

Logistic Mapping-Based Complex Network Modeling

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ABSTRACT

In this paper, a new topological approach for studying an integer sequence constructed from Logistic mapping is proposed. By evenly segmenting $[0,1]$ into N non-overlapping subintervals which is marked as $1, 2, \dots, N$, representing the nodes identities, a network is constructed for analysis. Wherein the undirected edges symbolize their relation of adjacency in an integer sequence obtained from the Logistic mapping and the top integral function. By observation, we find that nodes' degree changes with different values of λ instead of the initial value— X_0 , and the degree distribution presents the characteristics of scale free network, presenting power law distribution. The results presented in this paper provide some insight into degree distribution of the network constructed from integer sequence that may help better understanding of the nature of Logistic mapping.

Keywords: Complex Network; Logistic Mapping; Power Law Distribution

1. Introduction

Graphical representations are widely used for displaying relations among informational units because they help readers to visualize those relations and hence to understand them better. By representing individuals or organizations as nodes and interaction or relation between them with edges, complex networks can be constructed and analyzed [1-3]. Examples include the Internet, World Wide Web, social networks of acquaintance between individuals, metabolic networks, food webs, etc. [4-8].

Recently, the general theory of complex dynamical networks, which is an extension to the classical graph theory, has been reconsidered for a better understanding of the intrinsic relations, common properties and shared features of possibly all kinds of networks in the real world.

In this paper, a new topological approach is introduced to analyze an integer sequence with sufficiently long length generated by Logistic mapping and the top integral function. The integer sequence is used to construct a network, from which its properties are studied under a general complex network framework. We make research on the degree distribution through computer simulation, and come to the conclusion that the degree distribution

presents the characteristics of scale free network, presenting power law distribution.

2. Network Modeling Based on Logistic Mapping

2.1. Generating Nodes

Given a interval of $[0,1]$, we segment it evenly into N non-overlapping subintervals, each of which represents a unique node identity in the network to be built, wherein, the width of each subinterval is equal. In accordance with the order from left to right, the subintervals are recorded respectively as $1, 2, 3, \dots, N$, so there are a total of N nodes in the network to be built, the width of each subinterval is $1/N$, and the k -th subinterval— $(k-1/N, k/N]$ represents node k . For example, given $N=10$, there are 10 nodes in the network. In detail, evenly segment the interval of $[0,1]$ into 10 non-overlapping subintervals, thus the width of each subinterval is 0.1. The subinterval— $(0,0.1]$ represents node 1, $(0.1,0.2]$ represents node 2, $(0.2,0.3]$ represents node 3, \dots , and $(0.9,1]$ represents node 10. It should be noted that, where the interval is left-open and right-closed. In accordance with this principle, we get 10 nodes in the network we studied, as is shown in **Figure 1**.

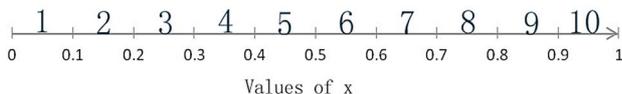


Figure 1. Segment of [0, 1] and nodes generation.

2.2. Logistic Mapping

One-dimensional Logistic mapping from a mathematical point of view is a very simple form of chaotic maps, and it is given by (1). Here

$$X_{n+1} = \lambda X_n (1 - X_n) \tag{1}$$

in which $\lambda \in [0, 4]$ is called Logistic parameter, and wherein $X_n \in [0, 1]$. Given different values of X_0 and λ , after several iterations, we will get different sequences. Studies have shown that the smaller the value of $|4 - \lambda|$ is, the more evenly the corresponding sequence distributed throughout the entire interval— $[0, 1]$. Moreover, only when $X_n \in [0, 1]$, the corresponding Logistic mapping sequence generated by X_0 is of non-periodic, non-convergence. If not, the corresponding sequence will converge at a certain specific value. In this paper, we analyze the integer sequence with sufficiently long length generated by Logistic mapping. Therefore, we only study the particular situation, in which $X_n \in [0, 1]$ and $\lambda \in [3.5699456, 4]$.

2.3. Generating the Integer Sequence

First, computer generates a random number, which belongs to the interval— $[0, 1]$, marked as X_0 . Then, for the given λ , N and the specific number of iterations, which is denoted as m , we use Logistic mapping iterations to generate X_1, X_2, \dots, X_{n-1} , which can be denoted as $\{X_k\}, k = 0, 1, 2, \dots, n-1$, wherein X_k means the value of the Logistic mapping at the k -th iteration. Seen from above, if m is given large enough, then there are plenty of X_k in the corresponding sequence, namely $\{X_k\}, k = 0, 1, 2, \dots, n-1$ is an integer sequence with sufficiently long length.

It is noteworthy that, for given N , each X_k in the sequence above corresponds to a unique node, which is denoted as Y_k . Based on the rule of node’s generation, for each X_k , we can get its corresponding node in the network, according to the subinterval it belongs to. For example, given $N = 10$, $X_m = 0.7$ and $X_n = 0.798$, since X_m and X_n respectively belong to the two sub-intervals— $(0.6, 0.7]$ and $(0.7, 0.8]$, so X_m and X_n respectively correspond to node 7 and node 8, namely, $Y_m = 7$ and $Y_n = 8$.

If we regard X_k as the independent variable, Y_k as the dependent variable, then we can use the top integral function to describe the correspondence relationship between them. The top integral function is the function that its value is the smallest integer greater than the inde-

pendent variable or equal to it, is given by (2). Here

$$Y_k = [N \cdot X_k] = \min \{m \in Z | N \cdot X_k \leq m\} \tag{2}$$

in which X_k represents the value of the Logistic mapping at the k -th iteration, Y_k represents the corresponding node of X_k , N means the total number of nodes in the network and Z represents the set of integers. Therefore, for any sequence of $\{X_k\}, k = 0, 1, 2, \dots, n-1$, we can use Equation (2) to obtain the corresponding integer sequence— $\{Y_k\}, k = 0, 1, 2, \dots, n-1$, through which we can get the rule of edge connection in the network we constructed.

2.4. Generating Networks

In the network we constructed, whether two nodes are connected or not, depends on their locations in $\{Y_k\}, k = 0, 1, 2, \dots, n-1$. In detail, it’s determined by their adjacency in the integer sequence above. For node X_k , it is just connected to its adjacent nodes—node X_{k-1} and node X_{k+1} . Note that the edge we studied is undirected and unweighted.

For example, given $N = 10$, $\lambda = 3.8$, $n = 10$ and $X_0 = 0.7$, we got the integer sequence, as is shown in **Table 1**.

Then we get the rule of edges’ generation as follow. The first edge is (7, 8), the second one is (8, 7), the third one is (7, 10) and the fourth one is (10, 4), \dots , and so on. Namely, the first edge is connected between node 7 node 8, the second is also connected between node 8 and node 7, the third is connected between node 7 and node 10, and so on. Wherein, reconnection is allowed. But in case of $Y_k = Y_{k+1}$, we skip over it, since it is not allowed that a node can be connected to itself. Connect other nodes in accordance with the rule above, and then we get the network topology, as is shown in **Figure 2**.

3. Parameters’ Influence on Nodes’ Degree

It mainly refers to two parameters in the process of the network’s generating, namely the initial value X_0 and

Table 1. Integer sequence.

0	0.7	7
1	0.798	8
2	0.6125448	7
3	0.901867938	10
4	0.336308208	4
5	0.84817899	9
6	0.489331286	5
7	0.949567478	10
8	0.181978513	2
9	0.565676868	6

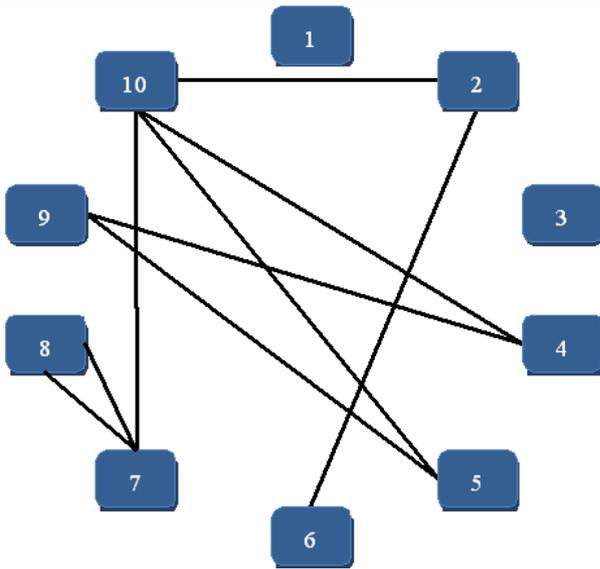


Figure 2. Network topology generated from sequence: 7, 8, 7, 10, 4, 9, 5, 10, 2, 6.

the Logistic parameter λ . Seen by the nature of the chaotic sequence, slight difference between the initial values will come to completely different chaotic sequences, and it's the same with the Logistic parameter. So, the complex networks obtained by different initial values and parameters are completely different. What about the nodes' degree? By simulation, we find that the nodes' degree in various networks showed particular properties as follow.

3.1. Initial Value's Influence on Nodes' Degree

Given $\lambda = 3.9$, $N = 200$ and $n = 1000$, we study nodes' degree influenced by 100 different initial values. We carry out a hundred of experiments through computer simulation, since there are a hundred of different values of X_0 .

In each experiment, the computer first generated a random number— X_0 , then carried out the iterations of a thousand times. Based on the result of the iterations and the top integral function, we get the final integer sequence of nodes. Then we carry out the connection of adjacent nodes according to the final sequence, and get the data of nodes' degree. The data is shown graphically in **Figure 3**.

As is shown in the figure above, there are 200 nodes in the network, wherein the horizontal axis represents the 200 nodes, while the vertical axis represents the 100 experiments. In the right side of this figure, we use a color bar to describe different values of the degree by different colors. The color bar shows that when the color changes gradually from blue to red, the corresponding value of the degree also synchronously increases. Wherein the color of dark blue represents the value of the degree is zero, namely the dark blue nodes in the figure are all isolated nodes.

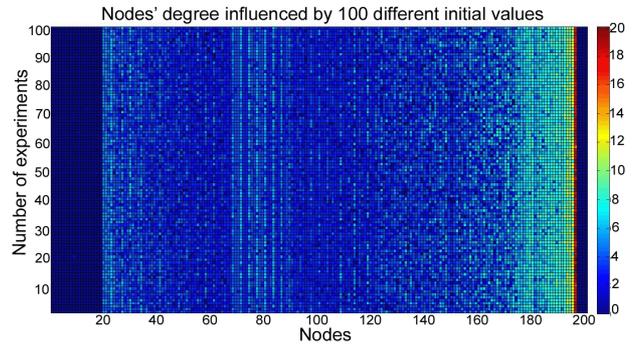


Figure 3. Nodes' degree influenced by 100 different initial values.

As is shown in **Figure 3**, in each of the 100 experiments, the value of each node's degree almost has no change although X_0 takes different values. In particular, the number of the isolated nodes in the figure is almost the same, which is the initial value has little effect on the number of isolated nodes. It reveals that in the condition of λ is given; the initial value has little effect on the nodes' degree.

3.2. Logistic Parameter's Influence on Nodes' Degree

Given $N = 200$ and $n = 1000$, we study the influence on nodes' degree caused by a total number of 100 different Logistic parameters. Since there are a hundred of different values of λ , so we carry out a hundred of experiments through computer simulation, with $\lambda_{i-1} < \lambda_i < \lambda_{i+1}$ herein the value of λ in the i th experiment is noted as λ_i , $1 \leq i \leq 100$. For each λ_i , we set, namely the value of λ_i increases with the increasing of i . And the value of each λ_i belongs to the interval that we obtained hereinabove— $[3.5699456, 4]$.

In each experiment, the computer first generated a random number— X_0 , then carry out a thousand of iterations according to each value of λ_i . Based on the result of the iterations and the top integral function, we get the final integer sequence of nodes. Then connect the adjacent nodes according to the final sequence, and get the data of nodes' degree. The data is shown graphically in **Figure 4**. Here in which the horizontal axis represents the 200 nodes, while the vertical axis represents the 100 experiments. As is shown in the figure, in most of the 100 experiments, the value of nodes' degree increase with the increase of λ . What's more, in the whole network, the total number of the isolated nodes decreases with the increase of the λ . It reveals that the larger λ is, the better the connectivity of the network is.

From what has been discussed above, we come to the conclusion that λ have more influence on nodes' degree and the connectivity of the network than X_0 , although it has important influence on the corresponding

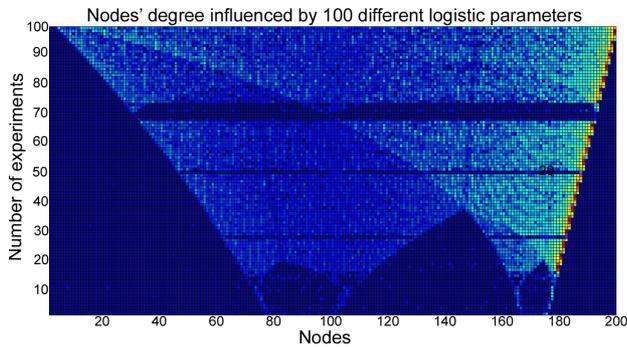


Figure 4. Nodes' degree influenced by 100 different Logistic parameters.

integer sequence. In detail, the larger λ is, the less the isolated nodes' number is, and the better the connectivity of the network is, while the initial value has little effect on the nodes' degree.

4. Degree Distribution

On the basis of the conclusion above, we conduct further research on the degree distribution by constructing six typical networks. In the network we constructed, we need to choose applicable parameters to decrease the number of isolated nodes, since the less the number of isolated nodes is, the better the connectivity is, and it is of little significance to the study of degree distribution.

Given $N = 100,000$, and the value of n is respectively as follow, 10,000, 50,000, 100,000, 500,000, 1,000,000, and 5,000,000. Therefore, there are ten thousands of nodes in the network, the total number of iterations is large enough, and the result of iteration is the corresponding integer sequence with sufficiently long length. Then we get the figure of degree distribution in each of the networks above through computer simulation, and the result is shown in **Figure 5**.

As is shown in **Figure 5**, in dual-logarithm coordinates system, the degree distributions of all the networks above approximate a straight line, presenting the characteristics of power law distribution. It is similar to the degree distribution of scale free network. Therefore, in terms of the degree distribution, the network we constructed from integer sequence presents the characteristics of scale free network.

The results presented above provide some insight into distributions of the integer sequence that may help better understanding of the nature of the Logistic mapping.

5. Conclusion

In this paper, we have studied the degree distribution of a network which is constructed from Logistic mapping. It is found that λ has more influence on nodes' degree and the connectivity of the network than X_0 . Further

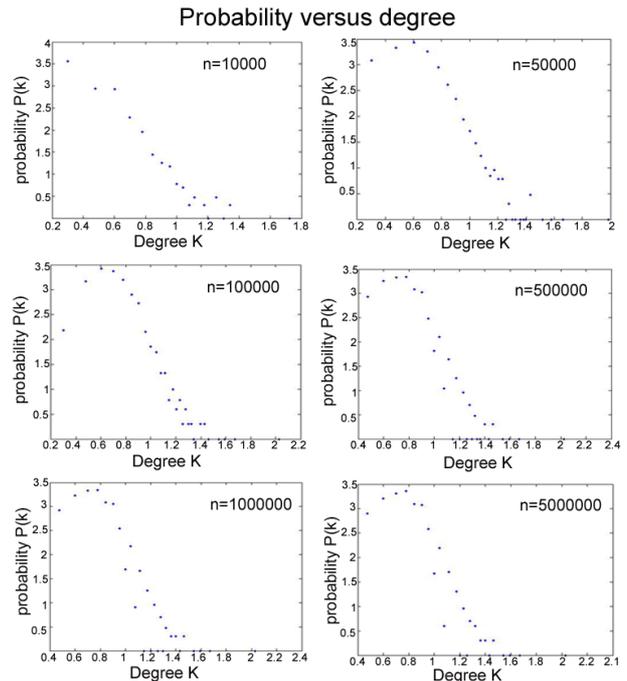


Figure 5. Probability versus degree.

more, the degree distribution of the network shows power law distribution, presenting the characteristics of scale free network. The results presented in this paper also provide some insight into how networks are constructed from integer sequences, as well as how integer sequence is generated from the Logistic mapping.

REFERENCES

- [1] R. Albert and A.-L. Barabasi, "Statistical Mechanics of Complex Networks," *Reviews of Modern Physics*, Vol. 74, No. 1, 2002, pp. 47-97. <http://dx.doi.org/10.1103/RevModPhys.74.47>
- [2] S. N. Dorogovtsev and J. F. F. Mendes, "Evolution of Networks," *Advances in Physics*, Vol. 51, No. 4, 2002, pp. 1079-1145. <http://dx.doi.org/10.1080/00018730110112519>
- [3] M. E. J. Newman, "The Structure and Function of Complex Networks," *SIAM Review*, Vol. 45, No. 2, 2003, pp. 167-224. <http://dx.doi.org/10.1137/S003614450342480>
- [4] R. Pastor-Satorras, A. Vazquez and A. Vespignani, "Dynamical and Correlation Properties of the Internet," *Physical Review Letters*, Vol. 87, No. 25, 2001, pp. 1-4. <http://dx.doi.org/10.1103/PhysRevLett.87.258701>
- [5] G. Bianconi and A. L. Barabasi, "Competition and Multiscaling in Evolving Networks," *Europhysics Letters*, Vol. 54, No. 4, 2001, pp. 436-442. <http://dx.doi.org/10.1209/epl/i2001-00260-6>
- [6] F. Liljeros, C. R. Edling, L. A. N. Amaral, H. E. Stanley and Y. Aberg, "The Web of Human Sexual Contacts," *Nature*, Vol. 411, No. 6840, 2001, pp. 907-908. <http://dx.doi.org/10.1038/35082140>

- [7] R. Albert, H. Jeong and A.-L. Barabasi, "Error and Attack Tolerance of Complex Networks," *Nature*, Vol. 406, No. 6794, 2000, pp. 378-382.
<http://dx.doi.org/10.1038/35019019>
- [8] M. Granovetter, "The Strength of Weak Ties," *American Journal of Sociology*, Vol. 78, No. 6, 1973, pp. 1360-1380.
<http://dx.doi.org/10.1086/225469>