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Some Mixed Soft Operations and Extremally Soft Disconnectedness via Two Soft Topologies

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ABSTRACT

In this paper, we study the concept of soft sets which is introduced by Molodtsov [5]. We give the definition of (τ_1,τ_2)-semi open soft (resp. (τ_1,τ_2)-pre open soft, (τ_1,τ_2)-α-open soft, (τ_1,τ_2)-β-open soft, $(\tau_1,\tau_2)'$-semi open soft, $(\tau_1,\tau_2)'$-pre open soft, $(\tau_1,\tau_2)'$-α-open soft, $(\tau_1,\tau_2)'$-β-open soft) set via two soft topologies. Also we introduce (τ_1,τ_2)-regular open soft and *ESDC* on two soft topologies. The aim of this paper is to investigate properties of some mixed soft operations and characterizations of *ESDC*. Finally, we study τ_2-*ESDC* soft topologies τ_1.

KEYWORDS

(τ_1,τ_2)-Semi Open Soft; (τ_1,τ_2)-Pre Open Soft; (τ_1,τ_2)-α-Open Soft; (τ_1,τ_2)-β-Open Soft; $(\tau_1,\tau_2)'$-Semi Open Soft; $(\tau_1,\tau_2)'$-Pre Open Soft; $(\tau_1,\tau_2)'$-α-Open Soft; $(\tau_1,\tau_2)'$-β-Open Soft; (τ_1,τ_2)-Regular Open Soft; *ESDC*

1. Introduction

Some set theories can be dealt with unclear concepts, for example theory of rough sets and theory of fuzzy sets. Unfortunately, these theories are not sufficient to deal with some difficulties and encounter some problems. In 2009, Ali, Feng, Liu, Min and Shabir [1] investigated several operations on soft sets and introduced some new notions such as the restricted intersection, the restricted union etc. In 2011, Hussain and Ahmad [2] researched some properties of soft topological space. Kandil *et al*. [3] had defined semi-open (pre-open, α-open, β-open) soft set and semi (pre, α, β)-soft continuity via these soft sets. In 2003, Maji, Biswas and Roy [4] introduced equality of two soft sets, subset of a soft set, null soft set, absolute soft set etc. In 1999, Molodtsov [5] introduced the soft theory as a general mathematical tool for dealing with these problems. He accomplished very significant applications of soft set theory such as solving some complications in economics, social science, medical science, engineering etc. In 2011, Shabir and Naz [6] defined and studied some notions such as soft topological space, soft interior, soft closure etc. In 2012, Zorlutuna *et al*. [7] introduced the concepts of soft interior point, soft interior, soft neighborhood, soft continuity and soft compactness. Later there has been an extensive study on the applications of soft set theory. Many people have studided soft theory and investigated some properties of this theory.

The aim of the present paper is to introduce and study notions of (τ_1,τ_2)-semi open soft (resp. (τ_1,τ_2)-pre open soft, (τ_1,τ_2)-α-open soft, (τ_1,τ_2)-β-open soft, $(\tau_1,\tau_2)'$-semi open soft, $(\tau_1,\tau_2)'$-pre-open soft,

$(\tau_1,\tau_2)'$-α-open soft, $(\tau_1,\tau_2)'$-β-open soft) set via two soft topologies. For this purpose, we consider two soft topologies τ_1 and τ_2 over X. Also we define (τ_1,τ_2)-regular open soft and *ESDC* on two soft topologies. Furthermore, we investigate some properties of some mixed soft operations and some characterizations of *ESDC*. Finally, we show τ_2-*ESDC* soft topologies τ_1.

2. Preliminaries

Soft sets and Soft Topology

Definition 2.1 [1]. The complement of a soft set (*F*,*A*) is defined as $(F,A)^c=(F^c,A)$, where $F^c(e)=(F(e))^c=X-F(e)$, for all $e\in A$.

Theorem 2.1 [2]. Let (X,τ,E) be a soft topological space over X, (F,E) and (G,E) are soft sets over X. Then

1) $\overline{\tilde{\varnothing}}=\tilde{\varnothing}$ and $\overline{\tilde{X}}=\tilde{X}$.

2) $(F,E)\tilde{\subseteq}\overline{(F,E)}$.

3) (F,E) is a closed set if and only if $(F,E)=\overline{(F,E)}$.

4) $\overline{\overline{(F,E)}}=\overline{(F,E)}$.

5) $(F,E)\tilde{\subseteq}(G,E)$ implies $\overline{(F,E)}\tilde{\subseteq}\overline{(G,E)}$.

6) $\overline{(F,E)\cup(G,E)}=\overline{(F,E)}\cup\overline{(G,E)}$.

7) $\overline{(F,E)\cap(G,E)}\tilde{\subseteq}\overline{(F,E)}\cap\overline{(G,E)}$.

Theorem 2.2 [2]. Let (X,τ,E) be a soft topological space over X and (F,E) and (G,E) are soft sets over X. Then

1) $\tilde{\varnothing}^o=\tilde{\varnothing}$ and $\tilde{X}^o=\tilde{X}$.

2) $(F,E)^o\tilde{\subseteq}(F,E)$.

3) $\left((F,E)^o\right)^o=(F,E)^o$.

4) (F,E) is a soft open set if and only if $(F,E)^o=(F,E)$.

5) $(F,E)\tilde{\subseteq}(G,E)$ implies $(F,E)^o\tilde{\subseteq}(G,E)^o$.

6) $(F,E)^o\cap(G,E)^o=((F,E)\cap(G,E))^o$.

7) $(F,E)^o\cup(G,E)^o\tilde{\subseteq}((F,E)\cup(G,E))^o$.

Theorem 2.3 [2]. Let (F,E) be a soft set of soft topological space over X. Then

1) $\left((F,E)^c\right)^o=\left(\overline{(F,E)}\right)^c$.

2) $\overline{(F,E)^c}=\left((F,E)^o\right)^c$.

3) $(F,E)^o=\left(\overline{(F,E)^c}\right)^c$.

Definition 2.2 [3]. Let (X,τ,E) be a soft topological space and $(F,E)\in SS(X)_E$. Then (F,E) is said to be

1) pre-open soft set if $(F,E)\tilde{\subseteq}int(cl(F,E))$;

2) semi-open soft set if $(F,E)\tilde{\subseteq}cl(int(F,E))$;

3) α-open soft set if $(F,E)\tilde{\subseteq}int(cl(int(F,E)))$;

4) β-open soft set if $(F,E)\tilde{\subseteq}cl(int(cl(F,E)))$.

Definition 2.3 [4]. Let (F,A) and (G,B) be two soft sets over a common universe *X*. Then (F,A) is said to be a soft subset of (G,B) if $A\subseteq B$ and $F(e)\subseteq G(e)$, for all $e\in A$. This relation is denoted by $(F,A)\tilde{\subseteq}(G,B)$.

Definition 2.4 [4]. A soft set (*F*,*A*) over *X* is said to be a null soft set if $F(e)=\varnothing$, for all $e\in A$. This is denoted by $\tilde{\varnothing}$.

Definition 2.5 [4]. A soft set (*F*,*A*) over *X* is said to be an absolute soft set if $F(e)=X$, for all $e \in A$. This denoted by $\tilde{X}$.

Definition 2.6 [4]. The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C) , where $C = A \bigcup B$ and $H(e)=F(e)$ if $e \in A-B$ or $H(e)=G(e)$ if $e \in B-A$ or $H(e)=F(e) \bigcup G(e)$ if $e \in A \bigcap B$ for all $e \in C$.

Definition 2.7 [4]. The intersection of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \bigcap B$ and for all $e \in C$, $H(e)=F(e) \bigcap G(e)$.

Definition 2.8 [5]. A pair (F,A), where F is mapping from A to $P(X)$, is called a soft set over X . The family of all soft sets on X is denoted by $SS(X)_E$.

(F,A) is said to be soft equal to (G,B) if $(F,A) \tilde{\subseteq} (G,B)$ and $(G,B) \tilde{\subseteq} (F,A)$. This relation is denoted by $(F,A)=(G,B)$.

Definition 2.9 [6]. Let τ be the collection of soft sets over *X*. Then τ is said to be a soft topology on *X* if

1) $\tilde{\varnothing}, \tilde{X} \in \tau$;

2) the intersection of any two soft sets in τ belongs to τ ;

3) the union of any number of soft sets in τ belongs to τ .

The triple (X,τ,E) is called a soft topological space over *X*. The members of τ are said to be $\tau -$ soft open sets or soft open sets in *X*. A soft set over *X* is said to be soft closed in *X* if its complement belongs to τ . The set of all soft open sets over X denoted by $OS(X,\tau,E)$ or $OS(X)$ and the set of all soft closed sets denoted by $CS(X,\tau,E)$ or $CS(X)$.

Definition 2.10 [6]. The difference of two soft sets (*F*,*A*) and (*G*,*A*) is defined by $(F,A)-(G,A)=(F-G,A)$, where $(F-G)(e)=F(e)-G(e)$, for all $e \in A$.

Definition 2.11 [6]. Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft closure of (F,E), denoted by $cl(F,E)$ is the intersection of all closed soft super sets of (F,E).

Definition 2.12 [7]. Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft interior of (F,E), denoted by $int(F,E)$ is the union of all open soft subsets of (F,E).

3. Some Properties of Some Mixed Soft Operations

In this section we investigated some properties of some mixed operations such as (τ_1,τ_2)-semi open soft, (τ_1,τ_2)-pre open soft. Also we will write int_1, cl_1, int_2, cl_2 for $int_{\tau_1}, cl_{\tau_1}, int_{\tau_2}, cl_{\tau_2}$, respectively.

Definition 3.1. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X . Then $(F,E) \in SS(X)_E$ is said to be

1) (τ_1,τ_2)-semi open soft if $(F,E) \tilde{\subseteq} cl_2(int_1(F,E))$;

2) (τ_1,τ_2)-pre open soft if $(F,E) \tilde{\subseteq} int_1(cl_2(F,E))$;

3) (τ_1,τ_2)- α -open soft if $(F,E) \tilde{\subseteq} int_1(cl_2(int_1(F,E)))$;

4) (τ_1,τ_2)- β -open soft if $(F,E) \tilde{\subseteq} cl_2(int_1(cl_2(F,E)))$.

The complement of (τ_1,τ_2)-semi open (resp. (τ_1,τ_2)-pre open, (τ_1,τ_2)- α -open, (τ_1,τ_2)- β -open) soft set is called (τ_1,τ_2)-semi closed (resp. (τ_1,τ_2)-pre closed, (τ_1,τ_2)- α -closed, (τ_1,τ_2)- β -closed) soft (See Figure 1).

Definition 3.2. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X . Then $(F,E) \in SS(X)_E$ is said to be

1) $(\tau_1,\tau_2)'$ -semi open soft if $(F,E) \tilde{\subseteq} cl_1(int_2(F,E))$;

2) $(\tau_1,\tau_2)'$ -pre open soft if $(F,E) \tilde{\subseteq} int_2(cl_1(F,E))$;

3) $(\tau_1,\tau_2)'$ - α -open soft if $(F,E) \tilde{\subseteq} int_2(cl_1(int_2(F,E)))$;

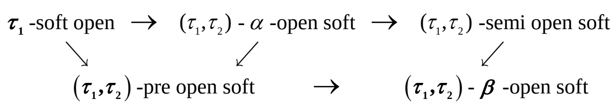

Figure 1. **The relations among τ_1 -soft open, (τ_1,τ_2) - α -open soft, (τ_1,τ_2) -semi open soft, (τ_1,τ_2) -pre open soft and (τ_1,τ_2) - β -open soft.**

4) $(\tau_1,\tau_2)'$-β-open soft if $(F,E)\tilde{\subseteq} cl_1(int_2(cl_1(F,E)))$.

The complement of $(\tau_1,\tau_2)'$-semi open (resp. $(\tau_1,\tau_2)'$-pre open, $(\tau_1,\tau_2)'$-α-open, $(\tau_1,\tau_2)'$-β-open) soft set is called $(\tau_1,\tau_2)'$-semi closed (resp. $(\tau_1,\tau_2)'$-pre closed, $(\tau_1,\tau_2)'$-α-closed, $(\tau_1,\tau_2)'$-β-closed) soft (See **Figure 2**).

Theorem 3.1. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) If $(F,E)\in SS(X)_E$ and $(G,E)\in\tau_1$, $(G,E)\tilde{\cap} cl_2(F,E)\tilde{\subseteq} cl_2((G,E)\tilde{\cap}(F,E))$.

2) If $(F,E)\in SS(X)_E$ and $(G,E)\in\tau_2$, $(G,E)\tilde{\cap} cl_1(F,E)\tilde{\subseteq} cl_1((G,E)\tilde{\cap}(F,E))$.

Proof. It is seen from Definition 2.11.

Theorem 3.2. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) (F,E) is a (τ_1,τ_2)-semi open soft set if and only if $cl_2(F,E)=cl_2(int_1(F,E))$.

2) (F,E) is a $(\tau_1,\tau_2)'$-semi open soft set if and only if $cl_1(F,E)=cl_1(int_2(F,E))$.

Proof. 1) *Necessity*. Let (F,E) be a (τ_1,τ_2)-semi open soft set. Since $(F,E)\tilde{\subseteq} cl_2(int_1(F,E))$, we have $cl_2(F,E)\tilde{\subseteq} cl_2(int_1(F,E))$. Also $cl_2(int_1(F,E))\tilde{\subseteq} cl_2(F,E)$. Hence $cl_2(F,E)=cl_2(int_1(F,E))$.

Sufficieny. Let $cl_2(F,E)=cl_2(int_1(F,E))$. Therefore $(F,E)\tilde{\subseteq} cl_2(F,E)=cl_2(int_1(F,E))$ and (F,E) is a (τ_1,τ_2)-semi open soft.

2) By a similar way.

Theorem 3.3. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) If (F,E) is a τ_1-soft open set and (G,E) is a (τ_1,τ_2)-pre open soft set, $(F,E)\tilde{\cap}(G,E)$ is a (τ_1,τ_2)-pre open soft.

2) If (F,E) is a τ_2-soft open set and (G,E) is a $(\tau_1,\tau_2)'$-pre open soft set, $(F,E)\tilde{\cap}(G,E)$ is a $(\tau_1,\tau_2)'$-pre open soft.

Proof. (1). Let (F,E) be τ_1-soft open and (G,E) be (τ_1,τ_2)-pre open soft set. Then

$$(F,E)\tilde{\cap}(G,E)\tilde{\subseteq} int_1(F,E)\tilde{\cap} int_1(cl_2(G,E))=int_1((F,E)\tilde{\cap}(G,E))\tilde{\subseteq} int_1(cl_2((F,E)\tilde{\cap}(G,E)))$$

from Theorem 3.1. Hence $(F,E)\tilde{\cap}(G,E)$ is a (τ_1,τ_2)-pre open soft.

(2). By a similar way.

Theorem 3.4. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) If either (F,E) is a (τ_1,τ_2)-semi open soft or (G,E) is a (τ_1,τ_2)-semi open soft set, $int_1(cl_2((F,E)\tilde{\cap}(G,E)))=int_1(cl_2(F,E))\tilde{\cap} int_1(cl_2(G,E))$.

2) If either (F,E) is a $(\tau_1,\tau_2)'$-semi open soft or (G,E) is a $(\tau_1,\tau_2)'$-semi open soft set, $int_2(cl_1((F,E)\tilde{\cap}(G,E)))=int_2(cl_1(F,E))\tilde{\cap} int_2(cl_1(G,E))$.

Proof. 1) Let $(F,E),(G,E)\in SS(X)_E$. We have

$$int_1(cl_2((F,E)\tilde{\cap}(G,E)))\tilde{\subseteq} int_1(cl_2(F,E))\tilde{\cap} int_1(cl_2(G,E)).$$

τ_2-soft open $\rightarrow$ $(\tau_1,\tau_2)'$-α-open soft $\rightarrow$ $(\tau_1,\tau_2)'$-semi open soft
$\searrow$ $\swarrow$ $\swarrow$
$(\tau_1,\tau_2)'$-pre open soft $\rightarrow$ $(\tau_1,\tau_2)'$-β-open soft

Figure 2. The relations among τ_1-soft open, $(\tau_1,\tau_2)'$-α-open soft, $(\tau_1,\tau_2)'$-semi open soft, $(\tau_1,\tau_2)'$-pre open soft and $(\tau_1,\tau_2)'$-β-open soft.

We assume that (F,E) is a (τ_1,τ_2)-semi open soft set. Then $cl_2(F,E)=cl_2(int_1(F,E))$ from Theorem 3.2. So

$$\begin{aligned}
&int_1(cl_2(F,E))\tilde{\cap} int_1(cl_2(G,E))=int_1\left[cl_2(F,E)\tilde{\cap} int_1(cl_2(G,E))\right]\\
&=int_1\left[cl_2(int_1(F,E))\tilde{\cap} int_1(cl_2(G,E))\right]\tilde{\subseteq} int_1\left(cl_2\left[int_1(F,E)\tilde{\cap} int_1(cl_2(G,E))\right]\right)\\
&=int_1\left(cl_2\left(int_1\left[int_1(F,E)\tilde{\cap} cl_2(G,E)\right]\right)\right)\tilde{\subseteq} int_1\left(cl_2\left(int_1\left(cl_2\left[int_1(F,E)\tilde{\cap}(G,E)\right]\right)\right)\right)\\
&\tilde{\subseteq} int_1\left(cl_2\left(int_1\left(cl_2\left[(F,E)\tilde{\cap}(G,E)\right]\right)\right)\right)=int_1\left(cl_2\left[(F,E)\tilde{\subseteq}(G,E)\right]\right)
\end{aligned}$$

from Theorem 3.1. Hence we have $int_1\left(cl_2\left((F,E)\tilde{\cap}(G,E)\right)\right)=int_1(cl_2(F,E))\tilde{\cap} int_1(cl_2(G,E))$.

2) By a similar way.

Theorem 3.5. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) If (F,E) is a τ_1-soft open set and (G,E) is a (τ_1,τ_2)-semi open soft set, $(F,E)\tilde{\cap}(G,E)$ is a (τ_1,τ_2)-semi open soft.

2) If (F,E) is a τ_2-soft open set and (G,E) is a $(\tau_1,\tau_2)'$-semi open soft set, $(F,E)\tilde{\cap}(G,E)$ is a $(\tau_1,\tau_2)'$-semi open soft.

Proof. 1) Let (F,E) be τ_1-open soft and (G,E) be (τ_1,τ_2)-semi open soft set. Then

$$(F,E)\tilde{\cap}(G,E)\tilde{\subseteq} int_1(F,E)\tilde{\cap} cl_2(int_1(G,E))\tilde{\subseteq} cl_2\left(int_1(F,E)\tilde{\cap} int_1(G,E)\right)=cl_2\left(int_1\left((F,E)\tilde{\cap}(G,E)\right)\right)$$

from Theorem 3.1. Therefore $(F,E)\tilde{\cap}(G,E)$ is a (τ_1,τ_2)-semi open soft.

2) By a similar way.

Theorem 3.6. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Let $(F,E),(G,E)\in SS(X)_E$, then

1) (F,E) is (τ_1,τ_2)-α-open soft set if and only if there exists a (G,E) τ_1-soft open set such that $(G,E)\tilde{\subseteq}(F,E)\tilde{\subseteq} int_1(cl_2(G,E))$.

2) If (F,E) is (τ_1,τ_2)-α-open soft set and $(F,E)\tilde{\subseteq}(G,E)\tilde{\subseteq} int_1(cl_2(F,E))$, then (G,E) is (τ_1,τ_2)-α-open soft.

3) (F,E) is $(\tau_1,\tau_2)'$-α-open soft set if and only if there exists a (G,E) τ_2-soft open set such that $(G,E)\tilde{\subseteq}(F,E)\tilde{\subseteq} int_2(cl_1(G,E))$.

4) If (F,E) is $(\tau_1,\tau_2)'$-α-open soft set and $(F,E)\tilde{\subseteq}(G,E)\tilde{\subseteq} int_2(cl_1(F,E))$, then (G,E) is $(\tau_1,\tau_2)'$-α-open soft.

Proof. 1) *Necessity*. Let $int_1(F,E)=(G,E)$. Then

$$(G,E)\tilde{\subseteq}(F,E)\tilde{\subseteq} int_1(cl_2(int_1(F,E)))=int_1(cl_2(F,E)).$$

Sufficiency. Let (G,E) is τ_1-soft open set and $(G,E)\tilde{\subseteq}(F,E)\tilde{\subseteq} int_1(cl_2(G,E))$. Then $int_1(G,E)=(G,E)\tilde{\subseteq} int_1(F,E)$. Hence $(F,E)\tilde{\subseteq} int_1(cl_2(int_1(G,E)))\tilde{\subseteq} int_1(cl_2(int_1(F,E)))$. Thus (F,E) is (τ_1,τ_2)-α-open soft.

2) Let (F,E) is (τ_1,τ_2)-α-open soft set and $(F,E)\tilde{\subseteq}(G,E)\tilde{\subseteq} int_1(cl_2(F,E))$. Hence

$$(F,E)\tilde{\subseteq}(G,E)\tilde{\subseteq} int_1\left(cl_2\left(int_1\left(cl_2\left(int_1(F,E)\right)\right)\right)\right)\tilde{\subseteq} int_1(cl_2(int_1(F,E)))\tilde{\subseteq} int_1(cl_2(int_1(G,E))).$$

Thus (G,E) is (τ_1,τ_2)-α-open soft.

3)-4) By a similar way.

Theorem 3.7. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) If (F,E) is (τ_1,τ_2) - α -open soft and (G,E) is (τ_1,τ_2) - β -open soft, then $(F,E)\tilde{\cap}(G,E)$ is (τ_1,τ_2) - β -open soft.

2) If (F,E) is $(\tau_1,\tau_2)'$ - α -open soft and (G,E) is $(\tau_1,\tau_2)'$ - β -open soft, then $(F,E)\tilde{\cap}(G,E)$ is $(\tau_1,\tau_2)'$ - β -open soft.

Proof. 1) Let (F,E) is (τ_1,τ_2) - α -open soft and (G,E) is (τ_1,τ_2) - β -open soft. Then

$$\begin{aligned}
&(F,E)\tilde{\subseteq}(G,E)\tilde{\subseteq} int_1\left(cl_2\left(int_1(F,E)\right)\right)\tilde{\subseteq} cl_2\left(int_1\left(cl_2(G,E)\right)\right)\\
&\tilde{\subseteq} cl_2\left[int_1\left(cl_2\left(int_1(F,E)\right)\right)\tilde{\cap} int_1\left(cl_2(G,E)\right)\right] = cl_2\left(int_1\left[cl_2\left(int_1(F,E)\right)\tilde{\cap} int_1\left(cl_2(G,E)\right)\right]\right)\\
&\tilde{\subseteq} cl_2\left(int_1\left(cl_2\left[int_1(F,E)\tilde{\cap} int_1\left(cl_2(G,E)\right)\right]\right)\right) = cl_2\left(int_1\left(cl_2\left(int_1\left[int_1(F,E)\tilde{\cap} cl_2(G,E)\right]\right)\right)\right)\\
&\tilde{\subseteq} cl_2\left(int_1\left(cl_2\left(int_1\left(cl_2\left[int_1(F,E)\tilde{\cap}(G,E)\right]\right)\right)\right)\right)\tilde{\subseteq} cl_2\left(int_1\left(cl_2\left(int_1\left(cl_2\left[(F,E)\tilde{\cap}(G,E)\right]\right)\right)\right)\right)\\
&\tilde{\subseteq} cl_2\left(int_1\left(cl_2\left[(F,E)\tilde{\cap}(G,E)\right]\right)\right)
\end{aligned}$$

from Theorem 3.1. Thus $(F,E)\tilde{\subseteq}(G,E)$ is (τ_1,τ_2) - β -open soft.

2) By a similar way.

Proposition 3.1. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. If $(F,E)\in SS(X)_E$, the following statements hold:

1) $int_1\left(cl_2(F,E)^c\right) = \tilde{X} - cl_1\left(int_2(F,E)\right)$.

2) $cl_2\left(int_1(F,E)^c\right) = \tilde{X} - int_2\left(cl_1(F,E)\right)$.

Proof. It is obvious from Definition 2.1., 2.11. and 2.12.

Theorem 3.8. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) $(F,E)\in SS(X)_E$ is a (τ_1,τ_2)-pre closed soft set if and only if $cl_1\left(int_2(F,E)\right)\tilde{\subseteq}(F,E)$.

2) $(F,E)\in SS(X)_E$ is a $(\tau_1,\tau_2)'$-pre closed soft set if and only if $cl_2\left(int_1(F,E)\right)\tilde{\subseteq}(F,E)$.

Proof. 1) *Necessity*. Let (F,E) be (τ_1,τ_2)-pre closed soft set. Then $(F,E)^c$ is a (τ_1,τ_2)-pre open soft set, that is

$$(F,E)^c \tilde{\subseteq} int_1\left(cl_2(F,E)^c\right) = int_1\left(cl_2\left(\tilde{X}-(F,E)\right)\right) = \tilde{X} - \left(cl_1\left(int_2(F,E)\right)\right).$$

Thus $cl_1\left(int_2(F,E)\right)\tilde{\subseteq}(F,E)$.

Sufficiency. Let $cl_1\left(int_2(F,E)\right)\tilde{\subseteq}(F,E)$, then $\tilde{X}-(F,E)\tilde{\subseteq}\tilde{X}-cl_1\left(int_2(F,E)\right) = int_1\left(cl_2\left(\tilde{X}-(F,E)\right)\right)$. Hence $\tilde{X}-(F,E)$ is a (τ_1,τ_2)-pre open soft set. Therefore (F,E) is a (τ_1,τ_2)-pre closed soft set.

2) By a similar way.

Theorem 3.9. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) $(F,E)\in SS(X)_E$ is a (τ_1,τ_2) - α -closed soft set if and only if $cl_1\left(int_2\left(cl_1(F,E)\right)\right)\tilde{\subseteq}(F,E)$.

2) $(F,E)\in SS(X)_E$ is a $(\tau_1,\tau_2)'$ - α -closed soft set if and only if $cl_2\left(int_1\left(cl_2(F,E)\right)\right)\tilde{\subseteq}(F,E)$.

Proof. 1) *Necessity*. Let (F,E) is a (τ_1,τ_2) - α -closed soft set. Then $(F,E)^c$ is a (τ_1,τ_2) - α -open soft, that is $(F,E)^c \tilde{\subseteq} int_1\left(cl_2\left(int_1(F,E)^c\right)\right) = int_1\left(cl_2\left(int_1\left(\tilde{X}-(F,E)\right)\right)\right) = \tilde{X} - cl_1\left(int_2\left(cl_1(F,E)\right)\right)$. Therefore, $cl_1\left(int_2\left(cl_1(F,E)\right)\right)\tilde{\subseteq}(F,E)$.

Sufficiency. Let $cl_1\left(int_2\left(cl_1(F,E)\right)\right)\tilde{\subseteq}(F,E)$, then

$$\tilde{X}-(F,E)\tilde{\subseteq}\tilde{X}-cl_1\left(int_2\left(cl_1(F,E)\right)\right) = int_1\left(cl_2\left(int_1(F,E)\right)\right).$$

Hence $\tilde{X}-(F,E)$ is a (τ_1,τ_2) - α -open soft set. Therefore (F,E) is a (τ_1,τ_2) - α -closed soft set.

2) By a similar way.

Theorem 3.10. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) $(F,E)\in SS(X)_E$ is a (τ_1,τ_2)-semi closed soft set if and only if $int_2(cl_1(F,E))\tilde{\subseteq}(F,E)$.

2) $(F,E)\in SS(X)_E$ is a $(\tau_1,\tau_2)'$-semi closed soft set if and only if $int_1(cl_2(F,E))\tilde{\subseteq}(F,E)$.

Proof. 1) *Necessity*. Let (F,E) is a (τ_1,τ_2)-semi closed soft set. Then $(F,E)^c$ is a (τ_1,τ_2)-semi open soft set, that is $(F,E)^c\tilde{\subseteq}cl_2\left(int_1(F,E)^c\right)=cl_2\left(int_1\left(\tilde{X}-(F,E)\right)\right)=\tilde{X}-int_2\left(cl_1(F,E)\right)$. Therefore, $int_2\left(cl_1(F,E)\right)\tilde{\subseteq}(F,E)$.

Sufficiency. Let $int_2\left(cl_1(F,E)\right)\tilde{\subseteq}(F,E)$, then $\tilde{X}-(F,E)\tilde{\subseteq}\tilde{X}-int_2\left(cl_1(F,E)\right)=cl_2\left(int_1\left(\tilde{X}-(F,E)\right)\right)$. Hence $\tilde{X}-(F,E)$ is a (τ_1,τ_2)-semi open soft set. Therefore (F,E) is a (τ_1,τ_2)-semi closed soft set.

2) By a similar way.

Theorem 3.11. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) If (F,E) is a (τ_1,τ_2)-α-open soft set and (G,E) is a (τ_1,τ_2)-semi open soft set, $(F,E)\tilde{\cap}(G,E)$ is a (τ_1,τ_2)-semi open soft.

2) If (F,E) is a $(\tau_1,\tau_2)'$-α-open soft set and (G,E) is a $(\tau_1,\tau_2)'$-semi open soft set, $(F,E)\tilde{\cap}(G,E)$ is a $(\tau_1,\tau_2)'$-semi open soft.

Proof. 1) Let (F,E) is a (τ_1,τ_2)-α-open soft and (G,E) is a (τ_1,τ_2)-semi open soft. Then

$$\begin{aligned}&(F,E)\tilde{\cap}(G,E)\\&\tilde{\subseteq}int_1\left(cl_2\left(int_1(F,E)\right)\right)\tilde{\cap}cl_2\left(int_1(G,E)\right)\tilde{\subseteq}cl_2\left[int_1\left(cl_2\left(int_1(F,E)\right)\right)\tilde{\cap}int_1(G,E)\right]\\&\tilde{\subseteq}cl_2\left(int_1\left[cl_2\left(int_1(F,E)\right)\tilde{\cap}int_1(G,E)\right]\right)\tilde{\subseteq}cl_2\left(int_1\left(cl_2\left[int_1(F,E)\tilde{\cap}int_1(G,E)\right]\right)\right).\\&\tilde{\subseteq}cl_2\left(int_1\left(cl_2\left(int_1\left[(F,E)\tilde{\cap}(G,E)\right]\right)\right)\right)\tilde{\subseteq}cl_2\left(int_1\left((F,E)\tilde{\cap}(G,E)\right)\right)\end{aligned}$$

Therefore $(F,E)\tilde{\cap}(G,E)$ is a (τ_1,τ_2)-semi open soft.

2) By a similar way.

Theorem 3.12. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then

1) $(F,E)\in SS(X)_E$ is a (τ_1,τ_2)-β-closed soft set if and only if $int_2\left(cl_1\left(int_2(F,E)\right)\right)\tilde{\subseteq}(F,E)$.

2) $(F,E)\in SS(X)_E$ is a $(\tau_1,\tau_2)'$-β-closed soft set if and only if $int_1\left(cl_2\left(int_1(F,E)\right)\right)\tilde{\subseteq}(F,E)$.

Proof. 1) *Necessity*. Let (F,E) is a (τ_1,τ_2)-β-closed set. Then $(F,E)^c$ is a (τ_1,τ_2)-β-open soft set, that is $(F,E)^c\tilde{\subseteq}cl_2\left(int_1\left(cl_2(F,E)^c\right)\right)=cl_2\left(int_1\left(cl_2\left(\tilde{X}-(F,E)\right)\right)\right)=\tilde{X}-int_2\left(cl_1\left(int_2(F,E)\right)\right)$. Therefore, $int_2\left(cl_1\left(int_2(F,E)\right)\right)\tilde{\subseteq}(F,E)$.

Sufficiency. Let $int_2\left(cl_1\left(int_2(F,E)\right)\right)\tilde{\subseteq}(F,E)$. Then

$$\tilde{X}-(F,E)\tilde{\subseteq}\tilde{X}-int_2\left(cl_1\left(int_2(F;E)\right)\right)=cl_2\left(int_1\left(cl_2\left(\tilde{X}-(F,E)\right)\right)\right).$$

Hence $(F,e)^c$ is a (τ_1,τ_2)-β-open soft set. Therefore, (F,E) is a (τ_1,τ_2)-β-closed soft.

2) By a similar way.

4. Extremally Soft Disconnectedness on Two Soft Topologies

In this section we introduced extremally soft disconnectedness (briefly, *ESDC*) via two soft topological spaces over X and investigated some characterizations of *ESDC*.

Definition 4.1. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. τ_1 is said to be τ_2-*ESDC* if $(F,E)\in\tau_1$ implies that $cl_2(F,E)\in\tau_1$.

Definition 4.2. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. (F,E) is said to be (τ_1,τ_2)-regular open soft set if $(F,E)=int_1(cl_2(F,E))$.

Theorem 4.1. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then the following statements are equivalent:

1) τ_1 is τ_2-*ESDC*.

2) If $(F,E)=cl_1(F,E)$, $int_2(F,E)=cl_1(int_2(F,E))$.

3) If $(F,E)\in SS(X)_E$, $cl_2(int_1(F,E))\tilde{\subseteq} int_1(cl_2(F,E))$.

4) Every (τ_1,τ_2)-semi open soft set is (τ_1,τ_2)-pre open soft.

5) If (F,E) is a (τ_1,τ_2)-β-open soft set, $cl_2(F,E)\in\tau_1$.

6) Every (τ_1,τ_2)-β-open soft set is (τ_1,τ_2)-pre open soft.

7) Every (τ_1,τ_2)-semi open soft set is (τ_1,τ_2)-α-open soft.

8) If (F,E) is a (τ_1,τ_2)-regular open soft set, $(F,E)=cl_2(F,E)$.

Proof. 1) $\Rightarrow$ 2) Let $(F,E)=cl_1(F,E)$. Then $\tilde{X}-(F,E)=int_1(\tilde{X}-(F,E))$, that is $(F,E)^c\in\tau_1$. From 1), $cl_2(F,E)^c\in\tau_1$. Hence $int_1\left(cl_2(F,E)^c\right)=cl_2(F,E)^c$. Therefore, $int_2(F,E)=cl_1(int_2(F,E))$.

2) $\Rightarrow$ 3) Let $(F,E)\in SS(X)_E$, Then $\tilde{X}-int_1(F,E)$ is τ_1-soft closed and from 2) $int_2(\tilde{X}-int_1(F,E))$ is τ_1-soft closed. Therefore, $cl_2(int_1(F,E))=\tilde{X}-int_2(\tilde{X}-int_1(F,E))$ is τ_1-soft open and since $cl_2(int_1(F,E))\tilde{\subseteq} cl_2(F,E)$, we have $cl_2(int_1(F,E))\tilde{\subseteq} int_1(cl_2(F,E))$.

3) $\Rightarrow$ 4) Let (F,E) is a (τ_1,τ_2)-semi open soft set. Then $(F,E)\tilde{\subseteq} cl_2(int_1(F,E))\tilde{\subseteq} int_1(cl_2(F,E))$ from 3). Hence, (F,E) is a (τ_1,τ_2)-pre open soft.

4) $\Rightarrow$ 5) Let (F,E) is a (τ_1,τ_2)-β-open soft set, that is $(F,E)\tilde{\subseteq} cl_2(int_1(cl_2(F,E)))$. Then $cl_2(F,E)\tilde{\subseteq} cl_2(int_1(cl_2(F,E)))$ so that $cl_2(F,E)$ is a (τ_1,τ_2)-semi open soft, and from 4) it is (τ_1,τ_2)-pre open soft. $cl_2(F,E)\tilde{\subseteq} int_1(cl_2(cl_2(F,E)))=int_1(cl_2(F,E))$. Hence, $cl_2(F,E)\in\tau_1$.

5) $\Rightarrow$ 6) Let (F,E) is a (τ_1,τ_2)-β-open soft set. From 5), $cl_2(F,E)\in\tau_1$ and $cl_2(F,E)=int_1(cl_2(F,E))$. Hence, $(F,E)\tilde{\subseteq} cl_2(F,E)=int_1(cl_2(F,E))$. Therefore, (F,E) is a (τ_1,τ_2)-pre open soft.

6) $\Rightarrow$ 7) Let (F,E) is a (τ_1,τ_2)-semi open soft set, then (F,E) is a (τ_1,τ_2)-β-open soft. From 6), (F,E) is a (τ_1,τ_2)-pre open soft. Hence, (F,E) is a (τ_1,τ_2)-α-open soft.

7) $\Rightarrow$ 1) Let $(F,E)\in\tau_1$. Then, $(F,E)=int_1(F,E)$ so that

$$cl_2(F,E)=cl_2(int_1(F,E))\tilde{\subseteq} cl_2(int_1(cl_2(F,E)))$$

and $cl_2(F,E)$ is (τ_1,τ_2)-semi open soft. From 7), $cl_2(F,E)$ is (τ_1,τ_2)-α-open soft. Hence

$$cl_2(F,E)\tilde{\subseteq} int_1(cl_2(int_1(cl_2(F,E))))=int_1(cl_2(F,E))\tilde{\subseteq} cl_2(F,E)$$

and $cl_2(F,E)=int_1(cl_2(F,E))$ so that $cl_2(F,E)\in\tau_1$.

1) $\Rightarrow$ 8) Let (F,E) is (τ_1,τ_2)-regular open soft. Then $(F,E)\in\tau_1$ and $cl_2(F,E)\in\tau_1$ from 1). Therefore, $cl_2(F,E)=int_1(cl_2(F,E))=(F,E)$.

8) $\Rightarrow$ 1) Let $(F,E)\in\tau_1$. Since $int_1(cl_2(F,E))=int_1(cl_2(int_1(cl_2(F,E))))$, the soft set $int_1(cl_2(F,E))$ is (τ_1,τ_2)-regular open soft. From 8), $int_1(cl_2(F,E))=cl_2(int_1(cl_2(F,E)))$. Since $(F,E)\tilde{\subseteq} cl_2(F,E)$,

$$(F,E)\tilde{\subseteq} int_1(cl_2(F,E))=cl_2(int_1(cl_2(F,E))).$$

Hence, we have $cl_2(F,E)\tilde{\subseteq} cl_2(int_1(cl_2(F,E)))=int_1(cl_2(F,E))$ so that $cl_2(F,E)\in\tau_1$.

Theorem 4.2. Let X be an initial universe and E be a set of parameters. Let τ_1 and τ_2 be two soft topologies on X. Then the following statements are equivalent:

1) τ_1 is τ_2-*ESDC*.

2) If (F,E) is (τ_1,τ_2)-semi open soft set, $cl_2(F,E)\in\tau_1$.

3) If (F,E) is (τ_1,τ_2)-pre open soft set, $cl_2(F,E)\in\tau_1$.

4) If (F,E) is (τ_1,τ_2)-regular open soft set, $cl_2(F,E)\in\tau_1$.

Proof. 1) ⇒ 2) Every (τ_1,τ_2)-semi open soft set is (τ_1,τ_2)-β-open soft so that $cl_2(F,E)\in\tau_1$ from Theorem 4.1.

2) ⇒ 4). Every (τ_1,τ_2)-regular open soft set is (τ_1,τ_2)-semi open soft set since

$$(F,E)=int_1(cl_2(F,E))\tilde{\subseteq} cl_2(int_1(cl_2(F,E)))=cl_2(int_1(int_1(cl_2(F,E))))=cl_2(int_1(F,E)).$$

From 2), $cl_2(F,E)\in\tau_1$.

1) ⇒ 3) Since every (τ_1,τ_2)-pre open soft set is (τ_1,τ_2)-β-open soft, it is obvious from Theorem 4.1.

3) ⇒ 4) Since every (τ_1,τ_2)-regular open soft set is (τ_1,τ_2)-pre open soft, $cl_2(F,E)\in\tau_1$.

4) ⇒ 1) If $(F,E)\in\tau_1$, since $int_1(cl_2(F,E))=int_1(cl_2(int_1(cl_2(F,E))))$ and we have $int_1(cl_2(F,E))$ is (τ_1,τ_2)-regular open soft. Also we obtain from 4) $cl_2(int_1(cl_2(F,E)))\in\tau_1$ and $(F,E)\tilde{\subseteq} int_1(cl_2(F,E))$ by $(F,E)\tilde{\subseteq} cl_2(F,E)$ so that

$$(F,E)\tilde{\subseteq} int_1(cl_2(F,E))\tilde{\subseteq} cl_2(int_1(cl_2(F,E)))=int_1(cl_2(int_1(cl_2(F,E))))=int_1(cl_2(F,E)),$$

$$cl_2(F,E)\tilde{\subseteq} cl_2(int_1(cl_2(F,E)))=int_1(cl_2(F,E)).$$

Therefore $cl_2(F,E)\in\tau_1$.

Lemma 4.1. $\tau_1\tilde{\subseteq}\tau_2$ implies $int_1(F,E)\tilde{\subseteq} int_2(F,E)$ and $cl_2(F,E)\tilde{\subseteq} cl_1(F,E)$ for $(F,E)\in SS(X)_E$.

Proof. Obvious.

Theorem 4.3. Let X be an initial universe and E be a set of parameters. Let τ_1, τ_2 be two soft topologies on X and $\tau_1\tilde{\subseteq}\tau_2$. Then the following statements are equivalent:

1) τ_1 is τ_2-*ESDC*.

2) If $(F,E)\in\tau_1$ and $(G,E)\in\tau_2$, $cl_2(F,E)\tilde{\bigcap} cl_1(G,E)\tilde{\subseteq} cl_1((F,E)\tilde{\bigcap}(G,E))$.

3) If $(F,E)\in\tau_1$, $(G,E)\in\tau_2$ and $(F,E)\tilde{\bigcap}(G,E)=\tilde{\varnothing}$, $cl_2(F,E)\tilde{\bigcap} cl_1(G,E)=\tilde{\varnothing}$.

4) If $(F,E)\in SS(X)_E$, $(G,E)\in\tau_2$ and $(F,E)\tilde{\bigcap}(G,E)=\tilde{\varnothing}$, $cl_2(int_1(cl_2(F,E)))\tilde{\bigcap} cl_1(G,E)=\tilde{\varnothing}$.

Proof. 1) ⇒ 2) Let $(F,E)\in\tau_1$ and $(G,E)\in\tau_2$. From (1), $cl_2(F,E)\in\tau_2$. Then

$$\begin{aligned}cl_2(F,E)\tilde{\bigcap} cl_1(G,E)&\tilde{\subseteq} cl_1(cl_2(F,E)\tilde{\bigcap}(G,E))\tilde{\subseteq} cl_1(cl_2((F,E)\tilde{\bigcap}(G,E)))\\&\tilde{\subseteq} cl_1(cl_1((F,E)\tilde{\bigcap}(G,E)))=cl_1((F,E)\tilde{\bigcap}(G,E))\end{aligned}.$$

2) ⇒ 3) Let $(F,E)\in\tau_1$, $(G,E)\in\tau_2$ and $(F,E)\tilde{\bigcap}(G,E)=\tilde{\varnothing}$. From (2),

$$cl_2(F,E)\tilde{\bigcap} cl_1(G,E)\tilde{\subseteq} cl_1((F,E)\tilde{\bigcap}(G,E))=\tilde{\varnothing}.$$

3) ⇒ 4) Let $(F,E)\in SS(X)_E$, $(G,E)\in\tau_2$ and $(F,E)\tilde{\bigcap}(G,E)=\tilde{\varnothing}$. From (3), $int_1(cl_2(F,E))\in\tau_1$, $(G,E)\in\tau_2$ and $cl_2(F,E)\tilde{\bigcap}(G,E)=\tilde{\varnothing}$, $int_1(cl_2(F,E))\tilde{\bigcap}(G,E)=\tilde{\varnothing}$. Hence

$$cl_2(int_1(cl_2(F,E)))\tilde{\bigcap} cl_1(G,E)=\tilde{\varnothing}.$$

4) ⇒ 3) Let $(F,E)\in\tau_1$, $(G,E)\in\tau_2$ and $(F,E)\tilde{\bigcap}(G,E)=\tilde{\varnothing}$. From (4),

$$cl_2(int_1(cl_2(F,E)))\tilde{\bigcap} cl_1(G,E)=\tilde{\varnothing}.$$

Then we have $(F,E)\tilde{\subseteq} int_1(cl_2(F,E))$ since $(F,E)\tilde{\subseteq} cl_2(F,E)$. Therefore,

$$cl_2(F,E)\tilde{\subseteq} cl_2(int_1(cl_2(F,E))).$$

Hence $cl_2(F,E)\tilde{\cap}cl_1(G,E)=\tilde{\varnothing}$.

3) ⇒ 1) Let $(F,E)\in\tau_1$. Then $\tilde{X}-cl_2(F,E)\in\tau_2$ and $(F,E)\tilde{\cap}(\tilde{X}-cl_2(F,E))=\tilde{\varnothing}$. From (3),

$$cl_2(F,E)\tilde{\cap}cl_1(\tilde{X}-cl_2(F,E))=\tilde{\varnothing}.$$

Thus $cl_2(F,E)\tilde{\subseteq}\tilde{X}-cl_1(\tilde{X}-cl_2(F,E))=int_1(cl_2(F,E))$. Therefore, $cl_2(F,E)\in\tau_1$.

5. τ_2-ESDC Soft Topologies τ_1

The family of all semi-open (resp. pre-open, α-open, β-open) soft sets is denoted by $SOS(\tau_1)$ (resp. $POS(\tau_1)$, $\alpha OS(\tau_1)$, $\beta OS(\tau_1)$). Also the family of all (τ_1,τ_2)-semi open (resp. (τ_1,τ_2)-pre open, (τ_1,τ_2)-α-open, (τ_1,τ_2)-β-open) soft sets is denoted by $SOS((\tau_1,\tau_2))$ (resp. $POS((\tau_1,\tau_2))$, $\alpha OS((\tau_1,\tau_2))$, $\beta OS((\tau_1,\tau_2))$.

Theorem 5.1. If $\mu_1=SOS((\tau_1,\tau_2))$ and $\mu_2=SOS(\tau_2)$, $\mu_1\tilde{\subseteq}\mu_2$ and $(F,E)\in\mu_1$ implies $cl_{\mu_2}(F,E)\in\mu_1$.

Proof. Let $(F,E)\in\mu_1$. As $(F,E)\tilde{\subseteq}cl_2(int_1(F,E))$, $(F,E)\tilde{\subseteq}cl_2(int_2(F,E))$ from Lemma 4.1. $cl_2(int_1(F,E))$ is μ_2-soft closed for $(F,E)\in SS(X)_E$ since

$$int_2(cl_2(cl_2(int_1(F,E))))\tilde{\subseteq}cl_2(cl_2(int_1(F,E)))=cl_2(int_1(F,E)).$$

Hence $(F,E)\tilde{\subseteq}cl_2(int_1(F,E))$ implies $cl_{\mu_2}(F,E)\tilde{\subseteq}cl_2(int_1(F,E))$ and

$$(F,E)\tilde{\subseteq}cl_{\mu_2}(F,E)\tilde{\subseteq}cl_2(int_1(F,E))\tilde{\subseteq}cl_2(int_1(cl_{\mu_2}(F,E))).$$

Therefore, we obtain $cl_{\mu_2}(F,E)\in\mu_1$.

Theorem 5.2. If $\mu_1=\beta OS((\tau_1,\tau_2))$ and $\mu_2=\beta OS(\tau_2)$, $\mu_1\tilde{\subseteq}\mu_2$ and $(F,E)\in\mu_1$ implies $cl_{\mu_2}(F,E)\in\mu_1$.

Proof. Let $(F,E)\in\mu_1$. As $(F,E)\tilde{\subseteq}cl_2(int_1(cl_2(F,E)))$, $(F,E)\tilde{\subseteq}cl_2(int_2(cl_2(F,E)))$ from Lemma 4.1. $cl_2(int_1(cl_2(F,E)))$ is μ_2-soft closed for $(F,E)\in SS(X)_E$ since

$$int_2(cl_2(int_2(cl_2(int_1(cl_2(F,E))))))\tilde{\subseteq}cl_2(cl_2(int_1(cl_2(F,E))))=cl_2(int_1(cl_2(F,E))).$$

Hence, $(F,E)\tilde{\subseteq}cl_2(int_1(cl_2(F,E)))$ implies

$$(F,E)\tilde{\subseteq}cl_{\mu_2}(F,E)\tilde{\subseteq}cl_2(int_1(cl_2(F,E)))\tilde{\subseteq}cl_2(int_1(cl_2(cl_{\mu_2}(F,E)))).$$

Therefore, we obtain $cl_{\mu_2}(F,E)\in\mu_1$.

6. Conclusion

We give the definition of (τ_1,τ_2)-semi open soft (resp. (τ_1,τ_2)-pre open soft, (τ_1,τ_2)-α-open soft, (τ_1,τ_2)-β-open soft, $(\tau_1,\tau_2)'$-semi open soft, $(\tau_1,\tau_2)'$-pre open soft, $(\tau_1,\tau_2)'$-α-open soft, $(\tau_1,\tau_2)'$-β-open soft) set via two soft topologies. Also we introduce (τ_1,τ_2)-regular open soft and *ESDC* on two soft topologies. Some properties of some mixed soft operations and characterizations of *ESDC* are investigated. These properties which are studied are very important for studying anymore.

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