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New Applications to Solitary Wave Ansatz

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Abstract

In this article, the solitary wave and shock wave solitons for nonlinear Ostrovsky equation and Potential Kadomstev-Petviashvili equations have been obtained. The solitary wave ansatz is used to carry out the solutions.

Keywords

Solitary Wave Solitons, Shock Wave Solitons, The Ostrovsky Equation, The Potential Kadomstev-Petviashvili Equation, Solitary Wave Ansatz

1. Introduction

Nonlinear wave phenomena appear in various scientific and engineering fields such as electrochemistry, electromagnetics, fluid dynamics, acoustics, cosmology, astrophysics and plasma physics. See references [1]-[4].

In recent time, the numerous approaches have been developed to obtain the solutions of nonlinear equations. For example the (G'/G)-expansion method [5] [6], the first integral method [7], the adomian decomposition method [8], the generalized differential transform method [9], Jacobi elliptic method [10], the automated tanh-function method [11] and the modified simple equation method [12] etc.

Nonlinear wave is one of the fundamental objects of nature and a growing interest has been given to the propagation of nonlinear waves in the dynamical system. The solitary wave ansatz method [13] [14] is rather heuristic and processes significant features that make it practical for the determination of single soliton solutions for a wide class of nonlinear evolution equations. The solitary wave and shock wave solitons have been obtained, using solitary wave ansatz method, for nonlinear Ostrovsky equation and Potential Kadomstev-Petviashvili (PKP) equation, and we clearly see the consistency, which has recently been applied successfully.

The Ostrovsky equation is, a model of ocean currents motion, read as

$$\left(u_t+\left(u^2\right)_x-\beta u_{xxx}\right)_x-\gamma u=0, \tag{1.1}$$

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where β and γ are constants. Parameter β determines the type of dispersion, namely, $\beta=-1$ (negative-dispersion) for surface and internal waves in the ocean and surface waves in a shallow channel with an uneven bottom; $\beta=1$ (positive dispersion) for capillary waves on the surface of liquid or for oblique magneto-acoustic waves. Parameter $\gamma>0$ measures the effect of rotation.

The Potential Kadomstev-Petviashvili (PKP) equation has been considered in the following manner

$$u_t+\left(u^3\right)_x+\left(u^2\right)_{xxx}=0. \tag{1.2}$$

2. Solitary Waves Solitons

In this section, the solitary wave solution or non-topological solution to the Ostrovsky Equation (1.1) and Potential Kadomstev-Petviashvili Equation (1.2) have been found using the following solitary wave ansatz. For this, we have

$$u(x,t)=\frac{A}{\cosh^p\xi},\text{ where } \xi=B(x-\nu t) \tag{2.3}$$

where A is the amplitude of the solitons, B is the inverse width of the solitons and ν is the velocity of the solitary wave.

2.1. OS-BBM Equation

From the Equation (2.3), it can be followed

$$u_t=\frac{AB\nu p\tanh\xi}{\cosh^p\xi} \tag{2.4}$$

$$\left(u^2\right)_x=\frac{-2A^2Bp\tanh\xi}{\cosh^{2p}\xi} \tag{2.5}$$

$$u_{xxx}=\frac{-AB^3p^3\tanh\xi}{\cosh^p\xi}+\frac{AB^3p(p+1)(p+2)\tanh\xi}{\cosh^{p+2}\xi} \tag{2.6}$$

$$u=\frac{A}{\cosh^p\xi} \tag{2.7}$$

$$\left(u_t+\left(u^2\right)_x-\beta u_{xxx}\right)=\frac{AB\nu p\tanh\xi}{\cosh^p\xi}+\frac{-2A^2Bp\tanh\xi}{\cosh^{2p}\xi}-\left\{\frac{-AB^3p^3\tanh\xi}{\cosh^p\xi}+\frac{AB^3p(p+1)(p+2)\tanh\xi}{\cosh^{p+2}\xi}\right\}$$

$$\begin{aligned}\left(u_t+\left(u^2\right)_x-\beta u_{xxx}\right)_x&=\frac{-AB^2\nu p^2}{\cosh^p\xi}+\frac{AB^2\nu p(p+1)}{\cosh^{p+2}\xi}+\frac{4A^2B^2p^2}{\cosh^{2p}\xi}-\frac{2A^2B^2p(2p+1)}{\cosh^{2p+2}\xi}-\frac{\beta AB^4p^4}{\cosh^p\xi}\\&+\frac{\beta AB^4p(p+1)\left\{p^2+(p+2)^2\right\}}{\cosh^{p+2}\xi}-\frac{\beta AB^4p(p+1)(p+2)(p+3)}{\cosh^{p+4}\xi}.\end{aligned} \tag{2.8}$$

After substituting Equations (2.4)-(2.8) into (1.1), the following equation is obtained

$$\begin{aligned}&\frac{-AB^2\nu p^2}{\cosh^p\xi}+\frac{AB^2\nu p(p+1)}{\cosh^{p+2}\xi}+\frac{4A^2B^2p^2}{\cosh^{2p}\xi}-\frac{2A^2B^2p(2p+1)}{\cosh^{2p+2}\xi}-\frac{\beta AB^4p^4}{\cosh^p\xi}+\frac{\beta AB^4p(p+1)\left\{p^2+(p+2)^2\right\}}{\cosh^{p+2}\xi}\\&-\frac{\beta AB^4p(p+1)(p+2)(p+3)}{\cosh^{p+4}\xi}-\frac{\gamma A}{\cosh^p\xi}=0.\end{aligned} \tag{2.9}$$

It may be noted that $p=2$ is being calculated when exponents $2p+2$ and $p+4$ are equated equal to each other. Furthermore, set the coefficients of the linearly independent terms to zero. Thus, we can write

$$-2A^2B^2p(p+1)-\beta AB^4p(p+1)(p+2)(p+3)=0,$$
$$-AB^2vp^2-\beta AB^4p^4-\gamma A=0.$$

Solving the above system of equations and also set $p=1$, then it can be written

$$A=-10\beta B, B=B, v=\frac{-(1)}{4}\frac{16\beta B^4+\gamma}{B^2}.$$

Hence, the solitary wave solution of the OS-BBM equation is given by

$$u(x,t)=\frac{A}{\cosh\{B(x-vt)\}}. \tag{2.10}$$

2.2. Potential Kadomstev-Petviashvili (PKP) Equation

It can, thus, be written from Equation (2.3) as follows

$$u_t=\frac{ABvp\tanh\xi}{\cosh^p\xi} \tag{2.11}$$

$$\left(u^3\right)_x=\frac{-3A^3Bp\tanh\xi}{\cosh^{3p}\xi} \tag{2.12}$$

$$\left(u^2\right)_{xxx}=\frac{-8A^2B^3p^3\tanh\xi}{\cosh^{2p}\xi}+\frac{2A^2B^3p(2p+1)(2p+2)\tanh\xi}{\cosh^{2p+2}\xi}. \tag{2.13}$$

After substituting Equations (2.11)-(2.13) into Equation (1.2), the following equation is obtained

$$\frac{ABvp\tanh\xi}{\cosh^p\xi}+\frac{-3A^3Bp\tanh\xi}{\cosh^{3p}\xi}+\frac{-8A^2B^3p^3\tanh\xi}{\cosh^{2p}\xi}+\frac{2A^2B^3p(2p+1)(2p+2)\tanh\xi}{\cosh^{2p+2}\xi}=0. \tag{2.14}$$

It may be noted that $p=2$ is being calculated when exponents $3p$ and $2p+2$ are equated equal to each other. Furthermore, set the coefficients of the linearly independent terms to zero. Thus, we can write

$$-3A^3Bp+2A^2B^3p(2p+1)(2p+2)=0.$$

Solving the above system of equations and also set $p=2$, then it can be written

$$A=20B^2, B=B, v=v.$$

Hence, the solitary wave solution of the Potential Kadomstev-Petviashvili (PKP) equation is given by

$$u(x,t)=\frac{A}{\cosh\{B(x-vt)\}}. \tag{2.15}$$

3. Shock Waves Solitons

In this section, the shock wave solution or topological solution to the Ostrovsky Equation (1.1) and Potential Kadomstev-Petviashvili Equation (1.2) have been found using the following solitary wave ansatz. For this, we can write

$$u(x,t)=A\tanh^p\xi, \text{where } \xi=B(x-vt) \text{ and } p>0 \tag{3.16}$$

where A and B are free parameters and are the amplitude and inverse width of the soliton, while v is the velocity of the soliton. The value of the exponent p is determined later.

3.1. OS-BBM Equation

Following Equation (3.16), it can be written

$$u_t = -ABvp\left\{\tanh^{p-1}\xi - \tanh^{p+1}\xi\right\} \tag{3.17}$$

$$\left(u^2\right)_x = 2A^2Bp\left\{\tanh^{2p-1}\xi - \tanh^{2p-1}\xi\right\} \tag{3.18}$$

$$\begin{aligned} u_{xxx} &= AB^3p(p-1)(p-2)\tanh^{p-3}\xi - AB^3p\left(3p^2-3p+2\right)\tanh^{p-1}\xi \\ &\quad + AB^3p\left(3p^2+3p+2\right)\tanh^{p+1}\xi - AB^3p(p+1)(p+2)\tanh^{p+3}\xi \end{aligned} \tag{3.19}$$

$$\begin{aligned} &\left(u_t + \left(u^2\right)_x - \beta u_{xxx}\right) \\ &= -ABvp\left\{\tanh^{p-1}\xi - \tanh^{p+1}\xi\right\} + 2A^2Bp\left\{\tanh^{2p-1}\xi - \tanh^{2p+1}\xi\right\} - \beta AB^3p(p-1)(p-2)\tanh^{p-3}\xi \\ &\quad + \beta AB^3p\left(3p^2-3p+2\right)\tanh^{p-1}\xi - \beta AB^3p\left(3p^2+3p+2\right)\tanh^{p+1}\xi + \beta AB^3p(p+1)(p+2)\tanh^{p+3}\xi \end{aligned}$$

$$\begin{aligned} &\left(u_t + \left(u^2\right)_x - \beta u_{xxx}\right)_x \\ &= -AB^2vp(p-1)\tanh^{p-2}\xi + 2AB^2vp^2\tanh^{p}\xi - AB^2vp(p+1)\tanh\ \tanh^{p+2}\xi + 2A^2B^2p(2p-1)\tanh^{2p-2}\xi \\ &\quad -8A^2B^2p^2\tanh^{2p}\xi + 2A^2B^2p(2p+1)\tanh^{2p+2}\xi - \beta AB^4p(p-1)(p-2)(p-3)\tanh^{p-4}\xi \\ &\quad +4\beta AB^4p(p-1)\left(p^2-2p+2\right)\tanh^{p-2}\xi - 2\beta AB^4p^2\left(3p^2+5\right)\tanh^{p}\xi \\ &\quad +4\beta AB^4p(p+1)\left(p^2+2p+2\right)\tanh^{p+2}\xi - \beta AB^4p(p+1)(p+2)(p+3)\tanh^{p+4}\xi. \end{aligned} \tag{3.20}$$

After substituting Equations (3.17)-(3.20) into (1.1), the following equation is obtained

$$\begin{aligned} &-AB^2vp(p-1)\tanh^{p-2}\xi + 2AB^2vp^2\tanh^{p}\xi - AB^2vp(p+1)\tanh^{p+2}\xi \\ &+2A^2B^2p(2p-1)\tanh^{2p-2}\xi - 8A^2B^2p^2\tanh^{2p}\xi + 2A^2B^2p(2p+1)\tanh^{2p+2}\xi \\ &-\beta AB^4p(p-1)(p-2)(p-3)\tanh^{p-4}\xi + 4\beta AB^4p(p-1)\left(p^2-2p+2\right)\tanh^{p-2}\xi \\ &-2\beta AB^4p^2\left(3p^2+5\right)\tanh^{p}\xi + 4\beta AB^4p(p+1)\left(p^2+2p+2\right)\tanh^{p+2}\xi \\ &-\beta AB^4p(p+1)(p+2)(p+3)\tanh^{p+4}\xi - \gamma A\tanh^{p}\xi = 0. \end{aligned}$$

It may be noted that $p = 2$ is being calculated when exponents $2p+2$ and $p+4$ are to be set equal to each other. Furthermore, set the coefficients of the linearly independent terms to zero. It can, thus, be written as

$$2A^2B^2p(2p+1) - \beta AB^4p(p+1)(p+2)(p+3) = 0,$$
$$2AB^2vp^2 + 2A^2B^2p(2p-1) - 2\beta AB^4p^2\left(3p^2+5\right) - \gamma A = 0.$$

Solving the above system of equations and also set $p = 2$, then it can be written

$$A = 6\beta B^2, B = B, v = \frac{1}{8}\left(\frac{64\beta B^4 + \gamma}{b^2}\right).$$

Hence, the solitary wave solution of the OS-BBM equation is given by

$$u(x,t) = A\tanh^2 B(x - vt),\ p > 0. \tag{3.21}$$

3.2. Potential Kadomstev-Petviashvili (PKP) Equation

From Equation (3.16), it can be followed

$$u_t = -ABvp\left\{\tanh^{p-1}\xi - \tanh^{p+1}\xi\right\} \tag{3.22}$$

$$\left(u^3\right)_x = 3A^3Bp\left\{\tanh^{3p-1}\xi - \tanh^{3p+1}\xi\right\} \tag{3.23}$$

$$\left(u^2\right)_{xxx} = 2A^2B^3p\left(2p-1\right)\left(2p-2\right)\tanh^{2p-3}\xi - 2A^2B^3p\left(12p^2-6p+2\right)\tanh^{2p-1}\xi$$
$$+2A^2B^3p\left(12p^2+6p+2\right)\tanh^{2p+1}\xi - 2A^2B^3p\left(2p+1\right)\left(2p+2\right)\tanh^{2p+3}\xi. \quad (3.24)$$

After substituting Equations (3.22)-(3.24) into (1.2), the following equation is obtained

$$-AB\nu p\left\{\tanh^{p-1}\xi - \tanh^{p+1}\xi\right\} + 3A^3Bp\left\{\tanh^{3p-1}\xi - \tanh^{3p+1}\xi\right\}$$
$$+2A^2B^3p\left(2p-1\right)\left(2p-2\right)\tanh^{2p-3}\xi - 2A^2B^3p\left(12p^2-6p+2\right)\tanh^{2p-1}\xi$$
$$+2A^2B^3p\left(12p^2+6p+2\right)\tanh^{2p+1}\xi - 2A^2B^3p\left(2p+1\right)\left(2p+2\right)\tanh^{2p+3}\xi = 0.$$

It may be noted that $p=2$ is being calculated when exponents $3p+1$ and $2p+3$ are to be set equal to each other. Furthermore, set the coefficients of the linearly independent terms to zero. It can, thus, be written as

$$-3A^3Bp - 2A^2B^3p\left(2p+1\right)\left(2p+2\right) = 0,$$
$$-AB\nu p + 2A^2B^3p\left(2p-1\right)\left(2p-2\right) = 0.$$

Solving the above system of equations and also set $p=2$, then it can be written

$$A = -20B^2, B = B, \nu = -240B^4.$$

Hence, the solitary wave solution of the Potential Kadomstev-Petviashvili (PKP) equation is given by

$$u\left(x,t\right) = A\tanh^p B\left(x-\nu t\right),\ p>0. \quad (3.25)$$

4. Conclusion

The growing interest of nonlinear waves has been given to the propagation in the dynamical system. The solitary wave ansatz method is rather heuristic and processes significant features that make it practical for the determination of single soliton solutions for a wide class of nonlinear evolution equations. The solitary wave and shock wave solitons have been constructed, using the solitary wave ansatz method, for Ostrovsky equation and Potential Kadomstev-Petviashvili equation and we clearly see the consistency, which has recently been applied successfully.

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