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Radiation Effects on Flow past a Stretching Plate with Temperature Dependent Viscosity

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ABSTRACT

The effect of radiation on the flow over a stretching plate of an optically thin gray, viscous and incompressible fluid is studied. The fluid viscosity is assumed to vary as an inverse linear function of the temperature. The partial differential equations (PDEs) and their boundary conditions, describing the problem under consideration, are dimensionalized and the numerical solution is obtained by using the finite volume discretization methodology which is suitable for fluid mechanics applications. The numerical results for the velocity and temperature profiles are shown for different dimensionless parameters entering the problem under consideration, such as the temperature parameter, θ_r, the radiation parameter, S, and the Prandtl number, Pr. The numerical results indicate a strong influence of these parameters on the non-dimensional velocity and temperature profiles in the boundary layer.

Keywords: Radiation; Stretching Plate; Optically Thin Gray Fluid; Viscosity as a Function of the Temperature

1. Introduction

At high temperature, radiation has significant effects on the flow field. These effects have substantial applications in many industrial areas, such as electrical power generation, solar power technology, and aerospace engineering.

There has been extensive research on the effects of radiation on fluid flow. The free convection flow in the presence of radiation has been previously studied by Ali *et al*. [1], Seddeek and Abdelmeguid [2], Raptis and Toki [3], and Malekzadeh *et al*. [4]. The magnetohydrodynamic (MHD) flow in the presence of radiation has been investigated by Chamkha *et al*. [5], Raptis *et al*. [6], Duwairi [7], Ouaf [8], Abd-El Aziz [9], Pal and Mondal [10] and Shit and Haldar [11]. The flow through a porous medium in the presence of radiation has been studied by Murthy *et al*. [12], Al-Harbi [13], Al-Odat *et al*. [14], Raptis and Perdikis [15], Duwairi [16] and Badruddin *et al*. [17], and Awad *et al*. [18]. Raptis [19], Datti *et al*. [20], Abel *et al*. [21], Siddheshwar and Mahabaleswar [22] and Khan [23] have investigated the effects of radiation on the viscoelastic flow. The above studies, however, are under the assumption that the fluid is considered to be a thick gray fluid.

Bestman and Adiepong [24] studied the unsteady hydromagnetic free-convection flow with radiative heat transfer in a rotating thin gray fluid. The unsteady flow under the radiation effect of a thin gray fluid over a moving vertical plate was studied by Raptis and Perdikis [25]. Rajesh [26] studied the radiation effects of a thin gray fluid on MHD free convective flow near a vertical plate with ramped wall temperature under small magnetic Reynolds number. Rajput and Kumar [27] investigated the rotation and radiation effects on MHD flow of a thin gray fluid past an impulsively started vertical plate with variable temperature. Raptis [28] studied the free convective oscillatory flow and mass transfer past a porous plate in the presence of radiation of an optically thin fluid.

In the present study, we determine the effect of radiation on the flow field over a stretching plate of an optically thin gray fluid. We consider the fluid as viscous and incompressible, with temperature dependent viscosity.

The presented results are obtained after dimensionalization of the PDEs using a numerical approach. This approach is based on the finite volume (FV) discretization scheme. The discretization was performed with the use of a specialized symbolic package created in *Mathematics*.

2. Governing Equations

We consider the flow of a viscous and incompressible fluid due to an isothermal stretching flat surface. The fluid properties are assumed to be isotropic and constant, except for the fluid dynamic viscosity. The *x*-axis is taken along the plate and the *y*-axis normal to it, as depicted in **Figure 1**. The radiation heat flux in the *x*-direction is considered negligible in comparison to that in the *y*-direction.

The equations governing the problem are given by:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0, \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

Energy equation

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}. \tag{3}$$

where u,υ are the components of the velocity in the *x* and *y* directions respectively, ρ is the fluid density, μ is the dynamic viscosity, *T* is the fluid temperature, *k* is the thermal conductivity, c_p is the specific heat of the fluid under constant pressure and q_r is the radiative heat flux.

The dynamic viscosity is assumed to be an inverse linear function of temperature [29].

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma\left(T-T_\infty\right)\right], \tag{4}$$

or

$$\frac{1}{\mu}=a\left(T-T_r\right), \tag{5}$$

where

$$a=\frac{\gamma}{\mu_\infty},\quad T_r=T_\infty-\frac{1}{\gamma},\quad \mu_\infty=\rho\nu_\infty, \tag{6}$$

γ is a constant, μ_∞ is the dynamic viscosity at infinity, ν_∞ is the kinematic viscosity at infinity, T_r is a reference temperature, T_∞ is the temperature at infinity, α is a constant which in general is positive for liquids and negative for gases.

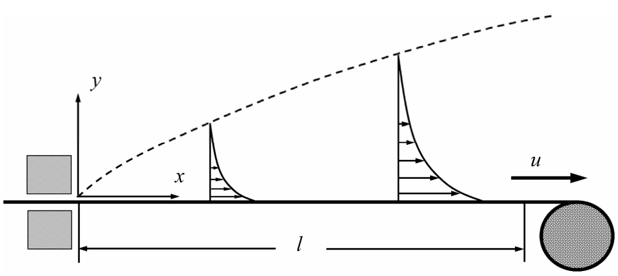

Figure 1. Physical model and co-ordinate system of the problem.

The boundary conditions are defined as follows:

$$\begin{aligned} &y=0,\quad u=cx,\quad \upsilon=0,\quad T=T_w,\\ &y\to\infty,\quad u\to 0,\quad T\to T_\infty, \end{aligned} \tag{7}$$

where c is a constant and T_w is the temperature of the stretching flat surface. In the case of an optically thin gray fluid the local radiant absorption is expressed as [8, 25,28],

$$-\frac{\partial q_r}{\partial y}=4a^*\sigma^*\left(T_\infty^4-T^4\right), \tag{8}$$

where a^* is the absorption coefficient and σ^* is the Stefan-Boltzman constant. We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4\cong 4T_\infty^3 T-3T_\infty^4. \tag{9}$$

Equation (8) through (9) takes the form:

$$-\frac{\partial q_r}{\partial y}=16a^*\sigma^*T_\infty^3\left(T_\infty-T\right). \tag{10}$$

Introducing the following transformations

$$\begin{aligned} &\eta=y\left(\frac{c}{\nu_\infty}\right)^{1/2},\ u=cxf'(\eta),\\ &\upsilon=-\left(\nu_\infty c\right)^{1/2}f(\eta),\ \theta=\frac{T-T_\infty}{T_w-T_\infty}\\ &\theta_r=\frac{T_r-T_\infty}{T_w-T_\infty},\ (\text{Temperature parameter})\\ &Pr=\frac{\rho\nu_\infty c_p}{k},\ (\text{Prandtl number})\\ &S=\frac{16a^*\sigma^*T_\infty^3\nu_\infty}{kc},\ (\text{Radiation parameter}) \end{aligned} \tag{11}$$

where a prime denotes differentiation with respect to η. In view of (10) and (11), Equation (1) is satisfied identically and Equations (2) and (3) reduce to

$$f'''-\frac{1}{\left(\theta-\theta_r\right)}\theta'f''+\frac{\left(\theta-\theta_r\right)}{\theta_r}\left(f'^2-ff''\right)=0, \tag{12}$$

$$\theta''+Prf\theta'-S\theta=0. \tag{13}$$

The boundary conditions (7) are transformed to

$$\begin{aligned} &f(0)=0,\quad f'(0)=1,\quad \theta(0)=1,\\ &f'(\infty)=0,\quad \theta(\infty)=0. \end{aligned} \tag{14}$$

3. Numerical Solution

The non-linear system of coupled differential Equations (12) and (13) subject to the boundary conditions (14) has been solved following a symbolic approach. For this purpose we have used the Computer Algebra System (CAS) *Mathematica* [30].

The analysis begins by obtaining the discretized form of the system of equations by using a symbolic package developed for that purpose [31]. To discretize the coupled set of ordinary differential equations the finite volume method on a collocated grid is used [32]. Having obtained the discretized system, we construct the system of algebraic equations. Then the system is solved algebraically by *Mathematica*'*s* function Solve in respect to the grid values of the functions f', θ. More details about the numerical approach can be found elsewhere [31]. Grid independence studies were performed to establish that the results are not grid dependent.

4. Results and Discussion

In the present study we numerically investigate the effect of radiation on the flow field over a stretching plate of an optically thin gray fluid, **Figure 1**. The fluid was considered viscous and incompressible. The viscosity was temperature dependent as shown in Equation (4). The results are presented in figures for the non-dimensional velocity, f', and non-dimensional temperature, θ in respect to the temperature parameter, θ_r, radiation parameter, S, and the Prantdl number, Pr.

In **Figure 2**, the effect of the temperature parameter θ_r $(\theta_r : -10, -1, -0.1)$ on the non-dimensional velocity, f', when $Pr = 0.5$ and $S = 5$ is presented. Velocity decreases with the increase of the temperature parameter. Especially for $\theta_r = -0.1$, the velocity is substantially reduced leading to a thinner boundary layer (**Figure 2, line 3**).

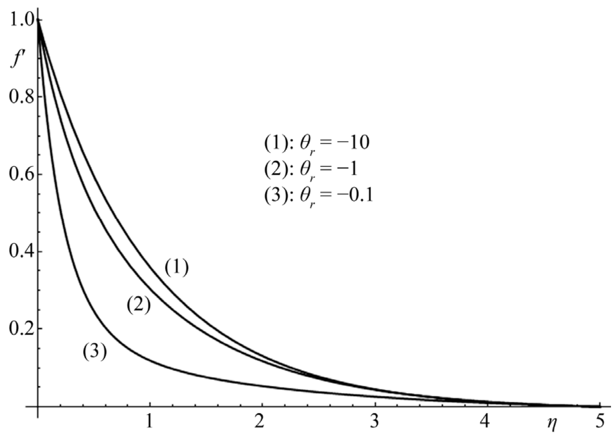

Figure 2. Velocity profiles for different values of the temperature parameter θ_r and for $Pr = 0.5$, $S = 5$.

The effect of the radiation parameter S (S: 0.1, 1, 7) on the non-dimensional velocity f' is shown in **Figure 3**, when $Pr = 0.5$ and $\theta_r = -1$. The velocity increases with the increase of the radiation parameter. This effect is more pronounced when the radiation parameter has larger values, as depicted in **Figure 3**.

The effect of the radiation parameter S (S: 0.1, 1, 7) on the non-dimensional temperature θ is shown in **Figure 4**, when $Pr = 0.5$ and $\theta_r = -1$. When the radiation parameter increases the temperature decreases, and this effect is more pronounced as the radiation parameter increases.

The effect of the Prandtl number, Pr, on the non-dimensional temperature θ is presented in **Figure 5** for three different values of Prandtl (Pr: 0.5, 0.7, 1), when $\theta_r = -1$ and $S = 1$. The increase of Prandtl number leads to a decrease of the temperature in the boundary layer.

Finally, **Figures 6** and **7** show the effect of the temperature parameter θ_r on the non-dimensional velocity f' and temperature θ when $Pr = 0.7$, $S = 1$ and for

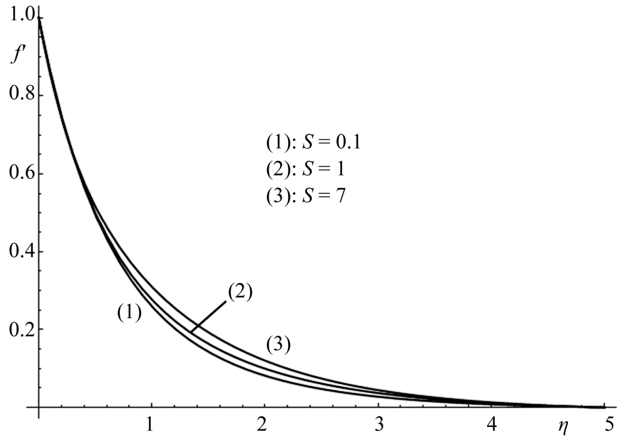

Figure 3. Velocity profiles for different values of the radiation parameter S and for $Pr = 0.5$, $\theta_r = -1$.

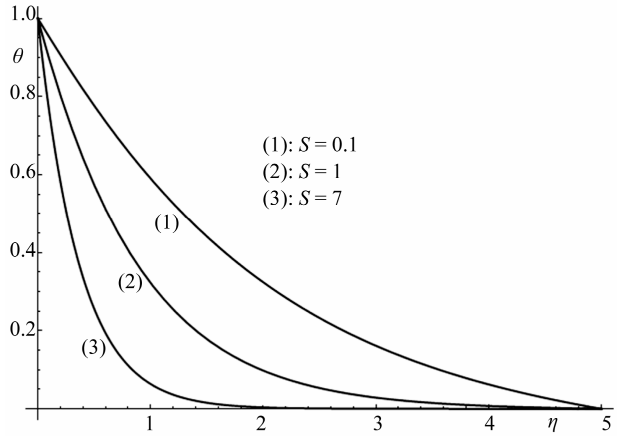

Figure 4. Temperature profiles for different values of the radiation parameter S and for $Pr = 0.5$, $\theta_r = -1$.

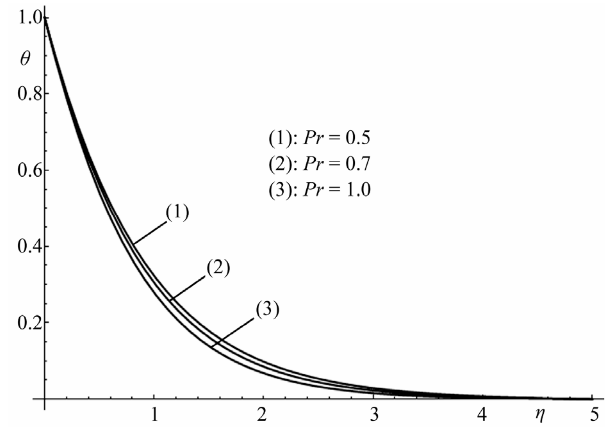

Figure 5. Temperature profiles for different values of the Prandtl number Pr and for $\theta_r = -1$ and $S = 1$.

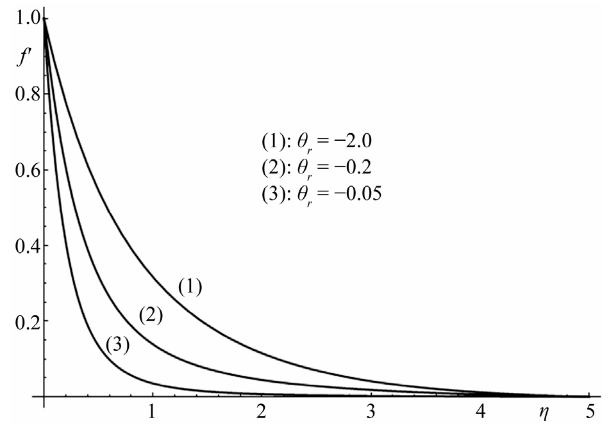

Figure 6. Velocity profiles for different values of the temperature parameter θ_r and for $Pr = 0.7$, $S = 1$.

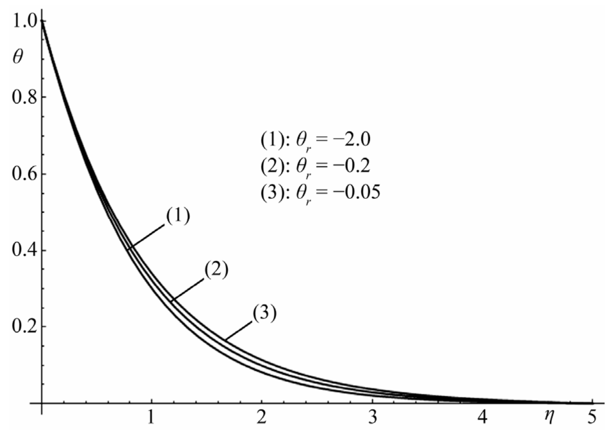

Figure 7. Temperature profiles for different values of the temperature parameter θ_r and for $Pr = 0.7$, $S = 1$.

three different values of the temperature parameter. Temperature increases with the increase of the temperature parameter, θ_r (θ_r: –2, –0.05, –0.01). However, the effect of the temperature parameter is more pronounced on the non-dimensional velocity f' and it decreases as the parameter θ_r increases, **Figure 6**.

The numerical results of this study could bring new insight on the effect of thermal radiation on the flow past a stretching plate with temperature dependent viscosity. These results could be utilized in many industrial and practical areas, including glass and semiconductor processing, atmospheric flows with application to global climate change, electrical power generation, solar power technology, and aerospace engineering.

5. Conclusion

The effects of thermal radiation on the flow and temperature fields over a stretching plate of an optically thin gray fluid were numerical investigated. The fluid was considered incompressible with temperature dependent viscosity. The main findings of this study could be summarized in the following: (1) Increase of the radiation parameter increases the velocity profile but decreases the temperature profiles. (2) On the other hand, increase of the temperature parameter decreases the velocity profile and increases the temperature profile in the boundary layer. (3) Increase of Prandtl number decreases the temperature profile in the boundary layer.

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