

# Application of Trial Equation Method for Solving the Getmanou Equation

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## Abstract

Under the travelling wave transformation, some nonlinear partial differential equations such as the Getmanou equation are transformed to ordinary differential equation. Then using trial equation method and combing complete discrimination system for polynomial, the classifications of all single traveling wave solution to this equation are obtained.

## Keywords

**The Nonlinear Partial Differential Equation, Trial Equation Method, Complete Discrimination System for Polynomia, Traveling Wave Transform, the Getmanou Equation**

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## 1. Introduction

Many problems in natural and engineering sciences are modeled by partial differential equations (PDE). Looking for the solutions of the equation, especially the exact solutions, is very important. These exact solutions can describe many important phenomena in physics and other fields and also help physicists to understand the mechanisms of the complicated physical phenomena. Many mathematicians and physicists work in the field, and a variety of powerful methods have been employed to study nonlinear phenomena, such as the inverse scattering transform [1], the Bäcklund transformation method [2], the Darboux transformation [3], the homogeneous balance method [4], the tanh function method [5], the exp-function method [6], the G'/G-expansion method [7], and so on.

Recently, Professor Liu proposed a powerful method named trial equation method [8] [9] for finding exact solutions to nonlinear differential equations. In this paper, I mainly use Liu's trial equation method and the theory of complete discrimination system for the fourth-order polynomial [10]-[12] to solve exact solutions of the Getmanou equation which has already been solved based on discrimination system of the fifth-order polynomial by Fan [13]. But as we can see, the solving process is very simple and clear with the trial equation method

combined with complete discrimination system for polynomial.

## 2. Application of Trial Equation Method

The Getmanou equation [12] reads as

$$u_{xt} + \frac{u_x u_t}{1-u^2} - u(1-u^2) = 0. \quad (1)$$

Or equivalently

$$(1-u^2)u_{xt} + u_x u_t - u(1-u^2)^2 = 0. \quad (2)$$

Taking the traveling wave transformation  $u = u(\xi_1)$  and  $\xi_1 = kx + \omega t$ , we can obtain the corresponding ODE

$$k\omega(1-u^2)u'' + k\omega(u')^2 - u^5 + 2u^3 - u = 0. \quad (3)$$

we take the trial equation as follows

$$u'' = a_0 + a_1 u + \dots + a_m u^m. \quad (4)$$

According to the trial equation method of rank homogeneous equation, balancing  $u^2 u''$  with  $u^5$  gets  $m = 3$ .

Equation (4) has the following specific form

$$u'' = a_3 u^3 + a_2 u^2 + a_1 u + a_0. \quad (5)$$

From Equation (5), we get

$$(u')^2 = \frac{1}{2} a_3 u^4 + \frac{2}{3} a_2 u^3 + a_1 u^2 + 2a_0 u + d. \quad (6)$$

Substituting Equations (5) and (6) into Equation (3), we have

$$r_5 u^5 + r_4 u^4 + r_3 u^3 + r_2 u^2 + r_1 u + r_0 = 0. \quad (7)$$

where

$$\begin{aligned} r_0 &= k\omega a_0 + k\omega d, \\ r_1 &= k\omega a_1 + 2k\omega a_0 - 1, \\ r_2 &= k\omega a_2 + k\omega a_1 - k\omega a_0, \\ r_3 &= k\omega a_3 + \frac{2}{3} k\omega a_2 - k\omega a_1 + 2, \\ r_4 &= \frac{1}{2} k\omega a_3 - k\omega a_2, \\ r_5 &= -k\omega a_3 - 1. \end{aligned} \quad (8)$$

Let the coefficient  $r_i$  ( $i = 0, 1, 2, 3, 4, 5$ ) be zero, we will yield nonlinear algebraic equations. Solving the equations, we will determine the values of  $a_0, a_1, a_2, a_3, d$ . We get

$$a_0 = \frac{1}{6k\omega}, a_1 = \frac{2}{3k\omega}, a_2 = -\frac{1}{2k\omega}, a_3 = -\frac{1}{k\omega}, d = \frac{1}{6k\omega}. \quad (9)$$

When  $a_3 > 0$ , we take transformations as follows

$$w = \left(\frac{1}{2} a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right), \xi = \left(\frac{1}{2} a_3\right)^{\frac{1}{4}} \xi_1. \quad (10)$$

Then Equation (6) becomes

$$(w')^2 = w^4 + pw^2 + qw + r. \quad (11)$$

where  $w$  is a function of  $\xi$  and

$$\begin{aligned} p &= a_1 \left( \frac{1}{2} a_3 \right)^{-\frac{1}{2}} - \frac{1}{3} \left( \frac{1}{2} a_3 \right)^{-\frac{1}{2}} a_2^2 a_3^{-1}. \\ q &= -\frac{2}{3} a_1 \left( \frac{1}{2} a_3 \right)^{-\frac{1}{4}} a_2 a_3^{-1} + \frac{4}{27} \left( \frac{1}{2} a_3 \right)^{-\frac{1}{4}} a_2^3 a_3^{-2} + 2a_0 \left( \frac{1}{2} a_3 \right)^{-\frac{1}{4}}. \\ r &= a_0 - \frac{2}{3} a_0 a_2 a_3^{-1} + \frac{1}{9} a_1 a_2^2 a_3^{-2} - \frac{1}{54} a_2^4 a_3^{-3}. \end{aligned} \quad (12)$$

When  $a_3 < 0$ , we take transformations as follows

$$w = \left( -\frac{1}{2} a_3 \right)^{\frac{1}{4}} \left( u + \frac{a_2}{3a_3} \right), \xi = \left( -\frac{1}{2} a_3 \right)^{\frac{1}{4}} \xi_1. \quad (13)$$

Then Equation (6) becomes

$$(w')^2 = -(w^4 + pw^2 + qw + r). \quad (14)$$

where  $w$  is a function of  $\xi$  and

$$\begin{aligned} p &= -a_1 \left( -\frac{1}{2} a_3 \right)^{-\frac{1}{2}} + \frac{1}{3} \left( -\frac{1}{2} a_3 \right)^{-\frac{1}{2}} a_2^2 a_3^{-1}. \\ q &= \frac{2}{3} a_1 \left( -\frac{1}{2} a_3 \right)^{-\frac{1}{4}} a_2 a_3^{-1} - \frac{4}{27} \left( -\frac{1}{2} a_3 \right)^{-\frac{1}{4}} a_2^3 a_3^{-2} - 2a_0 \left( -\frac{1}{2} a_3 \right)^{-\frac{1}{4}}. \\ r &= -a_0 + \frac{2}{3} a_0 a_2 a_3^{-1} - \frac{1}{9} a_1 a_2^2 a_3^{-2} + \frac{1}{54} a_2^4 a_3^{-3}. \end{aligned} \quad (15)$$

We write the complete discrimination system for polynomial  $F(w) = w^4 + pw^2 + qw + r$  as follows

$$D_1 = 4, D_2 = -p, D_3 = 8rp - 2p^3 - 9q^2, D_4 = 4p^4r - p^3q^2 + 36prq^2 - 32r^2p^2 - \frac{27}{4}q^4 + 64r^3, E_2 = 9q^2 - 32pr. \quad (16)$$

Then we consider the following ODE

$$(w')^2 = \varepsilon(w^4 + pw^2 + qw + r). \quad (17)$$

where  $\varepsilon = \pm 1$ . Rewrite Equation (17) by integral form as follows

$$\pm(\xi - \xi_0) = \int \frac{dw}{\sqrt{\varepsilon(w^4 + pw^2 + qw + r)}}. \quad (18)$$

### 3. Classification of the Traveling Wave Solutions

According to the complete discrimination system for the fourth order polynomial, we give the corresponding single traveling wave solutions to Equation (1).

Case 1.  $D_4 = 0$ ,  $D_3 = 0$ ,  $D_2 < 0$ . Then we have

$$F(w) = \left[ (w - l_1)^2 + s_1^2 \right]^2. \quad (19)$$

where  $l_1$  and  $s_1$  are real numbers,  $s_1 > 0$ .

When  $\varepsilon = 1$ , we have

$$w = s_1 \tan \left[ s_1 (\xi - \xi_0) \right] + l_1. \quad (20)$$

The corresponding solution is

$$u = \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left\{ s_1 \tan \left[ s_1 \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) \right] + l_1 \right\} - \frac{a_2}{3a_3}. \tag{21}$$

Case 2.  $D_4 = 0, D_3 = 0, D_2 = 0$ . Then we have

$$F(w) = w^4. \tag{22}$$

When  $\varepsilon = 1$ , we have

$$w = -\frac{1}{(\xi - \xi_0)}. \tag{23}$$

The corresponding solution is

$$u = -\left(\frac{1}{2}a_3\right)^{-\frac{1}{2}} \frac{1}{(\xi_1 - \xi_0)} - \frac{a_2}{3a_3}. \tag{24}$$

Case 3.  $D_4 = 0, D_3 = 0, D_2 > 0, E_2 = 0$ . Then we have

$$F(w) = (w - \alpha)^2 (w - \beta)^2. \tag{25}$$

where  $\alpha$  and  $\beta$  are real numbers,  $\alpha > \beta$ .

When  $\varepsilon = 1$

(i) If  $w > \alpha$  or  $w < \beta$ , we have

$$w = \frac{\beta - \alpha}{2} \left[ \coth \frac{\alpha - \beta}{2} (\xi - \xi_0) - 1 \right] + \beta. \tag{26}$$

The corresponding solution is

$$u = \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left\{ \frac{\beta - \alpha}{2} \left[ \coth \frac{\alpha - \beta}{2} \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) - 1 \right] + \beta \right\} - \frac{a_2}{3a_3}. \tag{27}$$

(ii) If  $\alpha > w > \beta$ , we have

$$w = \frac{\beta - \alpha}{2} \left[ \tanh \frac{\alpha - \beta}{2} (\xi - \xi_0) - 1 \right] + \beta. \tag{28}$$

The corresponding solution is

$$u = \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left\{ \frac{\beta - \alpha}{2} \left[ \tanh \frac{\alpha - \beta}{2} \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) - 1 \right] + \beta \right\} - \frac{a_2}{3a_3}. \tag{29}$$

Case 4.  $D_4 = 0, D_3 > 0, D_2 > 0$ . Then we have

$$F(w) = (w - \alpha)^2 (w - \beta)(w - \gamma). \tag{30}$$

where  $\alpha, \beta, \gamma$  are real numbers, and  $\beta > \gamma$ .

When  $\varepsilon = 1$

(i) If  $\alpha > \beta, w > \beta$  or if  $\alpha < \gamma, w < \gamma$ , we have

$$\pm(\xi - \xi_0) = \frac{1}{\sqrt{(\alpha - \beta)(\alpha - \gamma)}} \ln \frac{\left[ \sqrt{(w - \beta)(\alpha - \gamma)} - \sqrt{(\alpha - \beta)(w - \gamma)} \right]^2}{|w - \alpha|}. \tag{31}$$

The corresponding solution is

$$\begin{aligned} & \pm \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) \\ &= \frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}} \ln \frac{\left\{ \sqrt{\left[\left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \beta\right] (\alpha-\gamma) - \sqrt{(\alpha-\beta) \left[\left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \gamma\right]}} \right\}^2}{\left|\left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \alpha\right|}. \end{aligned} \quad (32)$$

(ii) If  $\alpha > \beta$ ,  $w < \gamma$  or if  $\alpha < \gamma$ ,  $w < \beta$ , we have

$$\pm(\xi - \xi_0) = \frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}} \ln \frac{\left[ \sqrt{(w-\beta)(\gamma-\alpha)} - \sqrt{(\beta-\alpha)(w-\gamma)} \right]^2}{|w-\alpha|}. \quad (33)$$

The corresponding solution is

$$\begin{aligned} & \pm \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) \\ &= \frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}} \ln \frac{\left\{ \sqrt{\left[\left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \beta\right] (\gamma-\alpha) - \sqrt{(\beta-\alpha) \left[\left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \gamma\right]}} \right\}^2}{\left|\left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \alpha\right|}. \end{aligned} \quad (34)$$

(iii) If  $\beta > \alpha > \gamma$ , we have

$$\pm(\xi - \xi_0) = \frac{1}{\sqrt{(\beta-\alpha)(\alpha-\gamma)}} \arcsin \frac{(w-\beta)(\alpha-\gamma) + (\alpha-\beta)(w-\gamma)}{|(w-\alpha)(\beta-\gamma)|}. \quad (35)$$

The corresponding solution is

$$\begin{aligned} & \pm \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) \\ &= \frac{1}{\sqrt{(\beta-\alpha)(\alpha-\gamma)}} \arcsin \frac{\left[ \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \beta \right] (\alpha-\gamma) + (\alpha-\beta) \left[ \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \gamma \right]}{\left[ \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \alpha \right] (\beta-\gamma)}. \end{aligned} \quad (36)$$

When  $\varepsilon = -1$

(i) If  $\alpha > \beta$ ,  $w > \beta$  or if  $\alpha < \gamma$ ,  $w < \gamma$ , we have the corresponding solution is

$$\pm(\xi - \xi_0) = \frac{1}{\sqrt{(\alpha-\beta)(\alpha-\gamma)}} \ln \frac{\left[ \sqrt{(w-\beta)(\alpha-\gamma)} - \sqrt{(\alpha-\beta)(w-\gamma)} \right]^2}{|w-\alpha|}. \quad (37)$$

The corresponding solution is

$$\pm \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) = \frac{1}{\sqrt{(\alpha - \beta)(\alpha - \gamma)}} \ln \frac{\left\{ \sqrt{\left[ \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \beta \right] (\alpha - \gamma) - \sqrt{(\alpha - \beta) \left[ \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \gamma \right]} \right\}^2}{\left| \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \alpha \right|} \quad (38)$$

(ii) If  $\alpha > \beta$ ,  $w < \gamma$  or if  $\alpha < \gamma$ ,  $w < \beta$ , we have

$$\pm (\xi - \xi_0) = \frac{1}{\sqrt{(\alpha - \beta)(\alpha - \gamma)}} \ln \frac{\left[ \sqrt{(w - \beta)(\gamma - \alpha)} - \sqrt{(\beta - \alpha)(w - \gamma)} \right]^2}{|w - \alpha|} \quad (39)$$

The corresponding solution is

$$\pm \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) = \frac{1}{\sqrt{(\alpha - \beta)(\alpha - \gamma)}} \ln \frac{\left\{ \sqrt{\left[ \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \beta \right] (\gamma - \alpha) - \sqrt{(\beta - \alpha) \left[ \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \gamma \right]} \right\}^2}{\left| \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \alpha \right|} \quad (40)$$

(iii) If  $\beta > \alpha > \gamma$ , we have

$$\pm (\xi - \xi_0) = \frac{1}{\sqrt{(\beta - \alpha)(\alpha - \gamma)}} \arcsin \frac{(w - \beta)(\alpha - \gamma) + (\alpha - \beta)(w - \gamma)}{|(w - \alpha)(\beta - \gamma)|} \quad (41)$$

The corresponding solution is

$$\pm \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) = \frac{1}{\sqrt{(\beta - \alpha)(\alpha - \gamma)}} \arcsin \frac{\left[ \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \beta \right] (\alpha - \gamma) + (\alpha - \beta) \left[ \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \gamma \right]}{\left[ \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \left(u + \frac{a_2}{3a_3}\right) - \alpha \right] (\beta - \gamma)} \quad (42)$$

Case 5.  $D_4 = 0$ ,  $D_3 = 0$ ,  $D_2 > 0$ ,  $E_2 = 0$ . Then we have

$$F(w) = (w - \alpha)^3 (w - \beta). \quad (43)$$

where  $\alpha$  and  $\beta$  are real numbers.

When  $\varepsilon = 1$ , if  $w > \alpha$ ,  $w > \beta$  or if  $w < \alpha$ ,  $w < \beta$ , we have

$$w = \frac{4(\alpha - \beta)}{(\alpha - \beta)^2 (\xi - \xi_0)^2 - 4}. \quad (44)$$

The corresponding solution is

$$u = \left(\frac{1}{2}a_3\right)^{-\frac{1}{4}} \frac{4(\alpha - \beta)}{(\alpha - \beta)^2 \left(\frac{1}{2}a_3\right)^{\frac{1}{2}} (\xi_1 - \xi_0)^2 - 4} - \frac{a_2}{3a_3}. \quad (45)$$

When  $\varepsilon = -1$ , if  $w > \alpha$ ,  $w < \beta$  or if  $w < \alpha$ ,  $w > \beta$ , we have

$$w = \frac{4(\beta - \alpha)}{(\alpha - \beta)^2 (\xi - \xi_0)^2 + 4}. \quad (46)$$

The corresponding solution is

$$u = \left(-\frac{1}{2}a_3\right)^{-\frac{1}{4}} \frac{4(\beta - \alpha)}{(\alpha - \beta)^2 \left(-\frac{1}{2}a_3\right)^{\frac{1}{2}} (\xi_1 - \xi_0)^2 + 4} - \frac{a_2}{3a_3}. \quad (47)$$

Case 6.  $D_4 = 0$ ,  $D_2 D_3 < 0$ . Then we have

$$F(w) = (w - \alpha)^2 \left[ (w - l_1)^2 + s_1^2 \right]. \quad (48)$$

where  $\alpha$ ,  $l_1$  and  $s_1$  are real numbers.

When  $\varepsilon = 1$ , we have

$$w = \frac{\exp \left[ \pm \sqrt{(\alpha - l_1)^2 + s_1^2} (\xi - \xi_0) \right] - \gamma + \sqrt{(\alpha - l_1)^2 + s_1^2}}{\left\{ \exp \left[ \pm \sqrt{(\alpha - l_1)^2 + s_1^2} (\xi - \xi_0) \right] - \gamma \right\}^2 - 1}. \quad (49)$$

The corresponding solution is

$$u = \left(\frac{1}{2}a_3\right)^{-\frac{1}{4}} \frac{\exp \left[ \pm \sqrt{(\alpha - l_1)^2 + s_1^2} \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) \right] - \gamma + \sqrt{(\alpha - l_1)^2 + s_1^2}}{\left\{ \exp \left[ \pm \sqrt{(\alpha - l_1)^2 + s_1^2} \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0) \right] - \gamma \right\}^2 - 1} - \frac{a_2}{3a_3}. \quad (50)$$

where  $\gamma = \frac{\alpha - 2l_1}{\sqrt{(\alpha - l_1)^2 + s_1^2}}$ .

Case 7.  $D_4 > 0$ ,  $D_3 > 0$ ,  $D_1 > 0$ . Then we have

$$F(w) = (w - \alpha_1)(w - \alpha_2)(w - \alpha_3)(w - \alpha_4). \quad (51)$$

where  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  and  $\alpha_4$  are real numbers, and  $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$ .

When  $\varepsilon = 1$

(i) If  $w > \alpha_1$  or  $w < \alpha_4$ , we have

$$w = \frac{\alpha_2 (\alpha_1 - \alpha_4) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} (\xi - \xi_0), m \right) - \alpha_1 (\alpha_2 - \alpha_4)}{(\alpha_1 - \alpha_4) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} (\xi - \xi_0), m \right) - (\alpha_2 - \alpha_4)}. \quad (52)$$

The corresponding solution is

$$u = \left(\frac{1}{2}a_3\right)^{-\frac{1}{4}} \frac{\alpha_2(\alpha_1 - \alpha_4) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) - \alpha_1(\alpha_2 - \alpha_4)}{(\alpha_1 - \alpha_4) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) - (\alpha_2 - \alpha_4)} - \frac{a_2}{3a_3}. \quad (53)$$

(ii) If  $\alpha_2 > w > \alpha_3$ , we have

$$w = \frac{\alpha_4(\alpha_2 - \alpha_3) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} (\xi - \xi_0), m \right) - \alpha_3(\alpha_2 - \alpha_4)}{(\alpha_2 - \alpha_3) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} (\xi - \xi_0), m \right) - (\alpha_2 - \alpha_4)}. \quad (54)$$

The corresponding solution is

$$u = \left(\frac{1}{2}a_3\right)^{-\frac{1}{4}} \frac{\alpha_4(\alpha_2 - \alpha_3) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) - \alpha_3(\alpha_2 - \alpha_4)}{(\alpha_2 - \alpha_3) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) - (\alpha_2 - \alpha_4)} - \frac{a_2}{3a_3}. \quad (55)$$

When  $\varepsilon = -1$

(i)  $\alpha_1 > w > \alpha_2$ , we have

$$w = \frac{\alpha_3(\alpha_1 - \alpha_2) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} (\xi - \xi_0), m \right) - \alpha_2(\alpha_1 - \alpha_3)}{(\alpha_1 - \alpha_2) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} (\xi - \xi_0), m \right) - (\alpha_1 - \alpha_3)}. \quad (56)$$

The corresponding solution is

$$u = \left(-\frac{1}{2}a_3\right)^{-\frac{1}{4}} \frac{\alpha_3(\alpha_1 - \alpha_2) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) - \alpha_2(\alpha_1 - \alpha_3)}{(\alpha_1 - \alpha_2) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) - (\alpha_1 - \alpha_3)} - \frac{a_2}{3a_3}. \quad (57)$$

(ii)  $\alpha_3 > w > \alpha_4$ , we have

$$w = \frac{\alpha_1(\alpha_3 - \alpha_4) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} (\xi - \xi_0), m \right) - \alpha_4(\alpha_1 - \alpha_3)}{(\alpha_3 - \alpha_4) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} (\xi - \xi_0), m \right) - (\alpha_3 - \alpha_4)}. \quad (58)$$

The corresponding solution is



$$u = \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \frac{\alpha_1(\alpha_3 - \alpha_4) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) - \alpha_4(\alpha_3 - \alpha_1)}{(\alpha_3 - \alpha_4) \operatorname{sn}^2 \left( \frac{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}{2} \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) - (\alpha_3 - \alpha_1)} - \frac{a_2}{3a_3}. \quad (59)$$

where  $m^2 = \frac{(\alpha_1 - \alpha_2)(\alpha_3 - \alpha_4)}{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}$ .

Case 8.  $D_4 < 0, D_2 D_3 \geq 0$ . Then we have

$$F(w) = (w - \alpha)(w - \beta) \left[ (w - l_1)^2 + s_1^2 \right]. \quad (60)$$

where  $\alpha, \beta, l_1, s_1$  are real numbers, and  $\alpha > \beta, s_1 > 0$ .

When  $\varepsilon = 1$ , we have

$$w = \frac{\operatorname{acn} \left( \frac{\sqrt{-2s_1 m_1 (\alpha - \beta)}}{2mm_1} (\xi - \xi_0), m \right) + b}{\operatorname{ccn} \left( \frac{\sqrt{-2s_1 m_1 (\alpha - \beta)}}{2mm_1} (\xi - \xi_0), m \right) + d}. \quad (61)$$

The corresponding solution is

$$u = \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} \frac{\operatorname{acn} \left( \frac{\sqrt{-2s_1 m_1 (\alpha - \beta)}}{2mm_1} \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) + b}{\operatorname{ccn} \left( \frac{\sqrt{-2s_1 m_1 (\alpha - \beta)}}{2mm_1} \left(\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) + d} - \frac{a_2}{3a_3}. \quad (62)$$

When  $\varepsilon = -1$ , we have

$$w = \frac{\operatorname{acn} \left( \frac{\sqrt{2s_1 m_1 (\alpha - \beta)}}{2mm_1} (\xi - \xi_0), m \right) + b}{\operatorname{ccn} \left( \frac{\sqrt{2s_1 m_1 (\alpha - \beta)}}{2mm_1} (\xi - \xi_0), m \right) + d}. \quad (63)$$

The corresponding solution is

$$u = \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} \frac{\operatorname{acn} \left( \frac{\sqrt{2s_1 m_1 (\alpha - \beta)}}{2mm_1} \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) + b}{\operatorname{ccn} \left( \frac{\sqrt{2s_1 m_1 (\alpha - \beta)}}{2mm_1} \left(-\frac{1}{2}a_3\right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) + d} - \frac{a_2}{3a_3}. \quad (64)$$

where

$$\begin{aligned} c &= a - l_1 - \frac{s_1}{m_1}, d = a - l_1 - s_1 m_1, a = \frac{1}{2} [(\alpha + \beta)c - (\alpha - \beta)d], \\ b &= \frac{1}{2} [(\alpha + \beta)d - (\alpha - \beta)c], E = \frac{s_1^2 + (\alpha - l_1)(\beta - l_1)}{s_1(\alpha - \beta)}, \\ m_1 &= E \pm \sqrt{E^2 + 1}, m^2 = \frac{1}{1 + m_1^2}. \end{aligned} \quad (65)$$

We choose  $m_1$  such that  $\varepsilon m_1 < 0$ .

Case 9.  $D_4 > 0$ ,  $D_2 D_3 \leq 0$ . Then we have

$$F(w) = \left[ (w-l_1)^2 + s_1^2 \right] \left[ (w-l_2)^2 + s_2^2 \right]. \quad (66)$$

where  $l_1, l_2, s_1, s_2$  are real numbers, and  $s_1 > s_2 > 0$ .

When  $\varepsilon = 1$ , we have

$$w = \frac{a \operatorname{sn}(\eta(\xi - \xi_0), m) + b \operatorname{cn}(\eta(\xi - \xi_0), m)}{\operatorname{csn}(\eta(\xi - \xi_0), m) + d \operatorname{cn}(\eta(\xi - \xi_0), m)}. \quad (67)$$

The corresponding solution is

$$u = \left( \frac{1}{2} a_3 \right)^{-\frac{1}{4}} \frac{a \operatorname{sn} \left( \eta \left( \frac{1}{2} a_3 \right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) + b \operatorname{cn} \left( \eta \left( \frac{1}{2} a_3 \right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right)}{\operatorname{csn} \left( \eta \left( \frac{1}{2} a_3 \right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right) + d \operatorname{cn} \left( \eta \left( \frac{1}{2} a_3 \right)^{\frac{1}{4}} (\xi_1 - \xi_0), m \right)} - \frac{a_2}{3a_3}. \quad (68)$$

where

$$c = -s_1 - \frac{s_2}{m_1}, d = l_1 - l_2, a = l_1 c + s_1 d, b = l_1 d - s_1 c, \quad (69)$$

$$E = \frac{s_1^2 + s_2^2 + (l_1 - l_2)^2}{2s_1 s_2}, m_1 = E + \sqrt{E^2 - 1}, m^2 = 1 - \frac{1}{m_1^2}, \eta = s_2 \sqrt{\frac{m_1^2 c^2 + d^2}{c^2 + d^2}}.$$

In Equations (21) (24) (27) (29) (32) (34) (36) (38) (40) (42) (45) (47) (50) (53) (55) (57) (59) (62) (64) and (68), the integration constant  $\xi_0$  has been rewritten, but we still use it. The classifications of all single traveling wave solution to this equation are obtained.

## 4. Conclusion

In this paper, the trial equation method combined with complete discrimination system for polynomial has been effectively used to solve the Getmanou equation. The obtained results emphasize that the method is completely useful. With the same method, some of other equations can be dealt with.

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