k_{21} \\
 & + \frac{10441216205004348754619335338392107876352}{145553752099236970162344160802889693} k_{22} \\
 & - \frac{158534462155308165933053765885443805544448}{1892198777290080612110474090437566009} k_{23} \\
 & + \frac{181432806517898598279257504999621181571072}{1892198777290080612110474090437566009} k_{24} \\
 & - \frac{204455623466237187949433368235556009410560}{1892198777290080612110474090437566009} k_{25} \\
 & + \frac{227630597162990649137095970448532840644608}{1892198777290080612110474090437566009} k_{26} \\
 & - \frac{250987774353157354529477143263658618388480}{1892198777290080612110474090437566009} k_{27} \\
 & + \frac{274561926946398993801243151845685068824576}{1892198777290080612110474090437566009} k_{28} \\
 & - \frac{298422598762038946301726526524115644317696}{1892198777290080612110474090437566009} k_{29} \\
 & + \frac{322856309022475525259644633395354469924864}{1892198777290080612110474090437566009} k_{30} \\
 & - \frac{371723729543348683175480847137832121139200}{1892198777290080612110474090437566009} k_{31} \\
 & + \frac{743447459086697366350961694275664242278400}{1892198777290080612110474090437566009} k_{32},
 \end{aligned}$$

Continued

$$\begin{aligned}
 r_{24} := & \frac{145773107328387787239495588762715617161641984}{630732925763360204036824696812522003} k_0 - \frac{134765685349588570290149688612291480739397632}{630732925763360204036824696812522003} k_1 \\
 & + \frac{9615713480120246784518403483387207759478784}{48517917366412323387448053600963231} k_2 - \frac{116408011101702803812430720217175956637859840}{630732925763360204036824696812522003} k_3 \\
 & + \frac{108747836915362164721541759501412687669788672}{630732925763360204036824696812522003} k_4 - \frac{101730916482220209658193550268140666410008576}{630732925763360204036824696812522003} k_5 \\
 & + \frac{95039435947755773606098145007971966215192576}{630732925763360204036824696812522003} k_6 - \frac{88346668352253490169654967760481370656997376}{630732925763360204036824696812522003} k_7 \\
 & + \frac{81323025266904993979063586830755995784839168}{630732925763360204036824696812522003} k_8 - \frac{73638123063840656748351293497328834686484480}{630732925763360204036824696812522003} k_9 \\
 & + \frac{64961419707995851679177999477506380815138816}{630732925763360204036824696812522003} k_{10} - \frac{4227876781491018815427531934689850116800512}{48517917366412323387448053600963231} k_{11} \\
 & + \frac{43310633939484060954607575895935643346796544}{630732925763360204036824696812522003} k_{12} - \frac{29675851548640485090500617540300569985417216}{630732925763360204036824696812522003} k_{13} \\
 & + \frac{13728000703916946284035196433024650450567168}{630732925763360204036824696812522003} k_{14} \\
 & + \frac{4862634610354909208900888129411935899222016}{630732925763360204036824696812522003} k_{15} \\
 & - \frac{26425284310288950835265579508058705799151616}{630732925763360204036824696812522003} k_{16} \\
 & + \frac{51288487200928294427805450637245641295134720}{630732925763360204036824696812522003} k_{17} \\
 & - \frac{79779823591639442260286655377655384600215552}{630732925763360204036824696812522003} k_{18} \\
 & + \frac{112225579759759682567972165651870111449481216}{630732925763360204036824696812522003} k_{19} \\
 & - \frac{148950344015546216832872275886834932520058880}{630732925763360204036824696812522003} k_{20} \\
 & + \frac{190276540325411647454137656363868830540759040}{630732925763360204036824696812522003} k_{21} \\
 & - \frac{18194146879850300162757304070941297647026176}{48517917366412323387448053600963231} k_{22} \\
 & + \frac{288008947013086673598382477846067281621680128}{630732925763360204036824696812522003} k_{23} \\
 & - \frac{345044307361772458647558631531936780201754624}{630732925763360204036824696812522003} k_{24} \\
 & + \frac{407938190019062363129678334131770787664232448}{630732925763360204036824696812522003} k_{25} \\
 & - \frac{47699369197824943975855477146373498932297728}{630732925763360204036824696812522003} k_{26} \\
 & + \frac{552507512160275363549537826612488007195295744}{630732925763360204036824696812522003} k_{27} \\
 & - \frac{634763553341379054121040897638735202174894080}{630732925763360204036824696812522003} k_{28} \\
 & + \frac{724329621074038350709895720442356863314427904}{630732925763360204036824696812522003} k_{29} \\
 & - \frac{828515741909808859333454046800727456773832704}{630732925763360204036824696812522003} k_{30} \\
 & + \frac{1036887983581349876580570699517468643692642304}{630732925763360204036824696812522003} k_{31} \\
 & - \frac{2073775967162699753161141399034937287385284608}{630732925763360204036824696812522003} k_{32},
 \end{aligned}$$

Continued

$$\begin{aligned}
 r_{25} := & \frac{30369535433400425692686935693374976}{523140386311882945012572322487577} k_0 - \frac{30548727162750800714204873557216492}{523140386311882945012572322487577} k_1 \\
 + & \frac{2341156218945525925121338143139748}{40241568177837149616351717114429} k_2 - \frac{30024872588068556611637930492565772}{523140386311882945012572322487577} k_3 \\
 + & \frac{29232599827905537079386264036365336}{523140386311882945012572322487577} k_4 - \frac{27842804442099501942911262650939152}{523140386311882945012572322487577} k_5 \\
 + & \frac{25459204842448720792168607543575520}{523140386311882945012572322487577} k_6 - \frac{21407452619120992442617832457534592}{523140386311882945012572322487577} k_7 \\
 + & \frac{14567206920061526753831976512856320}{523140386311882945012572322487577} k_8 - \frac{3099338359407216404603652645626368}{523140386311882945012572322487577} k_9 \\
 - & \frac{15984819201215476418236924769695744}{523140386311882945012572322487577} k_{10} + \frac{3653167927184696226890820477117440}{40241568177837149616351717114429} k_{11} \\
 - & \frac{99060197958369474007053504906299392}{523140386311882945012572322487577} k_{12} + \frac{182687880879359203066337060250460160}{523140386311882945012572322487577} k_{13} \\
 - & \frac{316952615901078579346220447703138304}{523140386311882945012572322487577} k_{14} \\
 + & \frac{530197867579982217365618357622456320}{523140386311882945012572322487577} k_{15} \\
 - & \frac{864958160190631396682799005983473664}{523140386311882945012572322487577} k_{16} \\
 + & \frac{1383923756297083969736486490216366080}{523140386311882945012572322487577} k_{17} \\
 - & \frac{2177695291120799781815050956642254848}{523140386311882945012572322487577} k_{18} \\
 + & \frac{3374461682473263310402763715600416768}{523140386311882945012572322487577} k_{19} \\
 - & \frac{5151536628577199013849272625507991552}{523140386311882945012572322487577} k_{20} \\
 + & \frac{7748434988578592750116470679442849792}{523140386311882945012572322487577} k_{21} \\
 - & \frac{883146802636934355272972131593551872}{40241568177837149616351717114429} k_{22} \\
 + & \frac{16755082077855360812570820857829883904}{523140386311882945012572322487577} k_{23} \\
 - & \frac{24080278412759568541138590877071179776}{523140386311882945012572322487577} k_{24} \\
 + & \frac{34077701242270576134585448479248777216}{523140386311882945012572322487577} k_{25} \\
 - & \frac{47480839583288513519265530714712506368}{523140386311882945012572322487577} k_{26} \\
 + & \frac{65136275473935535166839023743392514048}{523140386311882945012572322487577} k_{27} \\
 - & \frac{88116776994777844639578211944379580416}{523140386311882945012572322487577} k_{28} \\
 + & \frac{118567880306947849062027664075855265792}{523140386311882945012572322487577} k_{29} \\
 - & \frac{163960187201773243383897644068308189184}{523140386311882945012572322487577} k_{30} \\
 + & \frac{254744800991424032027637604053214035968}{523140386311882945012572322487577} k_{31} \\
 - & \frac{509489601982848064055275208106428071936}{523140386311882945012572322487577} k_{32},
 \end{aligned}$$

Continued

$$\begin{aligned}
r_{26} := & -\frac{532576647580652673203087099587540161536}{630732925763360204036824696812522003} k_0 - \frac{17892618423041325884716514266653926008}{7786826243992101284405243170524963} k_1 \\
& + \frac{44505801013510081263317990655872221688}{16172639122137441129149351200321077} k_2 - \frac{1672658978849833322237280836145112668200}{630732925763360204036824696812522003} k_3 \\
& + \frac{516858628044770681060100011005773225232}{210244308587786734678941565604174001} k_4 - \frac{473827854412874945611124169162681013808}{210244308587786734678941565604174001} k_5 \\
& + \frac{1292138011589718117529956743103176106592}{630732925763360204036824696812522003} k_6 - \frac{387633468742758261653150900325437399296}{210244308587786734678941565604174001} k_7 \\
& + \frac{114853965527070992712320085170737263616}{70081436195928911559647188534724667} k_8 - \frac{904474643717864850160291642596684798464}{630732925763360204036824696812522003} k_9 \\
& + \frac{86140436151357529754116173259486728448}{70081436195928911559647188534724667} k_{10} - \frac{16565465879121948241030372065412737280}{16172639122137441129149351200321077} k_{11} \\
& + \frac{516842352657515900808283672661766988288}{630732925763360204036824696812522003} k_{12} - \frac{12921047998595956144078257625580354048}{210244308587786734678941565604174001} k_{13} \\
& + \frac{86140125592125736075381144567574115328}{210244308587786734678941565604174001} k_{14} \\
& - \frac{129209081193113319790421238556112648192}{630732925763360204036824696812522003} k_{15} \\
& - \frac{855922860290834750886914685644800}{210244308587786734678941565604174001} k_{16} \\
& + \frac{4785731294025498979112326425977307136}{23360478731976303853215729511574889} k_{17} \\
& - \frac{258427696931956483440618642038062481408}{630732925763360204036824696812522003} k_{18} \\
& + \frac{43071306930394493399559667223004053504}{70081436195928911559647188534724667} k_{19} \\
& - \frac{172285797542331673982701672178729353216}{210244308587786734678941565604174001} k_{20} \\
& + \frac{646075170355437997575776455063803658240}{630732925763360204036824696812522003} k_{21} \\
& - \frac{19879380078007994839052436513703067648}{16172639122137441129149351200321077} k_{22} \\
& + \frac{301506743245555160679099524221938171904}{210244308587786734678941565604174001} k_{23} \\
& - \frac{1033749421566836849380460077081460473856}{630732925763360204036824696812522003} k_{24} \\
& + \frac{387661525703268081381862327465684238336}{210244308587786734678941565604174001} k_{25} \\
& - \frac{143580778607988633398265251857243832320}{70081436195928911559647188534724667} k_{26} \\
& + \frac{1421478126450221242462251569114767163392}{630732925763360204036824696812522003} k_{27} \\
& - \frac{57434798267534328613775401875081789440}{23360478731976303853215729511574889} k_{28} \\
& + \frac{560004842049606198832984011105131036672}{210244308587786734678941565604174001} k_{29} \\
& - \frac{1809316591378582508060632018355065192448}{630732925763360204036824696812522003} k_{30} \\
& + \frac{689306907279370110394663996144803119104}{210244308587786734678941565604174001} k_{31} \\
& - \frac{1378613814558740220789327992289606238208}{210244308587786734678941565604174001} k_{32}
\end{aligned}$$

Continued

$$\begin{aligned}
 r_{27} := & -\frac{244068804056062971184384802624086696960}{1892198777290080612110474090437566009} k_0 + \frac{245521992973420144792619170884545569064}{1892198777290080612110474090437566009} k_1 \\
 & - \frac{18818002780861340372080281508236661624}{145553752099236970162344160802889693} k_2 + \frac{241386155217670164379934096448071769272}{1892198777290080612110474090437566009} k_3 \\
 & - \frac{235109091584591796519536925401669864464}{1892198777290080612110474090437566009} k_4 + \frac{224107060741608931164832553996481610160}{1892198777290080612110474090437566009} k_5 \\
 & - \frac{205260277431798626342847967011474345760}{1892198777290080612110474090437566009} k_6 + \frac{173268106619364970686911686796305998080}{1892198777290080612110474090437566009} k_7 \\
 & - \frac{119343228131200414742433269507040572416}{1892198777290080612110474090437566009} k_8 + \frac{29097575415579745749550262518087516160}{1892198777290080612110474090437566009} k_9 \\
 & + \frac{120780124052006402899134663315586976000}{1892198777290080612110474090437566009} k_{10} - \frac{28280346398185899917770346094335946496}{145553752099236970162344160802889693} k_{11} \\
 & + \frac{770638150237652519851251085624617302528}{1892198777290080612110474090437566009} k_{12} - \frac{1422168184083610334083018337916074206720}{1892198777290080612110474090437566009} k_{13} \\
 & + \frac{2464519536666052652128858971369315666944}{1892198777290080612110474090437566009} k_{14} - \frac{4113260258774340391564396738165787318272}{1892198777290080612110474090437566009} k_{15} \\
 & + \frac{6689185183531792382450120946849544318976}{1892198777290080612110474090437566009} k_{16} - \frac{10660308483866487668880657477762188591104}{1892198777290080612110474090437566009} k_{17} \\
 & + \frac{16694655874267023044770741038570563403776}{1892198777290080612110474090437566009} k_{18} - \frac{25723177991669238020353294889882271219712}{1892198777290080612110474090437566009} k_{19} \\
 & + \frac{39010077435528022426519049144709607325696}{1892198777290080612110474090437566009} k_{20} - \frac{58225712707746316710402738568300498911232}{1892198777290080612110474090437566009} k_{21} \\
 & + \frac{6578154782971990678785047592218031030272}{145553752099236970162344160802889693} k_{22} - \frac{123562899340355793462446913145903217901568}{1892198777290080612110474090437566009} k_{23} \\
 & + \frac{175631366879817503109242552996541448060928}{1892198777290080612110474090437566009} k_{24} - \frac{245594474635033919818807930188334132363264}{1892198777290080612110474090437566009} k_{25} \\
 & + \frac{337914195318356698462408678816680277704704}{1892198777290080612110474090437566009} k_{26} - \frac{457603105962950783222262237749319730462720}{1892198777290080612110474090437566009} k_{27} \\
 & + \frac{610774992243607696902489655398674015977472}{1892198777290080612110474090437566009} k_{28} - \frac{809657425509910485111333028589586184929280}{1892198777290080612110474090437566009} k_{29} \\
 & + \frac{1099960952747505022377408312863614120755200}{1892198777290080612110474090437566009} k_{30} - \frac{1680568007222694096909558881411669992407040}{1892198777290080612110474090437566009} k_{31} \\
 & + \frac{3361136014445388193819117762823339984814080}{1892198777290080612110474090437566009} k_{32}.
 \end{aligned}$$

$$\eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z) = \delta(b_1) + \sum_{n=1}^{\infty} c(n)q^n,$$

where for $n \in \mathbb{N}$,

$$\begin{aligned}
 c(n) = & -c_1\sigma_{15}(n) - c_2\sigma_{15}\left(\frac{n}{2}\right) - c_3\sigma_{15}\left(\frac{n}{3}\right) - c_4\sigma_{15}\left(\frac{n}{4}\right) - c_6\sigma_{15}\left(\frac{n}{6}\right) \\
 & - c_{12}\sigma_{15}\left(\frac{n}{12}\right) + r_1f_1(n) + \dots + r_{27}f_{27}(n).
 \end{aligned}$$

In particular,

$$\begin{aligned}
 c(2n) = & -c_1\sigma_{15}(2n) - c_2\sigma_{15}(n) - c_4\sigma_{15}\left(\frac{n}{2}\right) - (16385c_3 + c_6)\sigma_{15}\left(\frac{n}{3}\right) \\
 & - (c_{12} - 16384c_3)\sigma_{15}\left(\frac{n}{6}\right) + r_1f_1(2n) + \dots + r_{14}f_{14}(2n),
 \end{aligned}$$

$$c(2n-1) = -c_1\sigma_{15}(2n-1) - c_3\sigma_{15}\left(\frac{2n-1}{3}\right) + r_{15}f_{15}(2n-1) + \dots + r_{27}f_{27}(2n-1),$$

for $n \in \mathbb{N}$.

Proof. It follows from (6)-(11) that

$$a_1 + 2a_2 + 3a_3 + 4a_4 + 6a_6 + 12a_{12} = 24b_1, \tag{17}$$

$$a_1 + a_2 + a_3 + a_4 + a_6 + a_{12} = 32, \tag{18}$$

$$-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - \frac{2a_4}{3} - \frac{a_6}{3} - \frac{2a_{12}}{3} = -b_1 - b_5.$$

Now we will use p-k parametrization of Alaca, Alaca and Williams, see [15]:

$$p(q) := \frac{\varphi^2(q) - \varphi^2(q^3)}{2\varphi^2(q^3)}, \quad k(q) := \frac{\varphi^3(q^3)}{\varphi(q)}, \tag{19}$$

where the theta function $\varphi(q)$ is defined by

$$\varphi(q) = \sum_{n=-\infty}^{\infty} q^{n^2}.$$

Setting $x = p$ in (12), and multiplying both sides by k^{16} we obtain

$$\begin{aligned} & \frac{k^{16}}{2^{b_1+b_5}} p^{b_1} (1-p)^{b_2} (1+p)^{b_3} (1+2p)^{b_4} (2+p)^{b_5} \\ &= (k_0 + k_1p + k_2p^2 + k_3p^3 + k_4p^4 + k_5p^5 + k_6p^6 + k_7p^7 + k_8p^8 + k_9p^9 + k_{10}p^{10} \\ & \quad + k_{11}p^{11} + k_{12}p^{12} + k_{13}p^{13} + k_{14}p^{14} + k_{15}p^{15} + k_{16}p^{16} + k_{17}p^{17} + k_{18}p^{18} + k_{19}p^{19} \\ & \quad + k_{20}p^{20} + k_{21}p^{21} + k_{22}p^{22} + k_{23}p^{23} + k_{24}p^{24} + k_{25}p^{25} + k_{26}p^{26} + k_{27}p^{27} + k_{28}p^{28} \\ & \quad + k_{29}p^{29} + k_{30}p^{30} + k_{31}p^{31} + k_{32}p^{32})k^{16}. \end{aligned}$$

Alaca, Alaca and Williams [16] have established the following representations in terms of p and k :

$$\eta(q) = 2^{-1/6} p^{1/24} (1-p)^{1/2} (1+p)^{1/6} (1+2p)^{1/8} (2+p)^{1/8} k^{1/2}, \tag{20}$$

$$\eta(q^2) = 2^{-1/3} p^{1/12} (1-p)^{1/4} (1+p)^{1/12} (1+2p)^{1/4} (2+p)^{1/4} k^{1/2}, \tag{21}$$

$$\eta(q^3) = 2^{-1/6} p^{1/8} (1-p)^{1/6} (1+p)^{1/2} (1+2p)^{1/24} (2+p)^{1/24} k^{1/2}, \tag{22}$$

$$\eta(q^4) = 2^{-2/3} p^{1/6} (1-p)^{1/8} (1+p)^{1/24} (1+2p)^{1/8} (2+p)^{1/2} k^{1/2}, \tag{23}$$

$$\eta(q^6) = 2^{-1/3} p^{1/4} (1-p)^{1/12} (1+p)^{1/4} (1+2p)^{1/12} (2+p)^{1/12} k^{1/2}, \tag{24}$$

$$\eta(q^{12}) = 2^{-2/3} p^{1/2} (1-p)^{1/24} (1+p)^{1/8} (1+2p)^{1/24} (2+p)^{1/6} k^{1/2}, \tag{25}$$

$$\begin{aligned} E_6(q) &:= 1 - 504 \sum_{n=1}^{\infty} \sigma_5(n) q^n \\ &= (1 - 246p - 5532p^2 - 38614p^3 - 135369p^4 - 276084p^5 - 348024p^6 \\ & \quad - 276084p^7 - 135369p^8 - 38614p^9 - 5532p^{10} - 246p^{11} + p^{12})k^6, \end{aligned}$$

$$\begin{aligned} E_4(q) &:= 1 + 240 \sum_{n=1}^{\infty} \sigma_3(n) q^n \\ &= (1 + 124p + 964p^2 + 2788p^3 + 3910p^4 + 2788p^5 + 964p^6 + 124p^7 + p^8)k^4. \end{aligned}$$

Therefore, since

$$E_{16}(q) = \frac{2000}{3617} E_6^2(q) E_4(q) + \frac{1617}{3617} E_4^4(q)$$

we immediately obtain:

$$E_{16}(q) = \left(p^{32} + \frac{66032}{3617} p^{31} + \frac{134229104}{3617} p^{30} + \frac{31279681360}{3617} p^{29} + \frac{1532766571768}{3617} p^{28} \right. \\ + \frac{33608470215088}{3617} p^{27} + \frac{431880430212432}{3617} p^{26} + \frac{3705935707430928}{3617} p^{25} \\ + \frac{22884510224189340}{3617} p^{24} + \frac{106630640734839600}{3617} p^{23} + \frac{387088285203107184}{3617} p^{22} \\ + \frac{1119620321590028304}{3617} p^{21} + \frac{2622064678835063496}{3617} p^{20} + \frac{5029685780950817520}{3617} p^{19} \\ + \frac{7966990925710242000}{3617} p^{18} + \frac{10477360107681574608}{3617} p^{17} + \frac{11475149226459112710}{3617} p^{16} \\ + \frac{10477360107681574608}{3617} p^{15} + \frac{7966990925710242000}{3617} p^{14} + \frac{5029685780950817520}{3617} p^{13} \\ + \frac{2622064678835063496}{3617} p^{12} + \frac{1119620321590028304}{3617} p^{11} + \frac{387088285203107184}{3617} p^{10} \\ + \frac{106630640734839600}{3617} p^9 + \frac{22884510224189340}{3617} p^8 + \frac{3705935707430928}{3617} p^7 \\ + \frac{431880430212432}{3617} p^6 + \frac{33608470215088}{3617} p^5 + \frac{1532766571768}{3617} p^4 \\ \left. + \frac{31279681360}{3617} p^3 + \frac{134229104}{3617} p^2 + \frac{66032}{3617} p + 1 \right) k^{16},$$

$$E_{16}(q^2) = \left(p^{32} + 16p^{31} + \frac{409184}{3617} p^{30} + \frac{1652680}{3617} p^{29} + \frac{37454668}{3617} p^{28} \right. \\ + \frac{473232088}{3617} p^{27} + \frac{6483541032}{3617} p^{26} + \frac{56895857928}{3617} p^{25} \\ + \frac{349784703630}{3617} p^{24} + \frac{1625990384280}{3617} p^{23} + \frac{5904566270904}{3617} p^{22} \\ + \frac{17085449814024}{3617} p^{21} + \frac{40012993999716}{3617} p^{20} + \frac{76746190229880}{3617} p^{19} \\ + \frac{121562792943720}{3617} p^{18} + \frac{159871907133288}{3617} p^{17} + \frac{175100498074755}{3617} p^{16} \\ + \frac{159871907133288}{3617} p^{15} + \frac{121562792943720}{3617} p^{14} + \frac{76746190229880}{3617} p^{13} \\ + \frac{40012993999716}{3617} p^{12} + \frac{17085449814024}{3617} p^{11} + \frac{5904566270904}{3617} p^{10} \\ + \frac{1625990384280}{3617} p^9 + \frac{349784703630}{3617} p^8 + \frac{56895857928}{3617} p^7 \\ + \frac{6483541032}{3617} p^6 + \frac{473232088}{3617} p^5 + \frac{37454668}{3617} p^4 + \frac{1652680}{3617} p^3 \\ \left. + \frac{409184}{3617} p^2 + 16p + 1 \right) k^{16},$$

$$\begin{aligned}
 E_{16}(q^3) = & \left(p^{32} + 16p^{31} + 112p^{30} + \frac{1593520}{3617}p^{29} + \frac{3661048}{3617}p^{28} + \frac{4034128}{3617}p^{27} \right. \\
 & + \frac{6253392}{3617}p^{26} + \frac{94495728}{3617}p^{25} + \frac{584550300}{3617}p^{24} + \frac{2266437840}{3617}p^{23} \\
 & + \frac{8171962224}{3617}p^{22} + \frac{26676060144}{3617}p^{21} + \frac{64864877256}{3617}p^{20} \\
 & + \frac{116064498960}{3617}p^{19} + \frac{175509366480}{3617}p^{18} + \frac{243644509488}{3617}p^{17} \\
 & + \frac{279458260230}{3617}p^{16} + \frac{243644509488}{3617}p^{15} + \frac{175509366480}{3617}p^{14} \\
 & + \frac{116064498960}{3617}p^{13} + \frac{64864877256}{3617}p^{12} + \frac{26676060144}{3617}p^{11} \\
 & + \frac{8171962224}{3617}p^{10} + \frac{2266437840}{3617}p^9 + \frac{584550300}{3617}p^8 + \frac{94495728}{3617}p^7 \\
 & \left. + \frac{6253392}{3617}p^6 + \frac{4034128}{3617}p^5 + \frac{3661048}{3617}p^4 + \frac{1593520}{3617}p^3 + 112p^2 + 16p + 1 \right) k^{16}
 \end{aligned}$$

$$\begin{aligned}
 E_{16}(q^4) = & \left(\frac{1}{65536}p^{32} + \frac{3107}{14815232}p^{31} + \frac{8373509}{14815232}p^{30} - \frac{426024575}{3703808}p^{29} + \frac{21371210189}{7407616}p^{28} \right. \\
 & - \frac{2780210933}{231488}p^{27} - \frac{26515040733}{925952}p^{26} + \frac{46668005109}{462976}p^{25} + \frac{318947423355}{1851904}p^{24} \\
 & - \frac{132759215175}{462976}p^{23} - \frac{243776400843}{462976}p^{22} + \frac{49438651083}{115744}p^{21} + \frac{265269995439}{231488}p^{20} \\
 & + \frac{31565989785}{115744}p^{19} - \frac{6536535195}{14468}p^{18} + \frac{20783548953}{57872}p^{17} + \frac{573453480645}{462976}p^{16} \\
 & + \frac{58547998863}{57872}p^{15} + \frac{24046100565}{57872}p^{14} + \frac{2526073485}{14468}p^{13} + \frac{4307305053}{28936}p^{12} \\
 & + \frac{751890993}{7234}p^{11} + \frac{152474619}{3617}p^{10} + \frac{22732365}{3617}p^9 - \frac{69921135}{14468}p^8 - \frac{15090777}{3617}p^7 \\
 & \left. - \frac{2659113}{3617}p^6 + \frac{3862768}{3617}p^5 + \frac{3632488}{3617}p^4 + 440p^3 + 112p^2 + 16p + 1 \right) k^{16}
 \end{aligned}$$

$$\begin{aligned}
 E_{16}(q^6) = & \left(p^{32} + 16p^{31} + 112p^{30} + 440p^{29} + 1004p^{28} + 1064p^{27} - \frac{2745048}{3617}p^{26} \right. \\
 & - \frac{15372552}{3617}p^{25} - \frac{20069490}{3617}p^{24} - \frac{2603160}{3617}p^{23} + \frac{25848504}{3617}p^{22} \\
 & + \frac{33201144}{3617}p^{21} + \frac{8374116}{3617}p^{20} - \frac{19626360}{3617}p^{19} - \frac{18930840}{3617}p^{18} \\
 & + \frac{4834968}{3617}p^{17} + \frac{18832515}{3617}p^{16} + \frac{4834968}{3617}p^{15} - \frac{18930840}{3617}p^{14} \\
 & - \frac{19626360}{3617}p^{13} + \frac{8374116}{3617}p^{12} + \frac{33201144}{3617}p^{11} + \frac{25848504}{3617}p^{10} \\
 & - \frac{2603160}{3617}p^9 - \frac{20069490}{3617}p^8 - \frac{15372552}{3617}p^7 - \frac{2745048}{3617}p^6 \\
 & \left. + 1064p^5 + 1004p^4 + 440p^3 + 112p^2 + 16p + 1 \right) k^{16}
 \end{aligned}$$

$$\begin{aligned}
 E_{16}(q^{12}) = & \left(\frac{1}{65536} p^{32} + \frac{1}{4096} p^{31} + \frac{7}{4096} p^{30} + \frac{24835}{3703808} p^{29} \right. \\
 & + \frac{112559}{7407616} p^{28} + \frac{3577}{231488} p^{27} + \frac{19407}{925952} p^{26} + \frac{179709}{462976} p^{25} \\
 & + \frac{4552575}{1851904} p^{24} + \frac{2646075}{462976} p^{23} - \frac{4566453}{462976} p^{22} - \frac{11870997}{115744} p^{21} \\
 & - \frac{64064691}{231488} p^{20} - \frac{24289965}{115744} p^{19} + \frac{40724415}{57872} p^{18} \\
 & + \frac{127224993}{57872} p^{17} + \frac{932409285}{462976} p^{16} - \frac{109346667}{57872} p^{15} \\
 & - \frac{386476065}{57872} p^{14} - \frac{82555605}{14468} p^{13} + \frac{67084983}{28936} p^{12} \\
 & + \frac{18339}{2} p^{11} + 7122 p^{10} - 735 p^9 - \frac{22215}{4} p^8 - 4251 p^7 \\
 & \left. - 759 p^6 + 1064 p^5 + 1004 p^4 + 440 p^3 + 112 p^2 + 16 p + 1 \right) k^{16}.
 \end{aligned}$$

It is easy to check the following expressions by (20)-(25)

$$\begin{aligned}
 f_1 := \sum_{n=0}^{\infty} f_1(n) &= \frac{\eta^{18}(4z)\eta^{16}(6z)\eta^{10}(12z)}{\eta^{12}(2z)} \\
 &= \left(-\frac{1}{524288} p^{27} - \frac{45}{1048576} p^{26} - \frac{29}{65536} p^{25} \right. \\
 &\quad - \frac{2897}{1048576} p^{24} - \frac{761}{65536} p^{23} - \frac{36143}{1048576} p^{22} \\
 &\quad - \frac{4809}{65536} p^{21} - \frac{115427}{1048576} p^{20} - \frac{55863}{524288} p^{19} \\
 &\quad - \frac{2653}{65536} p^{18} + \frac{1845}{32768} p^{17} + \frac{3859}{32768} p^{16} + \frac{1823}{16384} p^{15} \\
 &\quad \left. + \frac{133}{2048} p^{14} + \frac{49}{2048} p^{13} + \frac{21}{4096} p^{12} + \frac{1}{2048} p^{11} \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_2 = \sum_{n=0}^{\infty} f_2(n) &= \frac{\eta^{20}(4z)\eta^2(6z)\eta^{20}(12z)}{\eta^{10}(2z)} \\
 &= \left(-\frac{1}{8388608} p^{29} - \frac{49}{16777216} p^{28} - \frac{69}{2097152} p^{27} \right. \\
 &\quad - \frac{945}{4194304} p^{26} - \frac{4377}{4194304} p^{25} - \frac{57639}{16777216} p^{24} \\
 &\quad - \frac{68717}{8388608} p^{23} - \frac{58817}{4194304} p^{22} - \frac{34155}{2097152} p^{21} \\
 &\quad - \frac{5291}{524288} p^{20} + \frac{847}{262144} p^{19} + \frac{1959}{131072} p^{18} + \frac{1137}{65536} p^{17} \\
 &\quad \left. + \frac{765}{65536} p^{16} + \frac{159}{32768} p^{15} + \frac{19}{16384} p^{14} + \frac{1}{8192} p^{13} \right) k^{16},
 \end{aligned}$$

$$f_3 = \sum_{n=0}^{\infty} f_3(n) = \frac{\eta^{15}(4z)\eta^7(6z)\eta^{19}(12z)}{\eta^9(2z)}$$

$$= \left(-\frac{1}{2097152}p^{28} - \frac{43}{4194304}p^{27} - \frac{421}{4194304}p^{26} - \frac{619}{1048576}p^{25} - \frac{2425}{1048576}p^{24} \right. \\ \left. - \frac{26443}{4194304}p^{23} - \frac{50501}{4194304}p^{22} - \frac{32463}{2097152}p^{21} - \frac{2925}{262144}p^{20} + \frac{17}{16384}p^{19} \right. \\ \left. + \frac{1709}{131072}p^{18} + \frac{1075}{65536}p^{17} + \frac{187}{16384}p^{16} + \frac{79}{16384}p^{15} + \frac{19}{16384}p^{14} + \frac{1}{8192}p^{13} \right) k^{16},$$

$$f_4 = \sum_{n=0}^{\infty} f_4(n) = \frac{\eta^{18}(12z)\eta^{10}(4z)\eta^{12}(6z)}{\eta^8(2z)} = \left(-\frac{1}{524288}p^{27} - \frac{37}{1048576}p^{26} \right. \\ \left. - \frac{77}{262144}p^{25} - \frac{1517}{1048576}p^{24} - \frac{1219}{262144}p^{23} - \frac{10571}{1048576}p^{22} - \frac{3789}{262144}p^{21} \right. \\ \left. - \frac{12519}{1048576}p^{20} - \frac{513}{524288}p^{19} + \frac{2933}{262144}p^{18} + \frac{2027}{131072}p^{17} \right. \\ \left. + \frac{731}{65536}p^{16} + \frac{157}{32768}p^{15} + \frac{19}{16384}p^{14} + \frac{1}{8192}p^{13} \right) k^{16},$$

$$f_5 = \sum_{n=0}^{\infty} f_5(n) = \frac{\eta^5(4z)\eta^{17}(6z)\eta^{17}(12z)}{\eta^7(2z)} = \left(-\frac{1}{131072}p^{26} - \frac{31}{262144}p^{25} - \frac{213}{262144}p^{24} \right. \\ \left. - \frac{849}{262144}p^{23} - \frac{2145}{262144}p^{22} - \frac{3471}{262144}p^{21} - \frac{3263}{262144}p^{20} - \frac{739}{262144}p^{19} + \frac{2463}{262144}p^{18} \right. \\ \left. + \frac{1905}{131072}p^{17} + \frac{357}{32768}p^{16} + \frac{39}{8192}p^{15} + \frac{19}{16384}p^{14} + \frac{1}{8192}p^{13} \right) k^{16},$$

$$f_6 = \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{17}(4z)\eta^{17}(6z)\eta^5(12z)}{\eta^7(2z)}$$

$$= \left(\frac{1}{65536}p^{27} + \frac{11}{32768}p^{26} + \frac{881}{262144}p^{25} + \frac{5285}{262144}p^{24} + \frac{20991}{262144}p^{23} \right. \\ \left. + \frac{57213}{262144}p^{22} + \frac{105569}{262144}p^{21} + \frac{117767}{262144}p^{20} + \frac{31081}{262144}p^{19} - \frac{142257}{262144}p^{18} \right. \\ \left. - \frac{136127}{131072}p^{17} - \frac{14399}{16384}p^{16} - \frac{397}{2048}p^{15} + \frac{3249}{8192}p^{14} \right. \\ \left. + \frac{2103}{4096}p^{13} + \frac{161}{512}p^{12} + \frac{115}{1024}p^{11} + \frac{23}{1024}p^{10} + \frac{1}{512}p^9 \right) k^{16},$$

$$f_7 = \sum_{n=0}^{\infty} f_7(n) = \frac{\eta^{19}(4z)\eta^{15}(6z)\eta^{15}(12z)}{\eta^{17}(2z)}$$

$$= \left(\frac{1}{4194304}p^{27} + \frac{23}{4194304}p^{26} + \frac{61}{1048576}p^{25} + \frac{791}{2097152}p^{24} + \frac{7001}{4194304}p^{23} \right. \\ \left. + \frac{22363}{4194304}p^{22} + \frac{26577}{2097152}p^{21} + \frac{5967}{262144}p^{20} + \frac{1017}{32768}p^{19} + \frac{4189}{131072}p^{18} \right. \\ \left. + \frac{1603}{65536}p^{17} + \frac{221}{16384}p^{16} + \frac{83}{16384}p^{15} + \frac{19}{16384}p^{14} + \frac{1}{8192}p^{13} \right) k^{16},$$

$$\begin{aligned}
 f_8 &= \sum_{n=0}^{\infty} f_8(n) = \frac{\eta^{17}(2z)\eta^{17}(4z)\eta^5(12z)}{\eta^7(6z)} \\
 &= \left(\frac{1}{4096} p^{31} + \frac{3}{512} p^{30} + \frac{1031}{16384} p^{29} + \frac{6445}{16384} p^{28} + \frac{101019}{65536} p^{27} \right. \\
 &\quad + \frac{120001}{32768} p^{26} + \frac{984145}{262144} p^{25} - \frac{1537659}{262144} p^{24} - \frac{7102105}{262144} p^{23} \\
 &\quad - \frac{9302339}{262144} p^{22} + \frac{2219505}{262144} p^{21} + \frac{23214863}{262144} p^{20} + \frac{26270329}{262144} p^{19} \\
 &\quad - \frac{5377425}{262144} p^{18} - \frac{20003423}{131072} p^{17} - \frac{3918925}{32768} p^{16} + \frac{346941}{8192} p^{15} \\
 &\quad + \frac{2088461}{16384} p^{14} + \frac{512615}{8192} p^{13} - \frac{7401}{256} p^{12} - \frac{47239}{1024} p^{11} \\
 &\quad \left. - \frac{16583}{1024} p^{10} + \frac{2319}{512} p^9 + \frac{775}{128} p^8 + \frac{37}{16} p^7 + \frac{27}{64} p^6 + \frac{1}{32} p^5 \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_9 &= \sum_{n=0}^{\infty} f_9(n) = \frac{\eta^{20}(2z)\eta^{14}(4z)\eta^{14}(12z)}{\eta^{16}(6z)} \\
 &= \left(\frac{1}{16384} p^{32} + \frac{23}{16384} p^{31} + \frac{939}{65536} p^{30} + \frac{2753}{32768} p^{29} + \frac{78995}{262144} p^{28} \right. \\
 &\quad + \frac{161007}{262144} p^{27} + \frac{340117}{1048576} p^{26} - \frac{117361}{65536} p^{25} - \frac{5224329}{1048576} p^{24} \\
 &\quad - \frac{2039005}{524288} p^{23} + \frac{6297515}{1048576} p^{22} + \frac{4229337}{262144} p^{21} + \frac{9352981}{1048576} p^{20} \\
 &\quad - \frac{7365203}{524288} p^{19} - \frac{3159555}{131072} p^{18} - \frac{379685}{65536} p^{17} + \frac{1073567}{65536} p^{16} \\
 &\quad + \frac{507447}{32768} p^{15} + \frac{323}{2048} p^{14} - \frac{15125}{2048} p^{13} - \frac{16989}{4096} p^{12} \\
 &\quad \left. + \frac{203}{2048} p^{11} + \frac{529}{512} p^{10} + \frac{123}{256} p^9 + \frac{25}{256} p^8 + \frac{1}{128} p^7 \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{10} &= \sum_{n=0}^{\infty} f_{10}(n) = \frac{\eta^{16}(4z)\eta^{18}(6z)}{\eta^2(2z)} \\
 &= \left(-\frac{1}{8192} p^{27} - \frac{43}{16384} p^{26} - \frac{835}{32768} p^{25} - \frac{9601}{65536} p^{24} \right. \\
 &\quad - \frac{4477}{8192} p^{23} - \frac{44075}{32768} p^{22} - \frac{66467}{32768} p^{21} - \frac{4547}{4096} p^{20} \\
 &\quad + \frac{80587}{32768} p^{19} + \frac{216681}{32768} p^{18} + \frac{108333}{16384} p^{17} + \frac{46257}{65536} p^{16} \\
 &\quad - \frac{200503}{32768} p^{15} - \frac{30571}{4096} p^{14} - \frac{6751}{2048} p^{13} + \frac{2303}{2048} p^{12} \\
 &\quad \left. + \frac{2471}{1024} p^{11} + \frac{391}{256} p^{10} + \frac{67}{128} p^9 + \frac{25}{256} p^8 + \frac{1}{128} p^7 \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{11} &= \sum_{n=0}^{\infty} f_{11}(n) = \frac{\eta^{20}(2z)\eta^{14}(4z)\eta^8(6z)}{\eta^{10}(12z)} \\
 &= \left(-\frac{1}{32}p^{29} - \frac{43}{64}p^{28} - \frac{813}{128}p^{27} - \frac{8683}{256}p^{26} - \frac{54571}{512}p^{25} \right. \\
 &\quad - \frac{169029}{1024}p^{24} + \frac{166177}{2048}p^{23} + \frac{3773287}{4096}p^{22} + \frac{1665717}{1024}p^{21} \\
 &\quad + \frac{541157}{4096}p^{20} - \frac{8039515}{2048}p^{19} - \frac{11541531}{2048}p^{18} + \frac{52297}{128}p^{17} \\
 &\quad + \frac{19076567}{2048}p^{16} + \frac{17486919}{2048}p^{15} - \frac{13136125}{4096}p^{14} \\
 &\quad - \frac{11342417}{1024}p^{13} - \frac{22863279}{4096}p^{12} + \frac{8459683}{2048}p^{11} \\
 &\quad + \frac{6273043}{1024}p^{10} + \frac{828501}{512}p^9 - \frac{220715}{128}p^8 - \frac{98825}{64}p^7 \\
 &\quad \left. - \frac{10071}{32}p^6 + \frac{3281}{16}p^5 + \frac{2581}{16}p^4 + \frac{399}{8}p^3 + \frac{31}{4}p^2 + \frac{1}{2}p \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{12} &= \sum_{n=0}^{\infty} f_{12}(n) = \eta^{20}(2z)\eta^2(4z)\eta^8(6z)\eta^2(12z) \\
 &= \left(\frac{1}{64}p^{28} + \frac{15}{64}p^{27} + \frac{371}{256}p^{26} + \frac{569}{128}p^{25} + \frac{5211}{1024}p^{24} - \frac{9241}{1024}p^{23} \right. \\
 &\quad - \frac{153891}{4096}p^{22} - \frac{8919}{256}p^{21} + \frac{179109}{4096}p^{20} + \frac{251003}{2048}p^{19} + \frac{113727}{2048}p^{18} \\
 &\quad - \frac{30079}{256}p^{17} - \frac{329953}{2048}p^{16} - \frac{2349}{256}p^{15} + \frac{497249}{4096}p^{14} + \frac{42867}{512}p^{13} \\
 &\quad - \frac{63711}{4096}p^{12} - \frac{93897}{2048}p^{11} - \frac{19619}{1024}p^{10} + \frac{1587}{512}p^9 + \frac{1475}{256}p^8 \\
 &\quad \left. + \frac{293}{128}p^7 + \frac{27}{64}p^6 + \frac{1}{32}p^5 \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{14} &= \sum_{n=0}^{\infty} f_{14}(n) = \frac{\eta^{16}(2z)\eta^{10}(4z)\eta^{18}(12z)}{\eta^{12}(6z)} \\
 &= \left(-\frac{1}{32768}p^{31} - \frac{41}{65536}p^{30} - \frac{369}{65536}p^{29} - \frac{3755}{131072}p^{28} \right. \\
 &\quad - \frac{45365}{524288}p^{27} - \frac{142317}{1048576}p^{26} + \frac{2541}{262144}p^{25} + \frac{523731}{1048576}p^{24} \\
 &\quad + \frac{238005}{262144}p^{23} + \frac{289525}{1048576}p^{22} - \frac{358967}{262144}p^{21} - \frac{2160639}{1048576}p^{20} \\
 &\quad - \frac{178493}{524288}p^{19} + \frac{503675}{262144}p^{18} + \frac{246645}{131072}p^{17} - \frac{51}{32768}p^{16} \\
 &\quad - \frac{18129}{16384}p^{15} - \frac{5643}{8192}p^{14} + \frac{25}{4096}p^{13} + \frac{805}{4096}p^{12} + \frac{207}{2048}p^{11} \\
 &\quad \left. + \frac{23}{1024}p^{10} + \frac{1}{512}p^9 \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{15} &= \sum_{n=0}^{\infty} f_{15}(n) = \frac{\eta^{19}(4z)\eta^9(6z)\eta^{15}(12z)}{\eta^{11}(2z)} \\
 &= \left(-\frac{1}{2097152}p^{28} - \frac{47}{4194304}p^{27} - \frac{507}{4194304}p^{26} - \frac{1659}{2097152}p^{25} \right. \\
 &\quad - \frac{3663}{1048576}p^{24} - \frac{45843}{4194304}p^{23} - \frac{103387}{4194304}p^{22} - \frac{20741}{524288}p^{21} \\
 &\quad - \frac{44163}{1048576}p^{20} - \frac{2789}{131072}p^{19} + \frac{1981}{131072}p^{18} + \frac{87}{2048}p^{17} \\
 &\quad \left. + \frac{1449}{32768}p^{16} + \frac{453}{16384}p^{15} + \frac{177}{16384}p^{14} + \frac{5}{2048}p^{13} + \frac{1}{4096}p^{12} \right) k^{16}, \\
 f_{16} &= \sum_{n=0}^{\infty} f_{16}(n) = \frac{\eta^{14}(4z)\eta^{14}(6z)\eta^{14}(12z)}{\eta^{10}(2z)} \\
 &= \left(-\frac{1}{524288}p^{27} - \frac{41}{1048576}p^{26} - \frac{191}{524288}p^{25} - \frac{2133}{1048576}p^{24} \right. \\
 &\quad - \frac{3955}{524288}p^{23} - \frac{20323}{1048576}p^{22} - \frac{18149}{524288}p^{21} - \frac{42831}{1048576}p^{20} \\
 &\quad - \frac{1629}{65536}p^{19} + \frac{605}{65536}p^{18} + \frac{155}{4096}p^{17} + \frac{1379}{32768}p^{16} \\
 &\quad \left. + \frac{111}{4096}p^{15} + \frac{11}{1024}p^{14} + \frac{5}{2048}p^{13} + \frac{1}{4096}p^{12} \right) k^{16}, \\
 f_{17} &= \sum_{n=0}^{\infty} f_{17}(n) = \frac{\eta^{15}(4z)\eta^{13}(6z)\eta^{19}(12z)}{\eta^{15}(2z)} \\
 &= \left(\frac{1}{4194304}p^{27} + \frac{21}{4194304}p^{26} + \frac{101}{2097152}p^{25} + \frac{589}{2097152}p^{24} \right. \\
 &\quad + \frac{4645}{4194304}p^{23} + \frac{13073}{4194304}p^{22} + \frac{211}{32768}p^{21} + \frac{2591}{262144}p^{20} \\
 &\quad + \frac{1477}{131072}p^{19} + \frac{1235}{131072}p^{18} + \frac{23}{4096}p^{17} \\
 &\quad \left. + \frac{37}{16384}p^{16} + \frac{9}{16384}p^{15} + \frac{1}{16384}p^{14} \right) k^{16}, \\
 f_{18} &= \sum_{n=0}^{\infty} f_{18}(n) = \frac{\eta^9(4z)\eta^{19}(6z)\eta^{13}(12z)}{\eta^9(2z)} \\
 &= \left(-\frac{1}{131072}p^{26} - \frac{35}{262144}p^{25} - \frac{275}{262144}p^{24} - \frac{1275}{262144}p^{23} \right. \\
 &\quad - \frac{3843}{262144}p^{22} - \frac{7761}{262144}p^{21} - \frac{10205}{262144}p^{20} - \frac{7265}{262144}p^{19} \\
 &\quad + \frac{985}{262144}p^{18} + \frac{273}{8192}p^{17} + \frac{2619}{65536}p^{16} + \frac{435}{16384}p^{15} \\
 &\quad \left. + \frac{175}{16384}p^{14} + \frac{5}{2048}p^{13} + \frac{1}{4096}p^{12} \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{19} &= \sum_{n=0}^{\infty} f_{19}(n) = \frac{\eta^{18}(4z)\eta^{10}(6z)\eta^{10}(12z)}{\eta^6(2z)} \\
 &= \left(\frac{1}{262144} p^{28} + \frac{23}{262144} p^{27} + \frac{965}{1048576} p^{26} + \frac{3041}{524288} p^{25} \right. \\
 &\quad + \frac{25479}{1048576} p^{24} + \frac{18429}{262144} p^{23} + \frac{146279}{1048576} p^{22} + \frac{91313}{524288} p^{21} \\
 &\quad + \frac{83989}{1048576} p^{20} - \frac{41021}{262144} p^{19} - \frac{98171}{262144} p^{18} - \frac{1493}{4096} p^{17} \\
 &\quad - \frac{4043}{32768} p^{16} + \frac{1029}{8192} p^{15} + \frac{1647}{8192} p^{14} + \frac{275}{2048} p^{13} \\
 &\quad \left. + \frac{209}{4096} p^{12} + \frac{11}{1024} p^{11} + \frac{1}{1024} p^{10} \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{20} &= \sum_{n=0}^{\infty} f_{20}(n) = \frac{\eta^{20}(2z)\eta^8(4z)\eta^{20}(6z)}{\eta^{16}(12z)} \\
 &= \left(-\frac{1}{2} p^{27} - \frac{35}{4} p^{26} - \frac{521}{8} p^{25} - \frac{4111}{16} p^{24} - \frac{15975}{32} p^{23} + \frac{327}{64} p^{22} \right. \\
 &\quad + \frac{297757}{128} p^{21} + \frac{1195499}{256} p^{20} + \frac{106085}{128} p^{19} - \frac{685735}{64} p^{18} \\
 &\quad - \frac{62285}{4} p^{17} + \frac{435395}{256} p^{16} + \frac{1648595}{64} p^{15} + \frac{1378995}{64} p^{14} \\
 &\quad - \frac{1378995}{128} p^{13} - \frac{7434175}{256} p^{12} - \frac{1547495}{128} p^{11} + \frac{776305}{64} p^{10} \\
 &\quad + \frac{951275}{64} p^9 + \frac{734305}{256} p^8 - \frac{73567}{16} p^7 - \frac{56545}{16} p^6 - \frac{2247}{4} p^5 \\
 &\quad \left. + \frac{4203}{8} p^4 + 365 p^3 + 107 p^2 + 16 p + 1 \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{21} &= \sum_{n=0}^{\infty} f_{21}(n) = \frac{\eta^{20}(2z)\eta^{20}(4z)}{\eta^4(6z)\eta^4(12z)} \\
 &= \left(-\frac{1}{512} p^{31} - \frac{51}{1024} p^{30} - \frac{1169}{2048} p^{29} - \frac{15687}{4096} p^{28} - \frac{133119}{8192} p^{27} \right. \\
 &\quad - \frac{697473}{16384} p^{26} - \frac{1714923}{32768} p^{25} + \frac{3811059}{65536} p^{24} + \frac{11844819}{32768} p^{23} \\
 &\quad + \frac{18633153}{32768} p^{22} - \frac{456813}{32768} p^{21} - \frac{23799867}{16384} p^{20} - \frac{67065705}{32768} p^{19} \\
 &\quad + \frac{3013731}{32768} p^{18} + \frac{27673335}{8192} p^{17} + \frac{219131721}{65536} p^{16} - \frac{14731353}{16384} p^{15} \\
 &\quad - \frac{68551245}{16384} p^{14} - \frac{5076561}{2048} p^{13} + \frac{5524785}{4096} p^{12} + \frac{2546313}{1024} p^{11} \\
 &\quad + \frac{856899}{1024} p^{10} - \frac{40113}{64} p^9 - \frac{170541}{256} p^8 - \frac{10647}{64} p^7 \\
 &\quad \left. + \frac{4977}{64} p^6 + \frac{567}{8} p^5 + \frac{371}{16} p^4 + \frac{15}{4} p^3 + \frac{1}{4} p^2 \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{22} &= \sum_{n=0}^{\infty} f_{22}(n) = \frac{\eta^{20}(4z)\eta^{20}(12z)}{\eta^4(2z)\eta^4(6z)} \\
 &= \left(\frac{1}{4194304} p^{30} + \frac{25}{4194304} p^{29} + \frac{1145}{16777216} p^{28} \right. \\
 &\quad + \frac{495}{1048576} p^{27} + \frac{18341}{8388608} p^{26} + \frac{29669}{4194304} p^{25} \\
 &\quad + \frac{269025}{16777216} p^{24} + \frac{50735}{2097152} p^{23} + \frac{39853}{2097152} p^{22} \\
 &\quad - \frac{4411}{524288} p^{21} - \frac{46849}{1048576} p^{20} - \frac{3785}{65536} p^{19} \\
 &\quad - \frac{2115}{65536} p^{18} + \frac{67}{8192} p^{17} + \frac{1903}{65536} p^{16} + \frac{195}{8192} p^{15} \\
 &\quad \left. + \frac{85}{8192} p^{14} + \frac{5}{2048} p^{13} + \frac{1}{4096} p^{12} \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{23} &= \sum_{n=0}^{\infty} f_{23}(n) = \frac{\eta^{20}(4z)\eta^{20}(6z)}{\eta^4(2z)\eta^4(12z)} \\
 &= \left(-\frac{1}{8192} p^{27} - \frac{47}{16384} p^{26} - \frac{1007}{32768} p^{25} - \frac{12941}{65536} p^{24} \right. \\
 &\quad - \frac{27509}{32768} p^{23} - \frac{79891}{32768} p^{22} - \frac{154617}{32768} p^{21} - \frac{84655}{16384} p^{20} \\
 &\quad + \frac{7835}{32768} p^{19} + \frac{377855}{32768} p^{18} + \frac{162507}{8192} p^{17} + \frac{912921}{65536} p^{16} \\
 &\quad - \frac{77123}{16384} p^{15} - \frac{322787}{16384} p^{14} - \frac{18661}{1024} p^{13} - \frac{11199}{2048} p^{12} \\
 &\quad + \frac{2387}{512} p^{11} + \frac{3253}{512} p^{10} + \frac{229}{64} p^9 + \frac{293}{256} p^8 \\
 &\quad \left. + \frac{13}{64} p^7 + \frac{1}{64} p^6 \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{24} &= \sum_{n=0}^{\infty} f_{24}(n) = \frac{\eta^{15}(4z)\eta(6z)\eta^{19}(12z)}{\eta^3(2z)} \\
 &= \left(\frac{1}{1048576} p^{29} + \frac{11}{524288} p^{28} + \frac{877}{4194304} p^{27} + \frac{5205}{4194304} p^{26} \right. \\
 &\quad + \frac{10137}{2097152} p^{25} + \frac{26721}{2097152} p^{24} + \frac{92837}{4194304} p^{23} + \frac{89789}{4194304} p^{22} \\
 &\quad - \frac{725}{524288} p^{21} - \frac{39571}{1048576} p^{20} - \frac{7325}{131072} p^{19} - \frac{4619}{131072} p^{18} \\
 &\quad + \frac{9}{2048} p^{17} + \frac{885}{32768} p^{16} + \frac{381}{16384} p^{15} + \frac{169}{16384} p^{14} \\
 &\quad \left. + \frac{5}{2048} p^{13} + \frac{1}{4096} p^{12} \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{25} &= \sum_{n=0}^{\infty} f_{25}(n) = \frac{\eta^{20}(2z)\eta^{20}(4z)}{\eta^4(6z)\eta^4(12z)} \\
 &= \left(-\frac{1}{8}p^{25} - \frac{9}{16}p^{24} - \frac{7}{16}p^{23} + \frac{55}{32}p^{22} + \frac{435}{128}p^{21} \right. \\
 &\quad - \frac{101}{256}p^{20} - \frac{781}{128}p^{19} - \frac{33}{8}p^{18} + \frac{115}{32}p^{17} + \frac{1475}{256}p^{16} \\
 &\quad + \frac{63}{64}p^{15} - \frac{41}{16}p^{14} - \frac{229}{128}p^{13} - \frac{15}{256}p^{12} \\
 &\quad \left. + \frac{55}{128}p^{11} + \frac{7}{32}p^{10} + \frac{3}{64}p^9 + \frac{1}{256}p^8 \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{26} &= \sum_{n=0}^{\infty} f_{26}(n) = \frac{\eta^{20}(2z)\eta^{20}(6z)\eta^8(12z)}{\eta^4(4z)} \\
 &= \left(-\frac{1}{8}p^{28} - \frac{41}{16}p^{27} - \frac{735}{32}p^{26} - \frac{7369}{64}p^{25} - \frac{42461}{128}p^{24} \right. \\
 &\quad - \frac{108327}{256}p^{23} + \frac{261427}{512}p^{22} + \frac{3059933}{1024}p^{21} \\
 &\quad + \frac{4316289}{1024}p^{20} - \frac{1428553}{1024}p^{19} - \frac{12680767}{1024}p^{18} \\
 &\quad - \frac{6900279}{512}p^{17} + \frac{3095779}{512}p^{16} + \frac{13721761}{512}p^{15} \\
 &\quad + \frac{2279991}{128}p^{14} - \frac{14642143}{1024}p^{13} - \frac{29221507}{1024}p^{12} \\
 &\quad - \frac{9789933}{1024}p^{11} + \frac{13635953}{1024}p^{10} + \frac{921227}{64}p^9 \\
 &\quad + \frac{560229}{256}p^8 - \frac{153647}{32}p^7 - \frac{111071}{32}p^6 \\
 &\quad \left. - \frac{987}{2}p^5 + \frac{4387}{8}p^4 + \frac{1475}{4}p^3 + \frac{429}{4}p^2 + 16p + 1 \right) k^{16},
 \end{aligned}$$

$$\begin{aligned}
 f_{27} &= \sum_{n=0}^{\infty} f_{27}(n) = \frac{\eta^{19}(2z)\eta^{13}(4z)\eta^{15}(6z)}{\eta^{15}(12z)} \\
 &= \left(-\frac{1}{32}p^{26} - \frac{15}{64}p^{25} - \frac{9}{16}p^{24} - \frac{1}{128}p^{23} \right. \\
 &\quad + \frac{1095}{512}p^{22} + \frac{2949}{1024}p^{21} - \frac{1435}{1024}p^{20} \\
 &\quad - \frac{6273}{1024}p^{19} - \frac{3279}{1024}p^{18} + \frac{2105}{512}p^{17} + \frac{2811}{512}p^{16} \\
 &\quad + \frac{315}{512}p^{15} - \frac{169}{64}p^{14} - \frac{1767}{1024}p^{13} - \frac{15}{1024}p^{12} \\
 &\quad \left. + \frac{451}{1024}p^{11} + \frac{225}{1024}p^{10} + \frac{3}{64}p^9 + \frac{1}{256}p^8 \right) k^{16}.
 \end{aligned}$$

Obviously, f_1, \dots, f_{27} are functions of q , see (3), (19). We see that $\{f_1, \dots, f_{27}\} \setminus \{f_7, f_9, f_{17}, f_{20}, f_{25}, f_{26}, f_{27}\} \in S_{16}(\Gamma_0(12))$, $f_7, f_9, f_{17}, f_{20}, f_{25}, f_{26}, f_{27} \in M_{16}(\Gamma_0(12)) \setminus S_{16}(\Gamma_0(12))$ by [17]. Now

$$\begin{aligned}
 & \eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z) \\
 &= q^{b_1} \prod_{n=1}^{\infty} (1-q^n)^{a_1} (1-q^{2n})^{a_2} (1-q^{3n})^{a_3} (1-q^{4n})^{a_4} (1-q^{6n})^{a_6} (1-q^{12n})^{a_{12}} \\
 &= 2^{-\frac{a_1}{6}-\frac{a_2}{3}-\frac{a_3}{6}-\frac{2a_4}{3}-\frac{a_6}{3}-\frac{2a_{12}}{3}} p^{\frac{a_1}{24}+\frac{a_2}{12}+\frac{a_3}{8}+\frac{a_4}{6}+\frac{a_6}{4}+\frac{a_{12}}{2}} \\
 &\quad \cdot (1-p)^{\frac{a_1}{2}+\frac{a_2}{4}+\frac{a_3}{6}+\frac{a_4}{8}+\frac{a_6}{12}+\frac{a_{12}}{24}} (1+p)^{\frac{a_1}{6}+\frac{a_2}{12}+\frac{a_3}{2}+\frac{a_4}{24}+\frac{a_6}{4}+\frac{a_{12}}{8}} \\
 &\quad \cdot (1+2p)^{\frac{a_1}{8}+\frac{a_2}{4}+\frac{a_3}{24}+\frac{a_4}{8}+\frac{a_6}{12}+\frac{a_{12}}{24}} (2+p)^{\frac{a_1}{8}+\frac{a_2}{4}+\frac{a_3}{24}+\frac{a_4}{2}+\frac{a_6}{12}+\frac{a_{12}}{6}} \\
 &\quad \cdot k^{\frac{a_1+a_2+a_3+a_4+a_6+a_{12}}{2}} \\
 &= \frac{k^{16}}{2^{b_1+b_5}} p^{b_1} (1-p)^{b_2} (1+p)^{b_3} (1+2p)^{b_4} (2+p)^{b_5} \\
 &= k^{16} (k_0 + k_1 p + k_2 p^2 + k_3 p^3 + k_4 p^4 + k_5 p^5 \\
 &\quad + k_6 p^6 + k_7 p^7 + k_8 p^8 + k_9 p^9 + k_{10} p^{10} + k_{11} p^{11} \\
 &\quad + k_{12} p^{12} + k_{13} p^{13} + k_{14} p^{14} + k_{15} p^{15} + k_{16} p^{16} \\
 &\quad + k_{17} p^{17} + k_{18} p^{18} + k_{19} p^{19} + k_{20} p^{20} + k_{21} p^{21} \\
 &\quad + k_{22} p^{22} + k_{23} p^{23} + k_{24} p^{24} + k_{25} p^{25} + k_{26} p^{26} \\
 &\quad + k_{27} p^{27} + k_{28} p^{28} + k_{29} p^{29} + k_{30} p^{30} + k_{31} p^{31} + k_{32} p^{32}) \\
 &= \frac{3617c_1}{16320} \left(1 + \frac{16320}{3617} \sum_{n=1}^{\infty} \sigma_{15}(n) q^n \right) + \frac{3617c_2}{16320} \left(1 + \frac{16320}{3617} \sum_{n=1}^{\infty} \sigma_{15}(n) q^{2n} \right) \\
 &\quad + \frac{3617c_3}{16320} \left(1 + \frac{16320}{3617} \sum_{n=1}^{\infty} \sigma_{15}(n) q^{3n} \right) + \frac{3617c_4}{16320} \left(1 + \frac{16320}{3617} \sum_{n=1}^{\infty} \sigma_{15}(n) q^{4n} \right) \\
 &\quad + \frac{3617c_5}{16320} \left(1 + \frac{16320}{3617} \sum_{n=1}^{\infty} \sigma_{15}(n) q^{6n} \right) + \frac{3617c_6}{16320} \left(1 + \frac{16320}{3617} \sum_{n=1}^{\infty} \sigma_{15}(n) q^{12n} \right) \\
 &\quad + r_1 q^{11} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{18} (1-q^{6n})^{16} (1-q^{12n})^{10}}{(1-q^{2n})^{12}} + r_2 q^{13} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{20} (1-q^{6n})^2 (1-q^{12})^{20}}{(1-q^{2n})^{10}} \\
 &\quad + r_3 q^{13} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{15} (1-q^{6n})^7 (1-q^{12n})^{19}}{(1-q^{2n})^9} + r_4 q^{13} \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{10} (1-q^{4n})^{12} (1-q^{6n})^{18}}{(1-q^{12n})^8} \\
 &\quad + r_5 q^{13} \prod_{n=1}^{\infty} \frac{((1-q^{4n})^7 (1-q^{6n})^{17} (1-q^{12n})^{17}}{(1-q^{2n})^7} + r_6 q^9 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{17} (1-q^{6n})^{17} (1-q^{12n})^5}{(1-q^{2n})^7} \\
 &\quad + r_7 q^{13} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{19} (1-q^{6n})^{15} (1-q^{12n})^{15}}{(1-q^{2n})^{17}} + r_8 q^5 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{17} (1-q^{4n})^{17} (1-q^{12n})^5}{(1-q^{6n})^7} \\
 &\quad + r_9 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{20} (1-q^{4n})^{14} (1-q^{12n})^{14}}{(1-q^{6n})^{16}} + r_{10} q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{16} (1-q^{6n})^{18}}{(1-q^{2n})^2} \\
 &\quad + r_{11} q \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{20} (1-q^{4n})^{14} (1-q^{6n})^8}{(1-q^{12n})^{10}} + r_{12} q^5 \prod_{n=1}^{\infty} (1-q^{2n})^{20} (1-q^{4n})^2 (1-q^{6n})^8 (1-q^{12n})^2
 \end{aligned}$$

$$\begin{aligned}
 &+ r_{13}q^{13} \prod_{n=1}^{\infty} \frac{(1-q^{2n})^2 (1-q^{6n})^{14} (1-q^{12n})^{20}}{(1-q^{4n})^4} + r_{14}q^9 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{16} (1-q^{4n})^{10} (1-q^{12n})^{18}}{(1-q^{6n})^{12}} \\
 &+ r_{15}q^{12} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{19} (1-q^{6n})^9 (1-q^{12n})^{15}}{(1-q^{2n})^{11}} + r_{16}q^{12} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{14} (1-q^{6n})^{14} (1-q^{12n})^{14}}{(1-q^{2n})^{10}} \\
 &+ r_{17}q^{14} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{15} (1-q^{6n})^{13} (1-q^{12n})^{19}}{(1-q^{2n})^{15}} + r_{18}q^{12} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^9 (1-q^{6n})^{19} (1-q^{12n})^{13}}{(1-q^{2n})^9} \\
 &+ r_{19}q^{10} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{18} (1-q^{6n})^{10} (1-q^{12n})^{10}}{(1-q^{2n})^6} \\
 &+ r_{20} \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{20} (1-q^{4n})^8 (1-q^{6n})^{20}}{(1-q^{12n})^{16}} \\
 &+ r_{21}q^2 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{20} (1-q^{4n})^{20}}{(1-q^{6n})^4 (1-q^{12n})^4} \\
 &+ r_{22}q^{12} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{20} (1-q^{12n})^{20}}{(1-q^{2n})^4 (1-q^{6n})^4} \\
 &+ r_{23}q^6 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{20} (1-q^{6n})^{20}}{(1-q^{2n})^4 (1-q^{12n})^4} \\
 &+ r_{24}q^{12} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{15} (1-q^{6n})(1-q^{12n})^{19}}{(1-q^{2n})^3} \\
 &+ r_{25}q^8 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{20} (1-q^{4n})^{20}}{(1-q^{6n})^4 (1-q^{12n})^4} \\
 &+ r_{26}q^{12} \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{20} (1-q^{6n})^{20} (1-q^{12n})^8}{(1-q^{4n})^4} \\
 &+ r_{27} \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{19} (1-q^{4n})^{13} (1-q^{6n})^{15}}{(1-q^{12n})^{15}} \\
 &= \delta(b_1) - \sum_{n=1}^{\infty} \left(c_1 \sigma_{15}(n) + c_2 \sigma_{15}\left(\frac{n}{2}\right) + c_3 \sigma_{15}\left(\frac{n}{3}\right) + c_4 \sigma_{15}\left(\frac{n}{4}\right) \right. \\
 &\quad \left. + c_6 \sigma_{15}\left(\frac{n}{6}\right) + c_{12} \sigma_{15}\left(\frac{n}{12}\right) \right) + r_1 f_1(n) + \dots + r_{27} f_{27}(n),
 \end{aligned}$$

where

$$\delta(b_1) = \begin{cases} 0 & \text{if } b_1 \neq 0 \\ 1 & \text{if } b_1 = 0 \end{cases}$$

So

$$c(n) = -\left(c_1\sigma_{15}(n) + c_2\sigma_{15}\left(\frac{n}{2}\right) + c_3\sigma_{15}\left(\frac{n}{3}\right) + c_4\sigma_{15}\left(\frac{n}{4}\right) \right. \\ \left. + c_6\sigma_{15}\left(\frac{n}{6}\right) + c_{12}\sigma_{15}\left(\frac{n}{12}\right) \right) + r_1f_1(n) + \dots + r_{27}f_{27}(n).$$

Therefore, for $n = 1, 2, \dots$,

$$c(2n) = -c_1\sigma_{15}(2n) - c_2\sigma_{15}(n) - c_4\sigma_{15}\left(\frac{n}{2}\right) - (65537c_3 + c_6)\sigma_{15}\left(\frac{n}{3}\right) \\ - (c_{12} - 65536c_3)\sigma_{15}\left(\frac{n}{6}\right) + r_{15}f_{15}(2n) + \dots + r_{27}f_{27}(2n), \\ c(2n-1) = -c_1\sigma_{15}(2n-1) - c_3\sigma_{15}\left(\frac{2n-1}{3}\right) + r_1f_1(2n-1) + \dots + r_{14}f_{14}(2n-1),$$

since it is easy to see that

$$\sigma_k\left(\frac{2n}{3}\right) = (2^k + 1)\sigma_k\left(\frac{n}{3}\right) - 2^k\sigma_k\left(\frac{n}{6}\right)$$

hence,

$$\sigma_{15}\left(\frac{2n}{3}\right) = 65537\sigma_{15}\left(\frac{n}{3}\right) - 65536\sigma_{15}\left(\frac{n}{6}\right),$$

and, for $n = 1, 2, \dots$,

$$f_1(2n) = \dots = f_{14}(2n) = 0, \\ f_{15}(2n-1) = \dots = f_{27}(2n-1) = 0.$$

Remark 2 We have found 360 eta quotients, see [Table 2](#), such that, for $n = 1, 2, \dots$,

$$c(2n) = -c_1\sigma_{15}(2n) - c_2\sigma_{15}(n) - c_4\sigma_{15}\left(\frac{n}{2}\right) - (65537c_3 + c_6)\sigma_{15}\left(\frac{n}{3}\right) \\ - (c_{12} - 65536c_3)\sigma_{15}\left(\frac{n}{6}\right) \\ c(2n-1) = -c_1\sigma_{15}(2n-1) - c_3\sigma_{15}\left(\frac{2n-1}{3}\right) + r_1f_1(2n-1) \\ + \dots + r_{14}f_{14}(2n-1).$$

and 875 eta quotients, such that for $n = 1, 2, \dots$,

$$c(2n) = -c_1\sigma_{15}(2n) - c_2\sigma_{15}(n) - c_4\sigma_{15}\left(\frac{n}{2}\right) - c_6\sigma_{15}\left(\frac{n}{3}\right) \\ - c_{12}\sigma_{15}\left(\frac{n}{6}\right) + r_{15}f_{15}(2n) + \dots + r_{27}f_{27}(2n), \\ c(2n-1) = 0.$$

Remark 3 If f is an eta quotient, then $f(-q)$ is also an eta quotient, so the coefficients of $\frac{1}{2}(f(q) + f(-q))$ are exactly the even coefficients of f . In particular, it means that we have obtained all coefficients of some sum of 360 eta quotients.

Table 2. The eta quotients whose even coefficients can be explicitly calculated.

No	a_2	a_4	a_6	a_{12}	c_1	c_2	c_3	c_4	c_6	c_{12}	No
1	-46	92	14	-28	$\frac{1}{217408}$	$-\frac{32769}{217408}$	0	$\frac{512}{3397}$	0	0	1
2	-45	87	19	-29	$\frac{65}{14348928}$	$-\frac{64545}{434816}$	0	$\frac{16640}{112101}$	0	0	2
3	-44	82	24	-30	$\frac{1}{224202}$	$-\frac{993}{6794}$	0	$\frac{16384}{112101}$	0	0	3
4	-43	77	29	-31	$\frac{21}{4782976}$	$-\frac{62559}{434816}$	0	$\frac{5376}{37367}$	0	0	4
5	-42	72	34	-32	$\frac{31}{7174464}$	$-\frac{30783}{217408}$	0	$\frac{15872}{112101}$	0	0	5
6	-41	67	39	-33	$\frac{61}{14348928}$	$-\frac{60573}{434816}$	0	$\frac{15616}{112101}$	0	0	6
7	-40	62	44	-34	$\frac{5}{1195744}$	$-\frac{14895}{108704}$	0	$\frac{5120}{37367}$	0	0	7
8	-39	57	49	-35	$\frac{59}{14348928}$	$-\frac{58587}{434816}$	0	$\frac{15104}{112101}$	0	0	8
9	-38	52	54	-36	$\frac{29}{7174464}$	$-\frac{28797}{217408}$	0	$\frac{14848}{112101}$	0	0	9
10	-37	47	59	-37	$\frac{19}{4782976}$	$-\frac{56601}{434816}$	0	$\frac{4864}{37367}$	0	0	10
11	-36	42	64	-38	$\frac{7}{1793616}$	$-\frac{6951}{54352}$	0	$\frac{14336}{112101}$	0	0	11
12	-35	37	69	-39	$\frac{5}{1304448}$	$-\frac{54615}{434816}$	0	$\frac{1280}{10191}$	0	0	12
13	-34	32	74	-40	$\frac{9}{2391488}$	$-\frac{26811}{217408}$	0	$\frac{4608}{37367}$	0	0	13
14	-33	27	79	-41	$\frac{53}{14348928}$	$-\frac{52629}{434816}$	0	$\frac{13568}{112101}$	0	0	14
15	-32	22	84	-42	$\frac{13}{3587232}$	$-\frac{12909}{108704}$	0	$\frac{13312}{112101}$	0	0	15
16	-31	17	89	-43	$\frac{6971}{1961319096}$	$-\frac{2307401}{19811304}$	$\frac{1}{2187}$	$\frac{28553216}{245164887}$	$-\frac{3641}{243}$	$\frac{32768}{2187}$	16
17	-37	83	11	-25	$\frac{19}{4782976}$	$-\frac{56601}{434816}$	0	$\frac{4864}{37367}$	0	0	17
18	-36	78	16	-26	$\frac{7}{1793616}$	$-\frac{6951}{54352}$	0	$\frac{14336}{112101}$	0	0	18
19	-35	73	21	-27	$\frac{5}{1304448}$	$-\frac{54615}{434816}$	0	$\frac{1280}{10191}$	0	0	19
20	-34	68	26	-28	$\frac{9}{2391488}$	$-\frac{26811}{217408}$	0	$\frac{4608}{37367}$	0	0	20
21	-33	63	31	-29	$\frac{53}{14348928}$	$-\frac{52629}{434816}$	0	$\frac{13568}{112101}$	0	0	21

Continued

22	-32	58	36	-30	$\frac{13}{3587232}$	$\frac{12909}{108704}$	0	$\frac{13312}{112101}$	0	0	22
23	-31	53	41	-31	$\frac{17}{4782976}$	$\frac{50643}{434816}$	0	$\frac{4352}{37367}$	0	0	23
24	-30	48	46	-32	$\frac{25}{7174464}$	$\frac{24825}{217408}$	0	$\frac{12800}{112101}$	0	0	24
25	-29	43	51	-33	$\frac{49}{14348928}$	$\frac{48657}{434816}$	0	$\frac{12544}{112101}$	0	0	25
26	-28	38	56	-34	$\frac{1}{298936}$	$\frac{2979}{27176}$	0	$\frac{4096}{37367}$	0	0	26
27	-27	33	61	-35	$\frac{47}{14348928}$	$\frac{46671}{434816}$	0	$\frac{12032}{112101}$	0	0	27
28	-26	28	66	-36	$\frac{23}{7174464}$	$\frac{22839}{217408}$	0	$\frac{11776}{112101}$	0	0	28
29	-25	23	71	-37	$\frac{15}{4782976}$	$\frac{44685}{434816}$	0	$\frac{3840}{37367}$	0	0	29
30	-24	18	76	-38	$\frac{1}{326112}$	$\frac{10923}{108704}$	0	$\frac{1024}{10191}$	0	0	30
31	-23	13	81	-39	$\frac{653}{217924344}$	$\frac{216143}{2201256}$	$\frac{1}{243}$	$\frac{2674688}{27240543}$	$\frac{3641}{27}$	$\frac{32768}{243}$	31
32	-28	74	8	-22	$\frac{1}{298936}$	$\frac{2979}{27176}$	0	$\frac{4096}{37367}$	0	0	32
33	-27	69	13	-23	$\frac{47}{14348928}$	$\frac{46671}{434816}$	0	$\frac{12032}{112101}$	0	0	33
34	-26	64	18	-24	$\frac{23}{7174464}$	$\frac{22839}{217408}$	0	$\frac{11776}{112101}$	0	0	34
35	-25	59	23	-25	$\frac{15}{4782976}$	$\frac{44685}{434816}$	0	$\frac{3840}{37367}$	0	0	35
36	-24	54	28	-26	$\frac{1}{326112}$	$\frac{10923}{108704}$	0	$\frac{1024}{10191}$	0	0	36
37	-23	49	33	-27	$\frac{1}{333696}$	$\frac{993}{10112}$	0	$\frac{256}{2607}$	0	0	37
38	-22	44	38	-28	$\frac{7}{2391488}$	$\frac{20853}{217408}$	0	$\frac{3584}{37367}$	0	0	38
39	-21	39	43	-29	$\frac{41}{14348928}$	$\frac{40713}{434816}$	0	$\frac{10496}{112101}$	0	0	39
40	-20	34	48	-30	$\frac{5}{1793616}$	$\frac{4965}{54352}$	0	$\frac{10240}{112101}$	0	0	40
41	-19	29	53	-31	$\frac{13}{4782976}$	$\frac{38727}{434816}$	0	$\frac{3328}{37367}$	0	0	41
42	-18	24	58	-32	$\frac{19}{7174464}$	$\frac{18867}{217408}$	0	$\frac{9728}{112101}$	0	0	42
43	-17	19	63	-33	$\frac{37}{14348928}$	$\frac{36741}{434816}$	0	$\frac{9472}{112101}$	0	0	43

Continued

44	-16	14	68	-34	$\frac{3}{1195744}$	$\frac{8937}{108704}$	0	$\frac{3072}{37367}$	0	0	44
45	-15	9	73	-35	$\frac{59}{24213816}$	$\frac{19529}{244584}$	$\frac{1}{27}$	$\frac{241664}{3026727}$	$\frac{3641}{3}$	$\frac{32768}{27}$	45
46	-19	65	5	-19	$\frac{13}{4782976}$	$\frac{38727}{434816}$	0	$\frac{3328}{37367}$	0	0	46
47	-18	60	10	-20	$\frac{19}{7174464}$	$\frac{18867}{217408}$	0	$\frac{9728}{112101}$	0	0	47
48	-17	55	15	-21	$\frac{37}{14348928}$	$\frac{36741}{434816}$	0	$\frac{9472}{112101}$	0	0	48
49	-16	50	20	-22	$\frac{3}{1195744}$	$\frac{8937}{108704}$	0	$\frac{3072}{37367}$	0	0	49
50	-15	45	25	-23	$\frac{35}{14348928}$	$\frac{34755}{434816}$	0	$\frac{8960}{112101}$	0	0	50
51	-14	40	30	-24	$\frac{17}{7174464}$	$\frac{16881}{217408}$	0	$\frac{8704}{112101}$	0	0	51
52	-13	35	35	-25	$\frac{1}{434816}$	$\frac{32769}{434816}$	0	$\frac{256}{3397}$	0	0	52
53	-12	30	40	-26	$\frac{1}{448404}$	$\frac{993}{13588}$	0	$\frac{8192}{112101}$	0	0	53
54	-11	25	45	-27	$\frac{31}{14348928}$	$\frac{30783}{434816}$	0	$\frac{7936}{112101}$	0	0	54
55	-10	20	50	-28	$\frac{5}{2391488}$	$\frac{14895}{217408}$	0	$\frac{2560}{37367}$	0	0	55
56	-9	15	55	-29	$\frac{29}{14348928}$	$\frac{28797}{434816}$	0	$\frac{7424}{112101}$	0	0	56
57	-8	10	60	-30	$\frac{7}{3587232}$	$\frac{6951}{108704}$	0	$\frac{7168}{112101}$	0	0	57
58	-7	5	65	-31	$\frac{5}{2690424}$	$\frac{1655}{27176}$	$\frac{1}{3}$	$\frac{20480}{336303}$	-10923	$\frac{32768}{3}$	58
59	-10	56	2	-16	$\frac{5}{2391488}$	$\frac{14895}{217408}$	0	$\frac{2560}{37367}$	0	0	59
60	-9	51	7	-17	$\frac{29}{14348928}$	$\frac{28797}{434816}$	0	$\frac{7424}{112101}$	0	0	60
61	-8	46	12	-18	$\frac{7}{3587232}$	$\frac{6951}{108704}$	0	$\frac{7168}{112101}$	0	0	61
62	-7	41	17	-19	$\frac{9}{4782976}$	$\frac{26811}{434816}$	0	$\frac{2304}{37367}$	0	0	62
63	-6	36	22	-20	$\frac{13}{7174464}$	$\frac{12909}{217408}$	0	$\frac{6656}{112101}$	0	0	63
64	-5	31	27	-21	$\frac{25}{14348918}$	$\frac{24825}{434816}$	0	$\frac{6400}{112101}$	0	0	64
65	-4	26	32	-22	$\frac{1}{597872}$	$\frac{2979}{54352}$	0	$\frac{2048}{37367}$	0	0	65

Continued

66	-3	21	37	-23	$\frac{23}{14348928}$	$\frac{22839}{434816}$	0	$\frac{5888}{112101}$	0	0	66
67	-2	16	42	-24	$\frac{1}{652224}$	$\frac{10923}{217408}$	0	$\frac{512}{10191}$	0	0	67
68	-1	11	47	-25	$\frac{7}{4782976}$	$\frac{20853}{434816}$	0	$\frac{1792}{37367}$	0	0	68
69	0	6	52	-26	$\frac{5}{3587232}$	$\frac{4965}{108704}$	0	$\frac{5120}{112101}$	0	0	69
70	1	1	57	-27	$\frac{1}{896808}$	$\frac{993}{27176}$	3	$\frac{4096}{112101}$	-98307	98304	70
71	-1	47	-1	-13	$\frac{7}{4782976}$	$\frac{20853}{434816}$	0	$\frac{1792}{37367}$	0	0	71
72	0	42	4	-14	$\frac{5}{3587232}$	$\frac{4965}{108704}$	0	$\frac{5120}{112101}$	0	0	72
73	1	37	9	-15	$\frac{19}{14348928}$	$\frac{18867}{434816}$	0	$\frac{4864}{112101}$	0	0	73
74	2	32	14	-16	$\frac{3}{2391488}$	$\frac{8937}{217408}$	0	$\frac{1536}{37367}$	0	0	74
75	3	27	19	-17	$\frac{17}{14348928}$	$\frac{16881}{434816}$	0	$\frac{4352}{112101}$	0	0	75
76	4	22	24	-18	$\frac{1}{896808}$	$\frac{993}{27176}$	0	$\frac{4096}{112101}$	0	0	76
77	5	17	29	-19	$\frac{5}{4782976}$	$\frac{14895}{434816}$	0	$\frac{1280}{37367}$	0	0	77
78	6	12	34	-20	$\frac{7}{7174464}$	$\frac{6951}{217408}$	0	$\frac{3584}{112101}$	0	0	78
79	7	7	39	-21	$\frac{13}{14348928}$	$\frac{12909}{434816}$	0	$\frac{3328}{112101}$	0	0	79
80	8	2	44	-22	$\frac{1}{1195744}$	$\frac{2979}{108704}$	0	$\frac{1024}{37367}$	0	0	80
81	9	-3	49	-23	$\frac{1}{896808}$	$\frac{993}{27176}$	27	$\frac{4096}{112101}$	-884763	884736	81
82	8	38	-4	-10	$\frac{1}{1195744}$	$\frac{2979}{108704}$	0	$\frac{1024}{37367}$	0	0	82
83	9	33	1	-11	$\frac{1}{1304448}$	$\frac{10923}{434816}$	0	$\frac{256}{10191}$	0	0	83
84	10	28	6	-12	$\frac{5}{7174464}$	$\frac{4965}{217408}$	0	$\frac{2560}{112101}$	0	0	84
85	11	23	11	-13	$\frac{3}{4782976}$	$\frac{8937}{434816}$	0	$\frac{768}{37367}$	0	0	85
86	12	18	16	-14	$\frac{1}{1793616}$	$\frac{993}{54352}$	0	$\frac{2048}{112101}$	0	0	86
87	13	13	21	-15	$\frac{7}{14348928}$	$\frac{6951}{434816}$	0	$\frac{1792}{112101}$	0	0	87

Continued

88	14	8	26	-16	$\frac{1}{2391488}$	$-\frac{2979}{217408}$	0	$\frac{512}{37367}$	0	0	88
89	15	3	31	-17	$\frac{5}{14348928}$	$-\frac{4965}{434816}$	0	$\frac{1280}{112101}$	0	0	89
90	16	-2	36	-18	$\frac{1}{3587232}$	$-\frac{993}{108704}$	0	$\frac{1024}{112101}$	0	0	90
91	17	-7	41	-19	$-\frac{5}{298936}$	$\frac{14895}{27176}$	243	$-\frac{20480}{37367}$	-7962867	7962624	91
92	17	29	-7	-7	$\frac{1}{4782976}$	$-\frac{2979}{434816}$	0	$\frac{256}{37367}$	0	0	92
93	18	24	-2	-8	$\frac{1}{7174464}$	$-\frac{993}{217408}$	0	$\frac{512}{112101}$	0	0	93
94	19	19	3	-9	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	94
95	21	9	13	-11	$-\frac{1}{14348928}$	$\frac{993}{434816}$	0	$-\frac{256}{112101}$	0	0	95
96	22	4	18	-12	$-\frac{1}{7174464}$	$\frac{993}{217408}$	0	$-\frac{512}{112101}$	0	0	96
97	23	-1	23	-13	$-\frac{1}{4782976}$	$\frac{2979}{434816}$	0	$-\frac{256}{37367}$	0	0	97
98	24	-6	28	-14	$-\frac{1}{3587232}$	$\frac{993}{108704}$	0	$-\frac{1024}{112101}$	0	0	98
99	25	-11	33	-15	$-\frac{137}{896808}$	$\frac{136041}{27176}$	2187	$-\frac{561152}{112101}$	-71665803	71663616	99
100	26	20	-10	-4	$-\frac{1}{2391488}$	$\frac{2979}{217408}$	0	$-\frac{512}{37367}$	0	0	100
101	27	15	-5	-5	$-\frac{7}{14348928}$	$\frac{6951}{434816}$	0	$-\frac{1792}{112101}$	0	0	101
102	28	10	0	-6	$-\frac{1}{1793616}$	$\frac{993}{54352}$	0	$-\frac{2048}{112101}$	0	0	102
103	29	5	5	-7	$-\frac{3}{4782976}$	$\frac{8937}{434816}$	0	$-\frac{768}{37367}$	0	0	103
104	30	0	10	-8	$-\frac{5}{7174464}$	$\frac{4965}{217408}$	0	$-\frac{2560}{112101}$	0	0	104
105	31	-5	15	-9	$-\frac{1}{1304448}$	$\frac{10923}{434816}$	0	$-\frac{256}{10191}$	0	0	105
106	32	-10	20	-10	$-\frac{1}{1195744}$	$\frac{2979}{108704}$	0	$-\frac{1024}{37367}$	0	0	106
107	33	-15	25	-11	$-\frac{1231}{896808}$	$\frac{1222383}{27176}$	19683	$-\frac{5042176}{112101}$	-644992227	644972544	107
108	35	11	-13	-1	$-\frac{5}{4782976}$	$\frac{14895}{434816}$	0	$-\frac{1280}{37367}$	0	0	108
109	36	6	-8	2	$-\frac{1}{896808}$	$\frac{993}{27176}$	0	$-\frac{4096}{112101}$	0	0	109

Continued

110	37	1	-3	-3	$-\frac{17}{14348928}$	$\frac{16881}{434816}$	0	$-\frac{4352}{112101}$	0	0	110
111	38	-4	2	-4	$-\frac{3}{2391488}$	$\frac{8937}{217408}$	0	$-\frac{1536}{37367}$	0	0	111
112	39	-9	7	-5	$-\frac{19}{14348928}$	$\frac{18867}{434816}$	0	$-\frac{4864}{112101}$	0	0	112
113	40	-14	12	-6	$-\frac{5}{3587232}$	$\frac{4965}{108704}$	0	$-\frac{5120}{112101}$	0	0	113
114	41	-19	17	-7	$-\frac{3691}{298936}$	$\frac{10995489}{27176}$	177147	$-\frac{15118336}{37367}$	-5804930043	5804752896	114
115	44	2	-16	2	$-\frac{1}{597872}$	$\frac{2979}{54352}$	0	$-\frac{2048}{37367}$	0	0	115
116	45	-3	-11	1	$-\frac{25}{14348918}$	$\frac{24825}{434816}$	0	$-\frac{6400}{112101}$	0	0	116
117	46	-8	-6	0	$-\frac{13}{7174464}$	$\frac{12909}{217408}$	0	$-\frac{6656}{112101}$	0	0	117
118	47	-13	-1	-1	$-\frac{9}{4782976}$	$\frac{26811}{434816}$	0	$-\frac{2304}{37367}$	0	0	118
119	48	-18	4	-2	$-\frac{7}{3587232}$	$\frac{6951}{108704}$	0	$-\frac{7168}{112101}$	0	0	119
120	49	-23	9	-3	$-\frac{99647}{896808}$	$\frac{98949471}{27176}$	1594323	$-\frac{408154112}{112101}$	-52244370387	52242776064	120
121	53	-7	-19	5	$-\frac{1}{434816}$	$\frac{32769}{434816}$	0	$-\frac{256}{3397}$	0	0	121
122	54	-12	-14	4	$-\frac{17}{7174464}$	$\frac{16881}{217408}$	0	$-\frac{8704}{112101}$	0	0	122
123	55	-17	-9	3	$-\frac{35}{14348928}$	$\frac{34755}{434816}$	0	$-\frac{8960}{112101}$	0	0	123
124	56	-22	-4	2	$-\frac{3}{1195744}$	$\frac{8937}{108704}$	0	$-\frac{3072}{37367}$	0	0	124
125	57	-27	1	1	$-\frac{896809}{896808}$	$\frac{890531337}{27176}$	14348907	$-\frac{3673329664}{112101}$	-470199333483	470184984576	125
126	62	-16	-22	8	$-\frac{7}{2391488}$	$\frac{20853}{217408}$	0	$-\frac{3584}{37367}$	0	0	126
127	63	-21	-17	7	$-\frac{1}{333696}$	$\frac{993}{10112}$	0	$-\frac{256}{2607}$	0	0	127
128	64	-26	-12	6	$-\frac{1}{326112}$	$\frac{10923}{108704}$	0	$-\frac{1024}{10191}$	0	0	128
129	65	-31	-7	5	$-\frac{2690421}{298936}$	$\frac{8014764159}{27176}$	129140163	$-\frac{11019964416}{37367}$	-4231794001347	4231664861184	129
130	71	-25	-25	11	$-\frac{17}{4782976}$	$\frac{50643}{434816}$	0	$-\frac{4352}{37367}$	0	0	130
131	72	-30	-20	10	$\frac{13}{3587232}$	$-\frac{12909}{108704}$	0	$\frac{13312}{112101}$	0	0	131

Continued

132	73	-35	-15	9	$\frac{72641345}{896808}$	$\frac{72132855585}{27176}$	1162261467	$\frac{297538949120}{112101}$	-38086146012123	38084983750656	132
133	80	-34	-28	14	$\frac{5}{1195744}$	$\frac{14895}{108704}$	0	$\frac{5120}{37367}$	0	0	133
134	81	-39	-23	13	$\frac{653772079}{896808}$	$\frac{649195674447}{27176}$	10460353203	$\frac{2677850435584}{112101}$	-342775314109107	342764853755904	134
135	89	-43	-31	17	$\frac{1961316227}{298936}$	$\frac{5842761040233}{27176}$	94143178827	$\frac{8033551265792}{37367}$	-3084977826981963	3084883683803136	135
136	-42	84	10	-20	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	136
137	-41	79	15	-21	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	137
138	-40	74	20	-22	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	138
139	-39	69	25	-23	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	139
140	-38	64	30	-24	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	140
141	-37	59	35	-25	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	141
142	-36	54	40	-26	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	142
143	-35	49	45	-27	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	143
144	-34	44	50	-28	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	144
145	-33	39	55	-29	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	145
146	-32	34	60	-30	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	146
147	-31	29	65	-31	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	147
148	-30	24	70	-32	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	148
149	-29	19	75	-33	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	149
150	-28	14	80	-34	$\frac{547}{7845276384}$	$\frac{181057}{79245216}$	$\frac{1}{2187}$	$\frac{560128}{245164887}$	$\frac{3641}{243}$	$\frac{32768}{2187}$	150
151	-33	75	7	-17	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	151
152	-32	70	12	-18	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	152
153	-31	65	17	-19	$\frac{1}{14348928}$	$\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	153

Continued

154	-30	60	22	-20	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	154
155	-29	55	27	-21	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	155
156	-28	50	32	-22	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	156
157	-27	45	37	-23	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	157
158	-26	40	42	-24	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	158
159	-25	35	47	-25	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	159
160	-24	30	52	-26	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	160
161	-23	25	57	-27	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	161
162	-22	20	62	-28	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	162
163	-21	15	67	-29	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	163
164	-20	10	72	-30	$\frac{61}{871697376}$	$-\frac{20191}{8805024}$	$-\frac{1}{243}$	$\frac{62464}{27240543}$	$\frac{3641}{27}$	$-\frac{32768}{243}$	164
165	-24	66	4	-14	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	165
166	-23	61	9	-15	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	166
167	-22	56	14	-16	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	167
168	-21	51	19	-17	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	168
169	-20	46	24	-18	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	169
170	-19	41	29	-19	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	170
171	-18	36	34	-20	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	171
172	-17	31	39	-21	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	172
173	-16	26	44	-22	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	173
174	-15	21	49	-23	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	174
175	-14	16	54	-24	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	175

Continued

176	-13	11	59	-25	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	176
177	-12	6	64	-26	$\frac{7}{96855264}$	$-\frac{2317}{978336}$	$-\frac{1}{27}$	$\frac{7168}{3026727}$	$\frac{3641}{3}$	$-\frac{32768}{27}$	177
178	-15	57	1	-11	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	178
179	-14	52	6	-12	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	179
180	-13	47	11	-13	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	180
181	-12	42	16	-14	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	181
182	-11	37	21	-15	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	182
183	-10	32	26	-16	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	183
184	-9	27	31	-17	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	184
185	-8	22	36	-18	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	185
186	-7	17	41	-19	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	186
187	-6	12	46	-20	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	187
188	-5	7	51	-21	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	188
189	-4	2	56	-22	$\frac{1}{10761696}$	$-\frac{331}{108704}$	$-\frac{1}{3}$	$\frac{1024}{336303}$	10923	$-\frac{32768}{3}$	189
190	-6	48	-2	-8	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	190
191	-5	43	3	-9	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	191
192	-4	38	8	-9	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	192
193	-3	33	13	-11	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	193
194	-2	28	18	-12	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	194
195	-1	23	23	-13	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	195
196	0	18	28	-14	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	196
197	1	13	33	-15	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	197

Continued

198	2	8	38	-16	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	198
199	3	3	43	-17	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	199
200	4	-2	48	-18	$\frac{1}{3587232}$	$-\frac{993}{108704}$	-3	$\frac{1024}{112101}$	98307	-98304	200
201	3	39	-5	-5	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	201
202	4	34	0	-6	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	202
203	5	29	5	-7	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	203
204	6	24	10	-8	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	204
205	7	19	15	-9	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	205
206	8	14	20	-10	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	206
207	9	9	25	-11	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	207
208	10	4	30	-12	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	208
209	11	-1	35	-13	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	209
210	12	-6	40	-14	$\frac{7}{3587232}$	$-\frac{6951}{108704}$	-27	$\frac{7168}{112101}$	884763	-884736	210
211	12	30	-8	-2	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	211
212	13	25	-3	-3	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	212
213	14	20	2	-4	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	213
214	15	15	7	-5	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	214
215	16	10	12	-6	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	215
216	17	5	17	-7	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	216
217	18	0	22	-8	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	217
218	19	-5	27	-9	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	218
219	20	-10	32	-10	$\frac{61}{3587232}$	$-\frac{60573}{108704}$	-243	$\frac{62464}{112101}$	7962867	-7962624	219

Continued

220	21	21	-11	1	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	220
221	22	16	-6	0	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	221
222	23	11	-1	-1	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	222
223	24	6	4	-2	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	223
224	25	1	9	-3	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	224
225	26	-4	14	-4	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	225
226	27	-9	19	-5	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	226
227	28	-14	24	-6	$\frac{547}{3587232}$	$-\frac{543171}{108704}$	-2187	$\frac{560128}{112101}$	71665803	-71663616	227
228	30	12	-14	4	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	228
229	31	7	-9	3	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	229
230	32	2	-4	2	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	230
231	33	-3	1	1	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	231
232	34	-8	6	0	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	232
233	35	-13	11	-1	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	233
234	36	-18	16	-2	$\frac{4921}{3587232}$	$-\frac{4886553}{108704}$	-19683	$\frac{5039104}{112101}$	644992227	-644972544	234
235	39	3	-17	7	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	235
236	40	-2	-12	6	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	236
237	41	-7	-7	5	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	237
238	42	-12	-2	4	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	238
239	43	-17	3	3	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	239
240	44	-22	8	2	$\frac{44287}{3587232}$	$-\frac{43976991}{108704}$	-177147	$\frac{45349888}{112101}$	5804930043	-5804752896	240
241	48	-16	-20	10	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	241

Continued

242	49	-11	-15	9	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	242
243	50	-16	-10	8	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	243
244	51	-21	-5	7	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	244
245	52	-26	0	6	$\frac{398581}{3587232}$	$-\frac{395790933}{108704}$	-1594323	$\frac{408146944}{112101}$	52244370387	-52242776064	245
246	57	-15	-23	13	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	246
247	58	-20	-18	12	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	247
248	59	-25	-13	11	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	248
249	60	-30	-8	10	$\frac{3587227}{3587232}$	$-\frac{3562116411}{108704}$	-14348907	$\frac{3673320448}{112101}$	470199333483	-470184984576	249
250	66	-24	-26	16	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	250
251	68	-29	-21	15	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	251
252	68	-34	-16	14	$\frac{32285041}{3587232}$	$-\frac{32059045713}{108704}$	-129140163	$\frac{33059881984}{112101}$	4231794001347	-4231664861184	252
253	75	-33	-29	19	$\frac{1}{14348928}$	$-\frac{993}{434816}$	0	$\frac{256}{112101}$	0	0	253
254	76	-38	-24	18	$\frac{290565367}{3587232}$	$-\frac{288531409431}{108704}$	-1162261467	$\frac{297538935808}{112101}$	38086146012123	-38084983750656	254
255	84	-42	-32	22	$\frac{60816007}{83424}$	$-\frac{60390294951}{2528}$	-10460353203	$\frac{62275591168}{2607}$	342775314109107	-342764853755904	255
256	-25	11	71	-25	$-\frac{1}{31381105536}$	$\frac{331}{316980864}$	$\frac{1}{2187}$	$-\frac{256}{245164887}$	$-\frac{3641}{243}$	$\frac{32768}{2187}$	256
257	-17	7	63	-21	$-\frac{1}{3486789504}$	$\frac{331}{35220096}$	$\frac{1}{243}$	$-\frac{256}{27240543}$	$-\frac{3641}{27}$	$\frac{32768}{243}$	257
258	-9	3	55	-17	$-\frac{1}{387421056}$	$\frac{331}{3913344}$	$\frac{1}{27}$	$-\frac{256}{3026727}$	$-\frac{3641}{3}$	$\frac{32768}{27}$	258
259	-1	-1	47	-13	$-\frac{1}{43046784}$	$\frac{331}{434816}$	$\frac{1}{3}$	$-\frac{256}{336303}$	-10923	$\frac{32768}{3}$	259
260	7	-5	39	-9	$-\frac{1}{4782976}$	$\frac{2979}{434816}$	3	$-\frac{256}{37367}$	-98307	98304	260
261	15	-19	31	-5	$-\frac{9}{4782976}$	$\frac{26811}{434816}$	27	$-\frac{2304}{37367}$	-884763	884736	261
262	23	-13	23	-1	$-\frac{81}{4782976}$	$\frac{241299}{434816}$	243	$-\frac{20736}{37367}$	-7962867	7962624	262
263	31	-17	15	3	$-\frac{729}{4782976}$	$\frac{2171691}{434816}$	2187	$-\frac{186624}{37367}$	-71665803	71663616	263

Continued

264	39	-21	7	7	$\frac{6561}{4782976}$	$\frac{19545219}{434816}$	19683	$\frac{1679616}{37367}$	-644992227	644972544	264
265	47	-25	-1	11	$\frac{59049}{4782976}$	$\frac{175906971}{434816}$	177417	$\frac{15116544}{37367}$	-5804930043	5804752896	265
266	55	-29	-9	15	$\frac{531441}{4782976}$	$\frac{1583162739}{434816}$	1594323	$\frac{136048896}{37367}$	-52244370387	52242776064	266
267	63	-33	-17	19	$\frac{4782969}{4782976}$	$\frac{14248464651}{434816}$	14348907	$\frac{1224440064}{37367}$	-470199333483	470184984576	267
268	71	-37	-25	23	$\frac{43046721}{4782976}$	$\frac{128236181859}{434816}$	129140163	$\frac{11019960576}{37367}$	-4231794001347	4231664861184	268
269	79	-41	-33	27	$\frac{387420489}{4782976}$	$\frac{1154125636731}{434816}$	1162261467	$\frac{99179645184}{37367}$	-38086146012123	38084983750656	269
270	-22	8	62	-16	$\frac{1}{31381105536}$	$\frac{331}{316980864}$	$\frac{1}{2187}$	$\frac{256}{245164887}$	$\frac{3641}{243}$	$\frac{32768}{2187}$	270
271	-14	4	54	-12	$\frac{1}{3486789504}$	$\frac{331}{35220096}$	$\frac{1}{243}$	$\frac{256}{27240543}$	$\frac{3641}{27}$	$\frac{32768}{243}$	271
272	-6	0	46	-8	$\frac{1}{387421056}$	$\frac{331}{3913344}$	$\frac{1}{27}$	$\frac{256}{3026727}$	$\frac{3641}{3}$	$\frac{32768}{27}$	272
273	2	-4	38	-4	$\frac{1}{43046784}$	$\frac{331}{434816}$	$\frac{1}{3}$	$\frac{256}{336303}$	10923	$\frac{32768}{3}$	273
274	10	-8	30	0	$\frac{1}{4782976}$	$\frac{2979}{434816}$	-3	$\frac{256}{37367}$	98307	-98304	274
275	18	-12	22	4	$\frac{9}{4782976}$	$\frac{26811}{434816}$	-27	$\frac{2304}{37367}$	884763	-884736	275
276	26	-16	14	8	$\frac{81}{4782976}$	$\frac{241299}{434816}$	-243	$\frac{20736}{37367}$	7962867	-7962624	276
277	34	-20	6	12	$\frac{729}{4782976}$	$\frac{2171691}{434816}$	-2187	$\frac{186624}{37367}$	71665803	-71663616	277
278	42	-24	-2	16	$\frac{6561}{4782976}$	$\frac{19545219}{434816}$	-19683	$\frac{1679616}{37367}$	644992227	-644972544	278
279	50	-28	-10	20	$\frac{59049}{4782976}$	$\frac{175906971}{434816}$	-177147	$\frac{15116544}{37367}$	5804930043	-5804752896	279
280	58	-32	-18	24	$\frac{531441}{4782976}$	$\frac{1583162739}{434816}$	-1594323	$\frac{136048896}{37367}$	52244370387	-52242776064	280
281	66	-36	-26	28	$\frac{4782969}{4782976}$	$\frac{14248464651}{434816}$	-14348907	$\frac{1224440064}{37367}$	470199333483	-470184984576	281
282	74	-40	-34	32	$\frac{43046721}{4782976}$	$\frac{128236181859}{434816}$	-129140163	$\frac{11019960576}{37367}$	4231794001347	-4231664861184	282
283	-19	5	53	-7	$\frac{1}{31381105536}$	$\frac{331}{316980864}$	$\frac{1}{2187}$	$\frac{256}{245164887}$	$\frac{3641}{243}$	$\frac{32768}{2187}$	283
284	-11	1	45	-3	$\frac{1}{3486789504}$	$\frac{331}{35220096}$	$\frac{1}{243}$	$\frac{256}{27240543}$	$\frac{3641}{27}$	$\frac{32768}{243}$	284
285	-3	-3	37	1	$\frac{1}{387421056}$	$\frac{331}{3913344}$	$\frac{1}{27}$	$\frac{256}{3026727}$	$\frac{3641}{3}$	$\frac{32768}{27}$	285

Continued

286	5	-7	29	5	$-\frac{1}{387421056}$	$\frac{331}{434816}$	$\frac{1}{3}$	$-\frac{256}{336303}$	-10923	$\frac{32768}{3}$	286
287	13	-11	21	9	$-\frac{1}{4782976}$	$\frac{2979}{434816}$	3	$-\frac{256}{37367}$	-98307	98304	287
288	21	-15	13	13	$\frac{9}{4782976}$	$-\frac{26811}{434816}$	-27	$\frac{2304}{37367}$	884763	-884736	288
289	29	-19	5	17	$-\frac{81}{4782976}$	$\frac{241299}{434816}$	243	$-\frac{20736}{37367}$	-7962867	7962624	289
290	37	-23	-3	21	$-\frac{729}{4782976}$	$\frac{2171691}{434816}$	2187	$-\frac{186624}{37367}$	-71665803	71663616	290
291	45	-27	-11	25	$-\frac{6561}{4782976}$	$\frac{19545219}{434816}$	19683	$-\frac{1679616}{37367}$	-644992227	644972544	291
292	53	-31	-19	29	$-\frac{59049}{4782976}$	$\frac{175906971}{434816}$	177147	$-\frac{15116544}{37367}$	-5804930043	5804752896	292
293	61	-35	-27	33	$-\frac{531441}{4782976}$	$\frac{1583162739}{434816}$	1594323	$-\frac{136048896}{37367}$	-52244370387	52242776064	293
294	69	-39	-35	37	$-\frac{4782969}{4782976}$	$\frac{14248464651}{434816}$	14348907	$-\frac{1224440064}{37367}$	-470199333483	470184984576	294
295	-16	2	44	2	$\frac{1}{31381105536}$	$-\frac{331}{316980864}$	$-\frac{1}{2187}$	$\frac{256}{245164887}$	$\frac{3641}{243}$	$-\frac{32768}{2187}$	295
296	-8	-2	36	6	$\frac{1}{3486789504}$	$-\frac{331}{35220096}$	$-\frac{1}{243}$	$\frac{256}{27240543}$	$\frac{3641}{27}$	$-\frac{32768}{243}$	296
297	0	-6	28	10	$\frac{1}{387421056}$	$-\frac{331}{3913344}$	$-\frac{1}{27}$	$\frac{256}{3026727}$	$\frac{3641}{3}$	$-\frac{32768}{27}$	297
298	8	-10	20	14	$\frac{1}{43046784}$	$-\frac{331}{434816}$	$-\frac{1}{3}$	$\frac{256}{336303}$	10923	$-\frac{32768}{3}$	298
299	16	-14	12	18	$\frac{1}{4782976}$	$-\frac{2979}{434816}$	-3	$\frac{256}{37367}$	98307	-98304	299
300	24	-18	4	22	$\frac{9}{4782976}$	$-\frac{26811}{434816}$	-27	$\frac{2304}{37367}$	884763	-884736	300
301	32	-22	-4	26	$\frac{81}{4782976}$	$-\frac{241299}{434816}$	-243	$\frac{20736}{37367}$	7962867	-7962624	301
302	40	-26	-12	30	$\frac{729}{4782976}$	$-\frac{2171691}{434816}$	-2187	$\frac{186624}{37367}$	71665803	-71663616	302
303	48	-30	-20	34	$\frac{6561}{4782976}$	$-\frac{19545219}{434816}$	-19683	$\frac{1679616}{37367}$	644992227	-644972544	303
304	56	-34	-28	38	$\frac{59049}{4782976}$	$-\frac{175906971}{434816}$	-177147	$\frac{15116544}{37367}$	5804930043	-5804752896	304
305	64	-38	-36	42	$\frac{531441}{4782976}$	$-\frac{1583162739}{434816}$	-1594323	$\frac{136048896}{37367}$	52244370387	-52242776064	305
306	-13	-1	35	11	$-\frac{1}{31381105536}$	$\frac{331}{316980864}$	$\frac{1}{2187}$	$-\frac{256}{245164887}$	$-\frac{3641}{243}$	$\frac{32768}{2187}$	306
307	-5	-5	27	15	$-\frac{1}{3486789504}$	$\frac{331}{35220096}$	$\frac{1}{243}$	$-\frac{256}{27240543}$	$-\frac{3641}{27}$	$\frac{32768}{243}$	307

Continued

308	3	-9	19	19	$-\frac{1}{387421056}$	$\frac{331}{3913344}$	$\frac{1}{27}$	$-\frac{256}{3026727}$	$-\frac{3641}{3}$	$\frac{32768}{27}$	308
309	11	-13	11	23	$-\frac{1}{43046784}$	$\frac{331}{434816}$	$\frac{1}{3}$	$-\frac{256}{336303}$	-10923	$\frac{32768}{3}$	309
310	19	-17	3	27	$-\frac{1}{4782976}$	$\frac{2979}{434816}$	3	$-\frac{256}{37367}$	-98307	98304	310
311	27	-21	-5	31	$-\frac{9}{4782976}$	$\frac{26811}{434816}$	27	$-\frac{2304}{37367}$	-884763	884736	311
312	35	-25	-13	35	$-\frac{81}{4782976}$	$\frac{241299}{434816}$	243	$-\frac{20736}{37367}$	-7962867	7962624	312
313	43	-29	-21	39	$-\frac{729}{4782976}$	$\frac{2171691}{434816}$	2187	$-\frac{186624}{37367}$	-71665803	71663616	313
314	51	-33	-29	43	$-\frac{6561}{4782976}$	$\frac{19545219}{434816}$	19683	$-\frac{1679616}{37367}$	-644992227	644972544	314
315	59	-37	-37	47	$-\frac{59049}{4782976}$	$\frac{175906971}{434816}$	177147	$-\frac{15116544}{37367}$	-5804930043	5804752896	315
316	-10	-4	26	20	$\frac{1}{31381105536}$	$-\frac{331}{316980864}$	$-\frac{1}{2187}$	$\frac{256}{245164887}$	$\frac{3641}{243}$	$-\frac{32768}{2187}$	316
317	-2	-8	18	24	$\frac{1}{3486789504}$	$-\frac{331}{35220096}$	$-\frac{1}{243}$	$\frac{256}{27240543}$	$\frac{3641}{27}$	$-\frac{32768}{243}$	317
318	6	-12	10	28	$\frac{1}{387421056}$	$-\frac{331}{3913344}$	$-\frac{1}{27}$	$\frac{256}{3026727}$	$\frac{3641}{3}$	$-\frac{32768}{27}$	318
319	14	-16	2	32	$\frac{1}{43046784}$	$-\frac{331}{434816}$	$-\frac{1}{3}$	$\frac{256}{336303}$	10923	$-\frac{32768}{3}$	319
320	22	-20	-6	36	$\frac{1}{4782976}$	$-\frac{2979}{434816}$	-3	$\frac{256}{37367}$	98307	-98304	320
321	30	-24	-14	40	$\frac{9}{4782976}$	$-\frac{26811}{434816}$	-27	$\frac{2304}{37367}$	884763	-884736	321
322	35	-28	-22	44	$\frac{81}{4782976}$	$-\frac{241299}{434816}$	-243	$\frac{20736}{37367}$	7962867	-7962624	322
323	46	-32	-30	48	$\frac{729}{4782976}$	$-\frac{2171691}{434816}$	-2187	$\frac{186624}{37367}$	71665803	-71663616	323
324	54	-36	-38	52	$\frac{6561}{4782976}$	$-\frac{19545219}{434816}$	-19683	$\frac{1679616}{37367}$	644992227	-644972544	324
325	-7	-7	17	29	$-\frac{1}{31381105536}$	$\frac{331}{316980864}$	$\frac{1}{2187}$	$-\frac{256}{245164887}$	$-\frac{3641}{243}$	$\frac{32768}{2187}$	325
326	1	-11	9	33	$-\frac{1}{3486789504}$	$\frac{331}{35220096}$	$\frac{1}{243}$	$-\frac{256}{27240543}$	$-\frac{3641}{27}$	$\frac{32768}{243}$	326
327	9	-15	1	37	$-\frac{1}{387421056}$	$\frac{331}{3913344}$	$\frac{1}{27}$	$-\frac{256}{3026727}$	$-\frac{3641}{3}$	$\frac{32768}{27}$	327
328	17	-19	-7	41	$-\frac{1}{43046784}$	$\frac{331}{434816}$	$\frac{1}{3}$	$-\frac{256}{336303}$	-10923	$\frac{32768}{3}$	328
329	25	-23	-15	45	$-\frac{1}{4782976}$	$\frac{2979}{434816}$	3	$-\frac{256}{37367}$	-98307	98304	329

Continued

330	33	-27	-23	49	$-\frac{9}{4782976}$	$\frac{26811}{434816}$	27	$-\frac{2304}{37367}$	-884763	884736	330
331	41	-31	-31	53	$-\frac{81}{4782976}$	$\frac{241299}{434816}$	243	$-\frac{20736}{37367}$	-7962867	7962624	331
332	49	-35	-39	57	$-\frac{729}{4782976}$	$\frac{2171691}{434816}$	2187	$-\frac{186624}{37367}$	-71665803	71663616	332
333	-4	-10	8	38	$\frac{1}{31381105536}$	$-\frac{331}{316980864}$	$-\frac{1}{2187}$	$\frac{256}{245164887}$	$\frac{3641}{243}$	$-\frac{32768}{2187}$	333
334	4	-14	0	42	$\frac{1}{3486789504}$	$-\frac{331}{35220096}$	$-\frac{1}{243}$	$\frac{256}{27240543}$	$\frac{3641}{27}$	$-\frac{32768}{243}$	334
335	12	-18	-8	46	$\frac{1}{387421056}$	$-\frac{331}{3913344}$	$-\frac{1}{27}$	$\frac{256}{3026727}$	$\frac{3641}{3}$	$-\frac{32768}{27}$	335
336	20	-22	-16	50	$\frac{1}{43046784}$	$-\frac{331}{434816}$	$-\frac{1}{3}$	$\frac{256}{336303}$	10923	$-\frac{32768}{3}$	336
337	28	-26	-24	54	$\frac{1}{4782976}$	$-\frac{2979}{434816}$	-3	$\frac{256}{37367}$	98307	-98304	337
338	36	-30	-32	58	$\frac{9}{4782976}$	$-\frac{26811}{434816}$	-27	$\frac{2304}{37367}$	884763	-884736	338
339	44	-34	-40	62	$\frac{81}{4782976}$	$-\frac{241299}{434816}$	-243	$\frac{20736}{37367}$	7962867	-7962624	339
340	-1	-13	-1	47	$-\frac{1}{31381105536}$	$\frac{331}{316980864}$	$\frac{1}{2187}$	$-\frac{256}{245164887}$	$-\frac{3641}{243}$	$\frac{32768}{2187}$	340
341	7	-17	-9	51	$-\frac{1}{3486789504}$	$\frac{331}{35220096}$	$\frac{1}{243}$	$-\frac{256}{27240543}$	$-\frac{3641}{27}$	$\frac{32768}{243}$	341
342	15	-21	-17	55	$-\frac{1}{387421056}$	$\frac{331}{3913344}$	$\frac{1}{27}$	$-\frac{256}{3026727}$	$-\frac{3641}{3}$	$\frac{32768}{27}$	342
343	23	-25	-25	59	$-\frac{1}{43046784}$	$\frac{331}{434816}$	$\frac{1}{3}$	$-\frac{256}{336303}$	-10923	$\frac{32768}{3}$	343
344	31	-29	-33	63	$-\frac{1}{4782976}$	$\frac{2979}{434816}$	3	$-\frac{256}{37367}$	-98307	98304	344
345	39	-33	-41	67	$-\frac{9}{4782976}$	$\frac{26811}{434816}$	27	$-\frac{2304}{37367}$	-884763	884736	345
346	2	-16	-10	56	$\frac{1}{31381105536}$	$-\frac{331}{316980864}$	$-\frac{1}{2187}$	$\frac{256}{245164887}$	$\frac{3641}{243}$	$-\frac{32768}{2187}$	346
347	10	-20	-18	60	$\frac{1}{3486789504}$	$-\frac{331}{35220096}$	$-\frac{1}{243}$	$\frac{256}{27240543}$	$\frac{3641}{27}$	$-\frac{32768}{243}$	347

Continued

348	18	-24	-26	64	$\frac{1}{387421056}$	$-\frac{331}{3913344}$	$-\frac{1}{27}$	$\frac{256}{3026727}$	$\frac{3641}{3}$	$-\frac{32768}{27}$	348
349	26	-28	-34	68	$\frac{1}{43046784}$	$-\frac{331}{434816}$	$-\frac{1}{3}$	$\frac{256}{336303}$	10923	$-\frac{32768}{3}$	349
350	34	-32	-42	72	$\frac{1}{4782976}$	$-\frac{2979}{434816}$	-3	$\frac{256}{37367}$	98307	-98304	350
351	5	-19	-19	65	$-\frac{1}{31381105536}$	$\frac{331}{316980864}$	$\frac{1}{2187}$	$-\frac{256}{245164887}$	$-\frac{3641}{243}$	$\frac{32768}{2187}$	351
352	13	-23	-27	69	$-\frac{1}{3486789504}$	$\frac{331}{35220096}$	$\frac{1}{243}$	$-\frac{256}{27240543}$	$-\frac{3641}{27}$	$\frac{32768}{243}$	352
353	21	-27	-35	73	$-\frac{1}{387421056}$	$\frac{331}{3913344}$	$\frac{1}{27}$	$-\frac{256}{3026727}$	$-\frac{3641}{3}$	$\frac{32768}{27}$	353
354	29	-31	-43	77	$-\frac{1}{43046784}$	$\frac{331}{434816}$	$\frac{1}{3}$	$-\frac{256}{336303}$	-10923	$\frac{32768}{3}$	354
355	8	-22	-28	74	$\frac{1}{31381105536}$	$-\frac{331}{316980864}$	$-\frac{1}{2187}$	$\frac{256}{245164887}$	$\frac{3641}{243}$	$-\frac{32768}{2187}$	355
356	16	-26	-36	78	$\frac{1}{3486789504}$	$-\frac{331}{35220096}$	$-\frac{1}{243}$	$\frac{256}{27240543}$	$\frac{3641}{27}$	$-\frac{32768}{243}$	356
357	24	-30	-44	82	$\frac{1}{387421056}$	$-\frac{331}{3913344}$	$-\frac{1}{27}$	$\frac{256}{3026727}$	$\frac{3641}{3}$	$-\frac{32768}{27}$	357
358	11	-25	-37	83	$-\frac{1}{31381105536}$	$\frac{331}{316980864}$	$\frac{1}{2187}$	$-\frac{256}{245164887}$	$-\frac{3641}{243}$	$\frac{32768}{2187}$	358
359	19	-29	-45	87	$-\frac{1}{3486789504}$	$\frac{331}{35220096}$	$\frac{1}{243}$	$-\frac{256}{27240543}$	$-\frac{3641}{27}$	$\frac{32768}{243}$	359
360	14	-28	-46	92	$\frac{1}{31381105536}$	$-\frac{331}{316980864}$	$-\frac{1}{2187}$	$\frac{256}{245164887}$	$\frac{3641}{243}$	$-\frac{32768}{2187}$	360

Remark 4 $S_{16}(\Gamma_0(12))$ is 27 dimensional, $M_{16}(\Gamma_0(12))$ is 33 dimensional, see [18] (Chapter 3, p. 87 and Chapter 5, p. 197), and generated by

$$\begin{aligned} &\Delta_{1,16}, \Delta_{1,16}(2z), \Delta_{1,16}(3z), \Delta_{1,16}(4z), \Delta_{1,16}(6z), \Delta_{1,16}(12z), \\ &\Delta_{2,16}, \Delta_{2,16}(2z), \Delta_{2,16}(3z), \Delta_{2,16}(6z), \\ &\Delta_{3,16,1}, \Delta_{3,16,1}(2z), \Delta_{3,16,1}(4z), \Delta_{3,16,2}(z), \Delta_{3,16,2}(2z), \Delta_{3,16,2}(4z), \\ &\Delta_{4,16}, \Delta_{4,16}(3z), \\ &\Delta_{6,16,1}, \Delta_{6,16,1}(2z), \Delta_{6,16,2}, \Delta_{6,16,2}(2z), \Delta_{6,16,3}, \Delta_{6,16,3}(2z), \\ &\Delta_{12,16,1}, \Delta_{12,16,2}, \Delta_{12,16,3} \text{ (conjugate of } \Delta_{12,16,2}) \end{aligned}$$

where $\Delta_{1,16}$ is the unique newform in $S_{16}(\Gamma_0(1))$; $\Delta_{2,16}$ is the unique newform in $S_{16}(\Gamma_0(2))$; $\Delta_{3,16,1}, \Delta_{3,16,2}$ are the unique newforms in $S_{16}(\Gamma_0(3))$, $\Delta_{4,16}$ is the unique newform in $S_{16}(\Gamma_0(4))$, $\Delta_{6,14,1}, \Delta_{6,14,2}, \Delta_{6,14,3}$ are the unique newforms in $S_{14}(\Gamma_0(6))$ and $\Delta_{12,16,1}, \Delta_{12,16,2}, \Delta_{12,16,3}$ are the unique newforms in $S_{16}(\Gamma_0(12))$. By

Table 3. Expression of f_i in terms of newforms.

$$\begin{aligned}
 f_1 = & \frac{29}{2211840000} \Delta_{1,16}(z) - \frac{29}{10240000} \Delta_{1,16}(2z) + \frac{17997}{81920000} \Delta_{1,16}(3z) + \frac{29}{67500} \Delta_{1,16}(4z) \\
 & - \frac{485919}{10240000} \Delta_{1,16}(6z) + \frac{17997}{2500} \Delta_{1,16}(12z) - \frac{97}{4992000000} \Delta_{2,16}(z) - \frac{97}{39000000} \Delta_{2,16}(2z) \\
 & + \frac{297861}{1664000000} \Delta_{2,16}(3z) + \frac{297861}{13000000} \Delta_{2,16}(6z) - \frac{517}{23601217536} \Delta_{3,16,1}(z) - \frac{517}{327794688} \Delta_{3,16,1}(2z) \\
 & - \frac{517}{720252} \Delta_{3,16,1}(4z) + \frac{1861}{96111360000} \Delta_{3,16,2}(z) + \frac{24193}{5339520000} \Delta_{3,16,2}(2z) + \frac{29776}{46929375} \Delta_{3,16,2}(4z) \\
 & - \frac{11}{6523846656} \Delta_{4,16}(z) - \frac{598035}{2174615552} \Delta_{4,16}(3z) - \frac{77}{2723217408} \Delta_{6,16,1}(z) + \frac{77}{21275136} \Delta_{6,16,1}(2z) \\
 & + \frac{1367}{59050819584} \Delta_{6,16,2}(z) - \frac{1367}{461334528} \Delta_{6,16,2}(2z) + \frac{1547}{110592000000} \Delta_{6,16,3}(z) + \frac{1547}{864000000} \Delta_{6,16,3}(2z) \\
 & - \frac{3133}{50594512896} \Delta_{12,16,1}(z) + (10111t + 98584027470) \Delta_{12,16,2}(z) + (10111t + 99288359730) \Delta_{12,16,3}(z),
 \end{aligned}$$

$$\begin{aligned}
 f_2 = & \frac{6341}{2972712960000} \Delta_{1,16}(z) - \frac{6341}{13762560000} \Delta_{1,16}(2z) + \frac{1148571}{36700160000} \Delta_{1,16}(3z) \\
 & + \frac{6341}{90720000} \Delta_{1,16}(4z) - \frac{31011417}{4587520000} \Delta_{1,16}(6z) + \frac{1148571}{1120000} \Delta_{1,16}(12z) \\
 & - \frac{4643}{1916928000000} \Delta_{2,16}(z) - \frac{4643}{14976000000} \Delta_{2,16}(2z) + \frac{5019003}{212992000000} \Delta_{2,16}(3z) \\
 & + \frac{5019003}{1664000000} \Delta_{2,16}(6z) - \frac{4253}{1510477922304} \Delta_{3,16,1}(z) - \frac{4253}{20978860032} \Delta_{3,16,1}(2z) \\
 & - \frac{4253}{46096128} \Delta_{3,16,1}(4z) + \frac{59381}{24604508160000} \Delta_{3,16,2}(z) + \frac{771953}{1366917120000} \Delta_{3,16,2}(2z) \\
 & + \frac{59381}{750870000} \Delta_{3,16,2}(4z) - \frac{797}{1252578557952} \Delta_{4,16}(z) - \frac{5327451}{139175395328} \Delta_{4,16}(3z) \\
 & - \frac{157}{33888927744} \Delta_{6,16,1}(z) + \frac{157}{264757248} \Delta_{6,16,1}(2z) + \frac{191}{52489617408} \Delta_{6,16,2}(z) \\
 & - \frac{191}{410075136} \Delta_{6,16,2}(2z) + \frac{497}{294912000000} \Delta_{6,16,3}(z) + \frac{497}{2304000000} \Delta_{6,16,3}(2z) \\
 & - \frac{1}{117116928} \Delta_{12,16,1}(z) + \frac{5599t + 33630313230}{7373558809991577600} \Delta_{12,16,2}(z) \\
 & + \frac{-5599t + 34020339570}{7373558809991577600} \Delta_{12,16,3}(z),
 \end{aligned}$$

$$\begin{aligned}
 f_3 = & \frac{899}{495452160000} \Delta_{1,16}(z) - \frac{899}{2293760000} \Delta_{1,16}(2z) + \frac{512007}{18350080000} \Delta_{1,16}(3z) \\
 & + \frac{899}{15120000} \Delta_{1,16}(4z) - \frac{13824189}{2293760000} \Delta_{1,16}(6z) + \frac{512007}{560000} \Delta_{1,16}(12z) \\
 & - \frac{677}{319488000000} \Delta_{2,16}(z) - \frac{677}{2496000000} \Delta_{2,16}(2z) + \frac{2228151}{106496000000} \Delta_{2,16}(3z) \\
 & + \frac{2228151}{832000000} \Delta_{2,16}(6z) - \frac{217}{83915440128} \Delta_{3,16,1}(z) - \frac{217}{1165492224} \Delta_{3,16,1}(2z) \\
 & - \frac{217}{2560896} \Delta_{3,16,1}(4z) + \frac{2953}{1366917120000} \Delta_{3,16,2}(z) + \frac{38389}{75939840000} \Delta_{3,16,2}(2z) \\
 & + \frac{2953}{41715000} \Delta_{3,16,2}(4z) - \frac{107}{208763092992} \Delta_{4,16}(z) - \frac{2396487}{69587697664} \Delta_{4,16}(3z) \\
 & - \frac{611}{152500174848} \Delta_{6,16,1}(z) + \frac{611}{1191407616} \Delta_{6,16,1}(2z) + \frac{95}{29525409792} \Delta_{6,16,2}(z) \\
 & - \frac{95}{230667264} \Delta_{6,16,2}(2z) + \frac{7}{4608000000} \Delta_{6,16,3}(z) + \frac{7}{36000000} \Delta_{6,16,3}(2z) \\
 & - \frac{1535}{202378051584} \Delta_{12,16,1}(z) + \frac{10949t + 80222205690}{199086087869772559520} \Delta_{12,16,2}(z) \\
 & + \frac{-10949t + 80984913030}{199086087869772559520} \Delta_{12,16,3}(z),
 \end{aligned}$$

Continued

$$\begin{aligned}
f_4 = & \frac{1}{645120000} \Delta_{1,16}(z) - \frac{3}{8960000} \Delta_{1,16}(2z) + \frac{3533}{1143360000} \Delta_{1,16}(3z) + \frac{2}{39375} \Delta_{1,16}(4z) \\
& - \frac{95391}{17920000} \Delta_{1,16}(6z) + \frac{3533}{4375} \Delta_{1,16}(12z) - \frac{7}{3744000000} \Delta_{2,16}(z) - \frac{7}{29250000} \Delta_{2,16}(2z) \\
& + \frac{15419}{832000000} \Delta_{2,16}(3z) + \frac{15419}{6500000} \Delta_{2,16}(6z) - \frac{7}{2950152192} \Delta_{3,16,1}(z) - \frac{7}{40974336} \Delta_{3,16,1}(2z) \\
& - \frac{14}{180063} \Delta_{3,16,1}(4z) + \frac{47}{24027840000} \Delta_{3,16,2}(z) + \frac{611}{1334880000} \Delta_{3,16,2}(2z) + \frac{3008}{46929375} \Delta_{3,16,2}(4z) \\
& - \frac{1}{2446442496} \Delta_{4,16}(z) - \frac{16637}{543653888} \Delta_{4,16}(3z) - \frac{199}{57187565568} \Delta_{6,16,1}(z) + \frac{199}{446777856} \Delta_{6,16,1}(2z) \\
& + \frac{505}{177152458752} \Delta_{6,16,2}(z) - \frac{505}{1384003584} \Delta_{6,16,2}(2z) + \frac{151}{110592000000} \Delta_{6,16,3}(z) \\
& + \frac{151}{864000000} \Delta_{6,16,3}(2z) - \frac{1027}{151783538688} \Delta_{12,16,1}(z) + \frac{1613t + 16683315210}{4666080184447795200} \Delta_{12,16,2}(z) + \frac{-1613t + 16795676790}{4666080184447795200} \Delta_{12,16,3}(z),
\end{aligned}$$

$$\begin{aligned}
f_5 = & \frac{187}{139345920000} \Delta_{1,16}(z) - \frac{187}{645120000} \Delta_{1,16}(2z) + \frac{112591}{5160960000} \Delta_{1,16}(3z) \\
& + \frac{187}{4252500} \Delta_{1,16}(4z) - \frac{337773}{71680000} \Delta_{1,16}(6z) + \frac{112591}{157500} \Delta_{1,16}(12z) \\
& - \frac{151}{89856000000} \Delta_{2,16}(z) - \frac{151}{702000000} \Delta_{2,16}(2z) + \frac{496013}{29952000000} \Delta_{2,16}(3z) \\
& + \frac{496013}{234000000} \Delta_{2,16}(6z) - \frac{151}{70803652608} \Delta_{3,16,1}(z) - \frac{151}{983384064} \Delta_{3,16,1}(2z) \\
& - \frac{151}{2160756} \Delta_{3,16,1}(4z) + \frac{523}{288334080000} \Delta_{3,16,2}(z) + \frac{6799}{16018560000} \Delta_{3,16,2}(2z) \\
& + \frac{8368}{140788125} \Delta_{3,16,2}(4z) - \frac{19}{58714619904} \Delta_{4,16}(z) - \frac{532223}{19571539968} \Delta_{4,16}(3z) \\
& - \frac{29}{9531260928} \Delta_{6,16,1}(z) + \frac{29}{74462976} \Delta_{6,16,1}(2z) + \frac{37}{14762704896} \Delta_{6,16,2}(z) \\
& - \frac{37}{11533632} \Delta_{6,16,2}(2z) + \frac{11}{9216000000} \Delta_{6,16,3}(z) + \frac{11}{72000000} \Delta_{6,16,3}(2z) \\
& - \frac{229}{37945884672} \Delta_{12,16,1}(z) + \frac{811t + 29641066470}{9332160368895590400} \Delta_{12,16,2}(z) \\
& + \frac{-811t + 29697560730}{9332160368895590400} \Delta_{12,16,3}(z),
\end{aligned}$$

$$\begin{aligned}
f_6 = & -\frac{527}{46448640000} \Delta_{1,16}(z) + \frac{527}{215040000} \Delta_{1,16}(2z) - \frac{61137}{573440000} \Delta_{1,16}(3z) \\
& - \frac{527}{1417500} \Delta_{1,16}(4z) + \frac{1650699}{1680000} \Delta_{1,16}(6z) - \frac{61137}{17500} \Delta_{1,16}(12z) \\
& - \frac{629}{29952000000} \Delta_{2,16}(z) - \frac{629}{234000000} \Delta_{2,16}(2z) + \frac{587709}{3328000000} \Delta_{2,16}(3z) \\
& + \frac{587709}{26000000} \Delta_{2,16}(6z) - \frac{5}{23601217536} \Delta_{3,16,1}(z) - \frac{5}{327794688} \Delta_{3,16,1}(2z) \\
& - \frac{5}{720252} \Delta_{3,16,1}(4z) + \frac{1217}{96111360000} \Delta_{3,16,2}(z) + \frac{15821}{5339520000} \Delta_{3,16,2}(2z) \\
& + \frac{19472}{46929375} \Delta_{3,16,2}(4z) + \frac{15}{2174615552} \Delta_{4,16}(z) + \frac{3105}{2174615552} \Delta_{4,16}(3z) \\
& + \frac{23}{1059028992} \Delta_{6,16,1}(z) - \frac{23}{8273664} \Delta_{6,16,1}(2z) - \frac{25}{1640300544} \Delta_{6,16,2}(z) \\
& + \frac{25}{12814848} \Delta_{6,16,2}(2z) + \frac{371}{27648000000} \Delta_{6,16,3}(z) + \frac{371}{216000000} \Delta_{6,16,3}(2z) \\
& - \frac{5}{1405403136} \Delta_{12,16,1}(z) + \frac{2249t - 655556670}{345635569218355200} \Delta_{12,16,2}(z) \\
& - \frac{2249t + 498891330}{345635569218355200} \Delta_{12,16,3}(z),
\end{aligned}$$

Continued

$$\begin{aligned}
 f_7 = & -\frac{121859542812417387532763153}{47140015663246252474728429649920000} \Delta_{1,16}(z) \\
 & -\frac{951140139433279244737601494210315446579282556996701576273333095511}{6144107841139294607117668839427510483552119903913404486424985600000} \Delta_{1,16}(2z) \\
 & -\frac{21302225876428712615716637}{482700167554923278701690880000} \Delta_{1,16}(3z) \\
 & -\frac{379409075464544302185315894706225042187514699339478015118057635393799}{8538502808642107946656172137145584421995225454703187117164134400000} \Delta_{1,16}(4z) \\
 & -\frac{26407113627491667178436267902753339023532093370734285799292395083112121}{6144107841139294607117668839427510483552119903913404486424985600000} \Delta_{1,16}(6z) \\
 & +\frac{1969424271653554293105803759254452282916421075920629041644919622330879781}{5376094360996882781227960234499071673108104915924228925621862400000} \Delta_{1,16}(12z) \\
 & +\frac{36720058380953696452825273}{8473327360904972429388029952000000} \Delta_{2,16}(z) \\
 & +\frac{3319662368135994360962746630782643327556238540571350141802886489844913}{17971515435332436725819181355325468164389950718946708122793082880000000} \Delta_{2,16}(2z) \\
 & -\frac{145634444765492327012883569}{3663349485907899883003904000000} \Delta_{2,16}(3z) \\
 & -\frac{6360303256651524200423491734929785744997662518794838633048666208256499673}{1996835048370270747313242372813940907154438968771856458088120320000000} \Delta_{2,16}(6z) \\
 & +\frac{118647854757936555056855}{25979419509406884495097382043648} \Delta_{3,16,1}(z) \\
 & +\frac{83242702806757313708276633133965682061060259117149650553416928792793}{196680264924278187527365120752681923591083620668152773695847499038720} \Delta_{3,16,1}(2z) \\
 & -\frac{4975663774361885770024976058051545503547488196197091586599181725785319}{221265298039812960968285760846767164039969073251671870407828436418560} \Delta_{3,16,1}(4z) \\
 & -\frac{23062692139224271815795569}{5501398456967329570302282792960000} \Delta_{3,16,2}(z) \\
 & -\frac{7613847806508625869819880062082885878544900885490999765032138052460227}{12815072929656283726795677781835929991068834089587383407560913715200000} \Delta_{3,16,2}(2z) \\
 & -\frac{7281724628231679140155914143013882991757662371586746783126233676507081919}{57667828183453276770580550018261684959809753403143225334024111718400000} \Delta_{3,16,2}(4z) \\
 & +\frac{1692717589838997548293}{5958892194165451739291342340096} \Delta_{4,16}(z) + \frac{189860527089807200143294831}{3310495663425250966272967966720} \Delta_{4,16}(3z) \\
 & +\frac{5696552799715101190393631}{1022939643182896076994459417968640} \Delta_{6,16,1}(z) \\
 & +\frac{7634874489069267246353244951577999282250372020067519553124845005509999}{8578285400928133256001232574366973128934185609141740206580425534996480} \Delta_{6,16,1}(2z) \\
 & -\frac{181480184405315924801251}{39610068890164772906176436109312} \Delta_{6,16,2}(z) \\
 & -\frac{88332540284712745105418185210418212158613084627116337754328435011033}{332166690336690874198543968105187305368504179602104977923978883497984} \Delta_{6,16,2}(2z) \\
 & -\frac{38342834754288358036199}{12363804514939162105138176000000} \Delta_{6,16,3}(z) \\
 & -\frac{62612746747939787550994711742495477820597898524627498312231359186779}{129602274774031995618888327081674049262427529223173375885527040000000} \Delta_{6,16,3}(2z) \\
 & +\frac{3515277290858932535620781}{271502025596663394871461785370624} \Delta_{12,16,1}(z) \\
 & +\frac{-419382167695689085876267t - 3747578352221313324333197997030}{568267368514024907209190826420480245760} \Delta_{12,16,2}(z) \\
 & +\frac{419382167695689085876267t - 3776792514022995026055338756250}{568267368514024907209190826420480245760} \Delta_{12,16,3}(z) \\
 & +\frac{207767842132293}{12804139123145291671075245260800} E_{16}(z) \\
 & +\frac{389352657132658554776560538059030483705486122469087773}{7219369180216201846903536789484910023526820456630348097126400} E_{16}(2z) \\
 & -\frac{2343816}{190054142030133966098225} E_{16}(3z) \\
 & -\frac{12728309157919516667880818187040166611739623824482340625581}{14438738360432403693807073578969820047053640913260696194252800} E_{16}(4z) \\
 & -\frac{63628032768920311754117531280298428267819697026418673529}{61181094747594930905962176182075508673956105564663966924800} E_{16}(6z) \\
 & +\frac{6935936546517597174149866024128733438060610717383151483691}{3609684590108100923451768394742455011763410228315174048563200} E_{16}(12z)
 \end{aligned}$$

Continued

$$\begin{aligned}
 f_8 = & \frac{112591}{1720320000} \Delta_{1,16}(z) - \frac{1013319}{71680000} \Delta_{1,16}(2z) + \frac{99379467}{573440000} \Delta_{1,16}(3z) + \frac{112591}{52500} \Delta_{1,16}(4z) - \frac{2683245609}{71680000} \Delta_{1,16}(6z) + \frac{99379467}{17500} \Delta_{1,16}(12z) \\
 & + \frac{496013}{9984000000} \Delta_{2,16}(z) + \frac{496013}{78000000} \Delta_{2,16}(2z) - \frac{722228319}{3328000000} \Delta_{2,16}(3z) - \frac{722228319}{26000000} \Delta_{2,16}(6z) + \frac{1359}{32374784} \Delta_{3,16,1}(z) + \frac{12231}{4046848} \Delta_{3,16,1}(2z) \\
 & + \frac{1359}{988} \Delta_{3,16,1}(4z) + \frac{4707}{131840000} \Delta_{3,16,2}(z) + \frac{550719}{65920000} \Delta_{3,16,2}(2z) + \frac{75312}{64375} \Delta_{3,16,2}(4z) - \frac{532223}{6523846656} \Delta_{4,16}(z) - \frac{90876411}{2174615552} \Delta_{4,16}(3z) \\
 & + \frac{261}{4358144} \Delta_{6,16,1}(z) - \frac{261}{34048} \Delta_{6,16,1}(2z) + \frac{333}{6750208} \Delta_{6,16,2}(z) - \frac{333}{52736} \Delta_{6,16,2}(2z) + \frac{24057}{1024000000} \Delta_{6,16,3}(z) + \frac{24057}{8000000} \Delta_{6,16,3}(2z) \\
 & - \frac{687}{5783552} \Delta_{12,16,1}(z) + \frac{-811t - 29641066470}{474122865868800} \Delta_{12,16,1}(z) + \frac{811t - 29697560730}{474122865868800} \Delta_{12,16,1}(z), \\
 \\
 f_9 = & \frac{690031126457668339015748339309}{654722439767309062149005967360000} \Delta_{1,16}(z) - \frac{30313735897583273163741275269903192690148498286128910682492659596077}{1306390929722242515838394336983274416565269494569587628926112563200000} \Delta_{1,16}(2z) \\
 & + \frac{799899248786273165894110437249}{60337520944365409837711360000} \Delta_{1,16}(3z) + \frac{31962553103687717441752839695825821595082965832329637243402688332703}{960581565972237143998819365428878247444628636541085506809651200000} \Delta_{1,16}(4z) \\
 & - \frac{144452844689852200011973956319391343390078921158974824977270464465007161}{4838484924897194503105164211049164505797294424331806033059671600000} \Delta_{1,16}(6z) \\
 & + \frac{2690220406148028760084220115772288239153869280444851401304772473461239443}{6048106156121493128881455263811455632246618030414757541324595200000} \Delta_{1,16}(12z) \\
 & - \frac{1546936517995753160934757826897}{1529906329052286688639505408000000} \Delta_{2,16}(z) - \frac{7532172104972029966648742024068871056932259740563894309947589724319883}{60653864594246973949639737074223455054816083676445139914426654720000000} \Delta_{2,16}(2z) \\
 & + \frac{65334608698496719190284691103969}{5952942914600337309881344000000} \Delta_{2,16}(3z) + \frac{26575146197015259853654842934262429580989313999587648723602002250023277129}{20217954864748991316546579024741151684938694558815046638142218240000000} \Delta_{2,16}(6z) \\
 & - \frac{64995111791223278036289759}{57910228673232081350525714432} \Delta_{3,16,1}(z) - \frac{45667778306690858421169710010000976479289998837345895690984544836003}{663795894119438882904857282540301492119907219755015611223485309255680} \Delta_{3,16,1}(2z) \\
 & - \frac{3104496502589097270819711314202655408789815930753729362125470131197739}{82974486764929860363107160317537686514988402469376951402935663656960} \Delta_{3,16,1}(4z) + \frac{1403546857583570285578054947}{943312492621284219873505280000} \Delta_{3,16,2}(z) \\
 & + \frac{14329285972638643543629581091861689872685029904814422513734257246037697}{43250871137589957577935412513696263719857315052357419000518083788800000} \Delta_{3,16,2}(2z) \\
 & + \frac{976862495070941011542240911770100305605653284833136704510347782849426781}{21625435568794978788967706256848131859928657526178709500259041894400000} \Delta_{3,16,2}(4z) \\
 & - \frac{4000415172103677146280243293}{11669497213574009656112212082688} \Delta_{4,16}(z) + \frac{159030405291697987047886934497899}{19449162022623349426853686804480} \Delta_{4,16}(3z) \\
 & - \frac{6490615812812629574062797}{4872254816257506844395192320} \Delta_{6,16,1}(z) + \frac{69975062572553947633992259869926144901086892231804018604718993470651}{35742855837200555233384690598623880372257733714239175274184397291520} \Delta_{6,16,1}(2z) \\
 & + \frac{17411204753689859442393381}{12074399905552438014380867584} \Delta_{6,16,1}(z) - \frac{2659018362675562552390232862198047351785541111494604510810374860653}{13840278764028786424939332004382804390354340816754374080165786812416} \Delta_{6,16,1}(2z) \\
 & + \frac{2052975379400861685204768993}{1831674742953949941501952000000} \Delta_{6,16,1}(z) + \frac{18918595550246483991798052025975313572614147821471554417739143964763}{1458025591207859950712493679668833054202309703760070047871217920000000} \Delta_{6,16,1}(2z) \\
 & + \frac{13225597978971593272817739}{10345298948203909269603024896} \Delta_{12,16,1}(z) + \frac{-141909938796823710450122633r - 1086799943717402981637157352934210}{848084842254711255869023041513062400} \Delta_{12,16,1}(z) \\
 & + \frac{141909938796823710450122633r - 1096685390053989721307112895548990}{848084842254711255869023041513062400} \Delta_{12,16,1}(z) \\
 & + \frac{2211028162513}{4801552171179484376653216972800} E_{16}(z) + \frac{12430263719850791753713866960057397717948624552591579}{812179032774322707766478888170523776467673013709141609267200} E_{16}(2z) \\
 & - \frac{269049582472896}{190054142030133966098225} E_{16}(3z) - \frac{1219071476531555786560758077395113414472912334000592129889}{48730741966459362466598873329023142658806038082254849655603200} E_{16}(4z) \\
 & - \frac{1233051489489362034181672002119412749553178366203000607}{68828731591044297269207448204834947258200618760246962790400} E_{16}(6z) \\
 & + \frac{523016583759503546253828158663967200476615418394172630999}{12182685491614840616649718332255785664701509520563712413900800} E_{16}(12z)
 \end{aligned}$$

$$\begin{aligned}
 f_{10} = & \frac{53}{725760000} \Delta_{1,16}(z) - \frac{53}{3360000} \Delta_{1,16}(2z) + \frac{243}{8960000} \Delta_{1,16}(3z) + \frac{848}{354375} \Delta_{1,16}(4z) - \frac{6561}{1120000} \Delta_{1,16}(6z) \\
 & + \frac{3888}{4375} \Delta_{1,16}(12z) + \frac{659}{29952000000} \Delta_{2,16}(z) + \frac{659}{234000000} \Delta_{2,16}(2z) + \frac{17661}{3328000000} \Delta_{2,16}(3z) + \frac{176661}{26000000} \Delta_{2,16}(6z) \\
 & - \frac{1}{40974336} \Delta_{3,16,1}(z) - \frac{1}{569088} \Delta_{3,16,1}(2z) - \frac{16}{20007} \Delta_{3,16,1}(4z) - \frac{7}{166860000} \Delta_{3,16,2}(z) - \frac{91}{9270000} \Delta_{3,16,2}(2z) \\
 & - \frac{7168}{5214375} \Delta_{3,16,2}(4z) - \frac{27}{271826944} \Delta_{4,16}(z) - \frac{5589}{271826944} \Delta_{4,16}(3z) - \frac{1}{58834944} \Delta_{6,16,1}(z) + \frac{1}{459648} \Delta_{6,16,1}(2z) \\
 & - \frac{1}{45563904} \Delta_{6,16,2}(z) + \frac{1}{355968} \Delta_{6,16,2}(2z) + \frac{71}{6912000000} \Delta_{6,16,3}(z) + \frac{71}{54000000} \Delta_{6,16,3}(2z) + \frac{1}{19519488} \Delta_{12,16,1}(z) \\
 & + \frac{-7t + 1783074}{64006586892288} \Delta_{12,16,1}(z) + \frac{7t + 1295454}{64006586892288} \Delta_{12,16,1}(z),
 \end{aligned}$$

Continued

$$\begin{aligned}
 f_{11} = & \frac{836863}{6720000} \Delta_{1,16}(z) - \frac{7531767}{280000} \Delta_{1,16}(2z) + \frac{686090331}{2240000} \Delta_{1,16}(3z) \\
 & + \frac{53559232}{13125} \Delta_{1,16}(4z) - \frac{18524438937}{280000} \Delta_{1,16}(6z) + \frac{43909781184}{4375} \Delta_{1,16}(12z) \\
 & + \frac{12840549}{104000000} \Delta_{2,16}(z) + \frac{12840549}{812500} \Delta_{2,16}(2z) - \frac{60595434261}{104000000} \Delta_{2,16}(3z) \\
 & - \frac{60595434261}{812500} \Delta_{2,16}(6z) + \frac{11079}{126464} \Delta_{3,16,1}(z) + \frac{99711}{15808} \Delta_{3,16,1}(2z) \\
 & + \frac{709056}{247} \Delta_{3,16,1}(4z) + \frac{43101}{515000} \Delta_{3,16,2}(z) + \frac{5042817}{257500} \Delta_{3,16,2}(2z) \\
 & + \frac{176541696}{64375} \Delta_{3,16,2}(4z) + \frac{1062583}{12741888} \Delta_{4,16}(z) + \frac{52612659}{4247296} \Delta_{4,16}(3z) \\
 & + \frac{6855}{68096} \Delta_{6,16,1}(z) - \frac{6855}{532} \Delta_{6,16,1}(2z) + \frac{17601}{210944} \Delta_{6,16,2}(z) \\
 & - \frac{17601}{1648} \Delta_{6,16,2}(2z) + \frac{2024433}{32000000} \Delta_{6,16,3}(z) + \frac{2024433}{250000} \Delta_{6,16,3}(2z) \\
 & + \frac{1401}{11296} \Delta_{12,16,1}(z) + \frac{7193t + 58049091810}{926021222400} \Delta_{12,16,2}(z) \\
 & + \frac{-7193t + 58550156190}{926021222400} \Delta_{12,16,3}(z),
 \end{aligned}$$

$$\begin{aligned}
 f_{12} = & -\frac{473}{120960000} \Delta_{1,16}(z) + \frac{473}{560000} \Delta_{1,16}(2z) - \frac{102789}{4480000} \Delta_{1,16}(3z) \\
 & - \frac{15136}{118125} \Delta_{1,16}(4z) + \frac{2775303}{560000} \Delta_{1,16}(6z) - \frac{3289248}{4375} \Delta_{1,16}(12z) \\
 & + \frac{1457}{624000000} \Delta_{2,16}(z) + \frac{1457}{4875000} \Delta_{2,16}(2z) + \frac{452709}{208000000} \Delta_{2,16}(3z) \\
 & + \frac{452709}{1625000} \Delta_{2,16}(6z) - \frac{11}{6829056} \Delta_{3,16,1}(z) - \frac{11}{94848} \Delta_{3,16,1}(2z) \\
 & - \frac{352}{6669} \Delta_{3,16,1}(4z) + \frac{37}{27810000} \Delta_{3,16,2}(z) + \frac{481}{1545000} \Delta_{3,16,2}(2z) \\
 & + \frac{75776}{1738125} \Delta_{3,16,2}(4z) + \frac{1}{1225728} \Delta_{6,16,1}(z) - \frac{1}{9576} \Delta_{6,16,1}(2z) \\
 & + \frac{5}{1898496} \Delta_{6,16,2}(z) - \frac{5}{14832} \Delta_{6,16,2}(2z) - \frac{51}{32000000} \Delta_{6,16,3}(z) - \frac{51}{250000} \Delta_{6,16,3}(2z),
 \end{aligned}$$

$$\begin{aligned}
 f_{13} = & -\frac{43}{17418240000} \Delta_{1,16}(z) + \frac{43}{80640000} \Delta_{1,16}(2z) + \frac{1601}{645120000} \Delta_{1,16}(3z) \\
 & - \frac{86}{1063125} \Delta_{1,16}(4z) - \frac{4803}{8960000} \Delta_{1,16}(6z) + \frac{3202}{39375} \Delta_{1,16}(12z) \\
 & - \frac{113}{89856000000} \Delta_{2,16}(z) - \frac{113}{702000000} \Delta_{2,16}(2z) - \frac{83381}{29952000000} \Delta_{2,16}(3z) \\
 & - \frac{83381}{234000000} \Delta_{2,16}(6z) + \frac{7}{8850456576} \Delta_{3,16,1}(z) + \frac{7}{122923008} \Delta_{3,16,1}(2z) \\
 & + \frac{14}{540189} \Delta_{3,16,1}(4z) + \frac{53}{36041760000} \Delta_{3,16,2}(z) + \frac{689}{2002320000} \Delta_{3,16,2}(2z) \\
 & + \frac{6784}{140788125} \Delta_{3,16,2}(4z) - \frac{1}{611610624} \Delta_{4,16}(z) - \frac{23}{67956736} \Delta_{4,16}(3z) \\
 & + \frac{5}{4765630464} \Delta_{6,16,1}(z) - \frac{5}{37231488} \Delta_{6,16,1}(2z) + \frac{1}{3690676224} \Delta_{6,16,2}(z) \\
 & - \frac{1}{28833408} \Delta_{6,16,2}(2z) + \frac{1}{6912000000} \Delta_{6,16,3}(z) + \frac{1}{54000000} \Delta_{6,16,3}(2z) \\
 & + \frac{1}{1185808896} \Delta_{12,16,1}(z) + \frac{-619t + 137004570}{291630011527987200} \Delta_{12,16,2}(z) \\
 & + \frac{619t + 93885030}{291630011527987200} \Delta_{12,16,3}(z),
 \end{aligned}$$

Continued

$$\begin{aligned}
 f_{14} = & -\frac{5999}{737280000} \Delta_{1,16}(z) + \frac{17997}{10240000} \Delta_{1,16}(2z) - \frac{1712421}{81920000} \Delta_{1,16}(3z) \\
 & - \frac{5999}{22500} \Delta_{1,16}(4z) + \frac{46235367}{10240000} \Delta_{1,16}(6z) - \frac{1712421}{2500} \Delta_{1,16}(12z) \\
 & - \frac{99287}{14976000000} \Delta_{2,16}(z) - \frac{99287}{117000000} \Delta_{2,16}(2z) + \frac{51549777}{1664000000} \Delta_{2,16}(3z) \\
 & + \frac{51549777}{13000000} \Delta_{2,16}(6z) - \frac{517}{97124352} \Delta_{3,16,1}(z) - \frac{1551}{4046848} \Delta_{3,16,1}(2z) \\
 & - \frac{517}{2964} \Delta_{3,16,1}(4z) - \frac{1861}{395520000} \Delta_{3,16,2}(z) - \frac{7259}{65920000} \Delta_{3,16,2}(2z) \\
 & - \frac{29776}{193125} \Delta_{3,16,2}(4z) + \frac{199345}{19571539968} \Delta_{4,16}(z) + \frac{5845851}{2174615552} \Delta_{4,16}(3z) \\
 & - \frac{77}{11206656} \Delta_{6,16,1}(z) + \frac{77}{87552} \Delta_{6,16,1}(2z) - \frac{1367}{243007488} \Delta_{6,16,2}(z) \\
 & + \frac{1367}{1898496} \Delta_{6,16,2}(2z) - \frac{13923}{4096000000} \Delta_{6,16,3}(z) - \frac{13923}{32000000} \Delta_{6,16,3}(2z) \\
 & + \frac{3133}{208207872} \Delta_{12,16,1}(z) + \frac{10111t + 98584027470}{12801317378457600} \Delta_{12,16,2}(z) \\
 & + \frac{-10111t + 99288359730}{12801317378457600} \Delta_{12,16,3}(z),
 \end{aligned}$$

$$\begin{aligned}
 f_{15} = & -\frac{419}{362880000} \Delta_{1,16}(2z) + \frac{21017}{45360000} \Delta_{1,16}(4z) - \frac{82989}{4480000} \Delta_{1,16}(6z) + \frac{4081527}{560000} \Delta_{1,16}(12z) \\
 & - \frac{6229}{1872000000} \Delta_{2,16}(2z) + \frac{6495309}{208000000} \Delta_{2,16}(6z) - \frac{49}{1843845121} \Delta_{3,16,1}(2z) \\
 & - \frac{19313}{23048064} \Delta_{3,16,1}(4z) + \frac{1249}{750870000} \Delta_{3,16,2}(2z) + \frac{295697}{375435000} \Delta_{3,16,2}(4z) \\
 & + \frac{115}{33094656} \Delta_{6,16,1}(2z) - \frac{143}{51259392} \Delta_{6,16,2}(2z) + \frac{691}{288000000} \Delta_{6,16,3}(2z)
 \end{aligned}$$

$$\begin{aligned}
 f_{16} = & -\frac{181}{181440000} \Delta_{1,16}(2z) + \frac{1151}{2835000} \Delta_{1,16}(4z) - \frac{36411}{2240000} \Delta_{1,16}(6z) + \frac{227481}{35000} \Delta_{1,16}(12z) \\
 & - \frac{2771}{936000000} \Delta_{2,16}(2z) + \frac{2873691}{104000000} \Delta_{2,16}(6z) - \frac{23}{92192256} \Delta_{3,16,1}(2z) \\
 & - \frac{1067}{1440504} \Delta_{3,16,1}(4z) + \frac{551}{375435000} \Delta_{3,16,2}(2z) + \frac{32632}{46929375} \Delta_{3,16,2}(4z) \\
 & + \frac{461}{148925952} \Delta_{6,16,1}(2z) - \frac{577}{230667264} \Delta_{6,16,2}(2z) + \frac{103}{48000000} \Delta_{6,16,3}(2z)
 \end{aligned}$$

$$\begin{aligned}
 f_{17} = & \frac{3016467424350077822543980849801255783}{58457986859248401255323017927213440000} \Delta_{1,16}(2z) + \frac{108228459021795622087139221463494625081}{7307248357406050156915377240901680000} \Delta_{1,16}(4z) \\
 & + \frac{3101772145184949025052089352942949167419}{2165110624416607453900852515822720000} \Delta_{1,16}(6z) - \frac{33047294429459075615400456620997496510267}{270638828052075931737606564477840000} \Delta_{1,16}(12z) \\
 & - \frac{129972863878150771751944331208617492329}{2110982858806192267553331202927152000000} \Delta_{2,16}(2z) + \frac{747069455447250702545628283720848817140227}{70366095293539742251777067642384000000} \Delta_{2,16}(6z) \\
 & - \frac{29332800951290575846259872152295739797}{207923367660974713584932910163512763392} \Delta_{3,16,1}(2z) + \frac{194818674082894111350832567271094277033}{25990420957621839198116613770439095424} \Delta_{3,16,1}(4z) \\
 & + \frac{167683605463202612943147532114209875299}{846727403414426061932569337789482170000} \Delta_{3,16,2}(2z) + \frac{17818757149853870998254485587531747229597}{423363701707213030966284668894741085000} \Delta_{3,16,2}(4z) \\
 & - \frac{14234744479773888084180581806762011023}{47982315614071087750369133114656791552} \Delta_{6,16,1}(2z) + \frac{46113624946307175246214068266935663717}{520229316657823372451370601137857845248} \Delta_{6,16,2}(2z) \\
 & + \frac{17432668584289644612623510125778279197}{108255531220830372695042625791136000000} \Delta_{6,16,3}(2z) - \frac{1723086865299786724260468}{95851251601776892490402324919235} E_{16}(2z) \\
 & + \frac{28164675296010249628106929524}{95851251601776892490402324919235} E_{16}(4z) + \frac{6645452032544826975883600572}{19170250320355378498080464983847} E_{16}(6z) \\
 & - \frac{61390212371869084720800671916}{95851251601776892490402324919235} E_{16}(12z),
 \end{aligned}$$

Continued

$$f_{18} = -\frac{53}{60480000} \Delta_{1,16}(2z) + \frac{169}{472500} \Delta_{1,16}(4z) - \frac{95987}{6720000} \Delta_{1,16}(6z) + \frac{304751}{52500} \Delta_{1,16}(12z) - \frac{823}{312000000} \Delta_{2,16}(2z) \\ + \frac{7629047}{312000000} \Delta_{2,16}(6z) - \frac{7}{30730752} \Delta_{3,16,1}(2z) - \frac{157}{240084} \Delta_{3,16,1}(4z) + \frac{163}{125145000} \Delta_{3,16,2}(2z) + \frac{9616}{15643125} \Delta_{3,16,2}(4z) \\ + \frac{137}{49641984} \Delta_{6,16,1}(2z) - \frac{43}{19222272} \Delta_{6,16,2}(2z) - \frac{23}{12000000} \Delta_{6,16,3}(2z)$$

$$f_{19} = -\frac{19}{15120000} \Delta_{1,16}(2z) - \frac{517}{1890000} \Delta_{1,16}(4z) + \frac{5967}{560000} \Delta_{1,16}(6z) + \frac{86319}{70000} \Delta_{1,16}(12z) \\ + \frac{29}{78000000} \Delta_{2,16}(2z) - \frac{240327}{26000000} \Delta_{2,16}(6z) - \frac{11}{7682688} \Delta_{3,16,1}(2z) + \frac{181}{960336} \Delta_{3,16,1}(4z) \\ + \frac{13}{15643125} \Delta_{3,16,2}(2z) - \frac{547}{15643125} \Delta_{3,16,2}(4z) + \frac{1}{1378944} \Delta_{6,16,1}(2z) \\ - \frac{11}{8543232} \Delta_{6,16,2}(2z) - \frac{67}{144000000} \Delta_{6,16,3}(2z) + \frac{1327}{1122729984} \Delta_{12,16,3}(z),$$

$$f_{20} = -\frac{1789863301316506331707525830936396010397}{409205908014738808787261125490494080000} \Delta_{1,16}(2z) \\ + \frac{20439003505536392898840742774185573928471}{51150738501842351098407640686311760000} \Delta_{1,16}(4z) \\ - \frac{428118279232272593712814478376474672266921}{15155774370916252177305967610759040000} \Delta_{1,16}(6z) \\ + \frac{5913241812965588697969179203011819458687803}{1894471796364531522163245951344880000} \Delta_{1,16}(12z) \\ - \frac{56074319398445610016903265236664233705843}{18998845729255730407979980826344368000000} \Delta_{2,16}(2z) \\ - \frac{23018447436087765658975155678061034153732609}{6332948576418576802659993608781456000000} \Delta_{2,16}(6z) \\ - \frac{170361382134798200672098480702946097331}{207923367660974713584932910163512763392} \Delta_{3,16,1}(2z) \\ + \frac{4415573731700296730438764143151963140211}{25990420957621839198116613770439095424} \Delta_{3,16,1}(4z) \\ - \frac{68724602578552246188634619544976533298883}{13547638454630816990921109404631714720000} \Delta_{3,16,2}(2z) \\ - \frac{6118695863007742140857697195092294060311199}{6773819227315408495460554702315857360000} \Delta_{3,16,2}(4z) \\ + \frac{93796990846912781659918133557133233667}{111958736432832538084194643934199180288} \Delta_{6,16,1}(2z) \\ - \frac{78411221142131637201294384192952374287}{21676221527409307185473775047410743552} \Delta_{6,16,2}(2z) \\ - \frac{731234530042280556976123929402546347833}{182681208935151253922884431022542000000} \Delta_{6,16,3}(2z) \\ + \frac{2160213030883660555725551077}{9201720153770581679078623192246560} E_{16}(2z) \\ - \frac{211826553064868434036365263458453}{55210320922623490074471739153479360} E_{16}(4z) \\ - \frac{5873317909615535661610105283501}{2300430038442645419769655798061640} E_{16}(6z) \\ - \frac{70554644323491197590334687391203}{11042064184524698014894347830695872} E_{16}(12z),$$

$$f_{21} = \frac{4437}{80000} \Delta_{1,16}(2z) + \frac{8874}{625} \Delta_{1,16}(4z) + \frac{14348907}{80000} \Delta_{1,16}(6z) + \frac{28697814}{625} \Delta_{1,16}(12z) \\ + \frac{2618849}{6500000} \Delta_{2,16}(2z) - \frac{11809150461}{6500000} \Delta_{2,16}(6z) + \frac{243}{1664} \Delta_{3,16,1}(2z) + \frac{486}{13} \Delta_{3,16,1}(4z) \\ + \frac{243}{1250} \Delta_{3,16,2}(2z) + \frac{31104}{625} \Delta_{3,16,2}(4z) + \frac{50301}{250000} \Delta_{6,16,3}(2z)$$

Continued

$$\begin{aligned}
f_{22} = & \frac{29}{2211840000} \Delta_{1,16}(z) - \frac{29}{10240000} \Delta_{1,16}(2z) + \frac{17997}{81920000} \Delta_{1,16}(3z) \\
& + \frac{29}{67500} \Delta_{1,16}(4z) - \frac{485919}{10240000} \Delta_{1,16}(6z) + \frac{17997}{2500} \Delta_{1,16}(12z) \\
& - \frac{97}{4992000000} \Delta_{2,16}(z) - \frac{97}{390000000} \Delta_{2,16}(2z) + \frac{297861}{1664000000} \Delta_{2,16}(3z) \\
& + \frac{297861}{13000000} \Delta_{2,16}(6z) - \frac{517}{23601217536} \Delta_{3,16,1}(z) - \frac{517}{327794688} \Delta_{3,16,1}(2z) \\
& - \frac{517}{720252} \Delta_{3,16,1}(4z) + \frac{1861}{96111360000} \Delta_{3,16,2}(z) + \frac{24193}{5339520000} \Delta_{3,16,2}(2z) \\
& + \frac{29776}{46929375} \Delta_{3,16,2}(4z) - \frac{11}{6523846656} \Delta_{4,16}(z) - \frac{598035}{2174615552} \Delta_{4,16}(3z) \\
& - \frac{77}{2723217408} \Delta_{6,16,1}(z) + \frac{77}{21275136} \Delta_{6,16,1}(2z) + \frac{1367}{59050819584} \Delta_{6,16,2}(z) \\
& - \frac{1367}{461334528} \Delta_{6,16,2}(2z) + \frac{1547}{110592000000} \Delta_{6,16,3}(z) \\
& + \frac{1547}{864000000} \Delta_{6,16,3}(2z) - \frac{3133}{50594512896} \Delta_{12,16,1}(z) \\
& + \frac{10111t + 9858027470}{3110720122965196800} \Delta_{12,16,2}(z) \\
& + \frac{-10111t + 99288359730}{3110720122965196800} \Delta_{12,16,3}(z),
\end{aligned}$$

$$\begin{aligned}
f_{23} = & \frac{29}{2211840000} \Delta_{1,16}(z) - \frac{29}{10240000} \Delta_{1,16}(2z) + \frac{17997}{81920000} \Delta_{1,16}(3z) \\
& + \frac{29}{67500} \Delta_{1,16}(4z) - \frac{485919}{10240000} \Delta_{1,16}(6z) + \frac{17997}{2500} \Delta_{1,16}(12z) \\
& - \frac{97}{4992000000} \Delta_{2,16}(z) - \frac{97}{390000000} \Delta_{2,16}(2z) + \frac{297861}{1664000000} \Delta_{2,16}(3z) \\
& + \frac{297861}{13000000} \Delta_{2,16}(6z) - \frac{517}{23601217536} \Delta_{3,16,1}(z) - \frac{517}{327794688} \Delta_{3,16,1}(2z) \\
& - \frac{517}{720252} \Delta_{3,16,1}(4z) + \frac{1861}{96111360000} \Delta_{3,16,2}(z) + \frac{24193}{5339520000} \Delta_{3,16,2}(2z) \\
& + \frac{29776}{46929375} \Delta_{3,16,2}(4z) - \frac{11}{6523846656} \Delta_{4,16}(z) - \frac{598035}{2174615552} \Delta_{4,16}(3z) \\
& - \frac{77}{2723217408} \Delta_{6,16,1}(z) + \frac{77}{21275136} \Delta_{6,16,1}(2z) + \frac{1367}{59050819584} \Delta_{6,16,2}(z) \\
& - \frac{1367}{461334528} \Delta_{6,16,2}(2z) + \frac{1547}{110592000000} \Delta_{6,16,3}(z) \\
& + \frac{1547}{864000000} \Delta_{6,16,3}(2z) - \frac{3133}{50594512896} \Delta_{12,16,1}(z) \\
& + \frac{10111t + 98584027470}{3110720122965196800} \Delta_{12,16,2}(z) \\
& + \frac{-10111t + 99288359730}{3110720122965196800} \Delta_{12,16,3}(z),
\end{aligned}$$

$$\begin{aligned}
f_{24} = & \frac{47}{120960000} \Delta_{1,16}(2z) - \frac{1021}{15120000} \Delta_{1,16}(4z) + \frac{12771}{4480000} \Delta_{1,16}(6z) \\
& - \frac{36153}{560000} \Delta_{1,16}(12z) - \frac{23}{624000000} \Delta_{2,16}(2z) - \frac{354051}{208000000} \Delta_{2,16}(6z) \\
& - \frac{11}{61461504} \Delta_{3,16,1}(2z) + \frac{181}{7682688} \Delta_{3,16,1}(4z) - \frac{37}{250290000} \Delta_{3,16,2}(2z) \\
& - \frac{5461}{125145000} \Delta_{3,16,2}(4z) + \frac{1}{11031552} \Delta_{6,16,1}(2z) - \frac{5}{17086464} \Delta_{6,16,2}(2z) \\
& + \frac{17}{96000000} \Delta_{6,16,3}(2z)
\end{aligned}$$

Continued

$$\begin{aligned}
 f_{25} = & -\frac{878685640055594915565020963806423215499}{3682853172132649279085350129414446720000} \Delta_{1,16}(2z) - \frac{17148102875315060668515774406980627337693}{460356646516581159885668766176805840000} \Delta_{1,16}(4z) \\
 & - \frac{764441387353080174716126299059936723266257}{136401969338246269595753708496831360000} \Delta_{1,16}(6z) + \frac{8855772537293604258134340281862162119463101}{17050246167280783699469213562103920000} \Delta_{1,16}(12z) \\
 & + \frac{53075774328418042315481049968024046596569}{5170989611563301573671819827437099312000000} \Delta_{2,16}(2z) - \frac{251982361255733948524967012896498175697053597}{56996537187767191223939942479033104000000} \Delta_{2,16}(6z) \\
 & + \frac{259187560276944972438562034995924792999}{467827577237193105566099047867903717632} \Delta_{3,16,1}(2z) - \frac{1769292743442904696991373356477046214087}{116956894309298276391524761966975929408} \Delta_{3,16,1}(4z) \\
 & - \frac{710764737548767893239234786758666502816663}{975429968733418823346319877133483459840000} \Delta_{3,16,2}(2z) - \frac{72586214200056483796638296836978093625927939}{487714984366709411673159938566741729920000} \Delta_{3,16,2}(4z) \\
 & + \frac{30108033285015100885446651141776646637}{27989684108208134521048660983549795072} \Delta_{6,16,1}(2z) - \frac{135958201607280717584912238482447969503}{346819544438548914967580400758571896832} \Delta_{6,16,2}(2z) \\
 & - \frac{30559265484757601996764253053290796906477}{52612188173323561129790716134492096000000} \Delta_{6,16,3}(2z) + \frac{90291524865603541349728867831}{1325047702142963761787321739683504640} E_{16}(2z) \\
 & - \frac{4482405405080632376282908663946101}{3975143106428891285361965219050513920} E_{16}(4z) + \frac{4357238443934985195238757084499917}{1325047702142963761787321739683504640} E_{16}(6z) \\
 & - \frac{8589580801298920020057411776157143}{3975143106428891285361965219050513920} E_{16}(12z),
 \end{aligned}$$

$$\begin{aligned}
 f_{26} = & -\frac{17792737526473618862640692662183084879}{9820941792353731410894267011771857920000} \Delta_{1,16}(2z) - \frac{604268046694798092302561343487389700001}{409205908014738808787261125490494080000} \Delta_{1,16}(4z) \\
 & - \frac{2916755593395337307004937806014706713147}{363738584901990052255343222658216960000} \Delta_{1,16}(6z) + \frac{82564496730812223539805579771600831471307}{15155774370916252177305967610759040000} \Delta_{1,16}(12z) \\
 & + \frac{593550430198936540934438713918027951799}{455972297502137529791519539832264832000000} \Delta_{2,16}(2z) - \frac{7800901820729198328618141073459831300126837}{151990765834045843263839846610754944000000} \Delta_{2,16}(6z) \\
 & + \frac{2041414681166865480787119341583304897}{262640043361231217159915254943384543232} \Delta_{3,16,1}(2z) - \frac{9394248114814513474717929333256082789}{1094333514005130071499646895974355968} \Delta_{3,16,1}(4z) \\
 & - \frac{8144673349886004718952772233084099485337}{650286645822279215564213251422322306560000} \Delta_{3,16,2}(2z) - \frac{307946999365475981365841493188766727499087}{108381107637046535927368875237053717760000} \Delta_{3,16,2}(4z) \\
 & + \frac{8188454816410230827335594606759308077}{424264685429681196950632334908544262144} \Delta_{6,16,1}(2z) - \frac{22045961439581124404486299502237056595}{6242751799893880469416447213654294142976} \Delta_{6,16,2}(2z) \\
 & - \frac{368525190674577063898391405322386940923}{35074792115549040753193810756328064000000} \Delta_{6,16,3}(2z) + \frac{905023961291585293054669339}{883365134761975841191547826455669760} E_{16}(2z) \\
 & - \frac{2921161715984479748627613509695}{176673026952395168238309565291133952} E_{16}(4z) - \frac{64837155038755209796398929244811}{883365134761975841191547826455669760} E_{16}(6z) \\
 & + \frac{79442058594716316954243942123947}{883365134761975841191547826455669760} E_{16}(12z),
 \end{aligned}$$

$$\begin{aligned}
 f_{27} = & -\frac{1718117326894260275110190763342510895373}{409205908014738808787261125490494080000} \Delta_{1,16}(2z) + \frac{18748429703159920733684137268006789090839}{51150738501842351098407640686311760000} \Delta_{1,16}(4z) \\
 & - \frac{419804987919864607826025399977026475308889}{15155774370916252177305967610759040000} \Delta_{1,16}(6z) + \frac{5726686569858080308795744013462033853133627}{1894471796364531522163245951344880000} \Delta_{1,16}(12z) \\
 & - \frac{52187892132874314105286378229620058902387}{18998845729255730407979980826344368000000} \Delta_{2,16}(2z) + \frac{17326978349901625674142837699510091394810881}{6332948576418576802659993608781456000000} \Delta_{2,16}(6z) \\
 & - \frac{140885403710693675549133231988596900547}{207923367660974713584932910163512763392} \Delta_{3,16,1}(2z) - \frac{4375167571183184235491084873160673210483}{25990420957621839198116613770439095424} \Delta_{3,16,1}(4z) \\
 & - \frac{66902063768040144707537539347361592601347}{13547638454630816990921109404631714720000} \Delta_{3,16,2}(2z) - \frac{5991982143626077356701775004399608840684191}{6773819227315408495460554702315857360000} \Delta_{3,16,2}(4z) \\
 & + \frac{109650796883140948080361736018993518595}{111958736432832538084194643934199180288} \Delta_{6,16,1}(2z) - \frac{76078747466447625991206995777656558031}{2167221527409307185473775047410743552} \Delta_{6,16,2}(2z) \\
 & - \frac{713951426227343767144987679152365694297}{182681208935151253922884431022542000000} \Delta_{6,16,3}(2z) + \frac{2160213030883660555725551077}{9201720153770581679078623192246560} E_{16}(2z) \\
 & - \frac{21182653064868434036365263458453}{55210320922623490074471739153479360} E_{16}(4z) - \frac{5873317909615535661610105283501}{2300430038442645419769655798061640} E_{16}(6z) \\
 & + \frac{70554644323491197590334687391203}{11042064184524698014894347830695872} E_{16}(12z).
 \end{aligned}$$

taking t as a root of $x^2 - 69660x - 65283075900$, we see f_1, \dots, f_{27} as linear combinations in **Table 3**.

References

- [1] Williams, K.S. (2012) Fourier Series of a Class of Eta Quotients. *International Journal of Number Theory*, **8**, 993-1004. <http://dx.doi.org/10.1142/S1793042112500595>
- [2] Yao, O.X.M., Xia, E.X.W. and Jin, J. (2013) Explicit Formulas for the Fourier Coefficients of a Class of Eta Quotients. *International Journal of Number Theory*, **9**, 487-503. <http://dx.doi.org/10.1142/S179304211250145X>
- [3] Köhler, G. (2011) Eta Products and Theta Series Identities. Springer-Verlag, Berlin. <http://dx.doi.org/10.1007/978-3-642-16152-0>
- [4] Gordon, B. (1961) Some Identities in Combinatorial Analysis. *Quarterly Journal of Mathematics*, **12**, 285-290.
- [5] Kac, V.G. (1978) Infinite-Dimensional Algebras, Dedekind's η -Function, Classical Möbius Function and the Very Strange Formula. *Advances in Mathematics*, **30**, 85-136. [http://dx.doi.org/10.1016/0001-8708\(78\)90033-6](http://dx.doi.org/10.1016/0001-8708(78)90033-6)
- [6] Macdonald, I.G. (1972) Affine Root Systems and Dedekind's η -Function. *Inventiones Mathematicae*, **15**, 91-143. <http://dx.doi.org/10.1007/BF01418931>
- [7] Zucker, I.J. (1987) A Systematic Way of Converting Infinite Series into Infinite Products. *Journal of Physics A*, **20**, L13-L17. <http://dx.doi.org/10.1088/0305-4470/20/1/003>
- [8] Zucker, I.J. (1990) Further Relations amongst Infinite Series and Products: II. The Evaluation of Three-Dimensional Lattice Sums. *Journal of Physics A*, **23**, 117-132. <http://dx.doi.org/10.1088/0305-4470/23/2/009>
- [9] Kendirli, B. (2015) Evaluation of Some Convolution Sums by Quasimodular Forms. *European Journal of Pure and Applied Mathematics*, **8**, 81-110.
- [10] Kendirli, B. (2015) Evaluation of Some Convolution Sums and Representation Numbers of Quadratic Forms of Discriminant -135. *British Journal of Mathematics and Computer Science*, **6**, 494-531. <http://dx.doi.org/10.9734/BJMCS/2015/13973>
- [11] Kendirli, B. (2014) Evaluation of Some Convolution Sums and the Representation Numbers. *Ars Combinatoria*, **CXVI**, 65-91.
- [12] Kendirli, B. (2012) Cusp Forms in $S_4(\Gamma_0(79))$ and the Number of Representations of Positive Integers by Some Direct Sum of Binary Quadratic Forms with Discriminant -79. *Bulletin of the Korean Mathematical Society*, **49**, 529-572. <http://dx.doi.org/10.4134/BKMS.2012.49.3.529>
- [13] Kendirli, B. (2012) Cusp Forms in $S_4(\Gamma_0(47))$ and the Number of Representations of Positive Integers by Some Direct Sum of Binary Quadratic Forms with Discriminant -47. *International Journal of Mathematics and Mathematical Sciences*, **2012**, Article ID: 303492.
- [14] Kendirli, B. (2012) The Bases of $M_4(\Gamma_0(71))$, $M_6(\Gamma_0(71))$ and the Number of Representations of Integers. *Mathematical Problems in Engineering*, **2013**, Article ID: 695265.
- [15] Alaca, A., Alaca, S. and Williams, K.S. (2006) On the Two-Dimensional Theta Functions of Borweins. *Acta Arithmetica*, **124**, 177-195. <http://dx.doi.org/10.4064/aal24-2-4>
- [16] Alaca, A., Alaca, S. and Williams, K.S. (2006) Evaluation of the convolution sums $\sum_{l+2m=n} \sigma(l)\sigma(m)$ and $\sum_{3l+4m=n} \sigma(l)\sigma(m)$. *Advances in Theoretical and Applied Mathematics*, **1**, 27-48.
- [17] Gordon, B. and Robins, S. (1995) Lacunarity of Dedekind η -Products. *Glasgow Mathematical Journal*, **37**, 1-14. <http://dx.doi.org/10.1017/S0017089500030329>
- [18] Diamond, F. and Shurman, J. (2005) A First Course in Modular Forms. Springer Graduate Texts in Mathematics 228. Springer, New York.