Non-Traveling Wave Solutions for the (1 + 1)-Dimensional Burgers System by Riccati Equation Mapping Approach

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ABSTRACT

Starting from the symbolic computation system Maple and Riccati equation mapping approach and a linear variable separation approach, a new family of non-traveling wave solutions of the (1 + 1)-dimensional Burgers system is derived.

1. Introduction

Exact solutions of nonlinear partial differential equations (NPDEs) have been of a major concern for both mathematicians and physicists [1-4]. Many efforts have been made on the study of NPDEs [5-8]. In the past few decades, many significant methods have been presented such as Bäcklund transformation, Darboux transformation, the extended tanh-function method, and the F-expansion method, Lie group analysis, homogeneous balance method, Jacobi elliptic function method, and the mapping method, etc. [9-15]. The mapping approach is a kind of classic, efficient and well-developed method to solve nonlinear evolution equations, the remarkable characteristic of which is that we can have many different ansatzs and therefore, a large number of solutions. In the past, we have solved the exact solutions of some nonlinear systems via the Riccati equation $\left(\phi' = \sigma + \phi^2\right)$ mapping method, such as (1 + 1)-dimensional related Schrödinger equation, (2 + 1)-dimensional Generalized Breor-Kaup system, (3 + 1)-dimensional Burgers system, (3 + 1)-dimensional Jimbo-Miwa system, (2 + 1)-dimensional modified dispersive water-wave system, (2 + 1)-dimensional Boiti-Leon-Pempinelli system, (2 + 1)-dimensional Korteweg de Vries system, (2 + 1)-dimensional asymmetric Nizhnik-Novikov-Veselov system et [16-19]. In this paper, via a mapping equation we find some new non-traveling wave solutions of the (1 + 1)-dimensional Burgers equation:

$$Q_t + QQ_x - Q_{xx} = 0. \tag{1}$$

2. Non-Traveling Wave Solutions of the Burgers System

As is well known, to search for the solitary wave solutions for a nonlinear physical model, we can apply different approaches. One of the most efficient methods of finding soliton excitations of a physical model is the so-called mapping approach. The basic ideal of the algorithm is as follows. For a given nonlinear partial differential equation (NPDE) with the independent variables $x = \left(x_0 = t, x_1, x_2, \cdots, x_m\right)$, and the dependent variable u, in the form

$$P\left(Q, Q_t, Q_{x_i}, Q_{x_i x_j}, \cdots\right) = 0, \tag{2}$$

where P is in general a polynomial function of its arguments, and the subscripts denote the partial derivatives, the solution can be assumed to be in the form

$$Q = A + \sum_{i=1}^{n}\left\{B_i \phi^i\left(q\right)\right\} \tag{3}$$

with

$$\phi' = \sigma + \phi^2, \tag{4}$$

where σ is a constant and the prime denotes the differentiation with respect to q. To determine Q explicitly, one may substitute (3) and (4) into the given NPDE and collect coefficients of polynomials of ϕ, then eliminate each coefficient to derive a set of partial differential equations of A, B_i, and q, and solve the system of partial differential equations to obtain A, B_i, and q. Finally, as (4) is known to possess the solutions

$$\phi = \begin{cases} -\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}q\right), & \sigma < 0, \\ -\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}q\right), & \sigma < 0, \\ \sqrt{\sigma}\tan\left(\sqrt{\sigma}q\right), & \sigma > 0, \\ -\sqrt{\sigma}\cot\left(\sqrt{\sigma}q\right), & \sigma > 0, \\ \dfrac{-1}{q} & \sigma = 0, \end{cases} \quad (5)$$

Substituting A_i, B_i, q and (5) into (3), one obtains the exact solutions to the given NPDE.

Now we apply the projective equation approach to (1). By the balancing procedure, the ansatz (3) becomes

$$Q = f(x,t) + g(x,t)\phi(q(x,t)) + \frac{h(x,t)}{\phi(q(x,t))}, \quad (6)$$

where f, g, h and q are functions of (x,t) to be determined. Substituting (6) and (4) into (1) and collecting coefficients of polynomials of ϕ, then setting each coefficient to zero, we have

$$-2gq_x^2 + g^2 q_x = 0, \quad (7)$$

$$gg_x - 2g_x q_x + gq_t + fgq_x - gg_{xx} = 0, \quad (8)$$

$$g_t - g_{xx} + gf_x + g^2 q_x \sigma = 0, \quad (9)$$

$$\begin{aligned}&fgq_x\sigma + f_t - gq_{xx}\sigma + ff_x + gh_x - f_{xx} + 2h_x q_x + hg_x \\ &- hq_t - 2g_x q_x \sigma + gq_t \sigma - fhq_x + hq_x x = 0,\end{aligned} \quad (10)$$

$$hf_x - h^2 q_x - h_{xx} - 2hq_x^2\sigma + fh_x + h_t = 0, \quad (11)$$

$$-fhq_x\sigma - hq_t\sigma + 2h_x q_x\sigma + hq_{xx}\sigma + hh_x = 0, \quad (12)$$

$$-2hq_x^2\sigma^2 - h^2 q_x\sigma = 0, \quad (13)$$

Based on (7), (13) and (8), we have

$$f = \frac{q_{xx} - q_t}{q_x}, \quad g = 2q_x, \quad h = -2q_x\sigma, \quad (14)$$

with

$$q = \chi(x) + \tau(t), \quad (15)$$

where $\chi \equiv \chi(x)$, $\tau \equiv \tau(t)$ are two arbitrary variable separation functions of x and of t, respectively. Based on the solutions of (4), one thus obtains following exact solutions of Equation (1):

$$\begin{aligned}Q_1 = &-\frac{\tau_t - \chi_{xx}}{\chi_x} - 2\chi_x\tanh\left(\chi + \tau\right) \\ &- 2\frac{\chi_x}{\tanh\left(\chi + \tau\right)}, \quad \sigma = -1,\end{aligned} \quad (16)$$

$$\begin{aligned}Q_2 = &-\frac{\tau_t - \chi_{xx}}{\chi_x} - 2\chi_x\coth\left(\chi + \tau\right) \\ &- 2\frac{\chi_x}{\coth\left(\chi + \tau\right)}, \quad \sigma = -1,\end{aligned} \quad (17)$$

$$\begin{aligned}Q_3 = &-\frac{\tau_t - \chi_{xx}}{\chi_x} + 2\chi_x\tan\left(\chi + \tau\right) \\ &- 2\frac{\chi_x}{\tan\left(\chi + \tau\right)}, \quad \sigma = 1,\end{aligned} \quad (18)$$

$$\begin{aligned}Q_4 = &-\frac{\tau_t - \chi_{xx}}{\chi_x} - 2\chi_x\cot\left(\chi + \tau\right) \\ &+ 2\frac{\chi_x}{\cot\left(\chi + \tau\right)}, \quad \sigma = 1,\end{aligned} \quad (19)$$

$$Q_5 = -\frac{\tau_t - \chi_{xx}}{\chi_x} - 2\frac{\chi_x}{\chi + \tau}, \quad \sigma = 0. \quad (20)$$

3. Summary and Discussion

In summary, with the help of a projective equation $\left(\phi' = \sigma + \phi^2\right)$ and a linear variable separation method, we find some new exact solutions of the $(1 + 1)$-dimensional Burgers system. Because of wide applications of the Burgers equation in physics, more properties are worthy to be studied such as its Lax pair, symmetry reduction, bilinear form, and Darboux transformation, etc. All these properties are worthy of studying further.

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