

On Rayleigh Wave in Two-Temperature Generalized Thermoelastic Medium without Energy Dissipation

Baljeet Singh¹, Kiran Bala^{2,3}

¹Department of Mathematics, Post Graduate Government College, Chandigarh, India

²Department of Mathematics, Government College, Haryana, India

³Research Scholar in Mathematics, Singhania University, Rajasthan, India

Email: bsinghgc11@gmail.com, kirandulat@gmail.com

Received October 8, 2012; revised November 8, 2012; accepted November 15, 2012

ABSTRACT

In this paper, Rayleigh surface wave is studied at a stress free thermally insulated surface of a two-temperature thermoelastic solid half-space in absence of energy dissipation. The governing equations of two-temperature generalized thermoelastic medium without energy dissipation are solved for surface wave solutions. The appropriate particular solutions are applied to the required boundary conditions to obtain the frequency equation of the Rayleigh wave. Some special cases are also derived. The non-dimensional speed is computed numerically and shown graphically to show the dependence on the frequency and two-temperature parameter.

Keywords: Two-Temperature; Generalized Thermoelasticity; Rayleigh Wave; Energy Dissipation

1. Introduction

Lord and Shulman [1] and Green and Lindsay [2] extended the classical dynamical coupled theory of thermoelasticity to generalized thermoelasticity theories. These theories treat heat propagation as a wave phenomenon rather than a diffusion phenomenon and predict a finite speed of heat propagation. Ignaczak and Ostoj-Starzewski [3] explained in detail, the above theories in their book on “Thermoelasticity with Finite Wave Speeds”. The theory of thermoelasticity without energy dissipation is another generalized theory, which was formulated by Green and Naghdi [4]. It includes the isothermal displacement gradients among its independent constitutive variables and differs from the previous theories in that it does not accommodate dissipation of thermal energy. The representative theories in the range of generalized thermoelasticity are reviewed by Hetnarski and Ignaczak [5]. Wave propagation in thermoelasticity has many applications in various engineering fields. Some problems on wave propagation in coupled or generalized thermoelasticity are studied by various researchers, for example, Deresiewicz [6], Sinha and Sinha [7], Sinha and Elsibai [8,9], Sharma, *et al.* [10], Othman and Song [11], Singh [12,13], and many more.

Gurtin and Williams [14,15] suggested the second law of thermodynamics for continuous bodies in which the entropy due to heat conduction was governed by one temperature, that of the heat supply by another temperature. Based on this suggestion, Chen and Gurtin [16] and

Chen *et al.* [17,18] formulated a theory of thermoelasticity which depends on two distinct temperatures, the conductive temperature Φ and the thermodynamic temperature T . The two-temperature theory involves a material parameter $a^* > 0$. The limit $a^* \rightarrow 0$ implies that $\Phi \rightarrow T$ and the classical theory can be recovered from two-temperature theory. The two-temperature model has been widely used to predict the electron and phonon temperature distributions in ultrashort laser processing of metals. Warren and Chen [19] stated that these two temperatures can be equal in time-dependent problems under certain conditions, whereas Φ and T are generally different in particular problems involving wave propagation. Following Boley and Tolins [20], they studied the wave propagation in the two-temperature theory of coupled thermoelasticity. They showed that the two temperatures T and Φ , and the strain are represented in the form of a travelling wave plus a response, which occurs instantaneously throughout the body. Puri and Jordan [21] discussed the propagation of harmonic plane waves in two temperature theory. Quintanilla and Jordan [22] presented exact solutions of two initial-boundary value problems in the two temperature theory with dual-phase-lag delay. Youssef [23] formulated a theory of two-temperature generalized thermoelasticity. Kumar and Mukhopadhyay [24] extended the work of Puri and Jordan [21] in the context of the linear theory of two-temperature generalized thermoelasticity formulated by Youssef [23]. Magana and Quintanilla [25] studied the uniqueness and growth of solutions in two-temperature generalized ther-

moelastic theories. Recently, Youssef [26] presented a theory of two-temperature thermoelasticity without energy dissipation.

In the present paper, Youssef [26] theory is applied to study the Rayleigh wave at the thermally insulated stress-free surface of an isotropic two-temperature thermoelastic solid half-space without energy dissipation. The frequency equation of the Rayleigh wave is obtained. The frequency equation is also approximated by assuming small thermal coupling. The dependence of numerical values of non-dimensional speed of the Rayleigh wave on material parameters, frequency and two-temperature parameters is shown graphically for a particular material of the model.

2. Basic Equations

We consider a two-temperature thermoelastic solid half-space in absence of energy dissipation. Following Youssef [26], the governing equations for a two-temperature generalized thermoelastic half-space without energy dissipation are

i) The heat conduction equation

$$K^* \Phi_{,ii} = \rho c_E \ddot{\theta} + \gamma T_0 \ddot{e}_{kk}, \quad (1)$$

ii) The displacement-strain relation

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad (2)$$

iii) The equation of motion

$$\rho \ddot{u}_i = (\lambda + \mu) u_{j,ij} + \mu u_{i,ij} - \gamma \theta_{,i}, \quad (3)$$

iv) The constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \gamma \theta) \delta_{ij}, \quad (4)$$

where $\gamma = (3\lambda + 2\mu)\alpha_t$, is the coupling parameter and α_t , is the thermal expansion coefficient. λ and μ are called Lamé's elastic constants. δ_{ij} is the Kronecker delta. K^* is material characteristic constant. T is the mechanical temperature, $\Phi_0 = T_0$ is the reference temperature. $\theta = T - T_0$ with $|\theta/T_0| \ll 1$. σ_{ij} is the stress tensor. e_{ij} is the strain tensor. ρ is the mass density. c_E is the specific heat at constant strain. u_i are the components of the displacement vector. Φ is the conductive temperature and satisfies the relation

$$\Phi - \theta = a^* \Phi_{,ii}, \quad (5)$$

where $a^* > 0$ is the two-temperature parameter. The superposed dots in the above equations denote the time derivatives. The subscripts followed by comma in these equations denote the space derivatives.

3. Analytical 2D Solution

We consider a homogeneous and isotropic two-temperature thermoelastic medium without energy dissipation

of an infinite extent with Cartesian coordinates system (x, y, z) , which is previously at uniform temperature T_0 . The origin is taken on the plane surface $z = 0$ and the z -axis is taken normally into the medium $z \geq 0$. The surface $z = 0$ is assumed stress-free and thermally insulated. The present study is restricted to the plane strain parallel to x - z plane, with the displacement vector $u = (u_1, 0, u_3)$. Now, Equation (3) has the following two components in x - z plane

$$(\lambda + 2\mu)u_{1,11} + (\lambda + \mu)u_{3,13} + \mu u_{1,33} - \gamma \theta_{,1} = \rho \ddot{u}_1, \quad (6)$$

$$(\lambda + 2\mu)u_{3,33} + (\lambda + \mu)u_{1,13} + \mu u_{3,11} - \gamma \theta_{,3} = \rho \ddot{u}_3, \quad (7)$$

The heat conduction Equation (1) is written in x - z plane as

$$K^* (\Phi_{,11} + \Phi_{,33}) = \rho c_E \frac{\partial^2 \theta}{\partial t^2} + \gamma T_0 \frac{\partial^2}{\partial t^2} (u_{1,1} + u_{3,3}), \quad (8)$$

and, Equation (5) becomes,

$$\Phi - \theta = a^* (\Phi_{,11} + \Phi_{,33}). \quad (9)$$

The displacement components u_1 and u_3 are written in terms of scalar potentials q and ψ as

$$u_1 = \frac{\partial q}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial q}{\partial z} + \frac{\partial \psi}{\partial x}. \quad (10)$$

Using Equations (9) and (10) in Equations (6) to (8), we obtain

$$\frac{\partial^2 q}{\partial t^2} = c_1^2 \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right) - \frac{\gamma}{\rho} \left[\Phi - a^* \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \right], \quad (11)$$

$$\frac{\partial^2 \psi}{\partial t^2} = c_2^2 \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} \right) \quad (12)$$

$$\begin{aligned} & K^* \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \\ &= \rho c_E \frac{\partial^2 \Phi}{\partial t^2} - a^* \rho c_E \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \\ &+ \gamma T_0 \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right), \end{aligned} \quad (13)$$

where $c_1^2 = \frac{(\lambda + 2\mu)}{\rho}$, $c_2^2 = \frac{\mu}{\rho}$.

Using the following quantities

$$x' = \frac{x}{(c_1/\omega^*)}, \quad z' = \frac{z}{(c_1/\omega^*)}, \quad t' = t\omega^*,$$

$$\Phi' = \frac{\gamma \Phi}{\rho c_1^2}, \quad q' = \frac{q}{(c_1/\omega^*)^2}, \quad \psi' = \frac{\psi}{(c_1/\omega^*)^2},$$

$$a^* = \frac{a'}{(c_1/\omega^*)^2}$$

where $\omega^* = \rho c_E c_1^2 / K^*$, in Equations (11) to (13) and suppressing the primes, we obtain the Equations (11) to (13) in dimensionless form as

$$\frac{\partial^2 q}{\partial t^2} = \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right) - \Phi + a^* \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right), \quad (14)$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{1}{v^2} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} \right), \quad (15)$$

$$\begin{aligned} \ddot{\Phi} + \epsilon \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right) \\ = \left(\frac{1}{\omega^*} + a^* \frac{\partial^2}{\partial t^2} \right) \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right), \end{aligned} \quad (16)$$

where $v^2 = \frac{c_1^2}{c_2^2}$, and

$$\epsilon = \frac{\gamma^2 T_0}{\rho^2 c_E c_1^2}, \quad (17)$$

is the coefficient of thermoelastic coupling.

For thermoelastic surface waves in the half-space propagating in x -direction, the potential functions Φ, q and ψ are taken in the following form

$$(\Phi, q, \psi) = (\hat{\Phi}(z), \hat{q}(z), \hat{\psi}(z)) \exp i(\eta x - \chi t), \quad (18)$$

where $\chi^2 = \eta^2 c^2$, η is wave number and c is the phase velocity.

Substituting Equation (18) in Equations (14) and (16), we obtain

$$\left[a_1 + \chi^2 \epsilon D^2 \right] \hat{q} + \left[a_2 + \left(\frac{1}{\omega^*} - \chi^2 a^* \right) D^2 \right] \hat{\Phi} = 0, \quad (19)$$

$$\left[a_3 - D^2 \right] \hat{q} + \left[a_4 - a^* D^2 \right] \hat{\Phi} = 0, \quad (20)$$

where $D^2 = \frac{d^2}{dz^2}$, and

$$a_1 = -\chi^2 \epsilon \eta^2, \quad a_2 = \chi^2 - \left(\frac{1}{\omega^*} - \chi^2 a^* \right) \eta^2,$$

$$a_3 = (-\chi^2 + \eta^2), \quad a_4 = (1 + a^* \eta^2)$$

Eliminating $\hat{q}, \hat{\Phi}$ from Equations (19) and (20), we obtain the following auxiliary equation

$$A_0 D^4 + A_1 D^2 + A_2 = 0, \quad (21)$$

where

$$A_0 = \frac{1}{\omega^*} - \chi^2 a^* (1 + \epsilon),$$

$$A_1 = a_2 - a_1 a^* - \frac{a_3}{\omega^*} + \chi^2 (a_3 a^* + \epsilon a_4),$$

$$A_2 = a_1 a_4 - a_2 a_3.$$

With the help of Equation (21) and keeping in mind that $\hat{q}, \hat{\Phi} \rightarrow 0$ as $z \rightarrow \infty$ for surface waves, the solutions q, Φ are written as

$$q = [A \exp(-\eta \beta_1 z) + B \exp(-\eta \beta_2 z)] \exp i(\eta x - \chi t) \quad (22)$$

$$\Phi = [\eta_1 A \exp(-\eta \beta_1 z) + \eta_2 B \exp(-\eta \beta_2 z)] \exp i(\eta x - \chi t) \quad (23)$$

where

$$\beta_1^2 = \frac{1}{\eta^2} \left[\frac{-A_1 + \sqrt{A_1^2 - 4A_0 A_2}}{2A_0} \right], \quad (24)$$

$$\beta_2^2 = \frac{1}{\eta^2} \left[\frac{-A_1 - \sqrt{A_1^2 - 4A_0 A_2}}{2A_0} \right], \quad (25)$$

and

$$\eta_i = \frac{\eta^2 (1 - \beta_i^2) - \chi^2}{a^* \eta^2 (\beta_i^2 - 1) - 1}, \quad (i = 1, 2). \quad (26)$$

Substituting Equation (18) in Equation (15) and keeping in mind that $\hat{\psi} \rightarrow 0$ as $z \rightarrow \infty$ for surface waves, we obtain the following solution

$$\psi = C \exp[-\eta \beta_3 z + i(\eta x - \chi t)], \quad (27)$$

where

$$\beta_3^2 = 1 - c^2 v^2. \quad (28)$$

4. Derivation of Frequency Equation

The mechanical and thermal conditions at the thermally insulated surface $z = 0$ are

i) Vanishing of the normal stress component

$$\sigma_{zz} = 0, \quad (29)$$

ii) Vanishing of the tangential stress component

$$\sigma_{zx} = 0, \quad (30)$$

iii) Vanishing of the normal heat flux component

$$\frac{\partial \Phi}{\partial z} = 0, \quad (31)$$

where

$$\begin{aligned} \sigma_{zz} = \lambda \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right) + 2\mu \left(\frac{\partial^2 \psi}{\partial x \partial z} \right) + 2\mu \frac{\partial^2 q}{\partial z^2} \\ - \gamma \left[\Phi - a^* \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \right], \end{aligned} \quad (32)$$

$$\sigma_{zx} = \mu \left[2 \frac{\partial^2 q}{\partial x \partial z} - \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} \right]. \tag{33}$$

Equation (29) to (31) are written in non-dimensional form as

$$\lambda \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right) + 2\mu \left(\frac{\partial^2 \psi}{\partial x \partial z} + \frac{\partial^2 q}{\partial z^2} \right) - \rho c_1^2 \left[\Phi - a^* \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \right] = 0, \tag{34}$$

$$2 \frac{\partial^2 q}{\partial x \partial z} - \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} = 0, \tag{35}$$

$$\frac{\partial \Phi}{\partial z} = 0. \tag{36}$$

Making use of solutions (22), (23) and (27) for q, Φ, Ψ in the Equations (34) to (36), we obtain the following homogenous system of three equations in A, B and C

$$\begin{aligned} & \left[-\lambda \eta^2 + \eta^2 \beta_1^2 (\lambda + 2\mu) - \rho c_1^2 \eta_1 \{ 1 + a^* \eta^2 (1 - \beta_1^2) \} \right] A \\ & + \left[-\lambda \eta^2 + \eta^2 \beta_2^2 (\lambda + 2\mu) - \rho c_1^2 \eta_2 \{ 1 + a^* \eta^2 (1 - \beta_2^2) \} \right] B \\ & - 2i\mu \eta^2 \beta_3 C = 0, \end{aligned} \tag{37}$$

$$2i\beta_1 A + 2i\beta_2 B + (1 + \beta_3^2) C = 0, \tag{38}$$

$$\eta_1 \beta_1 A + \eta_2 \beta_2 B = 0, \tag{39}$$

The non-trivial solution of Equations (37) to (39) exists if the determinant of the coefficients of A, B and C vanishes, *i.e.*,

$$\begin{aligned} & 4\mu \eta^2 \beta_1 \beta_2 \beta_3 (\eta_2 - \eta_1) \\ & - (1 + \beta_3^2) \\ & \cdot \left[\eta^2 \{ -\lambda (\eta_2 \beta_2 - \eta_1 \beta_1) + (\lambda + 2\mu) \beta_1 \beta_2 (\eta_2 \beta_1 - \eta_1 \beta_2) \} \right. \\ & \left. - \rho c_1^2 \eta_1 \eta_2 (\beta_2 - \beta_1) \{ 1 + a^* \eta^2 (1 + \beta_1 \beta_2) \} \right] = 0, \end{aligned} \tag{40}$$

which is the the frequency equation of thermoelastic Rayleigh wave in a two-temperature generalized thermoelastic medium without energy dissipation.

5. Special Cases

5.1. Small Thermal Coupling

In order to have an idea of the effect of two-temperature parameter on the speed of propagation of Rayleigh wave, we consider the case of small thermoelastic coupling. For most of materials, ϵ is small at normal temperature. Hence we can approximate the frequency equation by

assuming $\epsilon \ll 1$. For $\epsilon \ll 1$, the Equations (24) and (25) are approximated as

$$\beta_1 = \frac{c}{\chi} \sqrt{\frac{-b_1 + b_5}{2b_2}} \left[1 + \frac{\epsilon}{2} \left(\frac{\chi^2 a^*}{b_2} + \frac{-\chi^2 a^* (\chi^2 + 1) + 1 + \frac{1}{2} \frac{b_4}{b_3}}{-b_1 + b_5} \right) \right], \tag{41}$$

$$\beta_2 = \frac{c}{\chi} \sqrt{\frac{-(b_1 + b_5)}{2b_2}} \left[1 + \frac{\epsilon}{2} \left(\frac{\chi^2 a^*}{b_2} - \frac{-\chi^2 a^* (\chi^2 + 1) + 1 + \frac{1}{2} \frac{b_4}{b_3}}{b_1 + b_5} \right) \right], \tag{42}$$

where

$$b_1 = \chi^2 + b_2 (\chi^2 - 2\eta^2), \quad b_2 = \frac{1}{\omega^*} - \chi^2 a^*,$$

$$b_3 = (\chi^2 - \eta^2)(\chi^2 - b_2 \eta^2)$$

$$b_4 = 2b_1 \{ \eta^2 a^* (\chi^2 + 1) + 1 \} + 4b_2 b_3 \chi^2 \left\{ \frac{a^*}{b_2} + \eta^2 \left(\frac{1 + a^* \eta^2}{b_3} \right) \right\},$$

$$b_5 = \sqrt{b_1^2 - 4b_2 b_3}.$$

With the help of these approximations for β_1 and β_2 , the coupling coefficients η_1 and η_2 are approximated and hence the frequency Equation (40) is approximated.

5.2. Isotropic Elastic Case

If we neglect thermal parameters, then the frequency Equation (40) reduces to

$$(2 - c^2 v^2)^2 = 4\sqrt{1 - c^2} \sqrt{1 - c^2 v^2}, \tag{43}$$

which is the frequency equation of Rayleigh wave for an isotropic elastic case.

6. Numerical Example

If we put $c^2 = c^{*2} + \epsilon (\xi_1 + i\xi_2)$, where c^* is the classical Rayleigh wave velocity and ξ_1 and ξ_2 are two reals, then

$$\eta = \frac{\chi}{c^*} \left(1 - \frac{\epsilon \xi_1}{2c^{*2}} - i \frac{\epsilon \xi_2}{2c^{*2}} \right) \tag{44}$$

The velocity of propagation is equal to $\left(c^* + \frac{\epsilon \xi_1}{2c^*} \right)$

and the amplitude-attenuation factor is equal to

$$\exp\left[\frac{\epsilon\chi\xi_2x}{2c^{*3}}\right] \text{ with } \xi_2 < 0. \text{ The non-dimensional speed}$$

of propagation is computed for the following material parameters $\lambda = 7.59 \times 10^{11} \text{ Dyn} \cdot \text{cm}^{-2}$,

$$\mu = 1.89 \times 10^{11} \text{ Dyn} \cdot \text{cm}^{-2}, \quad \rho = 2.7 \text{ g} \cdot \text{cm}^{-3},$$

$$c_E = 0.236 \text{ Cal} \cdot \text{g}^{-1} \cdot \text{C}^{-1}.$$

$$K^* = 0.492 \text{ Cal} \cdot \text{cm}^{-1} \cdot \text{s}^{-1} \cdot \text{C}^{-1}, \quad \gamma = 0.02, \quad T_0 = 20^\circ \text{C}, \quad x = 1 \text{ cm}, \quad c^* = 0.9554.$$

The non-dimensional speed of Rayleigh wave is shown graphically against the range $0.1 \leq \chi \leq 0.5$ of frequency in **Figure 1**, when two-temperature a^* is 0.75. With the increase in frequency, it increases very sharply at low frequency range and slowly for higher frequency range. The non-dimensional speed of Rayleigh wave is also shown graphically against the range $0 \leq a^* \leq 1$ of two-temperature parameter in **Figure 2**, when the frequency $\chi = 0.1$. With the increase in value of two-temperature parameter, it increases very slowly. It seems almost constant in **Figure 2**, but it increases for the whole range of the two-temperature parameter.

7. Conclusion

The appropriate solutions of the governing equations of two-temperature generalized thermoelastic medium without energy dissipation are applied at the boundary conditions at a thermally insulated free surface of a half-space to obtain the frequency equation of Rayleigh wave. The frequency equation is approximated for the case of small thermal coupling and reduced for isotropic elastic case. From frequency equation of Rayleigh wave, it is observed that the phase speed of Rayleigh wave depends on various material parameters including the two-temperature parameter. The dependence of numerical values

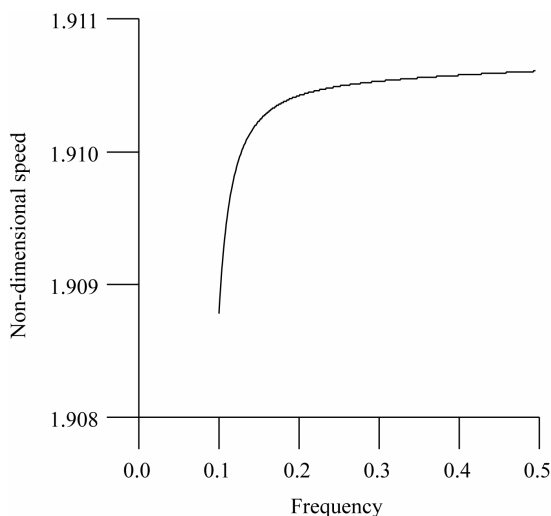


Figure 1. Variation of the non-dimensional speed of Rayleigh wave against frequency when $a^* = 0.75$.

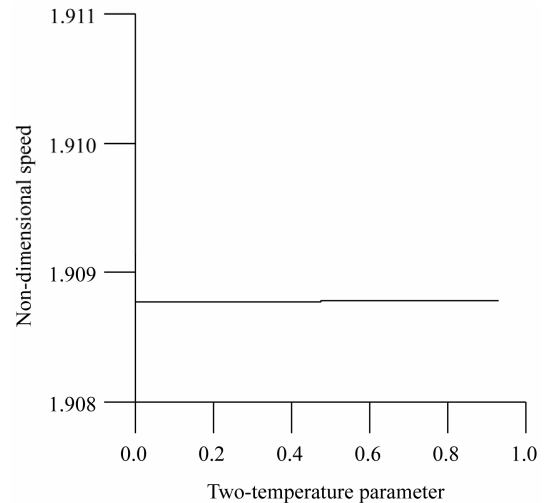


Figure 2. Variation of the non-dimensional speed of Rayleigh wave against two-temperature parameter when $\chi = 0.1$.

of non-dimensional speed on the frequency and two-temperature parameter is shown graphically for a particular material representing the model.

REFERENCES

- [1] H. Lord and Y. Shulman, "A Generalised Dynamical Theory of Thermoelasticity," *Journal of the Mechanics and Physics of Solids*, Vol. 15, No. 5, 1967, pp. 299-309. [doi:10.1016/0022-5096\(67\)90024-5](https://doi.org/10.1016/0022-5096(67)90024-5)
- [2] A. E. Green and K. A. Lindsay, "Thermoelasticity," *Journal of Elasticity*, Vol. 2, No. 1, 1972, pp. 1-7. [doi:10.1007/BF00045689](https://doi.org/10.1007/BF00045689)
- [3] J. Ignaczak and M. Ostoja-Starzewski, "Thermoelasticity with Finite Wave Speeds," Oxford University Press, Oxford, 2009.
- [4] A. E. Green and P. M. Naghdi, "Thermoelasticity without Energy Dissipation," *Journal of Elasticity*, Vol. 31, No. 3, 1993, pp. 189-208. [doi:10.1007/BF00044969](https://doi.org/10.1007/BF00044969)
- [5] R. B. Hetnarski and J. Ignaczak, "Generalized Thermoelasticity," *Journal of Thermal Stresses*, Vol. 22, No. 4-5, 1999, pp. 451-476. [doi:10.1080/014957399280832](https://doi.org/10.1080/014957399280832)
- [6] H. Deresiewicz, "Effect of Boundaries on Waves in a Thermo-Elastic Solid: Reflection of Plane Waves from Plane Boundary," *Journal of the Mechanics and Physics of Solids*, Vol. 8, No. 3, 1960, pp. 164-172. [doi:10.1016/0022-5096\(60\)90035-1](https://doi.org/10.1016/0022-5096(60)90035-1)
- [7] A. N. Sinha and S. B. Sinha, "Reflection of Thermoelastic Waves at a Solid Half Space with Thermal Relaxation," *Journal of Physics of the Earth*, Vol. 22, No. 2, 1974, pp. 237-244. [doi:10.4294/jpe1952.22.237](https://doi.org/10.4294/jpe1952.22.237)
- [8] S. B. Sinha and K. A. Elsibai, "Reflection of Thermoelastic Waves at a Solid Half-Space with Two Thermal Relaxation Times," *Journal of Thermal Stresses*, Vol. 19, No. 8, 1996, pp. 763-777. [doi:10.1080/01495739608946205](https://doi.org/10.1080/01495739608946205)

- [9] S. B. Sinha and K. A. Elsibai, "Reflection and Refraction of Thermoelastic Waves at an Interface of Two Semi-Infinite Media with Two Thermal Relaxation Times," *Journal of Thermal Stresses*, Vol. 20, No. 2, 1997, pp. 129-146. [doi:10.1080/01495739708956095](https://doi.org/10.1080/01495739708956095)
- [10] J. N. Sharma, V. Kumar and D. Chand, "Reflection of Generalized Thermoelastic Waves from the Boundary of a Half-Space," *Journal of Thermal Stresses*, Vol. 26, No. 10, 2003, pp. 925-942. [doi:10.1080/01495730306342](https://doi.org/10.1080/01495730306342)
- [11] M. I. A. Othman and Y. Song, "Reflection of Plane Waves from an Elastic Solid Half-Space under Hydrostatic Initial Stress without Energy Dissipation," *International Journal of Solids and Structures*, Vol. 44, No. 17, 2007, pp. 5651-5664. [doi:10.1016/j.ijsolstr.2007.01.022](https://doi.org/10.1016/j.ijsolstr.2007.01.022)
- [12] B. Singh, "Effect of Hydrostatic Initial Stresses on Waves in a Thermoelastic Solid Half-Space," *Applied Mathematics and Computation*, Vol. 198, No. 2, 2008, pp. 494-505. [doi:10.1016/j.amc.2007.08.072](https://doi.org/10.1016/j.amc.2007.08.072)
- [13] B. Singh, "Reflection of Plane Waves at the Free Surface of a Monoclinic Thermoelastic Solid Half-Space," *European Journal of Mechanics—A/Solids*, Vol. 29, No. 5, 2010, pp. 911-916. [doi:10.1016/j.euromechsol.2010.05.005](https://doi.org/10.1016/j.euromechsol.2010.05.005)
- [14] M. E. Gurtin and W. O. Williams, "On the Clausius-Duhem Inequality," *Zeitschrift für Angewandte Mathematik und Physik ZAMP*, Vol. 17, No. 5, 1966, pp. 626-633. [doi:10.1007/BF01597243](https://doi.org/10.1007/BF01597243)
- [15] M. E. Gurtin and W. O. Williams, "An Axiomatic Foundation/or Continuum Thermodynamics," *Archive for Rational Mechanics and Analysis*, Vol. 26, No. 2, 1967, pp. 83-117. [doi:10.1007/BF00285676](https://doi.org/10.1007/BF00285676)
- [16] P. J. Chen and M. E. Gurtin, "On a Theory of Heat Conduction Involving Two Temperatures," *Zeitschrift für Angewandte Mathematik und Physik ZAMP*, Vol. 19, No. 4, 1968, pp. 614-627. [doi:10.1007/BF01594969](https://doi.org/10.1007/BF01594969)
- [17] P. J. Chen, M. E. Gurtin and W. O. Williams, "A Note on Non-Simple Heat Conduction," *Zeitschrift für Angewandte Mathematik und Physik ZAMP*, Vol. 19, No. 4, 1968, pp. 969-970. [doi:10.1007/BF01602278](https://doi.org/10.1007/BF01602278)
- [18] P. J. Chen, M. E. Gurtin and W. O. Williams, "On the Thermodynamics of Non-Simple Elastic Materials with Two-Temperatures," *Zeitschrift für Angewandte Mathematik und Physik ZAMP*, Vol. 20, No. 1, 1969, pp. 107-112. [doi:10.1007/BF01591120](https://doi.org/10.1007/BF01591120)
- [19] W. E. Warren and P. J. Chen, "Wave Propagation in the Two-Temperature Theory of Thermoelasticity," *Acta Mechanica*, Vol. 16, No. 1-2, 1973, pp. 21-33. [doi:10.1007/BF01177123](https://doi.org/10.1007/BF01177123)
- [20] B. A. Boley and I. S. Tolins, "Transient Coupled Thermo-plastic Boundary Value Problems in the Half-Space," *Journal of Applied Mechanics*, Vol. 29, No. 4, 1962, pp. 637-646. [doi:10.1115/1.3640647](https://doi.org/10.1115/1.3640647)
- [21] P. Puri and P. M. Jordan, "On the Propagation of Harmonic Plane Waves under the Two-Temperature Theory," *International Journal of Engineering Science*, Vol. 44, No. 17, 2006, pp. 1113-1126. [doi:10.1016/j.ijengsci.2006.07.002](https://doi.org/10.1016/j.ijengsci.2006.07.002)
- [22] R. Quintanilla and P. M. Jordan, "A Note on the Two Temperature Theory with Dual-Phase-Lag Delay: Some Exact Solutions," *Mechanics Research Communications*, Vol. 36, No. 7, 2009, pp. 796-803. [doi:10.1016/j.mechrescom.2009.05.002](https://doi.org/10.1016/j.mechrescom.2009.05.002)
- [23] H. M. Youssef, "Theory of Two-Temperature Generalized Thermoelasticity," *IMA Journal of Applied Mathematics*, Vol. 71, No. 3, 2006, pp. 383-390. [doi:10.1093/imamat/hxh101](https://doi.org/10.1093/imamat/hxh101)
- [24] R. Kumar and S. Mukhopadhyay, "Effects of Thermal Relaxation Time on Plane Wave Propagation under Two-Temperature Thermoelasticity," *International Journal of Engineering Science*, Vol. 48, No. 2, 2010, pp. 128-139. [doi:10.1016/j.ijengsci.2009.07.001](https://doi.org/10.1016/j.ijengsci.2009.07.001)
- [25] A. Magaña and R. Quintanilla, "Uniqueness and Growth of Solutions in Two-Temperature Generalized Thermoelastic Theories," *Mathematics and Mechanics of Solids*, Vol. 14, No. 7, 2009, pp. 622-634. [doi:10.1177/1081286507087653](https://doi.org/10.1177/1081286507087653)
- [26] H. M. Youssef, "Theory of Two-Temperature Thermoelasticity without Energy Dissipation," *Journal of Thermal Stresses*, Vol. 34, No. 2, 2011, pp. 138-146. [doi:10.1080/01495739.2010.511941](https://doi.org/10.1080/01495739.2010.511941)