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Coupled Fixed Point Theorem for Weakly Compatible Mappings in Menger Spaces

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ABSTRACT

In this paper, first, we introduce the notion of weakly compatible maps for coupled maps and then prove a coupled fixed point theorem under more general *t*-norm(*H*-type norm) in Menger spaces. We support our theorem by providing a suitable example. At the end, we obtain an application.

Keywords: Menger Spaces; w-Compatible Maps; Phi-Contractive Conditions

1. Introduction

In 1942, Menger [1] introduced the notion of a probabilistic metric space (PM-space) which was, in fact, a generalization of metric space. The idea in probabilistic metric space is to associate a distribution function with a point pair, say (p,q), denoted by $F(p,q,t)$ where $t>0$ and interpret this function as the probability that distance between p and q is less than t, whereas in the metric space, the distance function is a single positive number. Sehgal [2] initiated the study of fixed points in probabilistic metric spaces. The study of these spaces was expanded rapidly with the pioneering works of Schweizer-Sklar [3].

In 1991, Mishra [4] introduced the notion of compatible mappings in the setting of probabilistic metric space. In 1996, Jungck [5] introduced the notion of weakly compatible mappings as follows:

Two self-mappings S and T are said to be weakly compatible if they commute at their coincidence points, *i.e.*, $Tu = Su$ for some $u \in X$, then $TSu = STu$.

Further, Singh and Jain [6] proved some results for weakly compatible in Menger spaces.

Fang [7] defined ϕ-contractive conditions and proved some fixed point theorems under ϕ-contractions for compatible and weakly compatible maps in Menger PM-spaces using t-norm of H-type, introduced by Hadžíc [8].

Recently, Bhaskar and Lakshmikantham [9], Lakshmikantham and Ćirić [10] gave some coupled fixed point theorems in partially ordered metric spaces.

Now, we prove a coupled fixed point theorem for a pair of weakly compatible maps satisfying ϕ-contractive conditions in Menger PM-space with a continuous t-norm of H-type. At the end, we derive a result for w-compatible maps, introduced by Abbas, Khan and Redenović [11].

2. Preliminaries

First, recall that a real valued function f defined on the set of real numbers is known as a distribution function if it is non-decreasing, left continuous and $\inf f(x) = 0$, $\sup f(x) = 1$. In what follows, $H(x)$ denotes the distribution function defined as follows:

$$H(x) = \begin{cases} 0, & \text{if } x \le 0, \\ 1, & \text{if } x > 0. \end{cases}$$

Definition 2.1. A probabilistic metric space (PM-space) is a pair $(X,)$ where X is a set and F is a function defined on $X \times X$ into the set of distribution functions such that if x, y and z are points of X, then

(F-1) $F(x,y;0) = 0$,
(F-2) $F(x,y;t) = H(t)$ iff $x = y$,

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(F-3) $(x,y;t) = F(y,x;t)$,

(F-4) if $F(x,y;s)=1$ and $F(y,z;t)=1$, then $F(x,z;s+t)=1$ for all $x,y,z \in X$ and $s,t \geq 0$.

For each x and y in X and for each real number ≥ 0, $F(x,y;t)$ is to be thought of as the probability that the distance between x and y is less than t.

It is interesting to note that, if (X,d) is a metric space, then the distribution function $F(x,y;t)$ defined by the relation $F(x,y;t) = H(t-d(x,y))$ induces a PM-space.

Definition 2.2. A t-norm t is a 2-place function, $t:[0,1]\times[0,1]\to[0,1]$ satisfying the following:

1) $t(0,0)=0$,
2) $t(a,1)=a$,
3) $t(a,b)=t(b,a)$,
4) if $a\leq c$, $b\leq d$, then $t(a,b)\leq t(c,d)$,
5) $t(t(a,b),c)=t(a,t(b,c))$ for all a, b, c in [0,1].

Definition 2.3. A Menger PM-space is a triplet (X,t) where (X,F) is a PM-space and t is a t-norm with the following condition:

(F-5) ($F(x,z;s+t)\geq t(F(x,y;s),F(y,z;t))$, for all $x,y,z\in X$ and $s,t\geq 0$.

This inequality is known as Menger's triangle inequality.

We consider (X,F,t) to be a Menger PM-space along with condition (F-6) $\lim_{n\to\infty} F(x,y,t)=1$, for all x,y in X.

Definition 2.4 [4]. Let $\sup_{0<t<1}\Delta(t,t)=1$. A t-norm Δ is said to be of H-type if the family of functions $\{\Delta^m(t)\}_{m=1}^{\infty}$ is equicontinuous at $t=1$, where

$\Delta^1(t)=\Delta t$, $\Delta^{m+1}(t)=\Delta(\Delta^m(t))$, $m=1,2,\cdots,t\in[0,1]$.

The t-norm Δ_M = min. is an example of t-norm of H type.

Remark 2.1. Δ is a H-type t-norm iff for any $\lambda\in(0,1)$, there exists $\delta(\lambda)\in(0,1)$ such that $\Delta^m(t)>(1-\lambda)$ for all $m\in N$, when $t>(1-\delta)$.

Definition 2.5. A sequence $\{x_n\}$ in a Menger PM space (X,F,t) is said

1) to converge to a point x in X if for every $\epsilon>0$ and $\lambda>0$, there is an integer n_0 such that $F(x_n,x,\epsilon)>1-\lambda$, for all $n\geq n_0$.

2) to be Cauchy if for each $\epsilon>0$ and $\lambda>0$, there is an integer n_0 such that $F(x_n,x_m,\epsilon)>1-\lambda$, for all $n,m\geq n_0$.

3) to be complete if every Cauchy sequence in it converges to a point of it.

Definition 2.6 [3]. Define $\Phi=\{\phi:R^+\to R^+\}$, where $R^+=[0,+\infty)$ and each $\phi\in\Phi$ satisfies the following conditions:

$(\phi-1)\phi$ is non-decreasing;

$(\phi-2)\phi$ is upper semicontinuous from the right;

$(\phi-3)\sum_{n=0}^{\infty}\phi^n(t)<+\infty$ for all $t>0$, where $\phi^{n+1}(t)=\phi(\phi^n(t)), n\in N$.

Clearly, if $\phi\in\Phi$, then $\phi(t)<t$ for all $t>0$.

Definition 2.7 [3]. An element $x\in X$ is called a common fixed point of the mappings

f : $X\times X\to X$ and g : $X\to X$ if

$$x=f(x,x)=g(x)$$

Definition 2.8 [6]. An element $(x,y)\in X\times X$ is called a

1) coupledfixed point of the mapping $f:X\times X\to X$ if $f(x,y)=x$, $f(y,x)=y$.

2) coupled coincidence point of the mappings $f:X\times X\to X$ and $g:X\to X$ if $f(x,y)=g(x)$ $f(y,x)=g(y)$.

3) common coupled fixed point of the mappings $f:X\times X\to X$ and $g:X\to X$ if $x=f(x,y)=g(x)$, $y=f(y,x)=g(y)$

Definition 2.9 [3]. The mappings $f:X\times X\to X$ and $g:X\to X$ are called commutative if

$g(f(x,y))=f(gx,gy)$, for all $x,y\in X$.

Abbas, Khan and Redenović [1] introduced the notion of w-compatible maps for coupled mappings as follows.

The mappings $F:X\times X\to X$ and $g:X\to X$ are called w-compatible if

$g(F(x,y))=F(gx,gy)$ whenever $F(x,y)=g(x)$, $F(y,x)=g(y)$.

In a similar mode, we state weakly compatible maps for coupled maps as follows:

Definition 2.10. The maps $f:X\times X\to X$ and $g:X\to X$ are called weakly compatible if $f(x,y)=g(x)$, $f(y,x)=g(y)$ implies $g(f(x,y))=f(gx,gy)$, $g(f(y,x))=f(gy,gx)$, for all $x,y\in X$.

We note that w-compatible are obviously weakly compatible maps.

3. Main Results

For convenience, we denote

(3.1)

$$[F(x,y,t)]^n=\underbrace{F(x,y,t)*F(x,y,t)*\cdots*F(x,y,t)}_{n},$$ for all $n\in N$.

Now we prove our main result.

Theorem 3.1. Let $(X,F,*)$ be Menger PM-Space, $*$ being continuous t – norm of H-type. Let $f:X\times X\to X$ and $g:X\to X$ be two mappings and there exists $\phi\in\Phi$ such that followings hold:

(3.2)

$$F(f(x,y),f(u,v),\phi(t))\geq F(gx,gu,t)*F(gy,gv,t),$$

for all x,y,u,v in X and $t>0$ and

1) Suppose that $f(X\times X)\subseteq g(X)$,
2) pair (f,g) is weakly compatible,
3) range space of one of the maps f or g is complete.

Then f and g have a coupled coincidence point. Moreover, there exists a unique point x in X such that $= f(x,y) = g(x)$.

Proof. Let x_0, y_0 be two arbitrary points in X. Since $f(X \times X) \subseteq g(X)$, we can choose x_1, y_1 in X such that $g(x_1) = f(x_0, y_0)$, $g(y_1) = f(y_0, x_0)$.

Continuing in this way we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $g(x_{n+1}) = f(x_n, y_n)$ and $g(y_{n+1}) = f(y_n, x_n)$ for all $n \ge 0$.

Step 1. We first show that $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences.

Since $*$ is a t-norm of H-type, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

(3.3) $\underbrace{(1-\delta)*(1-\delta)*\cdots*(1-\delta)}_{p} \ge (1-\epsilon)$, for all $p \in N$.

Since $\lim_{t\to\infty} F(x,y,t) = 1$, for all x, y in X, there exists $t_0 > 0$ such that $F(gx_0, gx_1, t_0) \ge (1-\delta)$ and $F(gy_0, gy_1, t_0) \ge 1-\delta$.

Since $\phi \in \Phi$ and using condition (ϕ-3), we have $\sum_{n=1}^{\infty} \phi^n(t_0) < \infty$. Then for any > 0, there exists $n_0 \in N$ such that

(3.4) $t > \sum_{k=n_0}^{\infty} \phi^k(t_0)$.

From (3.2), we have

$$F(gx_1, gx_2, \phi(t_0)) = F(f(x_0,y_0), f(x_1,y_1), \phi(t_0))$$
$$\ge F(gx_0, gx_1, t_0) * F(gy_0, gy_1, t_0)$$

$$F(gy_1, gy_2, \phi(t_0)) = F(f(y_0,x_0), f(y_1,x_1), \phi(t_0))$$
$$\ge F(gy_0, gy_1, t_0) * F(gx_0, gx_1, t_0).$$

Similarly, we can also get

$$F(gx_2, gx_3, \phi^2(t_0)) = F(f(x_1,y_1), f(x_2,y_2), \phi^2(t_0))$$
$$\ge F(gx_1, gx_2, \phi(t_0)) * F(gy_1, gy_2, \phi(t_0))$$

$$F(gy_2, gy_3, \phi^2(t_0)) = F(f(y_1,x_1), f(y_2,x_2), \phi^2(t_0))$$
$$\ge [F(gy_0, gy_1, t_0)]^2 * [F(gx_0, gx_1, t_0)]^2.$$

Continuing in this way, we can get

$$F(gx_n, gx_{n+1}, \phi^n(t_0))$$
$$\ge [F(gx_0, gx_1, t_0)]^{2^{n-1}} * [F(gy_0, gy_1, t_0)]^{2^{n-1}}$$

$$F(gy_n, gy_{n+1}, \phi^n(t_0))$$
$$\ge [F(gy_0, gy_1, t_0)]^{2^{n-1}} * [F(gx_0, gx_1, t_0)]^{2^{n-1}}.$$

So, from (3.3) and (3.4), for $m > n \ge n_0$, we have

$$F(gx_n, gx_m, t) \ge F\left(gx_n, gx_m, \sum_{k=n_0}^{\infty} \phi^k(t_0)\right)$$
$$\ge F\left(gx_n, gx_m, \sum_{k=n}^{m-1} \phi^k(t_0)\right)$$
$$\ge F(gx_n, gx_{n+1}, \phi^n(t_0)) * F(gx_{n+1}, gx_{n+2}, \phi^{n+1}(t_0)) * \cdots$$

$$*F(gx_{m-1}, gx_m, \phi^{m-1}(t_0)) \ge \left[\left[\left\{[F(gx_0,gx_1,t_0)]^{2^{n-1}} * [F(gy_0,gy_1,t_0)]^{2^{n-1}}\right\}\right.\right.$$
$$\left.\left. * \left\{[F(gx_0,gx_1,t_0)]^{2^n} * [F(gy_0,gy_1,t_0)]^{2^n}\right\} * \cdots * \left\{[F(gx_0,gx_1,t_0)]^{2^{m-2}} * [F(gy_0,gy_1,t_0)]^{2^{m-2}}\right\}\right]\right]$$
$$= [F(gx_0,gx_1,t_0)]^{2^{n-1}(2^{m-n}-1)} * [F(gy_0,gy_1,t_0)]^{2^{n-1}(2^{m-n}-1)}$$
$$\ge \underbrace{(1-\delta)*(1-\delta)*\cdots*(1-\delta)}_{2^n(2^{m-n}-1)} \ge (1-\epsilon)$$

which implies that $F(gx_n, gx_m, t) \ge (1-\epsilon)$, for all $m, n \in N$ with $m > n \ge n_0$ and $t > 0$.

So, $\{gx_n\}$ is a Cauchy sequence. Similarly, we can get that $\{gy_n\}$ is a Cauchy sequence.

Step 2. To show that f and g have a coupled coincidence point.

Without loss of generality, we assume that $g(X)$ is complete, then there exists points x, y in $g(X)$ so that $\lim_{n\to\infty} g(x_{n+1}) = x$, $\lim_{n\to\infty} g(y_{n+1}) = y$.

Again $x, y \in g(X)$ implies the existence of p, q in X so that $g(p) = x$, $g(q) = y$ and hence $\lim_{n\to\infty} g(x_{n+1}) = \lim_{n\to\infty} f(x_n, y_n) = g(p) = x$, $\lim_{n\to\infty} g(y_{n+1}) = \lim_{n\to\infty} f(y_n, x_n) = g(q) = y$.

From (3.2),

$$F(f(x_n,y_n), f(p,q), \phi(t))$$
$$\ge F(gx_n, g(p), t) * F(gy_n, g(q), t)$$

Taking limit as $n \to \infty$, we get $F(g(p), f(p,q), \phi(t)) = 1$ that is, $f(p,q) = g(p) = x$.

Similarly, $f(q,p) = g(q) = y$.

But f and g are weakly compatible, so that $f(p,q) = g(p) = x$ and $f(q,p) = g(q) = y$ implies $gf(p,q) = f(g(p), g(q))$ and $gf(q,p) = f(g(q), g(p))$, that is $g(x) = f(x,y)$

and $g(y)=f(y,x)$.

Hence f and g have a coupled coincidence point.

Step 3. To show that $g(x)=y$ and $g(y)=x$.

Since * is a t-norm of H-type, any $\epsilon>0$, there exists $\delta>0$ such that

$$\underbrace{(1-\delta)*(1-\delta)*\cdots*(1-\delta)}_{p}\geq(1-\epsilon) \quad \text{for all } p\in N.$$

Since $\lim_{t\to\infty}F(x,y,t)=1$, for all x,y in X, there exists $t_0>0$ such that

$F(gx,y,t_0)\geq(1-\delta)$ and $F(gy,x,t_0)\geq(1-\delta)$.

Since $\phi\in\Phi$ and using condition (ϕ-3), we have $\sum_{n=1}^{\infty}\phi^n(t_0)<\infty$. Then for any $t>0$, there exists $n_0\in N$ such that

$$t>\sum_{k=n_0}^{\infty}\phi^k(t_0)$$

Using condition (3.2), we have

$$F(gx,gy_{n+1},\phi(t_0))=F(f(x,y),f(y_n,x_n),\phi(t_0))$$
$$\geq F(gx,gy_n,t_0)*F(gy,gx_n,t_0),$$

letting $n\to\infty$, we get

$$F(gx,y,\phi(t_0))\geq F(gx,y,t_0)*F(gy,x,t_0),$$

By this way, we can get for all $n\in N$,

$$F(gx,y,\phi^n(t_0))\geq F(gx,y,\phi^{n-1}(t_0))*F(gy,x,\phi^{n-1}(t_0))$$
$$\geq[F(gx,y,t_0)]^{2^{n-1}}*[F(gy,x,t_0)]^{2^{n-1}}$$

thus, we have

$$F(gx,y,t)\geq F\left(gx,y,\sum_{k=n_0}^{\infty}\phi^k(t_0)\right)\geq F(gx,y,\phi^{n_0}(t_0))\geq[F(gx,y,t_0)]^{2^{n_0-1}}*[F(gy,x,t_0)]^{2^{n_0-1}}$$
$$\geq\underbrace{(1-\delta)*(1-\delta)*\cdots*(1-\delta)}_{2^{n_0}}\geq(1-\epsilon).$$

So, for any $\epsilon>0$, we have $F(gx,y,t)\geq(1-\epsilon)$, for all $t>0$.

This implies $g(x)=y$. Similarly, $g(y)=x$.

Step 4. Next we shall show that $=y$.

Since * is a *t*-norm of H-type, for any $\epsilon>0$, there exists $\delta>0$ such that

$$\underbrace{(1-\delta)*(1-\delta)*\cdots*(1-\delta)}_{p}\geq(1-\epsilon), \quad \text{for all } p\in N.$$

Since $\lim_{t\to\infty}F(x,y,t)=1$, for all x,y in X, there exists $t_0>0$ such that $F(x,y,t_0)\geq(1-\delta)$

Also, since $\phi\in\Phi$, using condition (ϕ-3), we have $\sum_{n=1}^{\infty}\phi^n(t_0)<\infty$. Then for any $t>0$, there exists $n_0\in N$ such that

$$t>\sum_{k=n_0}^{\infty}\phi^k(t_0).$$

Using condition (3.2), we have

$$F(gx_{n+1},gy_{n+1},\phi(t_0))=F(f(x_n,y_n),f(y_n,x_n),\phi(t_0))$$
$$\geq F(gx_n,gy_n,t_0)*F(gy_n,gx_n,t_0)$$

Letting $n\to\infty$, we get

$F(x,y,\phi(t_0))\geq F(x,y,t_0)*F(y,x,t_0)$. Thus we have

$$F(x,y,t)\geq F\left(x,y,\sum_{k=n_0}^{\infty}\phi^k(t_0)\right)\geq F(x,y,\phi^{n_0}(t_0))$$
$$\geq[F(x,y,t_0)]^{2^{n_0-1}}*[F(y,x,t_0)]^{2^{n_0-1}}$$
$$\geq\underbrace{(1-\delta)*(1-\delta)*\cdots*(1-\delta)}_{2^{n_0}}\geq(1-\epsilon)$$

which implies that $x=y$. Thus, we have proved that *f* and *g* have a common fixed point x in X.

Step 5. We now prove the uniqueness of x.

Let z be any point in X such that $z\neq x$ with $g(z)=z=f(z,z)$.

Since * is a t-norm of H-type, for any $\epsilon>0$, there exists $\delta>0$ such that

$$\underbrace{(1-\delta)*(1-\delta)*\cdots*(1-\delta)}_{p}\geq(1-\epsilon), \text{ for all } p\in N.$$

Since $\lim_{t\to\infty}F(x,y,t)=1$, for all x,y in X, there exists $t_0>0$ such that $F(x,z,t_0)\geq1-\delta$.

Also, since $\phi\in\Phi$, using condition (ϕ-3), we have $\sum_{n=1}^{\infty}\phi^n(t_0)<\infty$. Then for any $t>0$, there exists $n_0\in N$ such that $t>\sum_{k=n_0}^{\infty}\phi^k(t_0)$.

Using condition (3.2), we have

$$F(x,z,\phi(t_0))=F(f(x,x),f(z,z),\phi(t_0))$$
$$\geq F(g(x),g(z),t_0)*F(g(x),g(z),t_0)$$
$$=F(x,z,t_0)*F(x,z,t_0)[F(x,z,t_0)]^2.$$

Thus, we have

$$F(x,z,t)\geq F\left(x,z,\sum_{k=n_0}^{\infty}\phi^k(t_0)\right)\geq F(x,z,\phi^{n_0}(t_0))$$
$$\geq\geq\left([F(x,z,t_0)]^{2^{n_0-1}}\right)^2=(F(x,z,t_0))^{2^{n_0}}$$
$$\geq\underbrace{(1-\delta)*(1-\delta)*\cdots*(1-\delta)}_{2^{n_0}}\geq(1-\epsilon),$$

which implies that $x = y$.

Hence, *f* and *g* have a unique common fixed pointin X.

Next, we give an example in support of the Theorem 3.1.

Example 3.1. Let $X = [-2,2), a*b = ab$ for all $a,b \in [0,1]$ and $\varphi(t) = \frac{t}{t+1}$. Then $(X,F,*)$ is a Menger space, where

$F(x,y,t) = [\varphi(t)]^{|x-y|}$, for all x, y in X and $t > 0$.

Let $\varnothing(t) = \frac{t}{2}$, $g(x) = x$ and the mapping $f : X \times X \to X$ be defined by $f(x,y) = \frac{x^2}{16} + \frac{y^2}{16} - 2$.

It is easy to check that $f(X \times X) = [-2,-1] \subseteq [-2,2) = g(X)$. Further, $f(X \times X)$ is complete and the pair (f,g) is weakly compatible. We now check the condition (3.2),

$$F(f(x,y), f(u,v), \varnothing(t)) = F\left(f(x,y), f(u,v), \frac{t}{2}\right) = \left[\varphi\left(\frac{t}{2}\right)\right]^{|f(x,y)-f(u,v)|}$$

$$= \left[\frac{t}{t+2}\right]^{\left|x^2+y^2-u^2-v^2\right|/16} \geq \left[\frac{t}{t+2}\right]^{\left|x^2+y^2-u^2-v^2\right|/8}$$

$$\geq \left[\frac{t}{t+1}\right]^{|x-u|+|y-v|} = \left[\frac{t}{t+1}\right]^{|x-u|} \left[\frac{t}{t+1}\right]^{|y-v|} = F(x,u,t) * F(y,v,t),$$

for every $t > 0$.

Hence, all the conditions of theorem **3.1**, are satisfied. Thus f and g have a unique common coupled fixed point in X. Indeed, $x = 4(1-\sqrt{2})$ is a unique common coupled fixed point of *f* and *g*.

Theorem 3.2. Let $(X,F,*)$ be Menger PM - Space, $*$ being continuous t – norm of H-type. Let $f : X \times X \to X$ and $g : X \to X$ be two mappings and there exists $\phi \in \Phi$ satisfying (3.2).

Then there exists a unique point x in X such that $x = f(x,x) = g(x)$.

Proof. It follows immediately from Theorem 3.1.

Next we give an application of Theorem 3.1.

4. An Application

Theorem 4.1. Let $(X,F,*)$ be a Menger PM-space, $*$ being continuous t-norm defined by $a*b = \min.\{a,b\}$ for all a,b in X. Let M,N be weakly compatible self maps on X satisfying the following conditions:

(4.1) $M(X) \subseteq N(X)$,

(4.2) there exists $\phi \in \Phi$ such that

$F(Mx, My, \phi(t)) \geq F(Nx, Ny, t)$ for all x, y in X and $t > 0$.

If range space of any one of the maps M or N is complete, then M and N have a unique common fixed point in X.

Proof. By taking $f(x,y) = M(x)$ and $g(x) = N(x)$ for all $x, y \in X$ in Theorem 3.1, we get the desired result.

Taking $\phi(t) = kt, k \in (0,1)$, we have the following:

Cor. 4.2. Let $(X,F,*)$ be a Menger PM-space, $*$ being continuous t-norm defined by $a*b = \min.\{a,b\}$ for all a,b in X. Let M,N be weakly compatible self maps on X satisfying (4.1) and the following condition:

(4.3) there exists $k \in (0,1)$ such that

$F(Mx, My, kt) \geq F(Nx, Ny, t)$ for all x, y in X and $t > 0$.

If range space of any one of the maps M or N is complete, then M and N have a unique common fixed point in X.

Taking $N = I$, the identity map on X, we have the following:

Cor. 4.3. Let $(X,F,*)$ be a Menger PM-space, $*$ being continuous t-norm defined by $a*b = \min.\{a,b\}$ for all a,b in X. Let M,N be weakly compatible self maps on X satisfying (4.1) and the following condition:

(4.4) there exists $k \in (0,1)$ such that

$F(Mx, My, kt) \geq F(x, y, t)$ for all x, y in X and $t > 0$.

If range space of the map M is complete, then M and N have a unique common fixed point in X.

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