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Extension Error Set Based on Extension Set

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ABSTRACT

This paper gives the concepts of extension error set and fuzzy extension error set, discusses diverse extension error set and fuzzy extension error set based on extension set and error set, and puts forward the relevant propositions and operations. Finally, it provides proofs of the soundness and completeness for the propositions and operations.

Keywords: Extension Set; Error Set; Fuzzy Extension Error Set

1. Introduction

In the field of fuzzy mathematics, the research of set mainly concentrates on the static form of fuzzy set and its effective forms of reasoning and rule. However, the dynamic changes of the fuzzy set are important parts of set research. In this paper, firstly, we study extension error set and fuzzy extension error set's dynamic concept based on the theory of error eliminating and extenics. Then, we research diverse extension error set and fuzzy extension error set, and put forward the relevant propositions and operations. Finally, we provide proofs of the soundness and completeness for the propositions and operations. In one word, because of the study of extension error set, this paper has very important theoretical and practical significance in different fields.

2. Basic Definitions

2.1. Matter-Element [1-6]

Definition 2.1.1 An ordered triple composed of the measure v_m of O_m about c_m, with matter O_m as object, and cm as characteristic

$$M = (O_m, c_m, v_m)$$

As the fundamental element for matter description, it's referred to as 1-dimensional matter-element, and O_m, vm are referred to as the three key elements of matter-element M, within which, the two-tuples composed of cm and $v_m(c_m, v_m)$ is referred to as the characteristic-element of matter O_m.

For convenience, the whole matter-element is expressed as $\pounds(M)$, the whole matter is expressed as $\pounds(O_m)$, and whole characteristic as $\pounds(c_m)$. The domain of measure of characteristic c_m is expressed as $V(c_m)$, referred to as the domain of measure of c_m.

A matter with multiple characteristics, similar to 1-dimensional matter-element, can be defined as a multi-dimensional matter-element:

Definition 2.1.2 The array composed of matter O_m, n-names of characteristics of $c_{m1}, c_{m2}, \cdots, c_{mn}$ and the corresponding measure $v_{mi}(i = 1, 2, \cdots, n)$ of O_m about $c_{mi}(i = 1, 2, \cdots, n)$

$$M = \begin{bmatrix} O_m & c_{m1} & v_{m1} \\ & c_{m2} & v_{m2} \\ & \vdots & \vdots \\ & c_{mn} & v_{mn} \end{bmatrix} = (O_m, C_m, V_m)$$

is referred to as n-dimensional matter-element, wherein

$$C_m = \begin{bmatrix} c_{m1} \\ c_{m2} \\ \vdots \\ c_{mn} \end{bmatrix}, \quad V_m = \begin{bmatrix} v_{m1} \\ v_{m2} \\ \vdots \\ v_{mn} \end{bmatrix}.$$

2.2. Affair-Element

Interaction between matters is referred to as affair, described by affair-element.

Definition 2.2.1 The ordered triple composed of action O_a, action's characteristic c_a and the obtained measure v_a of O_a about c_a

$$A = (O_a, c_a, v_a)$$

is used as the fundamental element for affair description, referred to as 1-dimensional affair-element.

Basic characteristics of action include dominating object, acting object, receiving object, time, location, degree, mode, and tool, etc.

Definition 2.2.2 The array composed of action O_a, n-characteristics $c_{a1}, c_{a2}, \cdots, c_{an}$ and the obtained measure $v_{a1}, v_{a2}, \cdots, v_{an}$, of O_a about $c_{a1}, c_{a2}, \cdots, c_{an}$

$$\begin{bmatrix} O_a & c_{a1} & v_{a1} \\ & c_{a2} & v_{a2} \\ & \vdots & \vdots \\ & c_{an} & v_{an} \end{bmatrix} = (O_a, C_a, V_a) \overset{\Delta}{=} A$$

is referred to as n-dimensional affair-element, wherein

$$C_a = \begin{bmatrix} c_{a1} \\ c_{a2} \\ \vdots \\ c_{an} \end{bmatrix}, \quad V_a = \begin{bmatrix} v_{a1} \\ v_{a2} \\ \vdots \\ v_{an} \end{bmatrix}$$

2.3. Relation-Element

In the boundless universe, there is a network of relations among any matter, affair, person, information, knowledge and other matter, affair, person, information and knowledge. Because of interaction and interplay among these relations, the matter-element, affair-element and relation-element describing them also have various relations with other matter-elements, affair-elements and relation-elements, and the changes of theses relations will also be interacting and interplaying. Relation-element is a formalized tool to describe this kind of phenomena.

3. The Research of Extension Error Set

We research extension error set based on the theory of Extenics, and explore classical extension error set, fuzzy extension error set, multivariate extension error set. Moreover, we put forward the relevant propositions and operations. According to thses propositions and operations, we provide some proofs.

3.1. The Definition of Extension Error Set

Suppose $U(t)$ is an object set, $S(t)$ is a set of association rules, if

$$E = \Big\{ \big(\big(U(t), M(t), A(t), R(t) \big), x(t) = f\big(S \neq> u(t)\big) \big) \Big| \\ \big(U(t), M(t), A(t), R(t) \big) = u(t) \\ f \in U(t), r, x(t) = f\big(S \neq> u(t)\big) \Big\},$$

we call that "E" is an extension error set for association rule $S(t)$ in domain $U(t)$. In detail, U is a domain, $S(t)$ (incidence - standard) is a set of association rules, M refers to the matter-element, A representative affair-element, R represents the relationship between relation-element; $X(t) = f\big(S > u(t)\big)$ represents the correlation functions of extension error set, R is the real number field, T refers to the time. In this paper we take extension error set as a complex system, its' elements as subsystems.

So,

$$\underset{\cdot}{U}_+ = \big\{ u(t) \big| \big(u(t), x(t)\big) \in E, x(t) > 0 \big\},$$

$$\underset{\cdot}{U}_- = \big\{ u(t) \big| \big(u(t), x(t)\big) \in E, x(t) < 0 \big\},$$

$$\tilde{U} = \big\{ u(t) \big| \big(u(t), x(t)\big) \in E, x(t) = 0 \big\},$$

$$U_+ = \big\{ u(t) \big| \big(u(t), x(t)\big) \in E, x(t) \geq 0, \\ T\big(f\big(S \neq> u(t)\big)\big) < 0 \big\},$$

$$U_- = \big\{ u(t) \big| \big(u(t), x(t)\big) \in E, x(t) \leq 0, \\ T\big(f\big(S \neq> u(t)\big)\big) > 0 \big\},$$

$$\tilde{U} = \big\{ u(t) \big| \big(u(t), x(t)\big) \in E, T\big(f\big(S \neq> u(t)\big)\big) = 0 \big\},$$

are called extension error set's extension of the domain, negative extension field, extension, stable domain and negative stable region, critical region respectively.

$S(t) \neq> u(t)$

contain
1) $u(t)$ and $S(t)$ contradictious
2) $S(t)$ completely can not push out $u(t)$
3) $S(t)$ some part can not push out $u(t)$
4) $S(t)$ possible can not push out $u(t)$

In the definition $f\big(S \neq> u(t)\big)$ should be a general situation $f\big(u(t), S(t)\big)$.

Proposition 3.1.1 In U, if $S_1 = S_2$, $f_1 = f_2$, then

$$E_1 = \Big\{ \big(\big(U(t), M(t), A(t), R(t) \big), x(t) = f_1\big(S \neq> u(t)\big) \big) \Big| \\ \big(U(t), M(t), A(t), R(t) \big) = u(t) \in U(t), \\ f_1 \in U(t) r, \ x(t) = f_1\big(\neq> u(t)\big) \Big\}$$

$$E_2 = \Big\{ \big(\big(U(t), M(t), A(t), R(t) \big), x(t) = f_2\big(S \neq> u(t)\big) \big) \Big| \\ \big(U(t), M(t), A(t), R(t) \big) = u(t) \in U(t), \\ f_2 \in U(t) r, x(t) = f_2\big(S \neq> u(t)\big) \Big\}$$

have $E_1 = E_2$, vice versa.

Proof, when $S_1 = S_2$, $f_1 = f_2$,

$$\forall u(t) \in U, x(t) = f_1\big(S \neq> u(t)\big) = f_2\big(S \neq> u(t)\big) = y(t),$$

so, $\forall u(t) \in U$, when $(u(t), x(t)) \in E_1, (u(t), y(t)) \in E_2$ *have* $x(t) \neq y(t)$, so, $E_1 = E_2$ **,** conversely, if $E_1 = E_2$,we can know $S_1 = S_2$. Also if $f_1 \Rightarrow f_2$ Is true in *U*, then $\exists u(t) \in U$, have $f_1(S \neq> u(t)) \neq f_2(S \neq> u(t))$, so, for

$(u(t), x(t)) \in E_1, (u(t), y(t)) \in E_2$

$(u(t), x(t)) \in E_1, (u(t), y(t)) \in E_2$, have $x(t) \neq y(t)$,

This contradiction with $E_1 = E_2$. so, $f_1 = f_2$.

The end.

3.2. The Class of Extension Error Set

We according to the features of the elements can be divided:

1) Classic extension error set

$$E = \Big\{\big((U(t), M(t), A(t), R(t)), x(t) = f\,(S \neq> u(t))\big) \Big| \\ (U(t), M(t), A(t), R(t)) = u(t) \in U \\ f \in U'\{0,1\}, x(t) = f\,(S \neq> u(t))\Big\}$$

2) Fuzzy extension error set

$$E = \Big\{\big((U(t), M(t), A(t), R(t)), x(t) = f\,(S \neq> u(t))\big) \Big| \\ (U(t), M(t), A(t), R(t)) = u(t) \in U \\ f \in U'[0,1], x(t) = f\,(S \neq> u(t))\Big\}$$

3) Have critical point extension error set

$$E = \Big\{\big((U(t), M(t), A(t), R(t)), x(t) = f\,(S \neq> u(t))\big) \Big| \\ (U(t), M(t), A(t), R(t)) = u(t) \in U \\ f \in U(-\infty, +\infty), x(t) = f\,(S \neq> u(t))\Big\}$$

4. The Research of Fuzzy Extension Error Set

This section mainly research the definition, relation, operation of extension error set.

4.1. The Definition of Fuzzy Extension Error Set

Definition 4.1.1 Suppose *U* is object set, S is a set of association rules in *U*, if $\underset{\sim}{E} = \{(u,x) | u \in U, x = f\,(S \neq> u), f \subseteq U[0,1]\}$, we call that $\underset{\sim}{E}$ is a fuzzy extension error set for *S* in U.

4.2. The Relation between Fuzzy Extension Error Sets

4.2.1. Equation

Definition 4.2.1 Suppose

$\underset{\sim}{E}_1 = \{(u,x) | u \in U, x = f\,(S \neq> u), f \subseteq U[0,1]\}$,

$\underset{\sim}{E}_2 = \{(u,y) | u \in U, y = f_2\,(S_2 \neq> u), f_2 \subseteq U[0,1]\}$, if $\forall u \in U$, have $(u,x) \in \underset{\sim}{E}_1$, $(u,x) \in \underset{\sim}{E}_2$, make $x = y$ and $S_1 = S_2$, we call that $\underset{\sim}{E}_1 = \underset{\sim}{E}_2$ for association rule S_1 or S_2.

4.2.2. Subset

Definition 4.2.2 Suppose U_1 , U_2 are subset in *U*, and

$\underset{\sim}{E}_1 = \{(u,x) | u \in U, x = f\,(S \neq> u), f \subseteq U[0,1]\}$,

$\underset{\sim}{E}_2 = \{(u,y) | u \in U, y = f_2\,(S_2 \neq> u), f_2 \subseteq U[0,1]\}$, if $U_1 \subseteq U_2$ for association rule *S*, $\underset{\sim}{E}_1$ is the subset of $\underset{\sim}{E}_2$, so $\underset{\sim}{E}_1 \subseteq \underset{\sim}{E}_2$, or $\underset{\sim}{E}_2 \supseteq \underset{\sim}{E}_1$.

By definition, there are clearly established the following proposition:

Proposition 4.2.1 Suppose $\underset{\sim}{E}_1$, $\underset{\sim}{E}_2$, $\underset{\sim}{E}_3$ are subset for association rule *S*:

1) $\underset{\sim}{E}_1 \subseteq \underset{\sim}{E}_1$,

2) if $\underset{\sim}{E}_1 \subseteq \underset{\sim}{E}_2$, $\underset{\sim}{E}_1 \subseteq \underset{\sim}{E}_3$, then $\underset{\sim}{E}_2 \subseteq \underset{\sim}{E}_3$.

Proposition 4.2.3 Suppose $\underset{\sim}{E}_1$, $\underset{\sim}{E}_2$ are fuzzy sets for association rule S_1, S_2:

Then $\underset{\sim}{E}_1 = \{(u,x) | u \in U, x = f\,(S \neq> u), f \subseteq U[0,1]\}$,

$\underset{\sim}{E}_2 = \{(u,y) | u \in U, y = f_2\,(S_2 \neq> u), f_2 \subseteq U[0,1]\}$ If $\forall u \in U$, $(u,x) \in \underset{\sim}{E}_1$, $(u,x) \in \underset{\sim}{E}_2$, have $x \leq y$, then $\underset{\sim}{E}_1 \leq \underset{\sim}{E}_2$ or $\underset{\sim}{E}_2 \geq \underset{\sim}{E}_1$ for association rule in *U*.

Proposition 3.1.2.4 Suppose $\underset{\sim}{E}_1$, $\underset{\sim}{E}_2$, $\underset{\sim}{E}_3$ are fuzzy subsets for association rule S_1, S_2 ,S_3 in *U*

1) $\underset{\sim}{E}_1 \subseteq \underset{\sim}{E}_1$,

2) if $\underset{\sim}{E}_1 \subseteq \underset{\sim}{E}_2$, $\underset{\sim}{E}_1 \subseteq \underset{\sim}{E}_3$,then $\underset{\sim}{E}_2 \subseteq \underset{\sim}{E}_3$.

4.3. The Operations of between Fuzzy Extension Error Sets

4.3.1. The Union of Fuzzy Extension Error Set

If $f(x, y, S_1, S_2) \equiv 0$, then the definition of Fuzzy extension error set's union for association rule S_1, S_2:

Definition 4.3.1.1 Suppose $\underset{\sim}{E}_1$ and $\underset{\sim}{E}_2$ are fuzzy sets for association rule S_1, S_2 in *U*, and $\underset{\sim}{E}_3 = \{(u,z) | (u,x) \in \underset{\sim}{E}_1, (u,y) \in \underset{\sim}{E}_2, z = \max(x,y)\}$, then $\underset{\sim}{E}_3 = \underset{\sim}{E}_1 \vee \underset{\sim}{E}_2$, means union.

Proposition 3.2.2.1 Suppose $\underset{\sim}{E}_1$, $\underset{\sim}{E}_2$ are subsets for association rule S_1, S_2, then

1) $\underset{\sim}{E}_1 \vee \underset{\sim}{E}_1 = \underset{\sim}{E}_1$;

2) $\underset{\sim}{E}_1 \vee \underset{\sim}{E}_2 = \underset{\sim}{E}_2 \vee \underset{\sim}{E}_1$;

3) if $\underset{\sim}{E}_1 \leq \underset{\sim}{E}_2$, then $\underset{\sim}{E}_1 \leq \underset{\sim}{E}_1 \vee \underset{\sim}{E}_2 = \underset{\sim}{E}_2$.

4.3.2. The Intersection of Fuzzy Extension Error Set

Definition 4.3.2.1 Suppose $\underset{\sim}{E}_1$ and $\underset{\sim}{E}_2$ are fuzzy sets for association rule S_1, S_2 in *U* **and**

$\underset{\sim}{E}_3 = \{(u,z) | (u,x) \in \underset{\sim}{E}_1, (u,y) \in \underset{\sim}{E}_2, z = \min(x,y)\}$, then

$\underset{\sim}{E}_3 = \underset{\sim}{E}_1 \wedge \underset{\sim}{E}_2$ means intersection.

Proposition 4.3.2.1 Suppose $\underset{\sim}{E}_1$, $\underset{\sim}{E}_2$, $\underset{\sim}{E}_3$ are subsets for association rule S_1, S_2, S_3then

1) $\underset{\sim}{E}_1 \wedge \underset{\sim}{E}_1 = \underset{\sim}{E}_1$;
2) $\underset{\sim}{E}_1 \wedge \underset{\sim}{E}_2 = \underset{\sim}{E}_2 \wedge \underset{\sim}{E}_1$;
3) $\underset{\sim}{E}_1 \wedge \left(\underset{\sim}{E}_2 \wedge \underset{\sim}{E}_3\right) = \left(\underset{\sim}{E}_1 \wedge \underset{\sim}{E}_2\right) \wedge \underset{\sim}{E}_3$

5. Conclusion

Extenics and error eliminating theory have increasingly attracted the attention of academia and industry, especially in the fields of management and decision-making. So we study the extension error set and fuzzy extension error set. But, what we have done is not enough. It's in administrative before our theory is perfect. So, we call for more scholars from all over the world to do research about extenics and error eliminating theory. Only in this way, can they have wider value of applications in more fields.

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