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Regular Elements and Right Units of Semigroup $B_X(D)$ Defined Semilattice D for Which $V(D,\alpha)=Q\in\Sigma_3(X,8)$

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Abstract

In this paper we take $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\}$ subsemilattice of X-semilattice of unions D which satisfies the following conditions:

$$T_7\subset T_5\subset T_3\subset T_1\subset T_0,\ T_7\subset T_6\subset T_4\subset T_2\subset T_0,\ T_7\subset T_5\subset T_4\subset T_1\subset T_0,\ T_7\subset T_5\subset T_4\subset T_2\subset T_0,$$
$$T_7\subset T_6\subset T_4\subset T_1\subset T_0,\ T_5\setminus T_6\neq\varnothing,\ T_6\setminus T_5\neq\varnothing,\ T_4\setminus T_3\neq\varnothing,\ T_3\setminus T_4\neq\varnothing,\ T_2\setminus T_1\neq\varnothing,$$
$$T_1\setminus T_2\neq\varnothing,\ T_6\cup T_5=T_4,\ T_4\cup T_3=T_1,\ T_2\cup T_1=T_0.$$

We will investigate the properties of regular elements of the complete semigroup of binary relations $B_X(D)$ satisfying $V(D,\alpha)=Q$. For the case where X is a finite set we derive formulas by means of which we can calculate the numbers of regular elements and right units of the respective semigroup.

Keywords

Semilattice, Semigroup, Regular Element, Right Unit, Binary Relation

1. Introduction

Let X be an arbitrary nonempty set and D be an X-semilattice of unions, which means a nonempty set of subsets of the set X that is closed with respect to the set-theoretic operations of unification of elements from D. Let's denote an arbitrary mapping from X into D by f. For each f there exists a binary relation α_f on the set X that

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satisfies the condition $\alpha_f = \bigcup_{x\in X}\left(\{x\}\times f(x)\right)$. Let denote the set of all such α_f $(f: X\to D)$ by $B_X(D)$. It is not hard to prove that $B_X(D)$ is a semigroup with respect to the operation of multiplication of binary relations. $B_X(D)$ is called a complete semigroup of binary relations defined by a *X*-semilattice of unions *D* (see [1], Item 2.1), ([2], Item 2.1]).

An empty binary relation or an empty subset of the set *X* is denoted by $\varnothing$. The form $x\alpha y$ is used to express that $(x,y)\in\alpha$. Also, in this paper following conditions are used $x,y\in X$, $Y\subseteq X$, $\alpha\in B_X(D)$, $T\in D$, $\varnothing\neq D'\subseteq D$ and $t\in \breve{D} = \bigcup_{Y\in D} Y$. Moreover, following sets are denoted by given symbols:

$$y\alpha = \{x\in X \mid y\alpha x\},\ \ Y\alpha = \bigcup_{y\in Y} y\alpha,\ \ V(D,\alpha) = \{Y\alpha \mid Y\in D\},$$

$$X^* = \{T \mid \varnothing\neq T\subseteq X\},\ \ D'_t = \{Z'\in D' \mid t\in Z'\},\ \ D'_T = \{Z'\in D' \mid T\subseteq Z'\}.$$

$$\ddot{D}'_T = \{Z'\in D' \mid Z'\subseteq T\},\ \ l(D',T) = \cup(D'\setminus D'_T),\ \ Y_T^\alpha = \{x\in X \mid x\alpha = T\}.$$

And $\wedge(D,D_t)$ is an exact lower bound of the set D_t in the semilattice *D*.

Definition 1.1. Let $\varepsilon\in B_X(D)$. If $\varepsilon\circ\varepsilon=\varepsilon$ or $\alpha\circ\varepsilon=\alpha$ for any $\alpha\in B_X(D)$, then ε is called an idempotent element or called right unit of the semigroup $B_X(D)$ respectively (see [1]-[3]).

Definition 1.2. An element α taken from the semigroup $B_X(D)$ called a regular element of the semigroup $B_X(D)$ if in $B_X(D)$ there exists an element β such that $\alpha\circ\beta\circ\alpha=\alpha$ (see [1]-[4]).

Definition 1.3. We say that a complete *X*-semilattice of unions *D* is an *XI*-semilattice of unions if it satisfies the following two conditions:

1) $\wedge(D,D_t)\in D$ for any $t\in\breve{D}$;
2) $Z = \bigcup_{t\in Z}\wedge(D,D_t)$ for any nonempty element *Z* of *D* (see [1], definition 1.14.2), ([2] definition 1.14.2), [5] or [6].

Definition 1.4. Let *D* be an arbitrary complete *X*-semilattice of unions, $\alpha\in B_X(D)$ and $Y_T^\alpha = \{x\in X \mid x\alpha = T\}$. If

$$V[\alpha] = \begin{cases} V(X^*,\alpha), & \text{if } \varnothing\notin D,\\ V(X^*,\alpha), & \text{if } \varnothing\in V(X^*,\alpha),\\ V(X^*,\alpha)\cup\{\varnothing\}, & \text{if } \varnothing\notin V(X^*,\alpha) \text{ and } \varnothing\in D,\end{cases}$$

then it is obvious that any binary relation α of a semigroup $B_X(D)$ can always be written in the form $\alpha = \bigcup_{T\in V[\alpha]}\left(Y_T^\alpha\times T\right)$ the sequel, such a representation of a binary relation α will be called quasinormal.

Note that for a quasinormal representation of a binary relation α, not all sets Y_T^α $(T\in V[\alpha])$ can be different from an empty set. But for this representation the following conditions are always fulfilled:

1) $Y_T^\alpha\cap Y_{T'}^\alpha=\varnothing$, for any T, $T'\in D$ and $T\neq T'$;
2) $X = \bigcup_{T\in V[\alpha]} Y_T^\alpha$ (see [1], definition 1.11.1), ([2], definition 1.11.1).

Definition 1.5. We say that a nonempty element *T* is a nonlimiting element of the set *D*′ if $T\setminus l(D',T)\neq\varnothing$ and a nonempty element *T* is a limiting element of the set *D*′ if $T\setminus l(D',T)=\varnothing$ (see [1], definition 1.13.1 and definition 1.13.2), ([2], definition 1.13.1 and definition 1.13.2).

Definition 1.6. The one-to-one mapping φ between the complete *X*-semilattices of unions $\phi(Q,Q)$ and D'' is called a complete isomorphism if the condition

$$\varphi(\cup D_1) = \bigcup_{T=D_1}\varphi(T')$$

is fulfilled for each nonempty subset D_1 of the semilattice *D*′ (see [1], definition 6.3.2), ([2] definition 6.3.2) or [5]).

Definition 1.7. Let α be some binary relation of the semigroup $B_X(D)$. We say that the complete isomorphism φ between the complete semilattices of unions Q and D' is a complete α-isomorphism if

1) $Q = V(D,\alpha)$;

2) $\varphi(\varnothing) = \varnothing$ for $\varnothing \in V(D,\alpha)$ and $\varphi(T)\alpha = T$ for eny $T \in V(D,\alpha)$ (see [1], definition 6.3.3), ([2], definition 6.3.3).

Lemma 1.1. Let $Y = \{y_1, y_2, \cdots, y_k\}$ and $D_j = \{T_1, \cdots, T_j\}$ be any two sets. Then the number $s(k,j)$ of all possible mappings of Y into any subset D_j' of the set that D_j such that $T_j \in D_j'$ can be calculated by the formula $s(k,j) = j^k - (j-1)^k$ (see [1], Corollary 1.18.1), ([2], Corollary 1.18.1).

Lemma 1.2. Let D by a complete X-semilattice of unions. If a binary relation ε of the form $\varepsilon = \bigcup_{t \in \breve{D}} \left(\{t\} \times \wedge(D, D_t)\right) \cup \left((X \setminus \breve{D}) \times \breve{D}\right)$ is right unit of the semigroup $B_X(D)$, then ε is the greatest right unit of that semigroup (see [1], Lemma 12.1.2), ([2], Lemma 12.1.2).

Theorem 1.1. Let $D_j = \{T_1, T_2, \cdots, T_j\}$, X and Y- be three such sets, that $\varnothing \neq Y \subseteq X$. If f is such mapping of the set X, in the set D_j, for which $f(y) = T_j$ for some $y \in Y$, then the number s of all those mappings f of the set X in the set D_j is equal to $s = j^{|X \setminus Y|} \cdot \left(j^{|Y|} - (j-1)^{|Y|}\right)$ (see [1], Theorem 1.18.2), ([2], Theorem 1.18.2).

Theorem 1.2. Let $D = \{\breve{D}, Z_1, Z_2, \cdots, Z_{m-1}\}$ be some finite X-semilattice of unions and $C(D) = \{P_0, P_1, P_2, \cdots, P_{m-1}\}$ be the family of sets of pairwise nonintersecting subsets of the set X. If φ is a mapping of the semilattice D on the family of sets $C(D)$ which satisfies the condition $\varphi(\breve{D}) = P_0$ and $\varphi(Z_i) = P_i$ for any $i = 1, 2, \cdots, m-1$ and $\hat{D}_Z = D \setminus \{T \in D \mid Z \subseteq T\}$, then the following equalities are valid:

$$\breve{D} = P_0 \cup P_1 \cup P_2 \cup \cdots \cup P_{m-1}, \quad Z_i = P_0 \cup \bigcup_{T \in \hat{D}_{Z_i}} \varphi(T). \qquad (*)$$

In the sequel these equalities will be called formal.

It is proved that if the elements of the semilattice D are represented in the form (*), then among the parameters P_i $(i = 0, 1, 2, \cdots, m-1)$ there exist such parameters that cannot be empty sets for D. Such sets P_i $(0 < i \leq m-1)$ are called basis sources, whereas sets P_i $(0 \leq j \leq m-1)$ which can be empty sets too are called completeness sources.

It is proved that under the mapping φ the number of covering elements of the pre-image of a basis source is always equal to one, while under the mapping φ the number of covering elements of the pre-image of a completeness source either does not exist or is always greater than one (see [1], Item 11.4), ([2], Item 11.4) or [4]).

Theorem 1.3. Let D be a complete X-semilattice of unions. The semigroup $B_X(D)$ possesses a right unit iff D is an XI-semilattice of unions (see [1], Theorem 6.1.3, [2], Theorem 6.1.3, [7] or [8]).

Theorem 1.4. Let $\beta \in B_X(D)$. A binary relation β is a regular element of the semigroup $B_X(D)$ iff the complete X-semilattice of unions $D' = V(D,\beta)$ satisfies the following two conditions:

1) $V(X^*, \beta) \subseteq D'$;

2) D' is a complete XI-semilattice of unions (see [1] Theorem 6.3.1), ([2], Theorem 6.3.1).

Theorem 1.5. Let D be a finite X-semilattice of unions and $\alpha \circ \sigma \circ \alpha = \alpha$ for some α and σ of the semigroup $B_X(D)$; $D(\alpha)$ be the set of those elements T of the semilattice $Q = V(D,\alpha) \setminus \{\varnothing\}$ which are nonlimiting elements of the set $\ddot{Q}_T$. Then a binary relation α having a quasinormal representation of the form $\alpha = \bigcup_{T \in V(D,\alpha)} (Y_T^\alpha \times T)$ is a regular element of the semigroup $B_X(D)$ iff the set $V(D,\alpha)$ is a XI-semilattice of unions and for α-isomorphism φ of the semilattice $V(D,\alpha)$ on some X-subsemilattice D' of the semilattice D the following conditions are fulfilled:

1) $\varphi(T) = T\sigma$ for any $T \in V(D,\alpha)$;

2) $\bigcup_{T \in \ddot{D}(\alpha)_T} Y_T^\alpha \supseteq \varphi(T)$ for any $T \in D(\alpha)$;

3) $Y_T^\alpha \cap \varphi(T) \neq \varnothing$ for any element T of the set $\ddot{D}(\alpha)_T$ (see [1], Theorem 6.3.3), ([2], Theorem 6.3.3) or [5]).

2. Results

Let D be arbitrary X-semilattice of unions and $Q = \{T_7, T_6, T_5, T_4, T_3, T_2, T_1, T_0\} \subseteq D$, which satisfies the following conditions:

$$\begin{aligned}
&T_7 \subset T_5 \subset T_3 \subset T_1 \subset T_0,\ \ T_7 \subset T_6 \subset T_4 \subset T_2 \subset T_0,\\
&T_7 \subset T_5 \subset T_4 \subset T_1 \subset T_0,\ \ T_7 \subset T_5 \subset T_4 \subset T_2 \subset T_0,\\
&T_7 \subset T_6 \subset T_4 \subset T_1 \subset T_0,\ \ T_5 \setminus T_6 \neq \varnothing,\ \ T_6 \setminus T_5 \neq \varnothing,\\
&T_4 \setminus T_3 \neq \varnothing,\ \ T_3 \setminus Z_4 \neq \varnothing,\ \ T_2 \setminus T_1 \neq \varnothing,\ \ T_1 \setminus T_2 \neq \varnothing,\\
&T_6 \cup T_5 = T_4,\ \ T_4 \cup T_3 = T_1,\ \ T_2 \cup T_1 = T_0.
\end{aligned} \tag{1}$$

Figure 1 is a graph of semilattice Q, where the semilattice Q satisfies the conditions (1). The symbol $\Sigma_3(X,8)$ is used to denote the set of all X-semilattices of unions, whose every element is isomorphic to Q.

$P_7, P_6, P_5, P_4, P_3, P_2, P_1, P_0$ are pairwise disjoint subsets of the set X and let $C(Q)=\{P_7,P_6,P_5,P_4,P_3,P_2,P_1,P_0\}$ be a family sets, also

$$\psi = \begin{pmatrix} T_7 & T_6 & T_5 & T_4 & T_3 & T_2 & T_1 & T_0 \\ P_7 & P_6 & P_5 & P_4 & P_3 & P_2 & P_1 & P_0 \end{pmatrix}$$

is a mapping from the semilattice Q into the family sets $C(Q)$. Then we have following formal equalities of the semilattice Q:

$$\begin{aligned}
T_0 &= P_0 \cup P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5 \cup P_6 \cup P_7,\\
T_1 &= P_0 \cup P_2 \cup P_3 \cup P_4 \cup P_5 \cup P_6 \cup P_7,\\
T_2 &= P_0 \cup P_1 \cup P_3 \cup P_4 \cup P_5 \cup P_6 \cup P_7,\\
T_3 &= P_0 \cup P_2 \cup P_4 \cup P_5 \cup P_6 \cup P_7,\\
T_4 &= P_0 \cup P_3 \cup P_5 \cup P_6 \cup P_7,\\
T_5 &= P_0 \cup P_6 \cup P_7,\\
T_6 &= P_0 \cup P_3 \cup P_5 \cup P_7,\\
T_7 &= P_0.
\end{aligned} \tag{2}$$

Note that the elements P_1, P_2, P_3, P_6 are basis sources, the element P_0, P_4, P_5, P_7 is sources of completenes of the semilattice Q. Therefore $|X| \geq 4$ and $\delta = 4$ (see Theorem 1.2).

Theorem 2.1. *Let* $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\} \in \Sigma_3(X,8)$. *Then Q is XI-semilattice*

Proof. Let $t \in T_0$, $Q_t = \{T \in Q \mid t \in T\}$ and $\wedge(Q,Q_t)$ is the exact lower bound of the set Q_t in Q. Then from the formal equalities (2) we get that

$$Q_t = \begin{cases} T_0, & \text{if } t \in P_0,\\ \{T_2,T_0\}, & \text{if } t \in P_1,\\ \{T_3,T_1,T_0\}, & \text{if } t \in P_2,\\ \{T_6,T_4,T_2,T_1,T_0\}, & \text{if } t \in P_3,\\ \{T_3,T_2,T_1,T_0\}, & \text{if } t \in P_4,\\ \{T_6,T_4,T_3,T_2,T_1,T_0\}, & \text{if } t \in P_5,\\ \{T_5,T_4,T_3,T_2,T_1,T_0\}, & \text{if } t \in P_6,\\ \{T_6,T_5,T_4,T_3,T_2,T_1,T_0\}, & \text{if } t \in P_7, \end{cases} \qquad \wedge(Q,Q_t) = \begin{cases} T_7, & \text{if } t \in P_0,\\ T_2, & \text{if } t \in P_1,\\ T_3, & \text{if } t \in P_2,\\ T_6, & \text{if } t \in P_3,\\ T_5, & \text{if } t \in P_4,\\ T_7, & \text{if } t \in P_5,\\ T_5, & \text{if } t \in P_6,\\ T_7, & \text{if } t \in P_7, \end{cases}$$

We have $Q^{\wedge} = \{T_7,T_6,T_5,T_3,T_2\}$, $\wedge(Q,Q_t) \in Q$ for all t and $T_4 = T_6 \cup T_5$, $T_1 = T_6 \cup T_3$, $T_0 = T_3 \cup T_2$. The semilattice Q, which has diagram of **Figure 1**, is *XI*-semilattice, which follows from the Definition 1.3.

Theorem is proved.

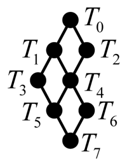

Figure 1. Diagram of Q.

Lemma 2.1. Let $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\}\in\Sigma_3(X,8)$. Then following equalities are true:

$$P_0\cup P_5\cup P_7=T_6\cap T_3,\ \ P_3=T_6\setminus T_3,\ \ P_4\cup P_6=\left(\left(T_3\cap T_2\right)\setminus T_6\right),$$
$$P_2=\left(T_3\setminus T_2\right),\ \ P_1=\left(T_2\setminus T_1\right).$$

Proof. This Lemma follows directly from the formal equalities (2) of the semilattice Q.

Lemma is proved.

Lemma 2.2. Let $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\}\in\Sigma_3(X,8)$. Then the binary relation

$$\varepsilon=\left(\left(T_6\cap T_3\right)\times T_7\right)\cup\left(\left(T_6\setminus T_3\right)\times T_6\right)\cup\left(\left(\left(T_3\cap T_2\right)\setminus T_6\right)\times T_5\right)$$
$$\cup\left(\left(T_3\setminus T_2\right)\times T_3\right)\cup\left(\left(T_2\setminus T_1\right)\times T_2\right)\cup\left(\left(X\setminus T_0\right)\times T_0\right)$$

is the largest right unit of the semigroup $B_X(D)$.

Proof. From preposition and from Theorem 2.1 we get that Q is *XI*-semilattice. To prove this Lemma we will use Lemma 1.2, lemma 2.1, and Theorem 1.3, from where we have that the following binary relation

$$\begin{aligned}\varepsilon&=\bigcup_{t\in\breve{D}}\left(\{t\}\times\wedge\left(Q,Q_t\right)\right)\cup\left(\left(X\setminus T_0\right)\times T_0\right)\\&=\left(\left(P_0\cup P_5\cup P_7\right)\times T_7\right)\cup\left(P_3\times T_6\right)\cup\left(\left(P_4\cup P_6\right)\times T_5\right)\cup\left(P_2\times T_3\right)\cup\left(P_1\times T_2\right)\cup\left(\left(X\setminus T_0\right)\times T_0\right)\\&=\left(\left(T_6\cap T_3\right)\times T_7\right)\cup\left(\left(T_6\setminus T_3\right)\times T_6\right)\cup\left(\left(\left(T_3\cap T_2\right)\setminus T_6\right)\times T_5\right)\cup\left(\left(T_3\setminus T_2\right)\times T_3\right)\\&\ \ \cup\left(\left(T_2\setminus T_1\right)\times T_2\right)\cup\left(\left(X\setminus T_0\right)\times T_0\right).\end{aligned}$$

is the largest right unit of the semigroup $B_X(D)$.

Lemma is proved.

Lemma 2.3. Let $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\}\in\Sigma_3(X,8)$. Binary relation α having quazinormal representation of the form

$$\alpha=\left(Y_7^\alpha\times T_7\right)\cup\left(Y_6^\alpha\times T_6\right)\cup\left(Y_5^\alpha\times T_5\right)\cup\left(Y_4^\alpha\times T_4\right)\cup\left(Y_3^\alpha\times T_3\right)\cup\left(Y_2^\alpha\times T_2\right)\cup\left(Y_1^\alpha\times T_1\right)\cup\left(Y_0^\alpha\times T_0\right)$$

where $Y_7^\alpha,\ Y_6^\alpha,\ Y_5^\alpha,\ Y_3^\alpha,\ Y_2^\alpha\notin\{\varnothing\}$ and $V(D,\alpha)=Q\in\Sigma_3(X,8)$ is a regular element of the semigroup $B_X(D)$ iff for some complete α-isomorphism $\varphi=\begin{pmatrix}T_7&T_6&T_5&T_4&T_3&T_2&T_1&T_0\\\overline{T}_7&\overline{T}_6&\overline{T}_5&\overline{T}_4&\overline{T}_3&\overline{T}_2&\overline{T}_1&\overline{T}_0\end{pmatrix}$ of the semilattice Q on some X-subsemilattice $Q'=\{\overline{T}_7,\overline{T}_6,\overline{T}_5,\overline{T}_4,\overline{T}_3,\overline{T}_2,\overline{T}_1,\overline{T}_0\}$ of the semilattice Q satisfies the following conditions:

$$Y_7^\alpha\supseteq\overline{T}_7,\ \ Y_7^\alpha\cup Y_6^\alpha\supseteq\overline{T}_6,\ \ Y_7^\alpha\cup Y_5^\alpha\supseteq\overline{T}_5,\ \ Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq\overline{T}_3,$$
$$Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq\overline{T}_2,\ \ Y_6^\alpha\cap\overline{T}_6\neq\varnothing,\ \ Y_5^\alpha\cap\overline{T}_5\neq\varnothing,$$
$$Y_3^\alpha\cap\overline{T}_3\neq\varnothing,\ \ Y_2^\alpha\cap\overline{T}_2\neq\varnothing.$$

Proof. It is easy to see, that the set $Q(\alpha)=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1\}$ is a generating set of the semilattice Q. Then the following equalities are hold:

$$\ddot{Q}(\alpha)_{T_7}=\{T_7\},\ \ \ddot{Q}(\alpha)_{T_6}=\{T_7,T_6\},\ \ \ddot{Q}(\alpha)_{T_5}=\{T_7,T_5\},\ \ \ddot{Q}(\alpha)_{T_4}=\{T_7,T_6,T_5,T_4\},$$
$$\ddot{Q}(\alpha)_{T_3}=\{T_7,T_5,T_3\},\ \ \ddot{Q}(\alpha)_{T_2}=\{T_7,T_6,T_5,T_4,T_2\},\ \ \ddot{Q}(\alpha)_{T_1}=\{T_7,T_6,T_5,T_4,T_3,T_1\}.$$

If we follow statement *b*) of the Theorem 1.5 we get that followings are true:

$$Y_7^\alpha\supseteq\overline{T}_7,\ \ Y_7^\alpha\cup Y_6^\alpha\supseteq\overline{T}_6,\ \ Y_7^\alpha\cup Y_5^\alpha\supseteq\overline{T}_5,\ \ Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\supseteq\overline{T}_4$$
$$Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq\overline{T}_3,\ \ Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq\overline{T}_2,$$
$$Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_3^\alpha\cup Y_1^\alpha\supseteq\overline{T}_1,$$

From the last conditions we have that following is true:

$$Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha=\left(Y_7^\alpha\cup Y_6^\alpha\right)\cup\left(Y_7^\alpha\cup Y_5^\alpha\right)\cup Y_4^\alpha\supseteq\overline{T}_6\cup\overline{T}_5\cup Y_4^\alpha=\overline{T}_4\cup Y_4^\alpha\supseteq\overline{T}_4,$$

$$Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_3^\alpha \cup Y_1^\alpha = \left(Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha\right) \cup \left(Y_7^\alpha \cup Y_6^\alpha\right) \cup Y_4^\alpha \cup Y_1^\alpha$$
$$\supseteq \overline{T}_3 \cup \overline{T}_6 \cup Y_4^\alpha \cup Y_1^\alpha = \overline{T}_1 \cup Y_4^\alpha \cup Y_1^\alpha \supseteq \overline{T}_1.$$

Moreover, the following conditions are true:

$$l\left(\ddot{Q}_{T_6}, T_6\right) = \cup\left(\ddot{Q}_{T_6} \setminus \{T_6\}\right) = T_7, \quad T_6 \setminus l\left(\ddot{Q}_{T_6}, T_6\right) = T_6 \setminus T_7 \neq \varnothing;$$

$$l\left(\ddot{Q}_{T_5}, T_5\right) = \cup\left(\ddot{Q}_{T_5} \setminus \{T_5\}\right) = T_7, \quad T_5 \setminus l\left(\ddot{Q}_{T_5}, T_5\right) = T_5 \setminus T_7 \neq \varnothing;$$

$$l\left(\ddot{Q}_{T_4}, T_4\right) = \cup\left(\ddot{Q}_{T_4} \setminus \{T_4\}\right) = \cup\{T_7, T_6, T_5\} = T_4, \quad T_4 \setminus l\left(\ddot{Q}_{T_4}, T_4\right) = T_4 \setminus T_4 = \varnothing;$$

$$l\left(\ddot{Q}_{T_3}, T_3\right) = \cup\left(\ddot{Q}_{T_3} \setminus \{T_3\}\right) = \cup\{T_7, T_5\} = T_5, \quad T_3 \setminus l\left(\ddot{Q}_{T_3}, T_3\right) = T_3 \setminus T_5 \neq \varnothing;$$

$$l\left(\ddot{Q}_{T_2}, T_2\right) = \cup\left(\ddot{Q}_{T_2} \setminus \{T_2\}\right) = \cup\{T_7, T_6, T_5, T_4\} = T_4, \quad T_2 \setminus l\left(\ddot{Q}_{T_2}, T_2\right) = T_2 \setminus T_4 \neq \varnothing;$$

$$l\left(\ddot{Q}_{T_1}, T_1\right) = \cup\left(\ddot{Q}_{T_1} \setminus \{T_1\}\right) = \cup\{T_7, T_6, T_5, T_4, T_3\} = T_1, \quad T_1 \setminus l\left(\ddot{Q}_{T_1}, T_1\right) = T_1 \setminus T_1 = \varnothing;$$

The elements T_6, T_5, T_3, T_2 are nonlimiting elements of the sets $\ddot{Q}(\alpha)_{T_6}$, $\ddot{Q}(\alpha)_{T_5}$, $\ddot{Q}(\alpha)_{T_3}$ and $\ddot{Q}(\alpha)_{T_2}$ respectively. The proof of condition $Y_6^\alpha \cap \overline{T}_6 \neq \varnothing$, $Y_5^\alpha \cap \overline{T}_5 \neq \varnothing$, $Y_3^\alpha \cap \overline{T}_3 \neq \varnothing$ and $Y_2^\alpha \cap \overline{T}_2 \neq \varnothing$ comes from the statement *c*) of the Theorem 1.5

Therefore the following conditions are hold:

$$Y_7^\alpha \supseteq \overline{T}_7, \quad Y_7^\alpha \cup Y_6^\alpha \supseteq \overline{T}_6, \quad Y_7^\alpha \cup Y_5^\alpha \supseteq \overline{T}_5, \quad Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha \supseteq \overline{T}_3,$$
$$Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_2^\alpha \supseteq \overline{T}_2, \quad Y_6^\alpha \cap \overline{T}_6 \neq \varnothing, \quad Y_5^\alpha \cap \overline{T}_5 \neq \varnothing,$$
$$Y_3^\alpha \cap \overline{T}_3 \neq \varnothing, \quad Y_2^\alpha \cap \overline{T}_2 \neq \varnothing.$$

Lemma is proved.

Definition 2.1. Assume that $Q' \in \Sigma_3(X, 8)$. Denote by the symbol $R(Q')$ the set of all regular elements α of the semigroup $B_X(D)$, for which the semilattices Q' and Q are mutually α-isomorphic and $V(D, \alpha) = Q'$.

Note that, $q = 1$, where q is the number of automorphism of the semilattice Q.

Theorem 2.2. Let $Q = \{T_7, T_6, T_5, T_4, T_3, T_2, T_1, T_0\} \in \Sigma_3(X, 8)$ and $\left|\Sigma_3(X, 8)\right| = m_0$. If X be finite set, and the *XI*-semilattice Q and $Q' = \left\{\overline{T}_7, \overline{T}_6, \overline{T}_5, \overline{T}_4, \overline{T}_3, \overline{T}_2, \overline{T}_1, \overline{T}_0\right\}$ (see **Figure 2**) are α-isomorphic, then

$$\left|R(Q')\right| = m_0 \cdot \left(2^{\left|\overline{T}_6 \setminus \overline{T}_3\right|} - 1\right) \cdot 2^{\left|\left(\overline{T}_3 \cap \overline{T}_2\right) \setminus \overline{T}_4\right|} \cdot \left(2^{\left|\overline{T}_5 \setminus \overline{T}_6\right|} - 1\right) \cdot \left(3^{\left|\overline{T}_3 \setminus \overline{T}_2\right|} - 2^{\left|\overline{T}_3 \setminus \overline{T}_2\right|}\right) \cdot \left(5^{\left|\overline{T}_2 \setminus \overline{T}_1\right|} - 4^{\left|\overline{T}_2 \setminus \overline{T}_1\right|}\right) \cdot 8^{\left|X \setminus \overline{T}_0\right|}$$

Proof. Assume that $\alpha \in R(Q')$. Then a quasinormal representation of a regular binary relation α has the form

$$\alpha = \left(Y_7^\alpha \times T_7\right) \cup \left(Y_6^\alpha \times T_6\right) \cup \left(Y_5^\alpha \times T_5\right) \cup \left(Y_4^\alpha \times T_4\right) \cup \left(Y_3^\alpha \times T_3\right) \cup \left(Y_2^\alpha \times T_2\right) \cup \left(Y_1^\alpha \times T_1\right) \cup \left(Y_0^\alpha \times T_0\right)$$

where Y_7^α, Y_6^α, Y_5^α, Y_3^α, $Y_2^\alpha \notin \{\varnothing\}$ and by Lemma 2.2 satisfies the conditions:

$$Y_7^\alpha \supseteq \overline{T}_7, \quad Y_7^\alpha \cup Y_6^\alpha \supseteq \overline{T}_6, \quad Y_7^\alpha \cup Y_5^\alpha \supseteq \overline{T}_5, \quad Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha \supseteq \overline{T}_3,$$
$$Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_2^\alpha \supseteq \overline{T}_2, \quad Y_6^\alpha \cap \overline{T}_6 \neq \varnothing, \quad Y_5^\alpha \cap \overline{T}_5 \neq \varnothing, \tag{3}$$
$$Y_3^\alpha \cap \overline{T}_3 \neq \varnothing, \quad Y_2^\alpha \cap \overline{T}_2 \neq \varnothing.$$

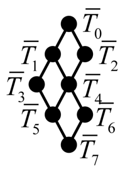

Figure 2. Diagram of Q'.

Father, let f_α is a mapping the set X in the semilattice Q satisfying the conditions $f_\alpha(t) = t\alpha$ for all $t \in X$. $f_{0\alpha}$, $f_{1\alpha}$, $f_{2\alpha}$, $f_{3\alpha}$, $f_{4\alpha}$ and $f_{5\alpha}$ are the restrictions of the mapping f_α on the sets $\bar{T}_6 \cap \bar{T}_3$, $\bar{T}_6 \setminus \bar{T}_3$, $(\bar{T}_3 \cap \bar{T}_2) \setminus \bar{T}_6$, $\bar{T}_3 \setminus \bar{T}_2$, $\bar{T}_2 \setminus \bar{T}_1$, $X \setminus \bar{T}_0$ respectively. It is clear, that the intersection disjoint elements of the set $\{\bar{T}_6 \cap \bar{T}_3, \bar{T}_6 \setminus \bar{T}_3, (\bar{T}_3 \cap \bar{T}_2) \setminus \bar{T}_6, \bar{T}_3 \setminus \bar{T}_2, \bar{T}_2 \setminus \bar{T}_1, X \setminus \bar{T}_0\}$ are empty set and

$\bar{T}_6 \cap \bar{T}_3 \cup \bar{T}_6 \setminus \bar{T}_3 \cup (\bar{T}_3 \cap \bar{T}_2) \setminus \bar{T}_6 \cup \bar{T}_3 \setminus \bar{T}_2 \cup \bar{T}_2 \setminus \bar{T}_1 \cup X \setminus \bar{T}_0 = X$.

We are going to find properties of the maps $f_{1\alpha}$, $f_{2\alpha}$, $f_{3\alpha}$, $f_{4\alpha}$, $f_{5\alpha}$, $f_{6\alpha}$.

1) $t \in \bar{T}_6 \cap \bar{T}_3$. Then by properties (3) we have $t \in \bar{T}_6 \cap \bar{T}_3 \subseteq (Y_7^\alpha \cup Y_6^\alpha) \cap (Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha) = Y_7^\alpha$, *i.e.*, $t \in Y_7^\alpha$ and $t\alpha = \bar{T}_7$ by definition of the set Y_7^α. Therefore $f_{1\alpha}(t) = T_7$ for all $t \in \bar{T}_6 \cap \bar{T}_3$.

2) $t \in \bar{T}_6 \setminus \bar{T}_3$. Then by properties (3) we have $t \in \bar{T}_6 \setminus \bar{T}_3 \subseteq Y_7^\alpha \cup Y_6^\alpha$, *i.e.*, $t \in Y_7^\alpha \cup Y_6^\alpha$ and $t\alpha = \{\bar{T}_7, \bar{T}_6\}$ by definition of the set Y_7^α and Y_6^α. Therefore $f_{2\alpha}(t) = \{T_7, T_6\}$ for all $t \in \bar{T}_6 \setminus \bar{T}_3$.

By suppose we have that $Y_6^\alpha \cap \bar{T}_6 \neq \varnothing$, *i.e.* $t_1\alpha = T_6$ for some $t_1 \in \bar{T}_6$. If $t_1 \in \bar{T}_3$. Then $t_1 \in Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha$. Therefore $t_1\alpha \in \{\bar{T}_7, \bar{T}_5, \bar{T}_3\}$. That is contradict of the equality $t_1\alpha = T_6$, while $T_6 \neq T_7$, $T_6 \neq T_5$ and $T_6 \neq T_3$ by definition of the semilattice *Q*. Therefore $f_{1\alpha}(t_1) = T_6$ for some $t \in \bar{T}_6 \setminus \bar{T}_3$.

3) $t \in (\bar{T}_3 \cap \bar{T}_2) \setminus \bar{T}_6$. Then by properties (3) we have

$$(\bar{T}_3 \cap \bar{T}_2) \setminus \bar{T}_6 \subseteq \bar{T}_3 \cap \bar{T}_2 \subseteq (Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha) \cap (Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_2^\alpha) = Y_7^\alpha \cup Y_5^\alpha$$

i.e., $t \in Y_7^\alpha \cup Y_5^\alpha$ and $t\alpha \in \{\bar{T}_7, \bar{T}_5\}$ by definition of the sets Y_7^α and Y_5^α. Therefore $f_{3\alpha}(t) \in \{T_7, T_5\}$ for all $t \in (\bar{T}_3 \cap \bar{T}_2) \setminus \bar{T}_6$.

By suppose we have, that $Y_5^\alpha \cap \bar{T}_5 \neq \varnothing$, *i.e.* $t_3\alpha = T_5$ for some $t_3 \in \bar{T}_5$. If $t_3 \in \bar{T}_6$ then $t_2 \in \bar{T}_6 \subseteq Y_7^\alpha \cup Y_6^\alpha$. Therefore $t_3\alpha \in \{T_7, T_6\}$. We have contradict of the equality $t_2\alpha = T_5$, since $T_5 \notin \{T_7, T_6\}$.

Therefore $f_{3\alpha}(t_3) = T_5$ for some $t_3 \in \bar{T}_5 \setminus \bar{T}_6$.

4) $t \in \bar{T}_3 \setminus \bar{T}_2$. Then by properties (3) we have $\bar{T}_3 \setminus \bar{T}_2 \subseteq \bar{T}_3 \subseteq Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha$, *i.e.*, $t \in Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha$ and $t\alpha \in \{\bar{T}_7, \bar{T}_5, \bar{T}_3\}$ by definition of the sets Y_7^α, Y_5^α, and Y_3^α. Therefore $f_{4\alpha}(t) \in \{T_7, T_5, T_3\}$ for all $t \in \bar{T}_3 \setminus \bar{T}_2$.

By suppose we have, that $Y_3^\alpha \cap \bar{T}_3 \neq \varnothing$, *i.e.* $t_4\alpha = T_3$ for some $t_4 \in \bar{T}_3$. If $t_4 \in \bar{T}_2$. Then $t_4 \in \bar{T}_2 \subseteq Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_2^\alpha$. Therefore $t_4\alpha \in \{T_7, T_6, T_5, T_4, T_2\}$. We have contradict of the equality $t_4\alpha = T_3$, since $T_3 \notin \{T_7, T_6, T_5, T_4, T_2\}$.

Therefore $f_{4\alpha}(t_4) = T_3$ for some $t \in \bar{T}_3 \setminus \bar{T}_2$.

5) $t \in \bar{T}_2 \setminus \bar{T}_1$. Then by properties (3) we have $\bar{T}_2 \setminus \bar{T}_1 \subseteq \bar{T}_2 \subseteq Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_2^\alpha$, *i.e.*, $t \in Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_2^\alpha$ and $t\alpha \in \{\bar{T}_7, \bar{T}_6, \bar{T}_5, \bar{T}_4, \bar{T}_2\}$ by definition of the sets Y_7^α, Y_6^α, Y_5^α, Y_4^α and Y_2^α. Therefore $f_{5\alpha}(t) \in \{T_7, T_6, T_5, T_4, T_2\}$ for all $t \in \bar{T}_2 \setminus \bar{T}_1$.

By suppose we have, that $Y_2^\alpha \cap \bar{T}_2 \neq \varnothing$, *i.e.* $t_5\alpha = T_2$ for some $t_5 \in \bar{T}_2$. If $t_5 \in \bar{T}_1$. Then $t_5 \in \bar{T}_1 \subseteq Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_3^\alpha \cup Y_1^\alpha$. Therefore $t_5\alpha \in \{T_7, T_6, T_5, T_4, T_3, T_1\}$. We have contradict of the equality $t_5\alpha = T_2$, since $T_2 \notin \{T_7, T_6, T_5, T_4, T_3, T_1\}$.

Therefore $f_{5\alpha}(t_5) = T_2$ for some $t \in \bar{T}_2 \setminus \bar{T}_1$.

6) $t \in X \setminus T_0$. Then by definition quasinormal representation binary relation α and by property (3) we have $t \in X \setminus T_0 \subseteq X = Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_3^\alpha \cup Y_2^\alpha \cup Y_1^\alpha \cup Y_0^\alpha$, *i.e.* $t\alpha \in \{\bar{T}_7, \bar{T}_6, \bar{T}_5, \bar{T}_4, \bar{T}_3, \bar{T}_2, \bar{T}_1, \bar{T}_0\}$ by definition of the sets Y_7^α, Y_6^α, Y_5^α, Y_4^α, Y_3^α, Y_2^α, Y_1^α, Y_0^α. Therefore $f_{6\alpha}(t) \in \{T_7, T_6, T_5, T_4, T_3, T_2, T_1, T_0\}$ for all $t \in X \setminus \breve{D}$.

Therefore for every binary relation α exist ordered system $(f_{1\alpha}, f_{2\alpha}, f_{3\alpha}, f_{4\alpha}, f_{5\alpha}, f_{6\alpha})$. It is obvious that for disjoint binary relations exist disjoint ordered systems.

Father, let

$$\begin{aligned} &f_1 : \bar{T}_6 \cap \bar{T}_3 \to T_7, && f_2 : \bar{T}_6 \setminus \bar{T}_3 \to \{T_7, T_6\}, \\ &f_3 : (\bar{T}_3 \cap \bar{T}_2) \setminus \bar{T}_6 \to \{T_7, T_5\}, && f_4 : \bar{T}_3 \setminus \bar{T}_2 \to \{T_7, T_5, T_3\}, \\ &f_5 : \bar{T}_2 \setminus \bar{T}_1 \to \{T_7, T_6, T_5, T_4, T_2\}, && f_6 : X \setminus \bar{T}_0 \to \{T_7, T_6, T_5, T_4, T_3, T_2, T_1, T_0\}. \end{aligned}$$

are such mappings, which satisfying the conditions:

7) $f_1(t) = T_7$ for all $t \in \bar{T}_6 \cap \bar{T}_3$;

8) $f_2(t) \in \{T_7, T_6\}$ for all $t \in \overline{T}_6 \setminus \overline{T}_3$ and $f_2(t_1) = T_6$ for some $t_1 \in \overline{T}_6 \setminus \overline{T}_3$;

9) $f_3(t) \in \{T_7, T_5\}$ for all $t \in (\overline{T}_3 \cap \overline{T}_2) \setminus \overline{T}_6$ and $f_3(t_2) = T_5$ for some $t_2 \in \overline{T}_5 \setminus \overline{T}_6$;

10) $f_4(t) \in \{T_7, T_5, T_3\}$ for all $t \in \overline{T}_3 \setminus \overline{T}_2$ and $f_4(t_3) = T_3$ for some $t_3 \in \overline{T}_3 \setminus \overline{T}_2$;

11) $f_5(t) \in \{T_7, T_6, T_5, T_4, T_2\}$ for all $t \in \overline{T}_2 \setminus \overline{T}_1$ and $f_5(t_4) = T_2$ for some $t_4 \in \overline{T}_2 \setminus \overline{T}_1$;

12) $f_6(t) \in \{T_6, T_5, T_4, T_3, T_2, T_1, T_0\}$ for all $t \in X \setminus \overline{T}_0$.

Now we define a map *f* of a set *X* in the semilattice *D*, which satisfies the condition:

$$f(t) = \begin{cases} f_1(t), & \text{if } t \in \overline{T}_6 \cap \overline{T}_3, \\ f_2(t), & \text{if } t \in \overline{T}_6 \setminus \overline{T}_3, \\ f_3(t), & \text{if } t \in (\overline{T}_3 \cap \overline{T}_2) \setminus \overline{T}_6, \\ f_4(t), & \text{if } t \in \overline{T}_3 \setminus \overline{T}_2, \\ f_5(t), & \text{if } \overline{T}_2 \setminus \overline{T}_1, \\ f_6(t), & \text{if } X \setminus \overline{T}_0. \end{cases}$$

Father, let $\beta = \bigcup_{x \in X} (\{x\} \times f(x))$, $Y_i^\beta = \{t \mid t\beta = T_i\}$ $(i = 1, 2, \cdots, 6)$. Then binary relation β my be representation by form

$$\beta = (Y_7^\beta \times T_7) \cup (Y_6^\beta \times T_6) \cup (Y_5^\beta \times T_5) \cup (Y_4^\beta \times T_4) \cup (Y_3^\beta \times T_3) \cup (Y_2^\beta \times T_2) \cup (Y_1^\beta \times T_1) \cup (Y_0^\beta \times T_0)$$

and satisfying the conditions:

$$\begin{gathered} Y_7^\beta \supseteq \overline{T}_7, \ Y_7^\beta \cup Y_6^\beta \supseteq \overline{T}_6, \ Y_7^\beta \cup Y_5^\beta \supseteq \overline{T}_5, \ Y_7^\beta \cup Y_5^\beta \cup Y_3^\beta \supseteq \overline{T}_3, \\ Y_7^\beta \cup Y_6^\beta \cup Y_5^\beta \cup Y_4^\beta \cup Y_2^\beta \supseteq \overline{T}_2, \ Y_6^\beta \cap \overline{T}_6 \neq \varnothing, \ Y_5^\beta \cap \overline{T}_5 \neq \varnothing, \\ Y_3^\beta \cap \overline{T}_3 \neq \varnothing, \ Y_2^\beta \cap \overline{T}_2 \neq \varnothing. \end{gathered}$$

(By suppose $f_2(t_1) = T_6$ for some $t_1 \in \overline{T}_6 \setminus \overline{T}_3$; $f_3(t_2) = T_5$ for some $t_2 \in \overline{T}_5 \setminus \overline{T}_6$; $f_4(t_3) = T_3$ for some $t_3 \in \overline{T}_3 \setminus \overline{T}_2$; $f_5(t_4) = T_2$ for some $t_4 \in \overline{T}_2 \setminus \overline{T}_1$. From this and by lemma 2.3 we have that $\beta \in R(Q')$.

Therefore for every binary relation $\alpha \in R(Q')$ and ordered system $(f_{1\alpha}, f_{2\alpha}, f_{3\alpha}, f_{4\alpha}, f_{5\alpha}, f_{6\alpha})$ exist one to one mapping.

By Theorem 1.1 the number of the mappings $f_{0\alpha}, f_{1\alpha}, f_{2\alpha}, f_{3\alpha}, f_{4\alpha}, f_{5\alpha}$ are respectively:

$$1, \ 2^{|\overline{T}_6 \setminus \overline{T}_3|} - 1, \ 2^{|(\overline{T}_3 \cap \overline{T}_2) \setminus (\overline{T}_5 \cup \overline{T}_6)|} \cdot \left(2^{|\overline{T}_5 \setminus \overline{T}_6|} - 1\right), \ 3^{|\overline{T}_3 \setminus \overline{T}_2|} - 2^{|\overline{T}_3 \setminus \overline{T}_2|}, \ 5^{|\overline{T}_2 \setminus \overline{T}_1|} - 4^{|\overline{T}_2 \setminus \overline{T}_1|}, \ 8^{|X \setminus \overline{T}_0|}$$

(see Lemma 1.1). The number of ordered system $(f_{1\alpha}, f_{2\alpha}, f_{3\alpha}, f_{4\alpha}, f_{5\alpha}, f_{6\alpha})$ or number idempotent elements of this case we my be calculated by formula

$$|R(Q')| = m_0 \cdot \left(2^{|\overline{T}_6 \setminus \overline{T}_3|} - 1\right) \cdot 2^{|(\overline{T}_3 \cap \overline{T}_2) \setminus \overline{T}_4|} \cdot \left(2^{|\overline{T}_5 \setminus \overline{T}_6|} - 1\right) \cdot \left(3^{|\overline{T}_3 \setminus \overline{T}_2|} - 2^{|\overline{T}_3 \setminus \overline{T}_2|}\right) \cdot \left(5^{|\overline{T}_2 \setminus \overline{T}_1|} - 4^{|\overline{T}_2 \setminus \overline{T}_1|}\right) \cdot 8^{|X \setminus \overline{T}_0|}$$

Theorem is proved.

Corollary 2.1. Let $Q = \{T_7, T_6, T_5, T_4, T_3, T_2, T_1, T_0\} \in \Sigma_3(X, 8)$, If *X* be a finite set and $E_X^{(r)}(Q)$ be the set of all right units of the semigroup $B_X(Q)$, then the following formula is true

$$\left|E_X^{(r)}(Q)\right| = \left(2^{|\overline{T}_6 \setminus \overline{T}_3|} - 1\right) \cdot 2^{|(\overline{T}_3 \cap \overline{T}_2) \setminus \overline{T}_4|} \cdot \left(2^{|\overline{T}_5 \setminus \overline{T}_6|} - 1\right) \cdot \left(3^{|\overline{T}_3 \setminus \overline{T}_2|} - 2^{|\overline{T}_3 \setminus \overline{T}_2|}\right) \cdot \left(5^{|\overline{T}_2 \setminus \overline{T}_1|} - 4^{|\overline{T}_2 \setminus \overline{T}_1|}\right) \cdot 8^{|X \setminus \overline{T}_0|}$$

Proof: This Corollary directly follows from the Theorem 2.2 and from the [2, 3 Theorem 6.3.7].

Corollary is proved.

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