Applied Mathematics, 2015, 6, 496-509
Published Online March 2015 in SciRes. http://www.scirp.org/journal/am
http://dx.doi.org/10.4236/am.2015.63047

Fuzzy Inventory Model for Deteriorating Items with Time Dependent Demand and Partial Backlogging

Sushil Kumar*, U. S. Rajput

Department of Mathematics & Astronomy, University of Lucknow, Lucknow, India
Email: *sushilmath4444@gmail.com

Received 10 February 2015; accepted 10 March 2015; published 11 March 2015

Abstract

In this paper we developed a fuzzy inventory model for deteriorating items with time dependent demand rate. Shortages are allowed and completely backlogged. The backlogging rate of unsatisfied demand is assumed to be a decreasing exponential function of waiting time. The demand rate, deterioration rate and backlogging rate are assumed as a triangular fuzzy numbers. The purpose of our study is to defuzzify the total profit function by signed distance method and centroid method. Further a numerical example is also given to demonstrate the developed crisp and fuzzy models. A sensitivity analysis is also given to show the effect of change of the parameters.

Keywords

Inventory, Deterioration, Shortages, Triangular Fuzzy Number, Signed Distance Method and Centroid Method

1. Introduction

In many inventory models uncertainty is due to fuzziness and fuzziness is the closed possible approach to reality. In recent years some researchers gave their attention towards a time dependent rate because the demand of newly launched products such as fashionable garments, electronic items, mobiles etc. increases with time and later it becomes constant. Deterioration is defined as damage, decay or spoilage of the items that are stored for future use always loose part of their value with passage of time, so deterioration cannot be avoided in any business scenarios. F. Harris (1915) [1] developed first inventory model. Lotfi A. Zadeh (1965) [2] introduced the concept of fuzzy set theory in inventory modeling. L. A. Zadeh [3] and R. E. Bellman (1970) considered an inven-

*Corresponding author.

How to cite this paper: Kumar, S. and Rajput, U.S. (2015) Fuzzy Inventory Model for Deteriorating Items with Time Dependent Demand and Partial Backlogging. *Applied Mathematics*, **6**, 496-509. http://dx.doi.org/10.4236/am.2015.63047

tory model on decision making in fuzzy environment. R. Jain (1976) [4] developed a fuzzy inventory model on decision making in the presence of fuzzy variables. D. Dubois and H. Prade (1978) [5] defined some operations on fuzzy numbers. J. Kacpryzk and P. Staniewski (1982) [6] developed an inventory model for long term inventory policy making through fuzzy decisions. H. J. Zimmerman (1983) [7] tried to use fuzzy sets in operational research. G. Urgeletti Tinarelli (1983) [8] considered the inventory control models and problems. K. S. Park (1987) [9] define the fuzzy set theoretical interpretation of an EOQ problem. M. Vujosevic, D. Petrovic and R. Petrovic (1996) [10] developed an EOQ formula by assuming inventory cost as a fuzzy number. J. S. Yao and H. M. Lee (1999) [11] developed a fuzzy inventory model by considering backorder as a trapezoidal fuzzy number. C. K. Kao and W. K. Hsu (2002) [12] developed a single period inventory model with fuzzy demand. C. H. Hsieh (2002) [13] developed an inventory model and give an approach of optimization of fuzzy production. J. S. Yao and J. Chiang (2003) [14] developed an inventory model without backorders and defuzzified the fuzzy holding cost by signed distance and centroid methods. Sujit D. Kumar, P. K. Kund and A. Goswami (2003) [15] developed an economic production quantity model with fuzzy demand and deterioration rate. J. K. Syed and L. A. Aziz (2007) [16] consider the signed distance method for a fuzzy inventory model without shortages. P. K. De and A. Rawat (2011) [17] developed a fuzzy inventory model without shortages by using triangular fuzzy number. C. K. Jaggi, S. Pareek, A. Sharma and Nidhi (2012) [18] developed a fuzzy inventory model for deteriorating items with time varying demand and shortages.

Sumana saha and Tripti Chakrabarty (2012) [19] developed a fuzzy EOQ model with time varying demand and shortages. D. Dutta and Pawan Kumar (2012) [20] considered a fuzzy inventory model without shortages using a trapezoidal fuzzy number. D. Dutta and Pawan Kumar (2013) [21] [22] considered an optimal replenishment policy for an inventory model without shortages by assuming fuzziness in demand, holding cost and ordering cost. Dipak Kumar Jana, Barun Das and Tapan Kumar Roy (2013) [23] give a fuzzy generic algorithm approach for an inventory model for deteriorating items with backorders under fuzzy inflation and discounting over random planning horizon.

In this paper we consider an inventory model for deteriorating items with time dependent demand rate and partial backlogging. Shortages are allowed and completely backlogged for the next replenishment cycle. The demand rate, deterioration rate and backlogging rate are assumed as triangular fuzzy numbers. The purpose of our study is to defuzzify the total profit function by signed distance method and centroid method and comparing the results of these two methods with the crisp model. **Figure 1** shows the developed model and **Figure 2** and **Figure 3** show the graphs of total profit function with respect to deterioration and backlogging rates.

2. Definitions and Preliminaries

When we are considering the fuzzy inventory model then the following definitions are needed.

(1) A fuzzy set $\tilde{A}$ on the given universal set X is denoted and defined by

$$\tilde{A} = \left\{ \left(x, \lambda_{\tilde{A}}(x) \right) : x \in X \right\}$$

where, $\lambda_{\tilde{A}} : X \to [0,1]$, is called the membership function,

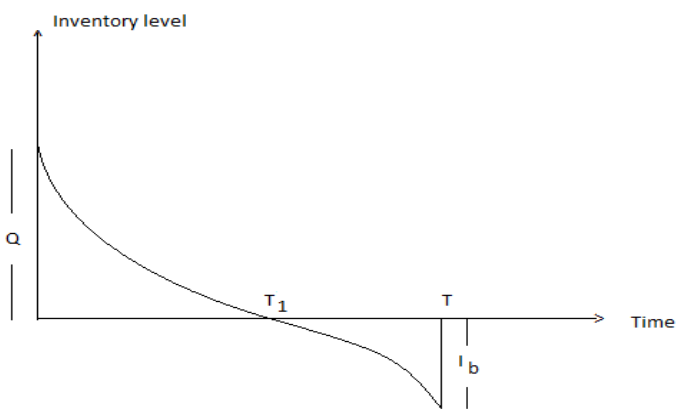

Figure 1. With respect to described model.

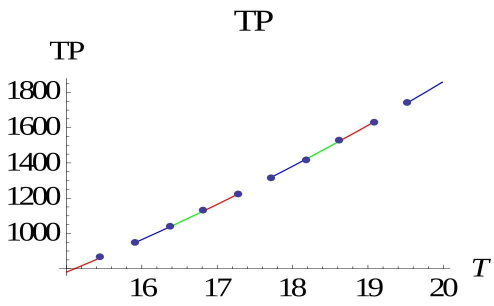

Figure 2. With respect σ.

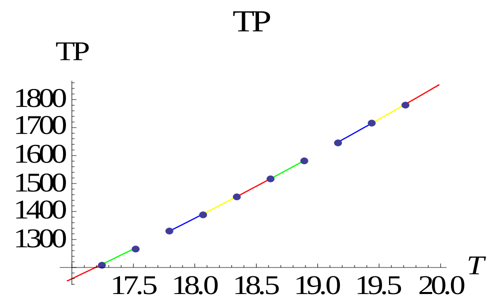

Figure 3. With respect θ.

And, $\lambda_{\tilde{A}}(x) =$ degree of x in $\tilde{A}$.

(2) A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, if its membership function $\lambda_{\tilde{A}}$ has the following properties

$\lambda_{\tilde{A}}(x)$ is upper semi continuous.

$\lambda_{\tilde{A}}(x) = 0$, outside some interval $[a_1, a_4]$.

$\exists$ real numbers a_2 and a_3, $a_1 \le a_2 \le a_3 \le a_4$ such that $\lambda_{\tilde{A}}(x)$ is increasing on $[a_1, a_2]$, decreasing on $[a_3, a_4]$ and $\lambda_{\tilde{A}}(x) = 1$, for each x in $[a_2, a_3]$.

(3) A triangular fuzzy number is specified by the triplet (a_1, a_2, a_3) where $a_1 \prec a_2 \prec a_3$ and defined by its continuous membership function $\lambda_{\tilde{A}} : X \to [0,1]$ as follows

$$\lambda_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2; \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3; \\ 0, & \text{otherwise.} \end{cases}$$

(4) Let $\tilde{A}$ be a fuzzy set defined on R, then the signed distance of $\tilde{A}$ is defined as

$$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\beta)] \mathrm{d}_\alpha$$

where, $A_\alpha = [A_L(\alpha) + A_R(\beta)] = [a + (b - a)\alpha, d - (d - c)\alpha]$, $\alpha \in [0,1]$ is an α cut of a fuzzy set $\tilde{A}$.

(5) If $\tilde{A} = (a, b, c)$ is a triangular fuzzy number then the signed distance of $\tilde{A}$ is defined as

$d(\tilde{A}, 0) = \frac{1}{4}(a + 2b + c)$.

(6) If $\tilde{A}=(a,b,c)$ is a triangular fuzzy number then the centroid method on $\tilde{A}$ is defined as $C(\tilde{A})=\frac{1}{3}(a+b+c)$.

3. Assumptions and Notations

We consider the following assumptions and notations.

The demand rate is $R(t)=\alpha t^{\beta-1}$ where α is a positive constant, for a increasing demand $\beta>1$, and for a decreasing demand $\beta<1$.

1. θ is the deterioration parameter.
2. σ is the backlogging parameter.
3. A is the ordering cost per order.
4. h_c is the holding cost per unit per unit time.
5. d_c is the deterioration cost per unit per unit time.
6. s_c is the shortages cost per unit per unit time.
7. p_c is the purchase cost per unit.
8. s_p is the selling price per unit, where $s_p \succ p_C$.
9. o_{pC} is the opportunity cost per unit due to lost sales.
10. T is the length of order cycle.
11. $\tilde{\theta}$ is the fuzzy deterioration parameter.
12. $\tilde{\sigma}$ is the fuzzy backlogging parameter.
13. $\tilde{\alpha}$ is the fuzzy demand parameter.
14. $\widetilde{TP}(T_1,T)$ is the total fuzzy profit per unit time.
15. T_1 is the time at which shortage starts.
16. $TP(T_1,T)$ is the total profit per unit time.
17. $I(t)$ is the inventory level at any time in $[0,T]$.
18. The inventory system consists only one item.
19. The time horizon T is infinite.
20. The lead time is zero.
21. The replenishment rate is infinite.

3.1. Mathematical Formulation

Suppose an inventory system consists Q units of the product in the beginning of each cycle. Due to demand and deterioration the inventory level decreases in $[0,T_1]$ and becomes zero at $t=T_1$. The interval $[T_1,T]$ is the shortages interval. During the shortages interval the unsatisfied demand is backlogged at a rate of $e^{-\sigma t}$, where t is the waiting time.

The instantaneous inventory level at any time t in $[0,T_1]$ are governed by the following differential equations

$$\frac{dI}{dt}+\theta I=-\alpha t^{\beta-1}, \quad 0\le t\le T_1 \tag{1}$$

with boundary condition $I(0)=Q$

$$\frac{dI}{dt}=-\alpha t^{\beta-1}e^{-\sigma t}, \quad T_1\le t\le T \tag{2}$$

with boundary condition $I(T_1)=0$

$$I(t)=Qe^{-\theta t}-\frac{\alpha t^{\beta}}{\beta}+\frac{\alpha\theta t^{\beta+1}}{\beta(\beta+1)}+\frac{\alpha\theta^2 t^{\beta+2}}{(\beta+1)} \tag{3}$$

The solution of Equation (1) is

$$I(t)=\mathrm{e}^{\theta(T_1-t)}\left\{\frac{\alpha T_1^{\beta}}{\beta}-\frac{\alpha\theta T_1^{\beta+1}}{\beta(\beta+1)}-\frac{\alpha\theta^2 T_1^{\beta+2}}{(\beta+1)}\right\}-\frac{\alpha t^{\beta}}{\beta}+\frac{\alpha\theta t^{\beta+1}}{\beta(\beta+1)}+\frac{\alpha\theta^2 t^{\beta+2}}{(\beta+1)} \quad (4)$$

The solution of Equation (2) is

$$I(t)=\frac{\alpha\sigma}{\beta+1}\left(t^{\beta+1}-T_1^{\beta+1}\right)+\frac{\alpha}{\beta}\left(T_1^{\beta}-t^{\beta}\right) \quad (5)$$

using $I(T_1)=0$, in Equation (3)

$$Q=\mathrm{e}^{\theta T_1}\left\{\frac{\alpha T_1^{\beta}}{\beta}-\frac{\alpha\theta T_1^{\beta+1}}{\beta(\beta+1)}-\frac{\alpha\theta^2 T_1^{\beta+2}}{(\beta+1)}\right\} \quad (6)$$

The ordering cost per cycle is

$$O_C=A \quad (7)$$

The holding cost per cycle is

$$H_C=h_C\int_0^{T_1} I(t)\,\mathrm{d}t;$$

$$H_C=h_C\left[\frac{\alpha T_1^{\beta+1}}{\beta+1}+\frac{\alpha\theta T_1^{\beta+2}}{\beta}\left\{\frac{1}{2}-\frac{1}{\beta+1}+\frac{1}{(\beta+1)(\beta+2)}\right\}-\frac{\alpha\theta^2 T_1^{\beta+3}}{(\beta+1)}\left\{1+\frac{1}{2\beta}-\frac{1}{\beta+3}\right\}\right]. \quad (8)$$

The deterioration cost per cycle is

$$D_C=d_C\left[Q-\int_0^{T_1}\alpha t^{\beta-1}\mathrm{d}t\right];$$

$$D_C=d_C\left[\frac{\alpha\theta T_1^{\beta+1}}{\beta+1}-\frac{\alpha\theta^2 T_1^{\beta+2}}{\beta}\right]. \quad (9)$$

The shortage cost per cycle is

$$S_C=s_C\int_{T_1}^{T}-I(t)\,\mathrm{d}t;$$

$$S_C=s_C\left[\frac{\alpha\sigma}{\beta+1}\left\{\frac{T^2}{\beta+2}+\frac{(\beta+1)T_1^{\beta+2}}{(\beta+2)}-TT_1^{\beta+1}\right\}+\frac{\alpha}{\beta}\left(TT_1^{\beta}-\frac{T_1^{\beta+1}}{\beta+1}-\frac{T^{\beta+1}}{\beta+1}\right)\right]. \quad (10)$$

The purchase cost per cycle in $[0,T_1]$ is

$$PC_1=p_C Q;$$

$$PC_1=p_C\left[\frac{\alpha T_1^{\beta}}{\beta}+\frac{\alpha\theta T_1^{\beta+1}}{\beta+1}-\frac{\alpha\theta^2 T_1^{\beta+2}}{\beta}\right]. \quad (11)$$

The purchase cost per cycle in $[T_1,T]$ is

$$PC_2=p_C\int_{T_1}^{T}\left(\alpha t^{\beta-1}\mathrm{e}^{-\sigma t}\right)\mathrm{d}t;$$

$$PC_2=\frac{p_C\alpha}{\beta}\left(T^{\beta}-T_1^{\beta}\right)-\frac{p_C\alpha^2}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right). \quad (12)$$

Due to lost sales the opportunity cost per cycle in $[T_1,T]$ is

$$CO_p = co_p \int_{T_1}^{T} \left(\alpha t^{\beta-1} - \alpha t^{\beta-1} e^{-\sigma t}\right) dt;$$
$$CO_p = \frac{co_p \alpha \sigma}{\beta+1}\left(T^{\beta+1} - T_1^{\beta+1}\right). \tag{13}$$

The sales revenue cost per cycle in $[0,T]$ is

$$S_R = s_R \left[\int_0^{T_1} \alpha t^{\beta-1} dt + \int_{T_1}^{T} \left(\alpha t^{\beta-1} e^{-\sigma t}\right) dt \right];$$
$$S_R = s_R \left[\frac{\alpha T^{\beta}}{\beta} - \frac{\alpha\sigma T^{\beta+1}}{\beta+1} + \frac{\alpha\sigma T_1^{\beta+1}}{\beta+1} \right]. \tag{14}$$

Therefore the total profit per unit time is

$$TP(T_1,T) = \frac{1}{T}\left[S_R - O_C - H_C - D_C - S_C - PC_1 - PC_2 - CO_P\right];$$
$$\begin{aligned} TP(T_1,T) = \frac{1}{T}\Bigg[& s_R\left\{\frac{\alpha T^{\beta}}{\beta} - \frac{\alpha\sigma T^{\beta+1}}{\beta+1} + \frac{\alpha\sigma T_1^{\beta+1}}{\beta+1}\right\} - A \\ & - h_C\left\{\frac{\alpha T_1^{\beta+1}}{\beta+1} + \frac{\alpha\theta T_1^{\beta+2}}{\beta}\left(\frac{1}{2} - \frac{1}{\beta+1} + \frac{1}{(\beta+1)(\beta+2)}\right) - \frac{\alpha\theta^2 T_1^{\beta+3}}{(\beta+1)}\left(1 + \frac{1}{2\beta} - \frac{1}{\beta+3}\right)\right\} \\ & - d_C\left\{\frac{\alpha\theta T_1^{\beta+1}}{\beta+1} - \frac{\alpha\theta^2 T_1^{\beta+2}}{\beta}\right\} + s_C\left\{\frac{\alpha\sigma}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2} + \frac{(\beta+1)T_1^{\beta+2}}{(\beta+2)} - TT_1^{\beta+1}\right) + \frac{\alpha}{\beta}\left(TT_1^{\beta} - \frac{T_1^{\beta+1}}{\beta+1} - \frac{T^{\beta+1}}{\beta+1}\right)\right\} \\ & - p_C\left\{\frac{\alpha T_1^{\beta}}{\beta} + \frac{\alpha\theta T_1^{\beta+1}}{\beta+1} - \frac{\alpha\theta^2 T_1^{\beta+2}}{\beta}\right\} - \frac{p_C\alpha}{\beta}\left(T^{\beta} - T_1^{\beta}\right) + \frac{p_C\alpha^2}{\beta+1}\left(T^{\beta+1} - T_1^{\beta+1}\right) \\ & - \frac{co_p\alpha\sigma}{\beta+1}\left(T^{\beta+1} - T_1^{\beta+1}\right)\Bigg]. \end{aligned} \tag{15}$$

For a Ist order approximation of $e^{-\theta t}$

$$\begin{aligned} TP(T_1,T) = \frac{1}{T}\Bigg[& s_R\left\{\frac{\alpha T^{\beta}}{\beta} - \frac{\alpha\sigma T^{\beta+1}}{\beta+1} + \frac{\alpha\sigma T_1^{\beta+1}}{\beta+1}\right\} - A - h_C\left\{\frac{\alpha T_1^{\beta+1}}{\beta+1} + \frac{\alpha\theta T_1^{\beta+2}}{2(\beta+2)}\right\} - \frac{\alpha\theta d_C T_1^{\beta+1}}{\beta+1} \\ & + s_C\left\{\frac{\alpha\sigma}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2} + \frac{(\beta+1)T_1^{\beta+2}}{(\beta+2)} - TT_1^{\beta+1}\right) + \frac{\alpha}{\beta}\left(TT_1^{\beta} - \frac{T_1^{\beta+1}}{\beta+1} - \frac{T^{\beta+1}}{\beta+1}\right)\right\} \\ & - p_C\left\{\frac{\alpha T_1^{\beta}}{\beta} + \frac{\alpha\theta T_1^{\beta+1}}{\beta+1}\right\} - \frac{p_C\alpha}{\beta}\left(T^{\beta} - T_1^{\beta}\right) + \frac{p_C\alpha}{\beta+1}\left(T^{\beta+1} - T_1^{\beta+1}\right) \\ & - \frac{co_p\alpha\sigma}{\beta+1}\left(T^{\beta+1} - T_1^{\beta+1}\right)\Bigg], \end{aligned} \tag{16}$$

The necessary condition for $TP(T_1,T)$ to be maximum is that $\frac{\partial TP(T_1,T)}{\partial T_1} = 0$ and $\frac{\partial TP(T_1,T)}{\partial T} = 0$, and solving these equations we find the optimum values of T_1 and T say T_1^* and T^* for which profit is maximum and the sufficient condition is

$$\left(\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 TP(T_1,T)}{\partial T^2}\right) - \left\{\frac{\partial^2 TP(T_1,T)}{\partial T_1 \partial T}\right\}^2 \succ 0 \text{ and } \left(\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right) \prec 0.$$

$$\frac{\partial TP(T_1,T)}{\partial T_1}=\frac{1}{T}\Bigg[p_C\alpha\sigma T_1^{\beta}-h_C\left\{\alpha T_1^{\beta}-\frac{\alpha\theta(\beta-4)T_1^{\beta+1}}{2\beta(\beta+1)}\right\}-\frac{d_C\alpha\theta}{\beta}\left\{\beta T_1^{\beta}-\theta(\beta+2)\right\}T_1^{\beta+1}$$
$$+s_C\left\{\sigma\alpha\left(T_1^{\beta+1}-TT_1^{\beta}\right)+\frac{\alpha}{\beta}\left(\beta TT_1^{\beta-1}-T_1^{\beta}\right)\right\}-p_C\alpha\left(T_1^{\beta-1}+\theta T_1^{\beta}\right)+p_C\alpha T_1^{\beta-1}$$
$$-p_C\alpha T_1^{\beta}+co_p\alpha\sigma T_1^{\beta}\Bigg], \quad (17)$$

$$\frac{\partial TP(T_1,T)}{\partial T}=\frac{1}{T}\left[s_P\alpha\left(T_1^{\beta-1}-\sigma T^{\beta}\right)+s_C\left\{\frac{\alpha\sigma\left(T^{\beta+1}-T_1^{\beta+1}\right)}{(\beta+1)}+\frac{\alpha}{\beta}\left(T_1^{\beta}-T^{\beta}\right)\right\}-p_C\alpha T^{\beta+1}+\alpha\left(p_C-co_p\sigma\right)T^{\beta}\right]$$
$$-\frac{1}{T^2}\left[TP(T_1,T)\right], \quad (18)$$

3.2. Fuzzy Model

Let us consider the inventory model in fuzzy environment due to uncertainty in parameters let us assume that the parameters θ, α and σ may change within some limits.

Let $\theta=(\theta_1,\theta_2,\theta_3)$, $\alpha=(\alpha_1,\alpha_2,\alpha_3)$ and $\sigma=(\sigma_1,\sigma_2,\sigma_3)$ are triangular fuzzy numbers then the total profit per unit time in fuzzy sense is

$$\widetilde{TP}(T_1,T)=\frac{1}{T}\Bigg[s_R\left\{\frac{\tilde{\alpha}T^{\beta}}{\beta}-\frac{\tilde{\alpha}\tilde{\sigma}T^{\beta+1}}{\beta+1}+\frac{\tilde{\alpha}\tilde{\sigma}T_1^{\beta+1}}{\beta+1}\right\}-A-h_C\left\{\frac{\tilde{\alpha}T_1^{\beta+1}}{\beta+1}+\frac{\tilde{\alpha}\tilde{\theta}T_1^{\beta+2}}{2(\beta+2)}\right\}-\frac{\tilde{\alpha}\tilde{\theta}d_CT_1^{\beta+1}}{\beta+1}$$
$$+s_C\left\{\frac{\tilde{\alpha}\tilde{\sigma}}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2}+\frac{(\beta+1)T_1^{\beta+2}}{(\beta+2)}-TT_1^{\beta+1}\right)+\frac{\tilde{\alpha}}{\beta}\left(TT_1^{\beta}-\frac{T_1^{\beta+1}}{\beta+1}-\frac{T^{\beta+1}}{\beta+1}\right)\right\} \quad (19)$$
$$-p_C\left\{\frac{\tilde{\alpha}T_1^{\beta}}{\beta}+\frac{\tilde{\alpha}\tilde{\theta}T_1^{\beta+1}}{\beta+1}\right\}-\frac{p_C\tilde{\alpha}}{\beta}\left(T^{\beta}-T_1^{\beta}\right)+\frac{p_C\tilde{\alpha}}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)-\frac{co_p\tilde{\alpha}\tilde{\sigma}}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)\Bigg],$$

Now we defuzzify the total profit $\widetilde{TP}(T_1,T)$ in two cases.

3.2.1. Signed Distance Method

By signed distance method the total profit per unit time is

$$\widetilde{TP}(T_1,T)=\frac{1}{4T}\left[\widetilde{TP}_1(T_1,T)+2\widetilde{TP}_2(T_1,T)+\widetilde{TP}_3(T_1,T)\right] \quad (20)$$

where,

$$\widetilde{TP}_1(T_1,T)=\frac{1}{4T}\Bigg[s_R\left\{\frac{\tilde{\alpha}_1T^{\beta}}{\beta}-\frac{\tilde{\alpha}_1\tilde{\sigma}_1T^{\beta+1}}{\beta+1}+\frac{\tilde{\alpha}_1\tilde{\sigma}_1T_1^{\beta+1}}{\beta+1}\right\}-A-h_C\left\{\frac{\tilde{\alpha}_1T_1^{\beta+1}}{\beta+1}+\frac{\tilde{\alpha}_1\tilde{\theta}_1T_1^{\beta+2}}{2(\beta+2)}\right\}$$
$$+s_C\left\{\frac{\tilde{\alpha}_1\tilde{\sigma}_1}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2}+\frac{(\beta+1)T_1^{\beta+2}}{(\beta+2)}-TT_1^{\beta+1}\right)+\frac{\tilde{\alpha}_1}{\beta}\left(TT_1^{\beta}-\frac{T_1^{\beta+1}}{\beta+1}-\frac{T^{\beta+1}}{\beta+1}\right)\right\}$$
$$-p_C\left\{\frac{\tilde{\alpha}_1T_1^{\beta}}{\beta}+\frac{\tilde{\alpha}_1\tilde{\theta}_1T_1^{\beta+1}}{\beta+1}\right\}-\frac{p_C\tilde{\alpha}_1}{\beta}\left(T^{\beta}-T_1^{\beta}\right)+\frac{p_C\tilde{\alpha}_1}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)$$
$$-\frac{\tilde{\alpha}_1\tilde{\theta}_1d_CT_1^{\beta+1}}{\beta+1}-\frac{co_p\tilde{\alpha}_1\tilde{\sigma}_1}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)\Bigg],$$

$$\widetilde{TP}_2(T_1,T)=\frac{1}{4T}\Bigg[s_R\left\{\frac{\tilde{\alpha}_2T^{\beta}}{\beta}-\frac{\tilde{\alpha}_2\tilde{\sigma}_2T^{\beta+1}}{\beta+1}+\frac{\tilde{\alpha}_2\tilde{\sigma}_2T_1^{\beta+1}}{\beta+1}\right\}-A-h_C\left\{\frac{\tilde{\alpha}_2T_1^{\beta+1}}{\beta+1}+\frac{\tilde{\alpha}_2\tilde{\theta}_2T_1^{\beta+2}}{2(\beta+2)}\right\}$$
$$+s_C\left\{\frac{\tilde{\alpha}_2\tilde{\sigma}_2}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2}+\frac{(\beta+1)T_1^{\beta+2}}{(\beta+2)}-TT_1^{\beta+1}\right)+\frac{\tilde{\alpha}_2}{\beta}\left(TT_1^{\beta}-\frac{T_1^{\beta+1}}{\beta+1}-\frac{T^{\beta+1}}{\beta+1}\right)\right\}$$
$$-p_C\left\{\frac{\tilde{\alpha}_2T_1^{\beta}}{\beta}+\frac{\tilde{\alpha}_2\tilde{\theta}_2T_1^{\beta+1}}{\beta+1}\right\}-\frac{p_C\tilde{\alpha}_2}{\beta}\left(T^{\beta}-T_1^{\beta}\right)+\frac{p_C\tilde{\alpha}_2}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)$$
$$-\frac{\tilde{\alpha}_2\tilde{\theta}_2d_CT_1^{\beta+1}}{\beta+1}-\frac{co_p\tilde{\alpha}_2\tilde{\sigma}_2}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)\Bigg],$$

$$\widetilde{TP}_3(T_1,T)=\frac{1}{4T}\Bigg[s_R\left\{\frac{\tilde{\alpha}_3T^{\beta}}{\beta}-\frac{\tilde{\alpha}_3\tilde{\sigma}_3T^{\beta+1}}{\beta+1}+\frac{\tilde{\alpha}_3\tilde{\sigma}_3T_1^{\beta+1}}{\beta+1}\right\}-A-h_C\left\{\frac{\tilde{\alpha}_3T_1^{\beta+1}}{\beta+1}+\frac{\tilde{\alpha}_3\tilde{\theta}_3T_1^{\beta+2}}{2(\beta+2)}\right\}$$
$$+s_C\left\{\frac{\tilde{\alpha}_3\tilde{\sigma}_3}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2}+\frac{(\beta+1)T_1^{\beta+2}}{(\beta+2)}-TT_1^{\beta+1}\right)+\frac{\tilde{\alpha}_3}{\beta}\left(TT_1^{\beta}-\frac{T_1^{\beta+1}}{\beta+1}-\frac{T^{\beta+1}}{\beta+1}\right)\right\}$$
$$-p_C\left\{\frac{\tilde{\alpha}_3T_1^{\beta}}{\beta}+\frac{\tilde{\alpha}_3\tilde{\theta}_3T_1^{\beta+1}}{\beta+1}\right\}-\frac{p_C\tilde{\alpha}_3}{\beta}\left(T^{\beta}-T_1^{\beta}\right)+\frac{p_C\tilde{\alpha}_3}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)$$
$$-\frac{\tilde{\alpha}_3\tilde{\theta}_3d_CT_1^{\beta+1}}{\beta+1}-\frac{co_p\tilde{\alpha}_3\tilde{\sigma}_3}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)\Bigg],$$

From Equation (20) we have

$$\widetilde{TP}(T_1,T)$$
$$=\frac{1}{4T}\Bigg[s_R\left\{\frac{(\alpha_1+2\alpha_2+\alpha_3)T^{\beta}}{\beta}-\frac{(\alpha_1\sigma_1+2\alpha_2\sigma_2+\alpha_3\sigma_3)T^{\beta+1}}{\beta+1}+\frac{(\alpha_1\sigma_1+2\alpha_2\sigma_2+\alpha_3\sigma_3)T_1^{\beta+1}}{\beta+1}\right\}-4A$$
$$-h_C\left\{\frac{(\alpha_1+2\alpha_2+\alpha_3)T_1^{\beta+1}}{\beta+1}+\frac{(\alpha_1\theta_1+2\alpha_2\theta_2+\alpha_3\theta_3)T_1^{\beta+2}}{2(\beta+2)}\right\}$$
$$+s_C\left\{\frac{(\alpha_1\sigma_1+2\alpha_2\sigma_2+\alpha_3\sigma_3)}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2}+\frac{(\beta+1)T_1^{\beta+2}}{(\beta+2)}-TT_1^{\beta+1}\right)+\frac{(\alpha_1+2\alpha_2+\alpha_3)}{\beta}\left(TT_1^{\beta}-\frac{T_1^{\beta+1}}{\beta+1}-\frac{T^{\beta+1}}{\beta+1}\right)\right\} \quad (21)$$
$$-p_C\left\{\frac{(\alpha_1+2\alpha_2+\alpha_3)T_1^{\beta}}{\beta}+\frac{(\alpha_1\theta_1+2\alpha_2\theta_2+\alpha_3\theta_3)T_1^{\beta+1}}{\beta+1}\right\}$$
$$-\frac{p_C(\alpha_1+2\alpha_2+\alpha_3)}{\beta}\left(T^{\beta}-T_1^{\beta}\right)+\frac{p_C(\alpha_1+2\alpha_2+\alpha_3)}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)$$
$$-\frac{(\alpha_1\theta_1+2\alpha_2\theta_2+\alpha_3\theta_3)d_CT_1^{\beta+1}}{\beta+1}-\frac{co_p(\alpha_1\sigma_1+2\alpha_2\sigma_2+\alpha_3\sigma_3)}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)\Bigg].$$

The necessary condition for $\widetilde{TP}(T_1,T)$ to be maximum is that $\frac{\partial\widetilde{TP}(T_1,T)}{\partial T_1}=0$ and $\frac{\partial\widetilde{TP}(T_1,T)}{\partial T}=0$, and solving these equations we find the optimum values of T_1 and T say T_1^* and T^* for which profit is maximum and the sufficient condition is

$$\left(\frac{\partial^2\widetilde{TP}(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2\widetilde{TP}(T_1,T)}{\partial T^2}\right)-\left\{\frac{\partial^2\widetilde{TP}(T_1,T)}{\partial T_1\partial T}\right\}^2\succ 0 \text{ and } \left(\frac{\partial^2\widetilde{TP}(T_1,T)}{\partial T_1^2}\right)\prec 0.$$

$$\frac{\partial \widetilde{TP}(T_1,T)}{\partial T_1}=\frac{1}{4T}\Big[p_C(\alpha_1\sigma_1+2\alpha_2\sigma_2+\alpha_3\sigma_3)T_1^{\beta}$$
$$-h_C\left\{(\alpha_1+2\alpha_2+\alpha_3)T_1^{\beta}-\frac{(\alpha_1\theta_1+2\alpha_2\theta_2+\alpha_3\theta_3)(\beta-4)T_1^{\beta+1}}{2\beta(\beta+1)}\right\}$$
$$-\frac{d_C(\alpha_1\theta_1+2\alpha_2\theta_2+\alpha_3\theta_3)}{\beta}\left\{\beta T_1^{\beta}-(\theta_1+2\theta_2+\theta_3)(\beta+2)\right\}T_1^{\beta+1} \tag{22}$$
$$+s_C\left\{(\alpha_1\sigma_1+2\alpha_2\sigma_2+\alpha_3\sigma_3)\left(T_1^{\beta+1}-TT_1^{\beta}\right)+\frac{(\alpha_1+2\alpha_2+\alpha_3)}{\beta}\left(\beta TT_1^{\beta-1}-T_1^{\beta}\right)\right\}$$
$$-p_C(\alpha_1+2\alpha_2+\alpha_3)\left(T_1^{\beta-1}+\theta T_1^{\beta}\right)+p_C(\alpha_1+2\alpha_2+\alpha_3)T_1^{\beta-1}-p_C(\alpha_1+2\alpha_2+\alpha_3)T_1^{\beta}$$
$$+co_p(\alpha_1\sigma_1+2\alpha_2\sigma_2+\alpha_3\sigma_3)T_1^{\beta}\Big],$$

$$\frac{\partial \widetilde{TP}(T_1,T)}{\partial T}=\frac{1}{4T}\Big[s_P(\alpha_1+2\alpha_2+\alpha_3)\left(T_1^{\beta-1}-\sigma T^{\beta}\right)$$
$$+s_C\left\{\frac{(\alpha_1\sigma_1+2\alpha_2\sigma_2+\alpha_3\sigma_3)\left(T^{\beta+1}-T_1^{\beta+1}\right)}{(\beta+1)}+\frac{(\alpha_1+2\alpha_2+\alpha_3)}{\beta}\left(T_1^{\beta}-T^{\beta}\right)\right\} \tag{23}$$
$$-p_C(\alpha_1+2\alpha_2+\alpha_3)T^{\beta-1}+(\alpha_1+2\alpha_2+\alpha_3)\left(p_C-co_p(\sigma_1+2\sigma_2+\sigma_3)\right)T^{\beta}\Big]$$
$$-\frac{1}{T^2}\left[TP(T_1,T)\right],$$

3.2.2. Centroid Method

By Centroid method the total profit per unit time is

$$\widetilde{TP}(T_1,T)=\frac{1}{3T}\left[\widetilde{TP}_1(T_1,T)+\widetilde{TP}_2(T_1,T)+\widetilde{TP}_3(T_1,T)\right] \tag{24}$$

$$\widetilde{TP}(T_1,T)$$
$$=\frac{1}{3T}\Bigg[s_R\left\{\frac{(\alpha_1+\alpha_2+\alpha_3)T^{\beta}}{\beta}-\frac{(\alpha_1\sigma_1+\alpha_2\sigma_2+\alpha_3\sigma_3)T^{\beta+1}}{\beta+1}+\frac{(\alpha_1\sigma_1+\alpha_2\sigma_2+\alpha_3\sigma_3)T_1^{\beta+1}}{\beta+1}\right\}-3A$$
$$-h_C\left\{\frac{(\alpha_1+\alpha_2+\alpha_3)T_1^{\beta+1}}{\beta+1}+\frac{(\alpha_1\theta_1+\alpha_2\theta_2+\alpha_3\theta_3)T_1^{\beta+2}}{2(\beta+2)}\right\}$$
$$+s_C\left\{\frac{(\alpha_1\sigma_1+\alpha_2\sigma_2+\alpha_3\sigma_3)}{\beta+1}\left(\frac{T^{\beta+2}}{\beta+2}+\frac{(\beta+1)T_1^{\beta+2}}{(\beta+2)}-TT_1^{\beta+1}\right)+\frac{(\alpha_1+\alpha_2+\alpha_3)}{\beta}\left(TT_1^{\beta}-\frac{T_1^{\beta+1}}{\beta+1}-\frac{T^{\beta+1}}{\beta+1}\right)\right\} \tag{25}$$
$$-p_C\left\{\frac{(\alpha_1+\alpha_2+\alpha_3)T_1^{\beta}}{\beta}+\frac{(\alpha_1\theta_1+\alpha_2\theta_2+\alpha_3\theta_3)T_1^{\beta+1}}{\beta+1}\right\}$$
$$-\frac{p_C(\alpha_1+\alpha_2+\alpha_3)}{\beta}\left(T^{\beta}-T_1^{\beta}\right)+\frac{p_C(\alpha_1+\alpha_2+\alpha_3)}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)$$
$$-\frac{(\alpha_1\theta_1+\alpha_2\theta_2+\alpha_3\theta_3)d_C T_1^{\beta+1}}{\beta+1}-\frac{co_p(\alpha_1\sigma_1+\alpha_2\sigma_2+\alpha_3\sigma_3)}{\beta+1}\left(T^{\beta+1}-T_1^{\beta+1}\right)\Bigg],$$

The necessary condition for $\widetilde{TP}(T_1,T)$ to be maximum is that $\frac{\partial \widetilde{TP}(T_1,T)}{\partial T_1}=0$ and $\frac{\partial \widetilde{TP}(T_1,T)}{\partial T}=0$, and solving these equations we find the optimum values of T_1 and T say T_1^* and T^* for which profit is maximum and the sufficient condition is

$$\left(\frac{\partial^2 \widetilde{TP}(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 \widetilde{TP}(T_1,T)}{\partial T^2}\right)-\left\{\frac{\partial^2 \widetilde{TP}(T_1,T)}{\partial T_1 \partial T}\right\}^2 \succ 0 \quad \text{and} \quad \left(\frac{\partial^2 \widetilde{TP}(T_1,T)}{\partial T_1^2}\right)\prec 0$$

$$\begin{aligned}\frac{\partial \widetilde{TP}(T_1,T)}{\partial T_1}=\frac{1}{3T}\Bigg[& p_C(\alpha_1\sigma_1+\alpha_2\sigma_2+\alpha_3\sigma_3)T_1^{\beta}-h_C\left\{(\alpha_1+\alpha_2+\alpha_3)T_1^{\beta}-\frac{(\alpha_1\theta_1+\alpha_2\theta_2+\alpha_3\theta_3)(\beta-4)T_1^{\beta+1}}{2\beta(\beta+1)}\right\}\\ &-\frac{d_C(\alpha_1\theta_1+\alpha_2\theta_2+\alpha_3\theta_3)}{\beta}\left\{\beta T_1^{\beta}-(\theta_1+\theta_2+\theta_3)(\beta+2)\right\}T_1^{\beta+1}\\ &+s_C\left\{(\alpha_1\sigma_1+\alpha_2\sigma_2+\alpha_3\sigma_3)\left(T_1^{\beta+1}-TT_1^{\beta}\right)+\frac{(\alpha_1+\alpha_2+\alpha_3)}{\beta}\left(\beta TT_1^{\beta-1}-T_1^{\beta}\right)\right\}\\ &-p_C(\alpha_1+\alpha_2+\alpha_3)\left(T_1^{\beta-1}+\theta T_1^{\beta}\right)+p_C(\alpha_1+\alpha_2+\alpha_3)T_1^{\beta-1}-p_C(\alpha_1+\alpha_2+\alpha_3)T_1^{\beta}\\ &+co_p(\alpha_1\sigma_1+\alpha_2\sigma_2+\alpha_3\sigma_3)T_1^{\beta}\Bigg],\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{\partial \widetilde{TP}(T_1,T)}{\partial T}=\frac{1}{3T}\Bigg[& s_P(\alpha_1+\alpha_2+\alpha_3)\left(T_1^{\beta-1}-\sigma T^{\beta}\right)\\ &+s_C\left\{\frac{(\alpha_1\sigma_1+\alpha_2\sigma_2+\alpha_3\sigma_3)\left(T^{\beta+1}-T_1^{\beta+1}\right)}{(\beta+1)}+\frac{(\alpha_1+\alpha_2+\alpha_3)}{\beta}\left(T_1^{\beta}-T^{\beta}\right)\right\}\\ &-p_C(\alpha_1+\alpha_2+\alpha_3)T^{\beta-1}+(\alpha_1+\alpha_2+\alpha_3)\left(p_C-co_p(\sigma_1+\sigma_2+\sigma_3)\right)T^{\beta}\Bigg]\\ &-\frac{1}{T^2}\left[TP(T_1,T)\right],\end{aligned} \tag{27}$$

3.3. Numerical Example

Let us consider an inventory system with the following parameters in appropriate units as

$$A=\text{Rs}200/\text{order} \quad s_p=\text{Rs}20/\text{unit/year}, \quad co_p=\text{Rs}6/\text{unit/year}, \quad \sigma=\text{Rs}3/\text{unit/year},$$

$$\alpha=8 \text{ unit/year}, \quad \beta=1 \text{ unit/year}, \quad \theta=0.01/\text{year}.$$

Table 1 shows that as we increase deterioration parameter θ then the total profit increases.
Table 2 shows that as we increase backlogging parameter σ then the total profit increases.
Table 3 shows that as we increase demand parameter α then the total profit increases.

3.3.1. Fuzzy Model

Let $\alpha=(5,10,20)$, $\sigma=(3,8,15)$ and $\theta=(0.001,0.005,0.150)$ are triangular fuzzy numbers.

The solution of the fuzzy inventory model can be determined by the following two methods.

3.3.2. Signed Distance Method

When α, σ and θ are triangular fuzzy numbers, then **Table 4** shows the value of total profit.
When α and σ are triangular fuzzy numbers, then **Table 5** shows the value of total profit.
When α and θ are triangular fuzzy numbers, then **Table 6** shows the value of total profit.

Table 1. Variation in total profit with respect to θ.

θ	T_1	T	TP
0.01	0.463225	19.98530	1852.37149
	9.18392	14.8322	−145.84282
0.05	0.46005	19.9855	1852.4180031
	9.20047	15.11240	−143.265133
0.10	0.456658	19.9873	1852.861567
	9.48808	16.9640	31.7336

Table 2. Variation in total profit with respect to σ.

σ	T_1	T	TP
3	0.463225	19.9853	1852.37149
	9.18392	14.8322	−145.84282
6	0.254734	19.7433	3012.687848
	8.07775	14.9421	−223.098494
9	0.175629	19.6626	4179.59984
	7.75299	15.0057	−344.806973

Table 3. Variation in total profit with respect to α.

α	T_1	T	TP
8	0.463225	19.9853	1852.37149
	9.18392	14.8322	−145.842829
10	0.46321	19.9869	2312.59975
	9.22675	14.8740	−182.252924
15	0.463191	19.9870	3477.162439
	9.28496	14.9307	−273.348202

Table 4. Variation in total profit with fuzzy numbers, α, σ and θ.

T_1	T	TP
161.6960	174.2120	−38818.36498
0.242176	20.3537	−13994.00000

Table 5. Variation in total profit with fuzzy numbers, α and σ.

T_1	T	TP
0.243607	20.3538	−13994.069903

3.3.3. Centroid Method

When α, σ and θ are triangular fuzzy numbers, then **Table 7** shows the value of total profit.

When α and σ are triangular fuzzy numbers, then **Table 8** shows the value of total profit.

When α and θ are triangular fuzzy numbers, then **Table 9** shows the value of total profit.

Table 6. Variation in total profit with fuzzy numbers, α and θ.

T_1	T	TP
0.0967304	0.868752	95.12730
338.3230	887.0950	−11109.943128

Table 7. Variation in total profit with fuzzy numbers, α, σ and θ.

T_1	T	TP
0.228336	20.2984	−15596.680854
133.3920	145.9400	−38288.109268

Table 8. Variation in total profit with fuzzy numbers, α and σ.

T_1	T	TP
0.22995	20.2984	−15596.702547

Table 9. Variation in total profit with fuzzy numbers, α and θ.

T_1	T	TP
0.646701	1.16303	105.622382
0.108538	0.855681	99.23360

4. Sensitivity Analysis

From **Table 1**, we see that as we increase the deterioration parameter θ then the optimal time period T_1, the optimal cycle time T and total profit increases.

From **Table 2**, we see that as we increase the backlogging parameter σ then the optimal time period T_1, the optimal cycle time T decreases and total profit increases.

From **Table 3**, we see that as we increase the demand rate parameter α then the optimal time period T_1 decreases and the optimal cycle time T and total profit increases.

In the case of crisp model we see that the backlogging parameter σ is more sensitive than the deterioration parameter θ and the demand rate parameter α.

From the tables for signed distance method and centroid method we see that the fuzzy variables $\tilde{\theta}$ and $\tilde{\alpha}$ are more sensitive than the fuzzy variable $\tilde{\sigma}$. As we increase the fuzzy variables $\tilde{\theta}$ and $\tilde{\alpha}$ in the signed distance method and centroid method than the total profit increases rapidly in centroid method. Therefore in the sense of fuzziness the centroid method is better one than the signed distance method.

5. Conclusion

In this paper we studied a fuzzy inventory model for deteriorating items with time dependent demand rate and partial backlogging. Shortages are allowed and completely backlogged. As we increase the parameters α, σ and θ in the crisp model then the total profit increases and due to the uncertainties in the demand rate, deterioration rate and backlogging rate the parameters α, σ and θ are consider as triangular fuzzy numbers. For defuzzification by signed distance method and centroid method it has been observed that the total profit decreases as the optimal cycle time decreases and the profit given by the signed distance method is minimum as compared to the centroid method. Further this model can be generalized by considering time dependent deterioration rate, holding cost, shortage cost and so many types.

Acknowledgements

The author would like to thank anonymous referees for their valuable comments and suggestions for the improvement of this paper.

References

[1] Harris, F. (1915) Operations and Cost. AW Shaw CO., Chicago.

[2] Zadeh, L.A. (1965) Fuzzy Set. *Information Control*, **8**, 338-353. http://dx.doi.org/10.1016/S0019-9958(65)90241-X

[3] Zadeh, L.A. and Bellman, R.E. (1970) Decision Making in a Fuzzy Environment. *Management Science*, **17**, 140-164.

[4] Jain, R. (1976) Decision Making in the Presence of Fuzzy Variables. *IIIE Transactions on Systems*, *Man and Cybernetics*, **17**, 698-703.

[5] Dubois, D. and Prade, H. (1978) Operations on Fuzzy Numbers. *International Journal of System Science*, **9**, 613-626. http://dx.doi.org/10.1080/00207727808941724

[6] Kacpryzk, J. and Staniewski, P. (1982) Long Term Inventory Policy Making through Fuzzy Decision Making Methods. *Fuzzy Sets and System*, **8**, 117-132. http://dx.doi.org/10.1016/0165-0114(82)90002-1

[7] Zimmerman, H.J. (1983) Using Fuzzy Sets in Operational Research. *European Journal of Operation Research*, **13**, 201-206. http://dx.doi.org/10.1016/0377-2217(83)90048-6

[8] Urgeletti Tinarelli, G. (1983) Inventory Control Models and Problems. *European Journal of Operation Research*, **14**, 1-12. http://dx.doi.org/10.1016/0377-2217(83)90283-7

[9] Park, K.S. (1987) Fuzzy Set Theoretical Interpretation of Economic Order Quantity. *IEEE Transactions on Systems*, *Man and Cybernetics*, **17**, 1082-1084. http://dx.doi.org/10.1109/TSMC.1987.6499320

[10] Vujosevic, M. and Petrovic, D. (1996) EOQ Formula When Inventory Cost Is Fuzzy. *International Journal of Production Economics*, **45**, 499-504. http://dx.doi.org/10.1016/0925-5273(95)00149-2

[11] Yao, J.S. and Lee, H.M. (1999) Fuzzy Inventory with or without Backorder for Fuzzy Order Quantity with Trapezoidal Fuzzy Number. *Fuzzy Sets and Systems*, **105**, 311-337. http://dx.doi.org/10.1016/S0165-0114(97)00251-0

[12] Yao, J.S. and Lee, H.M. (1999) Economic Order Quantity in Fuzzy Sense for Inventory without Backorder Model. *Fuzzy Sets and Systems*, **105**, 13-31.

[13] Kao, C.K. and Hsu, W.K. (2002) A Single Period Inventory Model with Fuzzy Demand. *Computers and Mathematics with Applications*, **43**, 841-848. http://dx.doi.org/10.1016/S0898-1221(01)00325-X

[14] Hsieh, C.H. (2002) Optimization of Fuzzy Production Inventory Models. *Information Sciences*, **146**, 29-40. http://dx.doi.org/10.1016/S0020-0255(02)00212-8

[15] Yao, J.S. and Chiang, J. (2003) Inventory without Backorder with Fuzzy Total Cost and Fuzzy Storing Cost Defuzzified by Centroid and Signed Distance. *European Journal of Operational Research*, **148**, 401-409. http://dx.doi.org/10.1016/S0377-2217(02)00427-7

[16] Kumar, Sujit De, Kundu, P.K. and Goswami, A. (2007) An Economic Production Quantity Inventory Model Involving Fuzzy Demand Rate and Fuzzy Deterioration Rate. *Journal of Applied Mathematics and Computing*, **12**, 251-260. http://dx.doi.org/10.1007/BF02936197

[17] Syed, J.K. and Aziz, L.A. (2007) Fuzzy Inventory Model without Shortages by Using Signed Distance Method. *Applied Mathematics and Information Sciences*, **1**, 203-209.

[18] De, P.K. and Rawat, A. (2011) A Fuzzy Inventory Model without Shortages Using Triangular Fuzzy Number. *Fuzzy Information and Engineering*, **3**, 59-68. http://dx.doi.org/10.1007/s12543-011-0066-9

[19] Jaggi, C.K., Pareek, S., Sharma, A. and Nidhi (2012) Fuzzy Inventory Model for Deteriorating Items with Time Varying Demand and Shortages. *American Journal of Operational Research*, **2**, 81-92. http://dx.doi.org/10.5923/j.ajor.20120206.01

[20] Saha, S. and Chakrabarti, T. (2012) Fuzzy EOQ Model with Time Dependent Demand and Deterioration with Shortages. *IOSR Journal of Mathematics*, **2**, 46-54.

[21] Dutta, D. and Kumar, P. (2012) Fuzzy Inventory Model without Shortage Using Trapezoidal Fuzzy Number with Sensitivity Analysis. *IOSR Journal of Mathematics*, **4**, 32-37.

[22] Dutta, D. and Kumar, P. (2013) Optimal Ordering Policy for an Inventory Model for Deteriorating Items without Shortages by Considering Fuzziness in Demand Rate, Ordering Cost and Holding Cost. *International Journal of Advanced Innovation and Research*, **2**, 320-325.

[23] Jain, D.K., Das, B. and Roy, T.K. (2013) A Fuzzy Generic Algorithm Approach for an Inventory Model for Deteriorating Items with Backorders under Fuzzy Inflation and Discounting over Random Planning Horizon. *Advances in Operations Research*, **2013**, 1-15.