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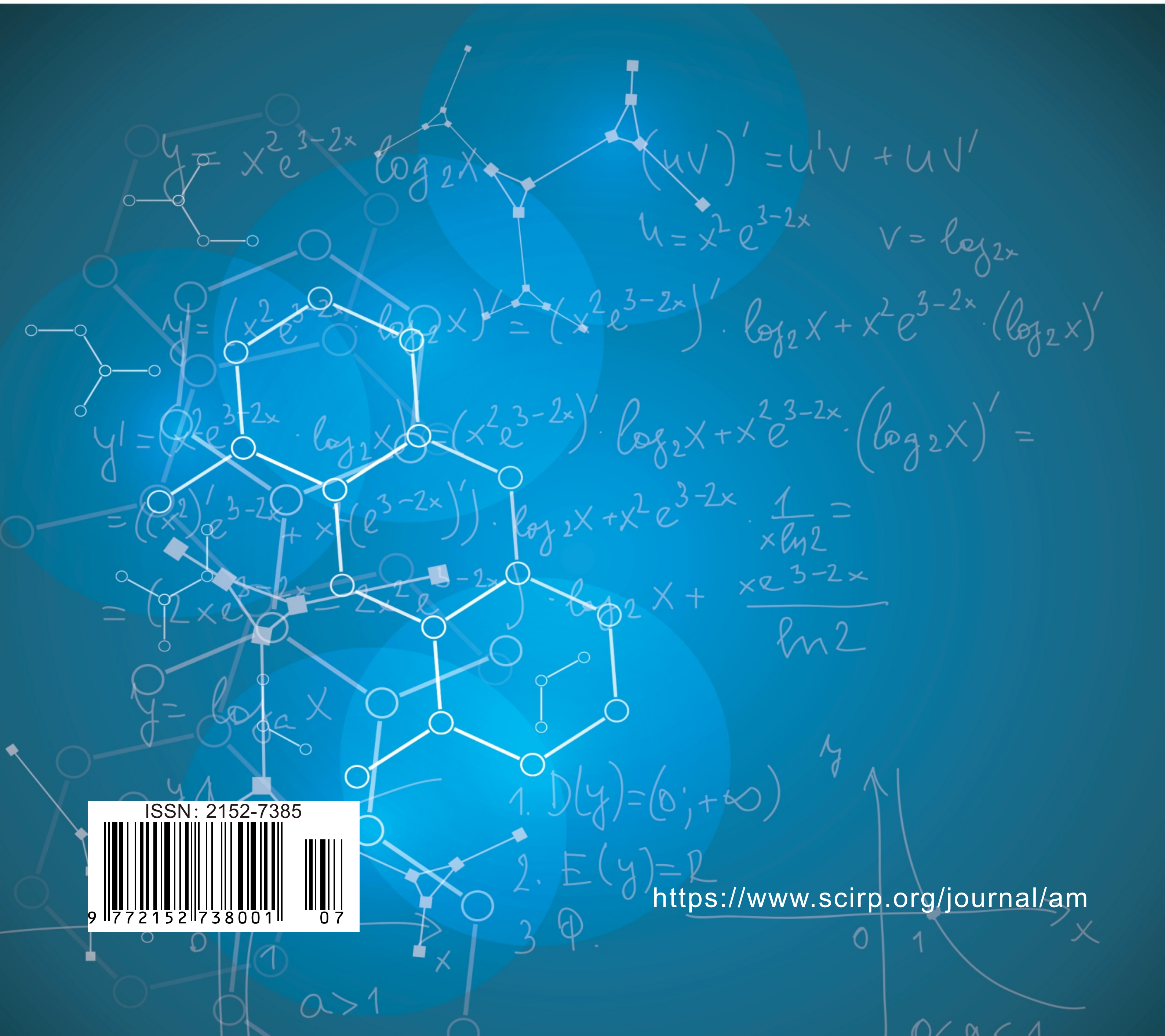

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Analysis of State Homicide Rates Using Statistical Ranking and Selection Procedures

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Abstract

Nonparametric and parametric subset selection procedures are used in the analysis of state homicide rates (SHRs), for the year 2005 and years 2014-2020, to identify subsets of states that contain the "best" (lowest SHR) and "worst" (highest SHR) rates with a prescribed probability. A new Bayesian model is developed and applied to the SHR data and the results are contrasted with those obtained with the subset selection procedures. All analyses are applied within the context of a two-way block design.

Keywords

Homicide Rates Analysis Reporting System, Probability of a Correct Selection, Bayesian Inference, WinBugs, Additive Model, Tukey One-Degree-of-Freedom Test for Additivity

1. Introduction

The United States experienced its biggest one-year increase on record in homicides in 2020, according to new figures released by the F.B.I. There is no simple explanation for the steep rise. A number of key factors are driving the violence, including the economic and social toll taken by the pandemic and a sharp increase in gun purchases. However, how does the homicide rate appear before 2020? As reported in a Wall Street Journal article [1], "the rate of 6.5 homicides per 100,000 residents is the highest since 1997, but still below historic highs of the early 1990s". This article further explores possible causes for the recent increasing trends in homicide rates.

This article focuses on the application of nonparametric (or distribution-free), parametric subset selection procedures and the Bayesian approach to analyze state homicide rate (SHR) data for the year 2005 and years 2014-2020. With the Bayesian approach, a probability distribution is derived over all possible permutations of the population means. Thus, the probability that any particular state is characterized by the largest (or smallest) mean can be easily obtained by appropriate summing of the permutation probabilities. The variability of SHRs is herein analyzed with advanced statistical techniques. While root causal analysis is also very important, it requires different investigative approaches.

The state homicide rate data is obtained from CDC:
https://www.cdc.gov/nchs/pressroom/sosmap/homicide_mortality/homicide.htm

There is an existing gap from the year 2005 to 2014 while addressing the data. The rate of 0.00 is not actually zero since we kept 2 decimal points for the data.

2. Formulation of Nonparametric Subset Selection Rules

The description of this selection rule will follow that given by Green and McDonald [2], Let $\Pi_1, \Pi_2, \cdots, \Pi_k$ be $k(\geq 2)$ independent pupulations. The associated random variables, X_{ij}, $j = 1, \cdots, n$; $i = 1, \cdots, k$, are assumed independent and to have a continuous distribution $F_j(x; \theta_i)$ where θ_i belong to some interval Θ on the real line. The basic model assumption is that $F_j(x; \theta_i)$ is a stochastically increasing family of distributions for each j. The additive model of the following form is used:

$$X_{ij} = \mu + \theta_i + \beta_j + \varepsilon_{ij} \tag{1}$$

where β_j indicates the particular block effect, θ_i indicates the population effect, and ε_{ij} is the random error. The distribution of ε_{ij} is any continuous distribution function $F_j(x)$ with mean 0. The distribution of X_{ij} will be stochastically ordered in θ as it is a location parameter in Equation (1). So, for example, $F_j(x)$ could be a normal distribution with mean 0 and standard deviation σ_j. The assumption of negligible interaction between population and block must be satisfied. Let $\theta_{[i]}$ denote the ith smallest unknown parameter, then for all x

$$F_j\left(x; \theta_{[1]}\right) \geq F_j\left(x; \theta_{[2]}\right) \geq \cdots \geq F_j\left(x; \theta_{[k]}\right) \tag{2}$$

where $\theta_{[1]}$ ($\theta_{[k]}$) characterizes the best (worst) population.

Let R_{ij} denote the rank of the observation X_{ij} among $X_{1j}, X_{2j}, \cdots, X_{kj}$. The variables R_{ij} take values from 1 to k. The selection procedures considered here are based on the rank sums, $T_i = \Sigma_j R_{ij}$, associated with Π_i, $i = 1, \cdots, k$. The structure for this process is outlined in **Table 1**.

Any subset selection procedure based on the rank sums should have the property that the probability that a correct selection (CS) occurs, *i.e.*, the worst population (or best population) is included in the selected subset, is bounded below by $P^*(k^{-1} < P^* < 1)$. That is, for a given selection rule R, the probability of a CS should satisfy the inequality,

$$\inf_{\Omega} P\left(\mathrm{CS} \mid \mathrm{R}\right) \geq P^*, \tag{3}$$

where $\Omega = \left\{\theta = \left(\theta_1, \cdots, \theta_k\right) : \theta_i \in \Theta, i = 1, \cdots, k\right\}$. In some cases, as noted later, inequality may only hold on a subspace Ω' of Ω.

Table 1. Structure for determining ranks and rank sums.

Block/Π	Π_1	Π_2	…	Π_k	SUM
Block 1	$X_{11} \approx R_{11}$	$X_{21} \approx R_{21}$		$X_{k1} \approx R_{k1}$	$k(k+1)/2$
Block 2	$X_{12} \approx R_{12}$	$X_{22} \approx R_{22}$		$X_{k2} \approx R_{k2}$	$k(k+1)/2$
.	.			.	.
.	.			.	.
.	.			.	.
Block n	$X_{1n} \approx R_{1n}$	$X_{2n} \approx R_{2n}$		$X_{kn} \approx R_{kn}$	$k(k+1)/2$
Rank SUMS (T_i)	T_1	T_2		T_k	$nk(k+1)/2$

The two selection rules for choosing a subset containing the worst population, as described in McDonald [3], are given by:

R_1: Select Π_i iff $T_i \geq \max(T_j) - b_1$

R_2: Select Π_i iff $T_i > b_2$.

Similarly, the two selection rules for choosing a subset containing the best population are given by:

R_3: Select Π_i iff $T_i \leq \min(T_j) + b_3$

R_4: Select Π_i iff $T_i < b_4$.

Note that the rules R_1 and R_2 could be written in the form that select Π_i iff $T_i > b$, where b is a stochastic quantity for R_1 and a deterministic quantity for R_2. A similar statement can be made for the rules R_3 and R_4.

As developed by McDonald [4] [5] [6], R_1 and R_3 are justified over a slippage space, Ω', where all parameters θ_i are equal with the possible exception of $\theta_{[k]}$ in case of rule R_1 or $\theta_{[1]}$ in case of rule R_3; and R_2 and R_4 are applicable over the entire parameter space. The constants b_1, b_3, and b_4 are chosen as small as possible and b_2 is chosen as large as possible preserving the probability goal. For large values of n, the selection rules are determined by the asymptotic formulae as described in McDonald [5] and are computed as:

$$b_1 = b_3 = h\left[nk\left(k+1\right)/6\right]^{1/2}, \tag{4}$$

$$b_2 = \left[n\left(k^2-1\right)/12\right]^{1/2}\Phi^{-1}\left(1-P^*\right)+n\left(k+1\right)/2, \tag{5}$$

$$b_4 = n\left(k+1\right)-b_2, \tag{6}$$

where the h-solution to be used in Equation (4) is given by:

$$\int_{-\infty}^{\infty}\Phi^{k-1}\left(x+h\sqrt{2}\right)\phi\left(x\right)\mathrm{d}x = P^*. \tag{7}$$

Here, Φ and ϕ represent the standard normal cumulative distribution function (CDF) and probability density function (PDF), respectively.

Taking P^* to be particular confidence level, the h-solution is given in **Table 1** of Gupta *et al.* [7], and can be used to determine the constants b_1 and b_3. The above integral can also be calculated to determine P^* for a given value of h, using a TI-83+ (or similar) calculator with numerical integration capability as shown in Green and McDonald [2]. The integral can be shown to be the probability that the maximum of U_i, $i = 1, \cdots, k$, is less than h where the U_i are normally distributed random variables with zero means, unit variances, and covariance of 0.5 (see Gupta *et al.* [7]). With confidence level P^*, it can be asserted (using these selection rules) that the chosen subset of the populations contains the one characterized by $\theta_{[k]}$ ($\theta_{[1]}$).

Since there is only one observation for each state for each year, there is no general test for additivity, *i.e.*, lack of interaction between states and years. Tukey developed a one degree-of-freedom test for nonadditivity when there is a single observation per cell, as given here. This test is used to establish the plausibility of model (1) for a power transformation of the SHRs. **Table 2** shows the Tukey one degree-of-freedom test for nonadditivity for the SHRs and for these rates raised to the 0.4 power. The test indicates significant evidence of interaction with the untransformed rates, and no significant evidence of interaction with the power transformation of the rates. The Tukey test is testing for interaction of the form $E\left(X_{ij}\right) = \mu + \theta_i + \beta_j + \lambda\theta_i\beta_j$. And the one degree of freedom test is given by testing for the one parameter λ. For the purpose of the nonparametric analyses to follow, the original SHR data will be used because ranks are invariant to monotone increasing transformations.

3. Nonparametric Subset Selection of States

The goal is now to choose a subset of the 50 states that can be asserted, with a specified confidence, to contain the state with the highest SHR (worst population), and similarly a state with the lowest SHR (best population) using the nonparametric ranking and selection procedures. Ranks $k = 1, \cdots, 50$ are assigned to states for each of $n = 8$ years, with a rank of "1" being the state with the lowest SHR. Based on these ranks, the selection procedure for choosing a subset of the 50 states asserts that the best state (or worst state) is contained with a specified confidence level P^*.

Table 2. Tukey's one degree-of-freedom test.

Tukey's 1 DF Test of Nonadditivity—SHR	Tukey's 1 DF Test of Nonadditivity—$SHR^{0.4}$
SS (Nonadditivity): 47.463	SS (Nonadditivity): 0.252
SS (Error): 211.477	SS (Error): 20.575
MS (Error): 0.722	MS (Error): 0.070
Significance Level: 0.050	Significance Level: 0.050
Test Statistic: 65.759	Test Statistic: 3.584
Critical Value: 3.873	Critical Value: 3.873
The test statistic is greater than the critical value, so there is significant evidence of interaction.	The test statistic is not greater than the critical value, so there is no significant evidence of interaction.

Similar to the structure as outlined in the second section, let R_{ij} denote the rank of the observation X_{ij} within the jth block. The variables R_{ij} take values from 1 to k and the selection procedure is based on the rank sums, $T_i = \sum_j R_{ij}$, associated with Π_i, $i = 1, \cdots, k$. In the case of ties, each tied state receives an average of their rank for that year. This is done for all 7 years. Ranks are then summed for each state and the rank sums T_i's, are orderd. The selection rule constants are determined by the asymptotic formulae as described in the second section.

Taking $P^* = 0.90$, the h-solution as given in Table 1 of Gupta *et al.* [7], is $h = 2.581$. This can be used to determine the constants b_1 and b_3. Using $n = 8$, $k = 50$, and $h = 2.581$, we obtain $b_1 = b_3 = 150.5$. Since $n = 8$ is not a particularly large sample size, the asymptotic values are compared with simulated values as described in McDonald [8]. The simulated value would yield $b_1 = b_3 = 148$ vs. 150.5 using the asymptotic formula. The other two constants are calculated to be $b_2 = 151.7$ and $b_4 = 256.3$. The data yields $\max(T_j) = 397$, and $\min(T_j) = 17.5$. The choice of $P^* = 0.90$ is determined by the degree of assurance one wishes to have concerning the goal of the chosen subset of states. This is similar to the choice of the level of confidence, an analyst would have concerning a confidence interval for a population parameter, e.g., the mean or variance.

With confidence level $P^* = 0.90$, it can be asserted that the following subsets of states contain that one characterized by $\theta_{[k]}$:

Rule R_1: Select the ith state iff $T_i \geq \max(T_j) - 150.5 = 246.5$. Twenty-one are chosen for "worst".

Rule R_2: Select the ith state iff $T_i > 151.7$. Thirty-two are chosen for "worst".

With the same 0.90 confidence level, it can be asserted that the following subset of states contain that one characterized by $\theta_{[1]}$:

Rule R_3: Select the ith state iff $T_i \leq \min(T_j) + 150.5 = 168.0$. Twenty-two are chosen for "best".

Rule R_4: Select the ith state iff $T_i < 256.3$. Thirty-one are chosen for "best".

The identification of the specific states chosen with these four selection rules is given in **Appendix B**.

4. Parametric Subset Selection of States

In this section, a normal means parametric selection procedure will be used to contrast the inference with that of the nonparametric approach. This approach to subset selection was developed by Gupta [9]. With the additive model (1)

$$E\left(X_{ij}\right) = \mu + \theta_i + \beta_j \tag{8}$$

Letting $\bar{X}_i = \left(\sum_j X_{ij}\right)/n$, then $E\left(\bar{X}_i\right) = \mu + \theta_i + \left(\sum_j \beta_j\right)/n$. Since the quantity $\mu + \left(\sum_j \beta_j\right)/n$ is constant for all i, inference on the ordered θ_i can be efficiently based on the ordering of the means, $\bar{X}_i$.

The additive model (1) will be used with X_{ij} replaced with $f\left(X_{ij}\right) = X_{ij}^{0.4}$ based on the results given in Table 2. Here, the ε_{ij} are assumed independent identically distributed normal variates with mean 0 and standard deviation σ.

Residual displays from a two-way additive analysis of variance (ANOVA) are given in Figure 1.

The residuals are with some outliers on the lower and upper ends. The data at lower ends looks piled up, however, as mentioned in section one, the rate data of 0.00 is not actually zero and not all the 0.00 are equal to each other. The "raw" data now will be the SHR to the 0.4 power. Since our interest is selection of "best" and "worst" subsets, we will retain the "outliers" and continue with a normal means selection process using the selection rule R_5 for the "worst" population subset and R_6 for the "best" population defined as follows:

R_5: Select the *i*th state iff $\bar{X}_i \geq \bar{X}_{[k]} - d$, $d > 0$

R_6: Select the *i*th state iff $\bar{X}_i \leq \bar{X}_{[1]} + c$, $c > 0$.

The $\bar{X}_i$'s are the respective sample means of the "raw" data and the $\bar{X}_{[i]}$'s are the ordered sample means. The positive constants d and c are chosen so that the $P(CS) \geq P^*$ for any configuration of the population (state) parameters, θ_i's. It can be shown that for a fixed P^*, $d = c$, and

$$d = h\sigma\left(2/n\right)^{1/2}, \tag{9}$$

where h is defined by the integral Equation (7).

For $k = 50$, $n = 8$, and $P^* = 0.90$, the constants $d = c = 1.2905\sigma$. The value of σ is chosen to be 0.261 based on the two-way additive ANOVA of the transformed SHRs (*i.e.*, the square root of the Mean Square for Error) as shown in Table 3.

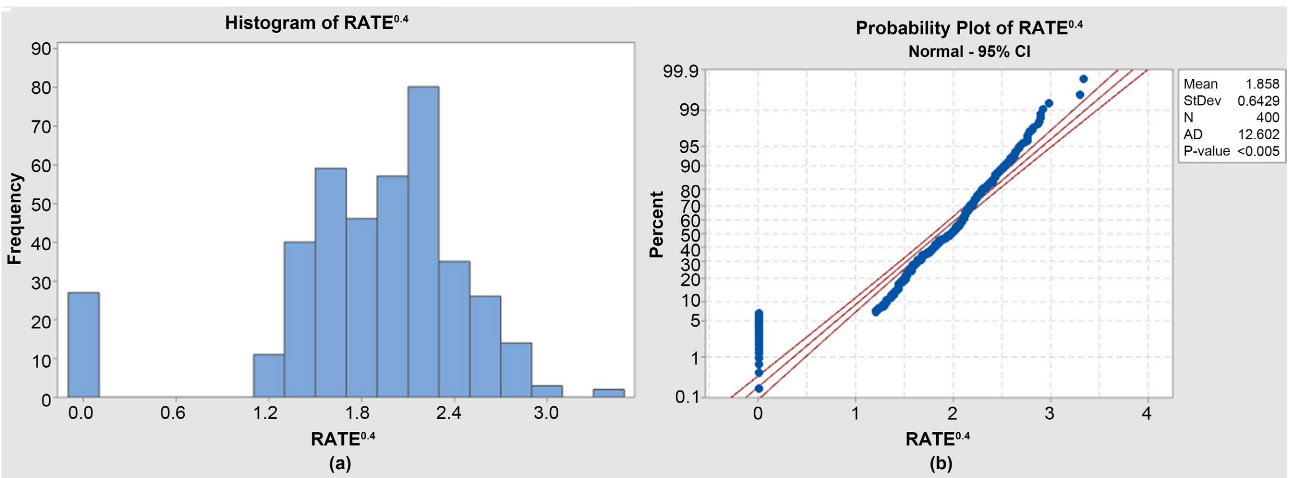

Figure 1. Residual plots for $SHR^{0.4}$ from a two-way additive ANOVA. (a) Distribution of transformed data; (b) Residual probability plot.

Table 3. Two-way ANOVA table for the transformed SHRs.

Source	DF	SS	MS	F Ratios	P Values
STATE	49	137.043	2.79679	41.07	0.000
YEAR	7	4.528	0.64681	9.50	0.000
Error	343	23.356	0.06809		
Total	399	164.926			

Both the factors state and year are shown to be highly significant in affecting the variability of the transformed SHRs, *i.e.*, their p-values are approximately zero.

Then $d = c = 1.2905 \times 0.261 = 0.34$. The means of the transformed rates are given in **Appendix C**. The maximum sample mean is 2.88 (LA) and the minimum sample mean is 0.00 (VT).

For selecting the "worst" subset,

R_5: Select the *i*th state iff $\bar{X}_i \geq \bar{X}_{[k]} - d = 2.88 - 0.34 = 2.54$.

The three states AL, MS, LA are chosen.

For selecting the "best" subset,

R_6: Select the *i*th state iff $\bar{X}_i \leq \bar{X}_{[1]} + c = 0.00 + 0.34 = 0.34$.

Only the state of VT is chosen for the selected subset.

An advantage of the parametric approach over the nonparametric approach is that the parametric analysis explicitly utilizes the magnitudes of the data rather than simply their rank values. Thus, in this analysis, the normal means parametric approach results in a dramatic reduction in the number of states chosen for the selected subsets. These results are displayed in Figure 2.

5. Bayesian Approach to the Selection Problem

In this section, a Bayesian approach is adopted and the population means are assumed to be stochastic. The idea is quite straightforward. A posterior distribution on the population means is used to simulate a large number of random draws, or realizations, of those means. With those draws, ordering probabilities of the population means can be estimated. And from these estimates, simple calculations can provide estimates of, e.g., the probability that a specific population

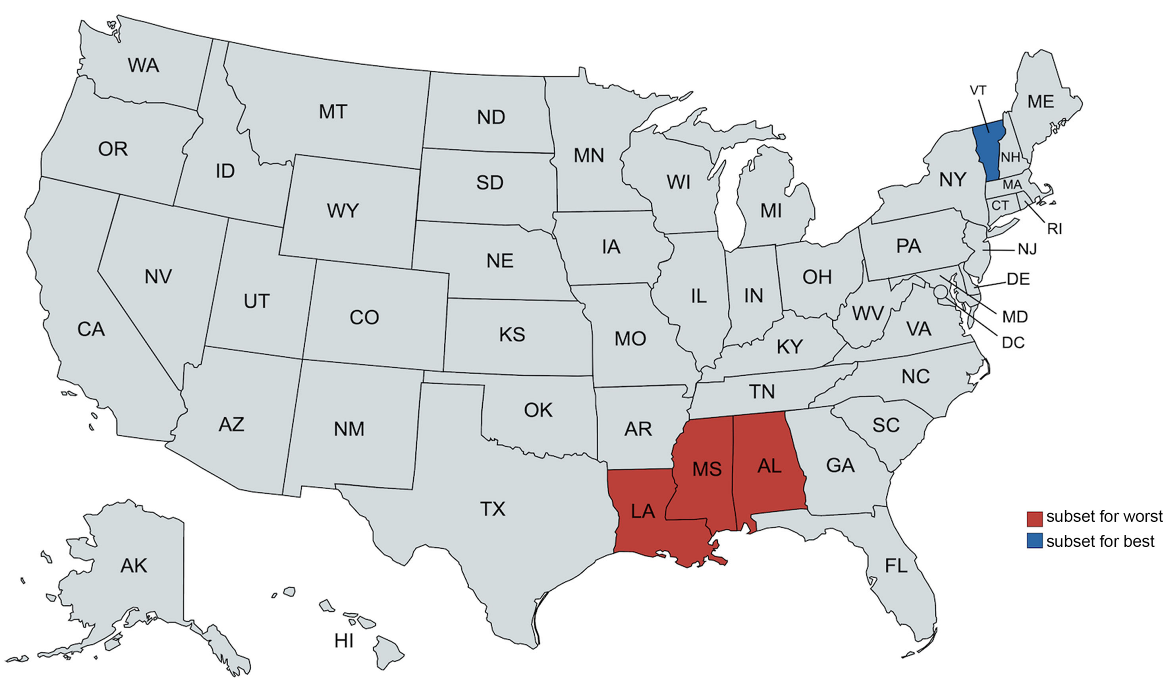

Figure 2. Selected states using parametric rules.

mean is greater than all the other population means. There are many choices that can be made for the posterior distributions. One such approach, utilizing flat (or noninformative) prior distributions on the population means is illustrated here.

As shown in Gill [10] and many other Bayesian texts, the posterior distribution of the mean of the *i*th population (state), μ_i, is normal with mean $\bar{X}_i$ and standard deviation $\sigma/\sqrt{n}$, $i = 1, \cdots, k$. In this situation, the Bayesian and frequentist results (via Central Limit Theorem) are very similar in form. The relevant calculations become

$$P\left(\mu_{m_1} \le \mu_{m_2} \le \cdots \le \mu_{m_k}\right), \tag{10}$$

where $(m_1, m_2, \cdots, m_k)$ is any of the k factorial permutations of the integers (1, 2, ⋯, k). For example, for $k = 4$, there would be 4! = 24 such probabilities to calculate. This can be easily handled with WinBugs (an MCMC simulator) or R. For frequentists, these calculations are meaningless. This approach is used in both of the following subsections. In section "Example with $k = 4$", all of the permutation probabilities (10) can be estimated with simulated draws of the posterior mean as k is small. In section "Bayesian Analysis of $SHR^{0.4}$" using R, with large k, applicable to the analysis of $SHR^{0.4}$, a convenient function in R is used to identify which population (state) realizes the largest and smallest posterior mean on each simulation pass.

5.1. Example with k = 4

Suppose we have $k = 4$ populations with three observations from each of the populations yielding sample means of 2, 3, 4, and 5. Assume a common known standard deviation equal to 1 and a flat (noninformative) prior distribution on the population means. Using, for example, WinBugs, all 24 values of the probabilities given in Equation (10) can be computed. By appropriate summing, the estimated values of $P[\mu_i = \max(\mu_j)]$, $i = 1, 2, 3, 4$, are obtained. **Table 4** gives the results of such computations for 6 of the 24 parameter permutations. These are the 6 permutations, where μ_4 is the largest of the four means.

The tabled values were generated with WinBugs using the model code given in **Appendix D** and specifying a large number (10^5) draws on the posterior means. The probabilities of the permutations, $P\left(\mu_{m_1} \le \mu_{m_2} \le \mu_{m_3} \le \mu_{m_4}\right)$, are denoted by P1.2.3.4 in **Table 4**.

Given the probabilities in **Table 4**, it now follows that $P(\mu_4 \text{ is max}) = 0.6844 + 0.1031 + \cdots + 0.0012 = 0.8873$, *i.e.*, the sum of the six probabilities in the Table. In a similar manner, the calculations yielded $P(\mu_1 \text{ is max}) = 0.00006$, $P(\mu_2 \text{ is max}) = 0.00364$, and $P(\mu_3 \text{ is max}) = 0.10900$. A complete probability distribution over all possible ordering of the population means is realized. This approach of calculating all the permutation probabilities is, from a practical vantage, limited to small values of k (say $k \le 5$ or 6). In our application to homicide rates where $k = 50$, another Bayesian approach is more useful as described in the next subsection.

Table 4. A sampling of WinBugs estimates for selection from four populations.

P1.2.3.4 = 0.6844
P1.3.2.4 = 0.1031
P2.1.3.4 = 0.09304
P2.3.1.4 = 0.00274
P3.1.2.4 = 0.00278
P3.2.1.4 = 0.0012

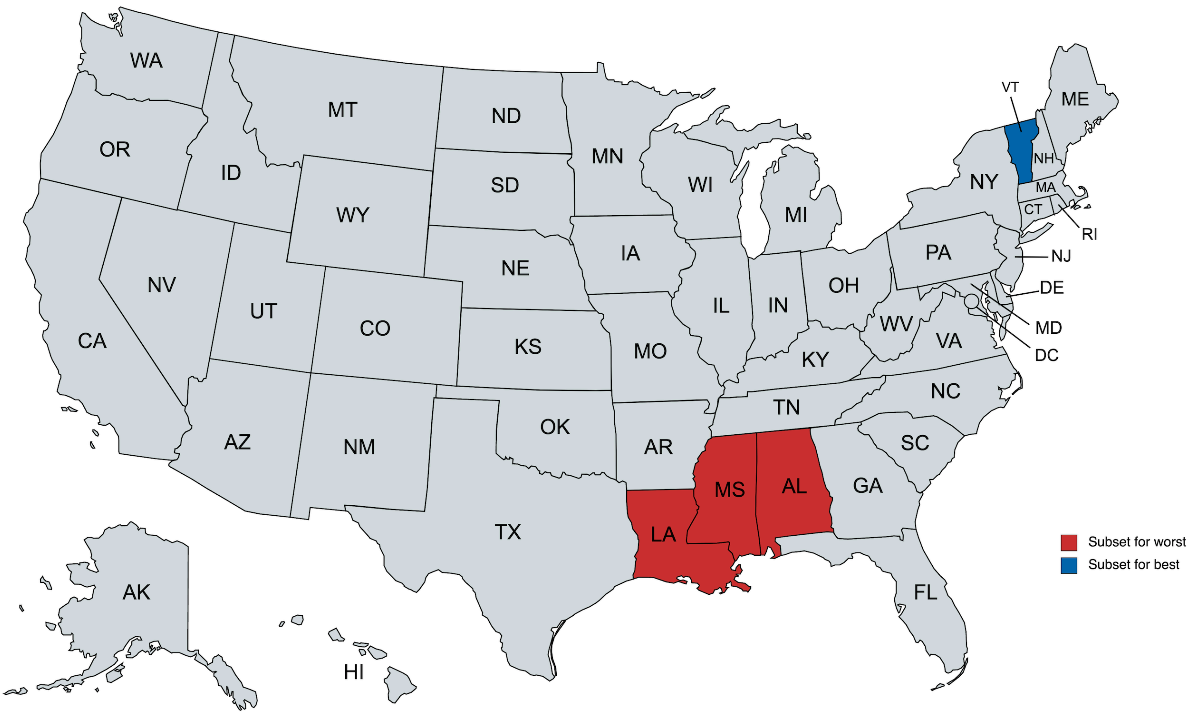

Figure 3. Selected states using Bayesian rules.

5.2. Bayesian Analysis of SHR$^{0.4}$ Using R

The power transformation makes plausible the negligible interaction assumption for the additive model. The "state effects", assuming flat normal priors, have a normal distribution centered at $\bar{X}_i$ and standard deviation $\sigma/\sqrt{n}$, $i = 1, \cdots, k$. We now simulate in R a draw from each state, rank the results (using "which.max" and "which.min"), and repeat a large number of times (e.g., 10^6) to obtain P(LA is worst) = 0.74702, P(MS is worst) = 0.20606, P(AL is worst) = 0.04209, P(VT is best) = 0.99768. These results are displayed in Figure 3. The R code for these calculations is given in **Appendix E**.

The results of the Bayesian analysis herein presented are in close agreement with the results given by the parametric selection procedure. This is as expected since the choice of a noninformative prior distribution results in an analysis based on the likelihood function as is the parametric selection procedure.

6. Concluding Remarks

The subset selection procedures, parametric or nonparametric, select a random number of populations to include in the subsets on which a confidence statement can be attached. Subset size is a random variable dependent on the observed data. Determining the constants required to implement the selection rules does require the determination of the Least Favorable Configuration (LFC), *i.e.*, the configuration of population parameters minimize the probability of a CS. With two of the procedures used in this article, R_1 and R_3, that determination has been made only in the situation where the underlying parameter space is a "slippage" space, *i.e.*, all population parameters are equal with the possible exception of one.

The nonparametric selection rules choose a much larger subset than the parametric procedures. And the conclusions from the Bayesian analyses are qualitatively closely aligned with those from the parametric selection procedures. This is not surprising as the nonparametric approach uses the ranks of the data, not the magnitudes. And as seen in **Figure 1**, there are outliers on the lower and upper end of the residual probability plot.

Bayesian procedures can yield a complete probability distribution over all orderings of the population parameters (e.g., means). There is a curse of dimensionality—$k!$ gets large very quickly. However, using simulation capability in WinBugs and R, it is straightforward to generate a probability distribution over the populations as to which has the maximum (minimum) parameter. This was illustrated with SHRs from $k = 50$ states.

The SHR results have been compared to MVTFRs conducted by McDonald [11]. The "worst" states selected for SHR are AL, MS, and LA while SC, MT, and MS for MVTFR. The "best" states selected for SHR is VT while MA for MVTFR. This is some consistency since the "worst" states are mostly from the Southeastern states and the "best" states are both from the Northeast.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix A. State Homicide Rates Raised to 0.4 Power

State	2005	2014	2015	2016	2017	2018	2019	2020
AK	1.93	1.86	2.30	2.21	2.57	2.24	2.59	2.21
AL	2.47	2.31	2.53	2.68	2.78	2.72	2.77	2.89
AR	2.30	2.26	2.23	2.38	2.49	2.42	2.45	2.79
AZ	2.41	1.90	1.98	2.09	2.13	2.06	2.03	2.24
CA	2.17	1.84	1.90	1.95	1.92	1.87	1.83	2.06
CO	1.71	1.61	1.69	1.79	1.84	1.86	1.79	2.02
CT	1.59	1.53	1.67	1.49	1.59	1.51	1.57	1.84
DE	2.13	2.13	2.24	2.18	2.17	2.15	2.06	2.50
FL	2.02	2.07	2.09	2.15	2.10	2.13	2.14	2.27
GA	2.19	2.13	2.21	2.29	2.29	2.26	2.31	2.56
HI	1.29	1.37	1.37	1.51	1.44	1.57	1.44	1.61
IA	1.21	1.44	1.44	1.51	1.63	1.49	1.49	1.67
ID	1.59	1.42	1.32	1.29	1.55	1.40	1.24	1.44
IL	2.15	2.07	2.17	2.43	2.41	2.30	2.31	2.63
IN	2.03	2.01	2.05	2.25	2.20	2.23	2.20	1.48
KS	1.72	1.67	1.86	1.95	2.11	2.03	1.89	2.18
KY	1.96	1.86	2.02	2.20	2.21	2.06	2.03	2.46
LA	2.77	2.67	2.74	2.90	2.91	2.82	2.93	3.31
MA	1.51	1.32	1.35	1.35	1.47	1.40	1.40	1.49
MD	2.55	2.14	2.54	2.52	2.53	2.44	2.51	2.65
ME	1.24	1.32	1.24	0.00	0.00	0.00	1.27	1.21
MI	2.17	2.09	2.10	2.14	2.09	2.11	2.11	2.38
MN	1.49	1.29	1.51	1.42	1.37	1.40	1.51	1.67
MO	2.21	2.24	2.47	2.50	2.64	2.65	2.59	2.87
MS	2.41	2.65	2.64	2.71	2.76	2.82	2.99	3.35
MT	1.63	1.53	1.74	1.79	1.79	1.78	1.69	2.13
NC	2.25	1.99	2.06	2.23	2.17	2.10	2.18	2.36
ND	0.00	0.00	1.57	0.00	0.00	1.44	1.57	1.81
NE	1.44	1.63	1.74	1.61	1.49	1.29	1.57	1.76
NH	0.00	0.00	0.00	0.00	0.00	1.27	1.51	0.00
NJ	1.92	1.81	1.83	1.84	1.76	1.69	1.63	1.79
NM	2.29	2.15	2.30	2.45	2.35	2.59	2.68	2.59
NV	2.27	2.09	2.14	2.23	2.25	2.26	1.98	2.21
NY	1.86	1.63	1.63	1.67	1.55	1.59	1.59	1.86
OH	1.99	1.93	2.05	2.11	2.24	2.15	2.13	2.42
OK	2.06	2.13	2.35	2.36	2.35	2.18	2.39	2.41
OR	1.53	1.42	1.63	1.61	1.57	1.44	1.55	1.71
PA	2.09	1.93	1.99	2.05	2.13	2.10	2.06	2.35
RI	1.57	1.44	1.51	1.40	0.00	0.00	1.44	1.55
SC	2.29	2.25	2.46	2.41	2.44	2.53	2.61	2.76
SD	1.53	1.57	1.78	1.86	1.78	1.72	1.67	2.11
TN	2.33	2.11	2.20	2.39	2.39	2.43	2.43	2.66

TX	2.11	1.93	1.99	2.05	2.02	1.96	2.03	2.25
UT	1.42	1.32	1.32	1.44	1.47	1.37	1.47	1.53
VA	2.10	1.76	1.83	1.98	1.96	1.92	1.95	2.10
VT	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WA	1.67	1.57	1.63	1.53	1.67	1.69	1.59	1.78
WI	1.79	1.55	1.83	1.87	1.69	1.72	1.78	2.06
WV	1.96	2.03	1.83	2.09	2.11	2.02	2.01	2.18
WY	0.00	1.81	0.00	0.00	0.00	1.76	1.81	1.89

Appendix B. State Rank Sums and Subsets of States Chosen by Nonparametric Rules

pstar = 0.9		Worst Selection Rules		Best Selection Rules	
STATE	RANK SUM	R1	R2	R3	R4
VT	17.5			X	X
NH	28			X	X
ME	30.5			X	X
MA	57.5			X	X
UT	58			X	X
ND	61.5			X	X
RI	62			X	X
ID	63			X	X
MN	67.5			X	X
HI	71			X	X
IA	83.5			X	X
WY	94.5			X	X
OR	98			X	X
NE	103.5			X	X
CT	113			X	X
WA	124.5			X	X
NY	132.5			X	X
SD	149.5			X	X
MT	152.5		X	X	X
NJ	155.5		X	X	X
CO	156.5		X	X	X
WI	157.5		X	X	X
KS	194.5		X		X
VA	196.5		X		X
CA	200.5		X		X
WV	217.5		X		X
TX	227		X		X
PA	240.5		X		X
KY	245.5		X		X
AZ	247	X	X		X
FL	255.5	X	X		X
OH	259	X	X		
MI	265	X	X		
NC	273.5	X	X		
IN	277.5	X	X		
NV	281	X	X		
AK	284	X	X		
DE	286.5	X	X		
OK	305.5	X	X		
GA	313	X	X		
IL	315	X	X		
TN	333	X	X		
AR	346	X	X		
NM	347.5	X	X		
SC	355.5	X	X		
MO	361.5	X	X		
MD	362	X	X		
AL	384	X	X		
MS	391.5	X	X		
LA	397	X	X		

Appendix C. Ordered Means of State SHR$^{0.4}$

pstar=0.9		Worst	Best
State	Mean	R5	R6
VT	0.00		x
NH	0.35		
ME	0.78		
ND	0.80		
WY	0.91		
RI	1.11		
ID	1.41		
MA	1.41		
UT	1.42		
HI	1.45		
MN	1.46		
IA	1.48		
OR	1.56		
NE	1.57		
CT	1.60		
WA	1.64		
NY	1.67		
SD	1.75		
MT	1.76		
NJ	1.78		
WI	1.79		
CO	1.79		
KS	1.93		
CA	1.94		
VA	1.95		
WV	2.03		
TX	2.04		
PA	2.09		
KY	2.10		
AZ	2.10		
FL	2.12		
OH	2.13		
MI	2.15		
NC	2.17		
NV	2.18		
IN	2.18		
DE	2.19		
AK	2.24		
OK	2.28		
GA	2.28		
IL	2.31		
TN	2.37		
AR	2.41		
NM	2.43		
SC	2.47		
MD	2.49		
MO	2.52		
AL	2.64	X	
MS	2.79	X	
LA	2.88	X	

Appendix D. WinBugs Code for Calculations Related to Table 4

```
# Ranking & Selection for k = 4 populations
model {
    for (i in 1:3) {
        x1[i] ~ dnorm(m1,tau1)
        x2[i] ~ dnorm(m2,tau2)
        x3[i] ~ dnorm(m3,tau3)
        x4[i] ~ dnorm(m4,tau4)
    }
    m1 ~ dnorm(a,b)
    m2 ~ dnorm(a,b)
    m3 ~ dnorm(a,b)
    m4 ~ dnorm(a,b)
    tau1 <- pow(sigma1,-2)
    tau2 <- pow(sigma2,-2)
    tau3 <- pow(sigma3,-2)
    tau4 <- pow(sigma4,-2)
    p1.2.3.4 <- step(m2-m1)*step(m3-m2)*step(m4-m3)
    p1.2.4.3 <- step(m2-m1)*step(m4-m2)*step(m3-m4)
    p1.3.2.4 <- step(m3-m1)*step(m2-m3)*step(m4-m2)
    p1.3.4.2 <- step(m3-m1)*step(m4-m3)*step(m2-m4)
    p1.4.2.3 <- step(m4-m1)*step(m2-m4)*step(m3-m2)
    p1.4.3.2 <- step(m4-m1)*step(m3-m4)*step(m2-m3)
    p2.1.3.4 <- step(m1-m2)*step(m3-m1)*step(m4-m3)
    p2.1.4.3 <- step(m1-m2)*step(m4-m1)*step(m3-m4)
    p2.3.1.4 <- step(m3-m2)*step(m1-m3)*step(m4-m1)
    p2.3.4.1 <- step(m3-m2)*step(m4-m3)*step(m1-m4)
    p2.4.1.3 <- step(m4-m2)*step(m1-m4)*step(m3-m1)
    p2.4.3.1 <- step(m4-m2)*step(m3-m4)*step(m1-m3)
    p3.1.2.4 <- step(m1-m3)*step(m2-m1)*step(m4-m2)
    p3.1.4.2 <- step(m1-m3)*step(m4-m1)*step(m2-m4)
    p3.2.1.4 <- step(m2-m3)*step(m1-m2)*step(m4-m1)
    p3.2.4.1 <- step(m2-m3)*step(m4-m2)*step(m1-m4)
    p3.4.1.2 <- step(m4-m3)*step(m1-m4)*step(m2-m1)
    p3.4.2.1 <- step(m4-m3)*step(m2-m4)*step(m1-m2)
    p4.1.2.3 <- step(m1-m4)*step(m2-m1)*step(m3-m2)
    p4.1.3.2 <- step(m1-m4)*step(m3-m1)*step(m2-m3)
    p4.2.1.3 <- step(m2-m4)*step(m1-m2)*step(m3-m1)
    p4.2.3.1 <- step(m2-m4)*step(m3-m2)*step(m1-m3)
    p4.3.1.2 <- step(m3-m4)*step(m1-m3)*step(m2-m1)
    p4.3.2.1 <- step(m3-m4)*step(m2-m3)*step(m1-m2)
    p[1] <- p1.2.3.4
    p[2] <- p1.2.4.3
```

```
    p[3] <- p1.3.2.4
    p[4] <- p1.3.4.2
    p[5] <- p1.4.2.3
    p[6] <- p1.4.3.2
    p[7] <- p2.1.3.4
    p[8] <- p2.1.4.3
    p[9] <- p2.3.1.4
    p[10]<- p2.3.4.1
    p[11]<- p2.4.1.3
    p[12]<- p2.4.3.1
    p[13]<- p3.1.2.4
    p[14]<- p3.1.4.2
    p[15]<- p3.2.1.4
    p[16]<- p3.2.4.1
    p[17]<- p3.4.1.2
    p[18]<- p3.4.2.1
    p[19]<- p4.1.2.3
    p[20]<- p4.1.3.2
    p[21]<- p4.2.1.3
    p[22]<- p4.2.3.1
    p[23]<- p4.3.1.2
    p[24]<- p4.3.2.1
    p.sum <- sum(p[])
}
list(a=0,b=0.001,x1=c(1,2,3),x2=c(2,3,4),x3=c(3,4,5),x4=c(4,5,6),
    sigma1=1,sigma2=1,sigma3=1,sigma4=1)
```

Appendix E. R Code for Bayesian Simulations Described in the Bayesian Analysis of $SHR^{0.4}$ Section

```
# R-code for Bayesian simulations of Rate^0.4
# k = number of populations; n = number of simulations
# sigma = model sd ; m = number of years
k=50; n=100000; sigma=0.261; m=8
# sigma value is estimate from two-way ANOVA of Rate^0.4
# mu values are means of (Rate^0.4)
x <- c(rep(0,k))
y <- c(rep(0,n))
z <- c(rep(0,n))
err <- sigma/sqrt(m)
mu <- c(2.24, 2.64, 2.41, 2.10, 1.94, 1.79, 1.60, 2.19, 2.12, 2.28, 1.45, 1.48,
1.41, 2.31, 2.18, 1.93, 2.10, 2.88, 1.41, 2.49, 0.78, 2.15, 1.46, 2.52, 2.79,
1.76, 2.17, 0.80, 1.57, 0.35, 1.78, 2.43, 2.18, 1.67, 2.13, 2.28, 1.56, 2.09,
1.11, 2.47, 1.75, 2.37, 2.04, 1.42, 1.95, 0.00, 1.64, 1.79, 2.03, 0.91)
```

```
names(mu) <- c("AK","AL","AR","AZ","CA","CO","CT","DE",
               "FL","GA","HI","IA","ID","IL","IN","KS",
               "KY","LA","MA","MD","ME","MI","MN","MO",
               "MS","MT","NC","ND","NE","NH","NJ","NM",
               "NV","NY","OH","OK","OR","PA","RI","SC",
               "SD","TN","TX","UT","VA","VT","WA","WI",
               "WV","WY")
mu
for (i in 1:n){
  for (j in 1:k) {x[j] <- rnorm(1, mean = mu[j], sd = err)}
  y[i] <- which.min(x)
  z[i] <- which.max(x)
}
table(y)
table(z)
```

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Goal Achieving Probabilities of Mean-Variance Strategies in a Market with Regime-Switching Volatility

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Abstract

In this paper, we establish properties for the switch-when-safe mean-variance strategies in the context of a Black-Scholes market model with stochastic volatility processes driven by a continuous-time Markov chain with a finite number of states. More precisely, expressions for the goal-achieving probabilities of the terminal wealth are obtained and numerical comparisons of lower bounds for these probabilities are shown for various market parameters. We conclude with asymptotic results when the Markovian changes in the volatility parameters appear with either higher or lower frequencies.

Keywords

First Passage Time Probabilities, Mean-Variance Strategy, Regime-Switching Model

1. Introduction

In the financial world, an investor is routinely subjected to finding strategies that offer higher returns with reduced risks. In his seminal paper [1], Nobel prize laureate Markowitz introduced the myopic (single period) mean-variance portfolio management problem where one calibrates the amount of wealth invested in risky assets (stocks) and a riskless asset (bond) in such a way that it minimizes the variance of a terminal wealth while targeting an average end return. Since then, scores of innovative research problems arose related to his original static model as well as dynamic extensions in both discrete and continuous time, as seen for example in the following recent papers: [2] [3] [4].

It's worth noting that since the unconstrained mean-variance approach is solely based on averaged return, then an investor might experience undesired marked scenarios such as returns below a safe investment in a bank account with guaranteed interest rate or even worst events such as bankruptcy. In an effort to reduce the probability of encountering these undesired scenarios while still aiming for the target wealth at the end of the investment horizon, Zhou and Li [5] devised a hybrid strategy that we will call here the switch-when-safe strategy. More precisely, in a continuous-time setting under a Black-Scholes market model with deterministic parameters, the investor follows the optimal unconstrained mean-variance strategy up to the first (random) moment, if it occurs, where he could reinvest all of his cumulative wealth in a riskless asset so that it would generate the desired wealth at the end of the investment horizon. In their paper, they discovered the following astonishing properties:

- The goal-achieving probability depends on neither the initial wealth nor the desired terminal wealth;
- The goal-achieving probability has an explicit expression in terms of market parameters and time horizon;
- The goal-achieving probability has a universal lower bound of 0.80, which depend on neither the market parameters nor the time horizon.

Still, in the context of deterministic market parameters in a Black Scholes model with stock prices driven by Brownian motions, these same properties were also uncovered when one considers cone-constrained mean-variance strategies such as no short-selling strategies [6] [7]. In this paper, we wish to explore if these properties carry on to more general market models for example by considering a Black-Scholes model with added randomness, more precisely, while maintaining deterministic interest for the riskless asset and deterministic drift parameters for the risky asset, we will allow the volatility parameter of the risky asset to change, depending on the state of a continuous-time Markov chain, independent of the stock prices driven by Brownian motions.

2. Market Model and Regime-Switching Mean-Variance Strategy

The market model is composed of a riskless asset and m risky assets with a volatility matrix $\sigma_{\alpha(t)}$ depending of an independent Markov chain α. The price $S_0(t)$ of the riskless asset a time t follow the dynamics given by the ODE:

$$\mathrm{d}S_0(t) = r(t) S_0(t) \mathrm{d}t$$

while the price of the risky assets follow the dynamics given by the SDEs:

$$\mathrm{d}S_i(t) = S_i(t)\left[\mu_i(t)\mathrm{d}t + \sum_{j=1}^{m} \sigma_{ij,\alpha(t)} \mathrm{d}W_j(t)\right], \quad i = 1,\cdots,m$$

where W_j are independent standard Brownian motions and $\{\alpha(t): t \geq 0\}$ is a continuous-time Markov chain with a finite set of states $\{1,\cdots,S\}$.

Let $W(t) = [W_j(t)]_{m\times 1}$, $\sigma_{\alpha(t)} = [\sigma_{ij,\alpha(t)}]_{m\times m}$, $B(t) = [\mu_i(t) - r(t)]_{1\times m}$ and $\pi(t) = [\pi_i(t)]_{m\times 1}$ be the investor portfolio: $\pi_i(t)$ is the amount invested in the i^{th} stock at time t. Then the self-financing wealth process X of the investor is driven by the SDE

$$\mathrm{d}X(t)=\left[r(t)X(t)+B(t)\pi(t)\right]\mathrm{d}t+\pi(t)\sigma_{\alpha(t)}(t)\mathrm{d}W(t),\quad X(0)=x_0.$$

A *mean-variance strategy* $\pi_{MV}(t)$ is one that minimizes the variance of the terminal wealth $Var(X(T))$ under the constraint that the expected terminal wealth satisfies $E(X(T))=z$ where $z>x_0\mathrm{e}^{\int_0^T r(s)\mathrm{d}s}$.

Zhou and Yin [8] showed that, for a regime-switching volatility model, this optimal strategy is given by

$$\pi_{MV}(t)=-\left[\sigma_{\alpha(t)}(t)\sigma_{\alpha(t)}(t)\right]^{-1}B(t)\left[X(t)+\lambda\mathrm{e}^{-\int_t^T r(s)\mathrm{d}s}\right]$$

where the Lagrange multiplier λ is given by

$$\lambda=\frac{z-x_0P(0,\alpha(0))\mathrm{e}^{-\int_0^T r(s)\mathrm{d}s}}{P(0,\alpha(0))\mathrm{e}^{-2\int_0^T r(s)\mathrm{d}s}-1}$$

and $P(t,k)$ is the solution to the following ODE system

$$\frac{\partial P(t,k)}{\partial t}=\left[\theta_k(t)\theta_k(t)-2r(t)\right]P(t,k)-\sum_{\ell=1}^{S}q_{k\ell}P(t,\ell)$$

$$P(T,k)=1$$

where $\theta_k(t)=B(t)\sigma_k^{-1}$ and $Q=\left[q_{k\ell}\right]_{S\times S}$ is the infinitesimal generator of Markov chain $\{\alpha(t):t\geq 0\}$.

Consequently, following Itô's formula, the wealth process $X_{MV}(t)$ of the mean-variance startegy can be expressed as

$$X_{MV}(t)=\left(x_0\mathrm{e}^{\int_0^T r(s)\mathrm{d}s}+\lambda\right)Z(t)-\lambda\mathrm{e}^{-\int_t^T r(s)\mathrm{d}s}$$

where

$$Z(t)=\exp\left\{-\frac{3}{2}\int_0^t\left\|\theta_{\alpha(s)}(s)\right\|^2\mathrm{d}s-\int_0^t\theta_{\alpha(s)}(s)\mathrm{d}W(s)\right\}.$$

This form is well-suited to the computations in the next section.

3. Switch-When-Safe Mean-Variance Strategy and Goal Achieving Probabilities

Consider the following stopping time:

$$\tau_z=\inf\left\{0\leq t\leq T:X_{MV}(t)\mathrm{e}^{\int_t^T r(s)\mathrm{d}s}=z\right\}.$$

This time, if it exists, is the first moment at which the wealth is such that, invested in the riskless asset, it would have a final value equal to the targeted expected terminal wealth of the mean-variance strategy.

The *switch-when-safe mean-variance strategy* of [5] is defined as

$$\pi_{SWS}(t)=\begin{cases}\pi_{MV}(t) & \text{if } t\leq\tau_z\wedge T,\\ 0 & \text{otherwise.}\end{cases}$$

Now observe that

$$X_{MV}(t)-z\mathrm{e}^{-\int_t^T r(s)\mathrm{d}s}=\frac{x_0\mathrm{e}^{\int_0^T r(s)\mathrm{d}s}-z}{1-P(0,\alpha(0))\mathrm{e}^{-2\int_0^T r(s)\mathrm{d}s}}\left[Z(t)-P(0,\alpha(0))\mathrm{e}^{-2\int_0^T r(s)\mathrm{d}s}\right].$$

Since $x_0\mathrm{e}^{\int_0^T r(s)\mathrm{d}s}-z<0$, it follows that the equality $X_{MV}(t)\mathrm{e}^{\int_t^T r(s)\mathrm{d}s}=z$ is verified if and only if

$$\frac{3}{2}\int_0^t\left\|\theta_{\alpha(s)}(s)\right\|^2\mathrm{d}s+\int_0^t\theta_{\alpha(s)}(s)\mathrm{d}W(s)=2\int_0^T r(s)\mathrm{d}s-\ln P(0,\alpha(0)).$$

From this condition, we see that, as it is the case in [5], the goal-achieving probability of the switch-when-safe mean-variance strategy for a regime-switching volatility model does not depend on either the initial wealth or the desired terminal wealth.

Now let us find an expression for the goal-achieving probabilities in the case of a model with one risky asset that is W is reduced to a one-dimensional brownian motion, let

$$Y(t)=\int_0^t\frac{3}{2}\theta_{\alpha(s)}^2(s)\mathrm{d}s+\int_0^t\theta_{\alpha(s)}(s)\mathrm{d}W(s)$$

First, according to Buffington and Elliottt [9], the characteristic function of the diffusion process Y is given by

$$\phi_t(u)=\left\langle\exp\left(Qt+\begin{bmatrix}\left(\frac{3}{2}iu-\frac{1}{2}u^2\right)\theta_1^2 & 0 & \cdots\\ 0 & \cdots & 0\\ \cdots & 0 & \left(\frac{3}{2}iu-\frac{1}{2}u^2\right)\theta_S^2\end{bmatrix}t\right)\pi(0),1_S\right\rangle$$

where 1_S is a S-dimensional vector of ones.

Let $a=2\int_0^T r(s)\mathrm{d}s-\ln P(0,1)$ represent the barrier, if T_a is the stopping time defined by

$$T_a=\inf\{0\le t\le T:Y(t)=a\}$$

then

$$\Pr(\tau_z\le T)=\Pr(T_a\le T).$$

By introducing the Wiener-Hopf factorization of the process $(Y(t),\alpha(t))$ that is to say, the couple (Q_+,Q_-) which solves for every $u>0$

$$\Xi(-Q_+)=\Xi(Q_-)=0$$

where

$$\Xi(P)=\frac{1}{2}\Sigma^2P^2+VP+Q-uI_S$$

with $\Sigma=diag(\theta_1,\cdots,\theta_S)$, $V=diag\left(\frac{3}{2}\theta_1^2,\cdots,\frac{3}{2}\theta_S^2\right)$, Q the infinitesimal generator and I_S the $S\times S$ identity matrix, then, following Jiang and Pistorius [10], the associated Laplace transform Ψ_a of the random variable T_a is given by

$$\Psi_a = \pi(0)\exp(aQ_+)1_S$$

therefore, through Laplace transform inversion, we deduce

$$\Pr(T_a \le T) = \frac{\Psi_a(0)}{2} - \frac{1}{\pi}\mathrm{Re}\left(\int_0^\infty \mathrm{e}^{-iu}\frac{\Psi_a(-iu)}{iu}\mathrm{d}u\right).$$

Moreover, since the ratio $\frac{\frac{3}{2}\theta_i^2}{(\theta_i)^2} = \frac{3}{2}$ is constant for $i = 1,\cdots,S$ then according to Hieber [11] the last expression is reduced to

$$\Pr(T_a \le T) = \frac{1+\exp(3a)}{2} - \frac{1}{\pi}\mathrm{Re}\left(\int_0^\infty \frac{\exp(3a+iua)-\exp(-iua)}{iu}\phi_T(u)\mathrm{d}u\right).$$

Both expressions can easily be evaluated numerically. However, it is worth mentioning that, even if one could find explicit forms for the exponential matrices (which is the case for $S = 2$ for example) appearing in these expressions, searching for possible closed-formed formulas for the integrals involved could prove to be quite challenging.

One notable exception is the trivial case where all possible values of the volatility matrix are reduced to a single constant matrix. Then we have $\theta_{\alpha(t)} \equiv \theta$ that is $Y(t)$ revert to a standard Brownian motion with drift and according to [5]:

$$\Pr(T_a \le T) = \Phi\left(\frac{1}{2}\|\theta\|\sqrt{T}\right) + \mathrm{e}^{3\|\theta\|^2 T}\Phi\left(-\frac{5}{2}\|\theta\|\sqrt{T}\right).$$

Figure 1 shows the probabilities in this case as a function of $x = \|\theta\|\sqrt{T}$.

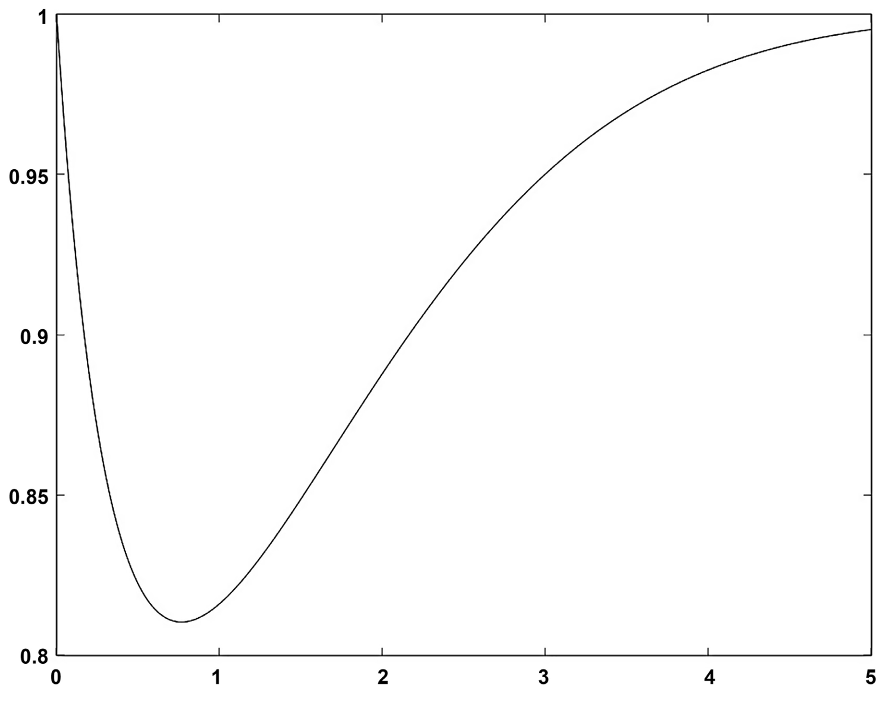

Figure 1. Goal-achieving probabilities as a function of $x = \|\theta\|\sqrt{T}$.

For the numerical study of lower bound probabilities, we will suppose hereon that we have one risky asset, the parameters r and μ are constant, and the volatility parameter σ follows a 2-state continuous-time Markov chain with an infinitesimal generator Q taking the form

$$Q = \begin{bmatrix} q_{11} & -q_{11} \\ -q_{22} & q_{22} \end{bmatrix}$$

where $q_{11}, q_{22} < 0$. In this case, the constant interest r allows us to have the explicit solution to the ODE system

$$\begin{bmatrix} P(t,1) \\ P(t,2) \end{bmatrix} = \exp\left(-(T-t)(M-Q)\right)\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

with

$$M = \begin{bmatrix} \theta_1^2 - 2r & 0 \\ 0 & \theta_2^2 - 2r \end{bmatrix}.$$

Furthermore, since α is 2-state Markov chain, the matrix Q_+ can be written explicitly [11] as

$$Q_+ = \begin{bmatrix} \dfrac{-\beta_{3,u}\beta_{4,u} + 2(q_{11}-u)/\theta_1^2}{\beta_{3,u}+\beta_{4,u}+3} & \dfrac{-2q_{11}/\theta_1^2}{\beta_{3,u}+\beta_{4,u}+3} \\ \dfrac{-2q_{22}/\theta_2^2}{\beta_{3,u}+\beta_{4,u}+3} & \dfrac{-\beta_{3,u}\beta_{4,u} + 2(q_{22}-u)/\theta_2^2}{\beta_{3,u}+\beta_{4,u}+3} \end{bmatrix}$$

where $\beta_{3,u} < \beta_{4,u}$ are the real positive roots of the quartic equation

$$\left(\frac{1}{2}\theta_1^2\beta^2 + \frac{3}{2}\theta_1^2\beta + q_{11} - u\right)\left(\frac{1}{2}\theta_2^2\beta^2 + \frac{3}{2}\theta_2^2\beta + q_{22} - u\right) - q_{11}q_{22} = 0.$$

Following the Cayley-Hamilton theorem we then have

$$\exp(aQ_+) = \left(\frac{\beta_{3,u}\mathrm{e}^{-a\beta_{4,u}} - \beta_{4,u}\mathrm{e}^{-a\beta_{3,u}}}{\beta_{3,u}-\beta_{4,u}}\right)I_2 + \left(\frac{\mathrm{e}^{-a\beta_{3,u}} - \mathrm{e}^{-a\beta_{4,u}}}{\beta_{3,u}-\beta_{4,u}}\right)Q_+$$

which leads us to an explicit expression for the Laplace transform Ψ_a. We will use it in our numerical computation of the goal-achieving probabilities $P(T_a \le T)$.

As an example, consider a market model with a single asset and a two-state volatility:

$$\mu = 0.10, \quad r = 0.01, \quad Q = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}.$$

We can compute the goal-achieving probabilities $P(T_a \le T)$ like in **Figure 1** and find a lower bound for them as a function of the different values of the stock's regime-switching volatility. **Table 1** gives the lower bound probabilities, assuming the initial regime-switching state is $\alpha(0) = 1$.

Clearly one observes that, in presence of a true regime-switching volatility model ($\sigma_1 \ne \sigma_2$), the lower bound probabilities cross the threshold of its deterministic model counterpart. Moreover, as σ_1 takes on larger values while σ_2 takes on lower values the lower bound probabilites gets fairly small, for example

Table 1. Lower bounds of goal-achieving probabilities.

σ_1/σ_2	0.05	0.10	0.15	0.20	0.25
0.05	0.810	0.795	0.783	0.778	0.776
0.10	0.771	0.810	0.800	0.788	0.777
0.15	0.706	0.797	0.810	0.806	0.800
0.20	0.645	0.776	0.806	0.810	0.808
0.25	0.594	0.753	0.798	0.808	0.810

if we take $\sigma_1 = 0.50$ and $\sigma_1 = 0.01$ the lower bound probability decreases to a mere 0.166.

4. Limit Cases of Goal Achieving Probabilities

Assume now that Q depends on a parameter k:

$$Q(k) = k\begin{bmatrix} q_{11} & -q_{11} \\ -q_{22} & q_{22} \end{bmatrix}$$

with $k > 0$. We will study the first passage time probabilities when either $k \uparrow \infty$ or $k \downarrow 0$, which corresponds respectively to the case where the regime-switching jumps appear with high frequency or are scarce.

- $k \uparrow \infty$ (average time to next jump tends to zero)

$$\lim_{k\to\infty} \exp\left(-T\left(M-Q\right)\right) = \begin{bmatrix} \frac{q_{22}}{q_{11}+q_{22}} & \frac{q_{11}}{q_{11}+q_{22}} \\ \frac{q_{22}}{q_{11}+q_{22}} & \frac{q_{11}}{q_{11}+q_{22}} \end{bmatrix} e^{-\left(\frac{q_{22}}{q_{11}+q_{22}}M_{11}+\frac{q_{11}}{q_{11}+q_{22}}M_{22}\right)T}$$

and therefore

$$\begin{bmatrix} P(0,1) \\ P(0,2) \end{bmatrix} \to \begin{bmatrix} e^{-\left(\frac{q_{22}}{q_{11}+q_{22}}M_{11}+\frac{q_{11}}{q_{11}+q_{22}}M_{22}\right)T} \\ e^{-\left(\frac{q_{22}}{q_{11}+q_{22}}M_{11}+\frac{q_{11}}{q_{11}+q_{22}}M_{22}\right)T} \end{bmatrix}.$$

The barrier $a \to 2rT - \ln P\left(0,\alpha(0)\right) = \theta_\infty^{(2)} T$ where

$$\theta_\infty^{(2)} = \frac{q_{22}}{q_{11}+q_{22}}\theta_1^2 + \frac{q_{11}}{q_{11}+q_{22}}\theta_2^2.$$

We can also show after tedious calculations that

$$\lim_{k\to\infty} \Psi_a = e^{\frac{\lambda}{\nu}\left(1-\sqrt{1+\frac{2\nu^2}{\lambda}u}\right)}$$

where $\nu = \frac{2}{3}T$ and $\lambda = \theta_\infty^{(2)} T^2$.

This expression corresponds to the Laplace transform of the inverse Gaussian (or Wald) density with mean ν and shape parameter λ, therefore

$$\Pr\left(T_a \le T\right) \to \Phi\left(\sqrt{\frac{\lambda}{T}}\left(\frac{T}{\nu}-1\right)\right) + e^{\frac{2\lambda}{\nu}}\Phi\left(-\sqrt{\frac{\lambda}{T}}\left(\frac{T}{\nu}+1\right)\right)$$

$$= \Phi\left(\frac{1}{2}\sqrt{\theta_\infty^{(2)}T}\right) + e^{3\theta_\infty^{(2)}T}\Phi\left(-\frac{5}{2}\sqrt{\theta_\infty^{(2)}T}\right)$$

For the single asset model of the previous section, with regime-switching volatilities $\sigma_1 = 0.10$ and $\sigma_2 = 0.20$, Figure 2 below shows the goal-achieving probabilites for increasing values of *k*.

- $k \downarrow 0$ (average time to next jump tends to infinity)

$$\lim_{k \to 0^+} \exp(-T(M-Q)) = \exp(-TM)$$

and therefore

$$\begin{bmatrix} P(0,1) \\ P(0,2) \end{bmatrix} \to \begin{bmatrix} e^{-M_{11}T} \\ e^{-M_{22}T} \end{bmatrix}.$$

The barrier $a \to 2rT - \ln P(0, \alpha(0)) = \theta_{\alpha(0)}T$.

In this case, we can obtain the limit of the passage-time probability in a straightforward manner. Since the average time to the next jumps tends towards infinity, the Markov chain $\alpha(t)$ will have a tendency to stay at its intital state $\alpha(0)$, thus

$$Y(t) \to \frac{3}{2}\theta_{\alpha(0)}^2 t + \theta_{\alpha(0)}W(t)$$

and therefore

$$\Pr(T_a \le T) \to \Phi\left(\frac{1}{2}\theta_{\alpha(0)}\sqrt{T}\right) + e^{3\theta_{\alpha(0)}^2 T}\Phi\left(-\frac{5}{2}\theta_{\alpha(0)}\sqrt{T}\right).$$

For the same example as above, Figure 3 illustrates this result for decreasing values of *k*.

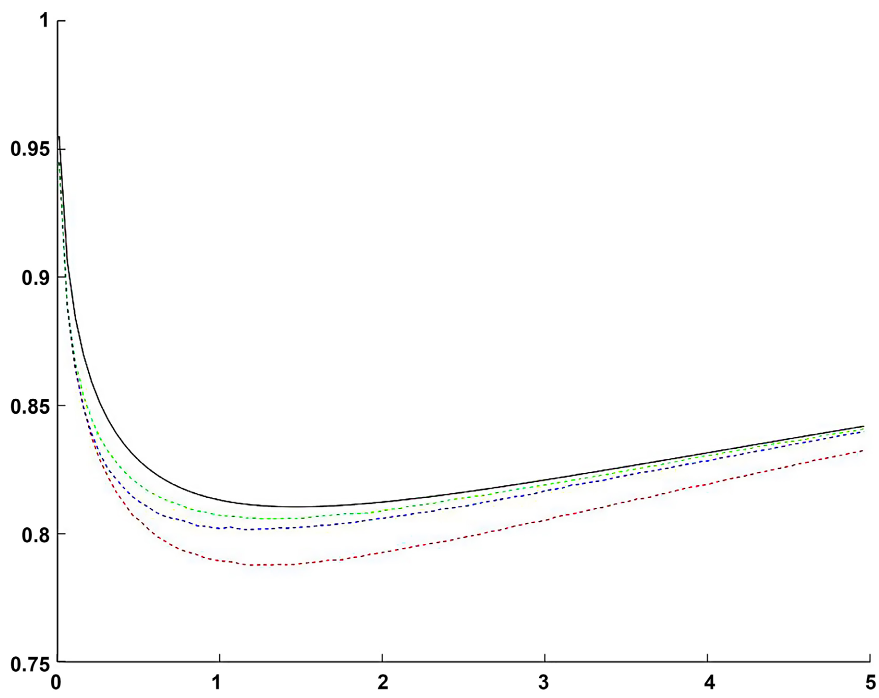

Figure 2. Goal-achieving probabilities for increasing *k*.

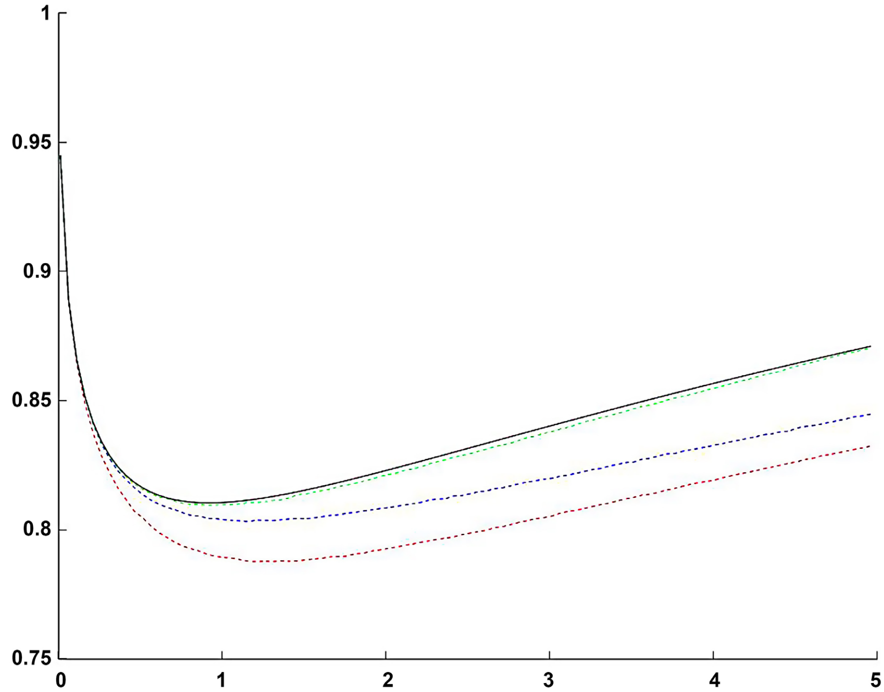

Figure 3. Goal-achieving probabilities for decreasing *k*.

5. Conclusion

In the context of a Black-Scholes market model with stochastic volatility processes driven by a continuous-time Markov chain with finite states, we obtained tractable expressions for the goal-achieving probabilities of switch-when-safe strategies as first introduced by Zhou and Li [5]. We observed that the goal-achieving probabilities are independent of the value of the initial wealth and targeted terminal mean wealth, a property shared with the standard Black-Scholes market counterpart. Unfortunately, it appears that a universal lower bound for these probabilities does not exist for the set of all possible market parameters and infinitesimal generators of the Markovian process as illustrated by our numerical studies. Finally, when the Markovian regime is allowed to either attain higher or lower frequencies than the first-passage time probabilities expressions converge to closed-form formulas.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Phenomenological Models of the Global Demographic Dynamics and Their Usage for Forecasting in 21st Century

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Abstract

A great discovery made by H. von Foerster, P. M. Mora and L. W. Amiot was published in a 1960 issue of "Science". The authors showed that existing data for calculating the Earth's population in the new era (from 1 to 1958) could be described with incredibly high proximity by a hyperbolic function with the point of singularity on 13 November 2026. Thus, empirical regularity of the rise of the human population was established, which was marked by explosive demographic growth in the 20th century when during only one century it almost quadrupled: from 1.656 billion in 1900 to 6.144 billion in 2000. Nowadays, the world population has already overcome 7.8 billion people. Immediately after 1960, an active search for phenomenological models began to explain the mechanism of the hyperbolic population growth and the following demographic transition designed to stabilize its population. A significant role in explaining the mechanism of the hyperbolic growth of the world population was played by S. Kuznets (1960) and E. Boserup (1965), who found out that the rates of technological progress historically increased in proportion to the Earth's population. It meant that the growth of the population led to raising the level of life-supporting technologies, and the latter in its turn enlarged the carrying capacity of the Earth, making it possible for the world population to expand. Proceeding from the information imperative, we have developed the model of the demographic dynamics for the 21st century for the first time. The model shows that with the development and spread of Intelligent Machines (IM), the number of the world population reaching a certain maximum will then irreversibly decline. Human depopulation will largely touch upon the most developed countries, where IM is used intensively nowadays. Until a certain moment in time, this depopulation in developed countries will be compensated by the explosive growth of the population in African countries located south of the Sahara. Calculations in our model reveal that the peak of the human population of 8.52 billion people will be reached in 2050, then it will irreversibly go down to 7.9 billion people by 2100, if developed countries do not take timely effective measures to overcome the process of information depopulation.

Keywords

Explosive Population Growth, Demographic Transition, Demographic, Technological and Information Imperatives, Phenomenological Models of The Demographic Dynamics, Demographic Forecast in the Age of Intelligent Machines

1. Introduction: Global Demographic Transition as the Main Driving Force of the 21st Century Civilisation

Unprecedented economic growth, rampant scientific and technological progress and spacious geopolitical changes in the 20th century were caused by an amazing demographic growth when during only one century the world population almost quadrupled: from 1.656 billion people in 1900 to 6.144 billion people in 2000 [according to the United Nations: https://population.un.org/wpp/Download/Standard/Population/]. At the same time humanity, beginning with the 1960s, has made a significant demographic transition, the essence of which is to change the explosive demographic growth lasting until 1960, into the mode of its slowdown with the following stabilization of the world population by means of a simultaneous decrease in birth and death rates. Moreover, as was stated by S.P. Kapitsa [1], the process of the demographic transition of endogenous origin only is going to last up to the 2050s. It should be noted that humanity has already been going through the demographic transition for the last 235 years. Most developed countries and leading developing countries such as China, India and others have already successfully gone through the demographic transition. In many developing countries, this process is only beginning but it is expected that it is going to be on the fast track, and it will last for 35 - 45 years and not 70 - 90 as it used to be.

The demographic transition in all countries was preceded by a rapid increase in speed and rate of population growth, which then was replaced by an equally rapid slowdown of growth rate, although speed continued to increase for decades. Due to the process of the demographic transition, in a short historical period—several decades only—the country's population has increased several times—from three to seven times, and then striven to stabilize in its number. As a rule, the demographic transition is accompanied by a significant increase in scientific and technological progress, productive forces and economy, the rise of culture and education, and great masses of migrants from the country to towns. There is no doubt that the demographic dynamics will remain the main driving force of the world development in the 21st century. After the end of the demographic transition, there is a sharp shift in the population age, and the proportion of elderly people grows significantly, as is the case with Japan. On the other hand, it all required an ever-increasing production and food and energy consumption, as well as mineral resources, which led to ever-increasing pressure on terrestrial biosphere and environmental pollution. Although developed and leading developing countries have successfully overcome these problems, they have used not only their internal capacities, but also cheap resources drained from the rest of the world. Besides, there was an absolute growth of population during the demographic transition which counted tens and hardly ever hundreds of millions of people (China and India). Now the growth of population amounts to more than a billion people from poor developing countries. Therefore, the question arises: is the world going to cope with such an invasion? Unfortunately, this question remains underexplored.

Indeed, the demographic transition is going on in African, Asian and partly Latin American developing countries. The explosive population growth is characteristic of African countries to the south of the Sahara Desert. According to the estimates of UN experts, in the next 30 years, about 60% of the world population growth will fall on Africa, *i.e.* more than 800 million people, and this number is bigger than the EU population ($\cong$ 730 million people). The population of Nigeria only by 2050 will exceed 400 million people. Rapid population growth of developing countries going through the demographic transition poses difficult questions to their governments, threatening economic growth, environmental quality, population safety and prosperity. Hardly would they be able to create millions of new working places, give sound education to a great number of young people and prepare them for productive activity to earn "demographic dividends". The Arab Spring that happened in the early 2010s reminds us of the fact that the population with a large proportion of unemployed young people poses great risks to the social and political stability in societies that do not meet high expectations in terms of standards of living.

The demographic explosion that is in high gear now in developing countries may be the main factor determining trends of the world development in the 21st century. A swelling demographic tide generated by developing countries may shake the whole world just like a tsunami if humanity does not take drastic measures to neutralize the negative consequences of this largely uncontrolled process. Indeed, in 2050 the world population will likely exceed 9 billion people (in contrast to 7.8 billion people now), *i.e.* there will be one more China. Such population growth will not remain unnoticed by the environment and climate changes. By 2050, humanity will require twice as many resources as our planet can provide. We will have to double food production to satisfy population needs, while productive land resources plummet, and fish resources in the world ocean are already running dry. It is taken into consideration here that nowadays around 2.8 billion people live in poverty and are malnourished. In addition, ecosystem destruction puts food and especially water safety at threat.

There will be five main problems accompanying the global demographic transition: 1) inequality of the demographic process; 2) increase in international migration (refugees); 3) population ageing; 4) urbanization acceleration; 5) escalation of climate warming and ecosystem destruction. It has already been noted above that demographic processes have an extremely irregular nature both in countries and world regions. In decades to follow many developed countries will see population decline, while in many developing African, Asian and Latin American countries there will be explosive or rapid population growth. For example, according to the UN medium-term forecast, by 2050 the EU population will shrink from 731 million people to 664 million people. The explosive population growth in developing countries and its decline in most developed countries will inspire international migration as well as refugees from the countries where violent social conflicts will arise. However, many developed countries see unregulated chaotic migration as a threat to national identity, social cohesion, domestic security and economic welfare.

Increasing life expectancy and population ageing provoked by it can be seen all over the world, but first and foremost it has a negative influence on the economy of most developed countries. Nowadays average life expectancy in the world is 71 years. And by 2050 it will increase to 76 years. Urbanization in developing countries going through the demographic transition will increase since the population growth and urban sprawl are closely intertwined. Now more than half of the world population lives in towns – around 4 billion people, and by 2050 their number will amount for two-thirds, *i.e.* more than 6 billion people. Economic, cultural and political gains connected to the population concentration in towns are evident. Towns are centers of innovations, culture and education, a melting pot of civilization, and the source of wealth and power. Moreover, towns and global urbanization have become the source of the greatest problems on our planet: among them, there are crimes, pollution, shantytowns, poverty and diseases, and above that, they are centers of extreme energy and environmental resources consumption. High energy expenditure has inevitable devastating consequences for the terrestrial biosphere.

Since all aspects of human life and activity are connected to energy, the explosive population growth has led to the massive increase in energy use and production in the 20th century. Besides, power industry is the main source of environmental pollution—soil, atmosphere and water—and it has led to unprecedented degradation of the human habitat. Besides, terrestrial biosphere disintegrates drastically, partially losing the most important function of environmental stabilization. Greenhouse gases emitted into the atmosphere in the process of combustion in power plants of fossil hydrocarbon fuels – coal, oil and natural gas have engendered global warming, which has already exceeded 1.2˚C in comparison with preindustrial level (1850). It is known that the exceedance of global warming level by 2˚C may lead to the global economic crisis with unpredictable negative consequences for human existence. That is why the UN acknowledged climate stabilization as the main ecological imperative and in 2015 at the UN Climate change conference in Paris, it adopted a historic agreement demanding detention of average global temperature rise at the level below 1.5˚C - 2˚C, which, in turn, requires reducing greenhouse gases emission two or three times by 2050.

Thus, the world goes through the most significant demographic shift in human history. The explosive population growth in the poorest developing countries with the following demographic transition and population aging provoked by it, a possible unpredictable international migration and refugee increase, urbanization acceleration in developing countries, further climate warming and terrestrial biosphere destruction foreshadow drastic social and economic consequences both on the regional and global levels. Besides, the rapid development of digital technologies and intellectual machines (IM), computers and robots with elements of artificial intelligence (AI) lead not only to the technological substitution of working places, depriving intermediate-level people of them, but also to population decline, which is to be seen first and foremost in the most developed OECD countries, which in any case expect a demographic decline. However, as is demonstrated in our work, all key models used at present for forecasting the global demographic dynamics do not take this factor into consideration and thus make a wrong forecast for the 21st century. The present work is concluded with an examination of a model for calculating population decline in the information and digital age (1980-2050), driven by the ever-growing usage of IM almost in all spheres of human activity.

2. Global Demographic Process. Phenomenological Models of the Demographic Dynamics

The graph of the growth curve representing the world population, calculated according to Kapitsa's evolutionary demographic model [1] is presented in **Figure 1** along with the actual data of the demographic dynamics (represented as dots) between 1 and 2008 A.D. [2]. As is seen from examining the actual trajectory of the world population growth shown in the figure, it had been accelerating for the last 500 years and slowed in the 1960s only. Moreover, the accelerating character of growth was expressed by the annual absolute growth of population and rates of this growth which increased with time. The highest growth rate was in 1962-1963 reaching 2.2%. Then it started to recede characterising the beginning of the global demographic transition. The highest annual world population growth hit 88 million people in 1989 characterising the passage of the flexible crossing point in the trajectory of the global demographic dynamics. It led to explosive demographic growth in the 20th century, when during only one century the world population quadrupled from 1.656 billion people in 1900 to 6.144 billion people in 2000. Over the past 20 years, the population has increased by almost 1.65 billion people and now the Earth's population exceeds 7.8 billion people [according to the United Nations:

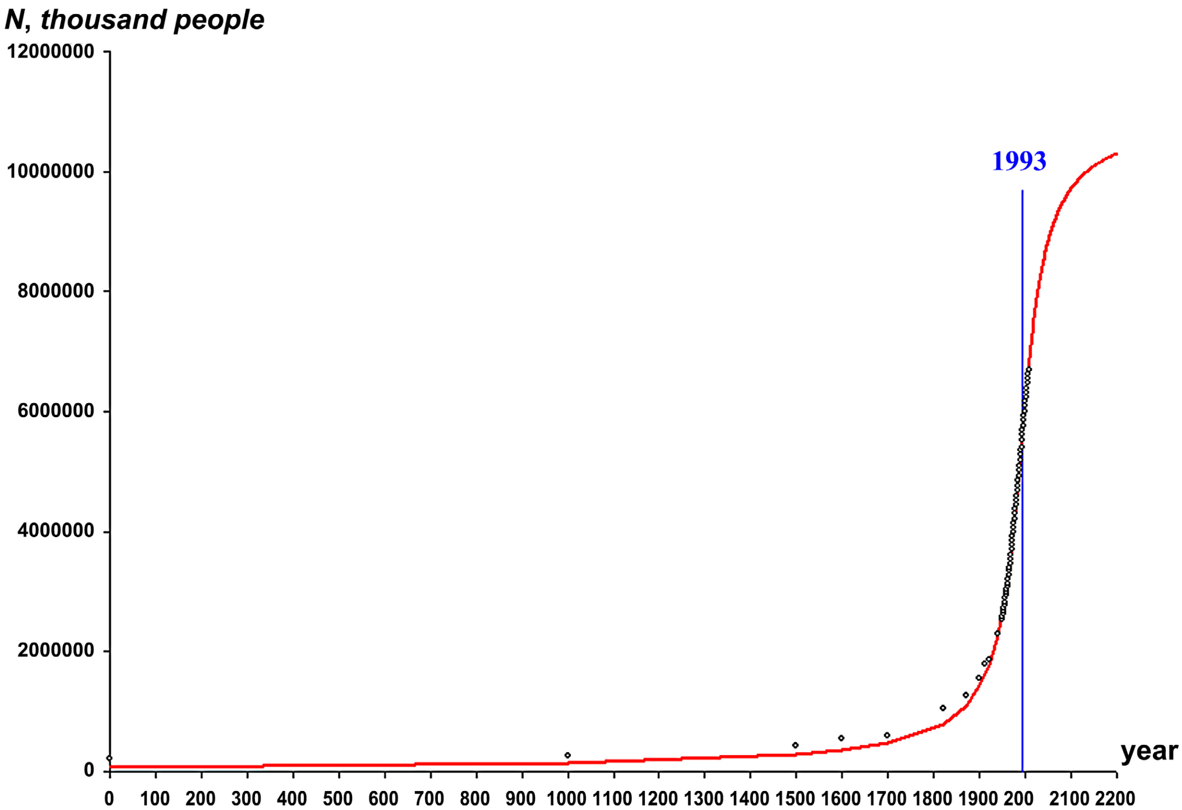

Figure 1. Global demographic dynamics [2] and evolutionary model of the population growth in World-System [1].

https://population.un.org/wpp/Download/Standard/Population/] and it continues to grow. It is evident that the Earth's population will continue to grow as long as the demographic transition continues.

The world population growth has been accompanied by economic growth caused by technological progress. The key role in the demographic dynamics was played by real mid-year per capita income. In **Figure 2**, you can see graphs of per capita income growth in the Industrial era for 5 advanced countries – Australia, Denmark, Netherlands, France and Sweden and one average country – Chile. Strong correlation between the demographic dynamics (**Figure 1**) and the dynamics of growth of per capita income (**Figure 2**) immediately strikes the eye. Secondly, advanced highly profitable countries constitute clear cluster of countries with approximately equal rates of per capita income. It is possible to show that average and low-profitable (developing) countries also form corresponding clusters. Thirdly, a sharp rise of per capita income in advanced countries becomes visible only with the start of the global demographic transition, although demographic transitions actually started in these countries 180 - 100 years earlier, while in Chile it coincided with the actual demographic transition.

The demographic transition occurred in different parts of the world at different time. This phenomenon consisting in the change of accelerating population by a slowdown mode with further stabilization, was first introduced by the French demographer Adolphe Landry [3] in respect to the French population

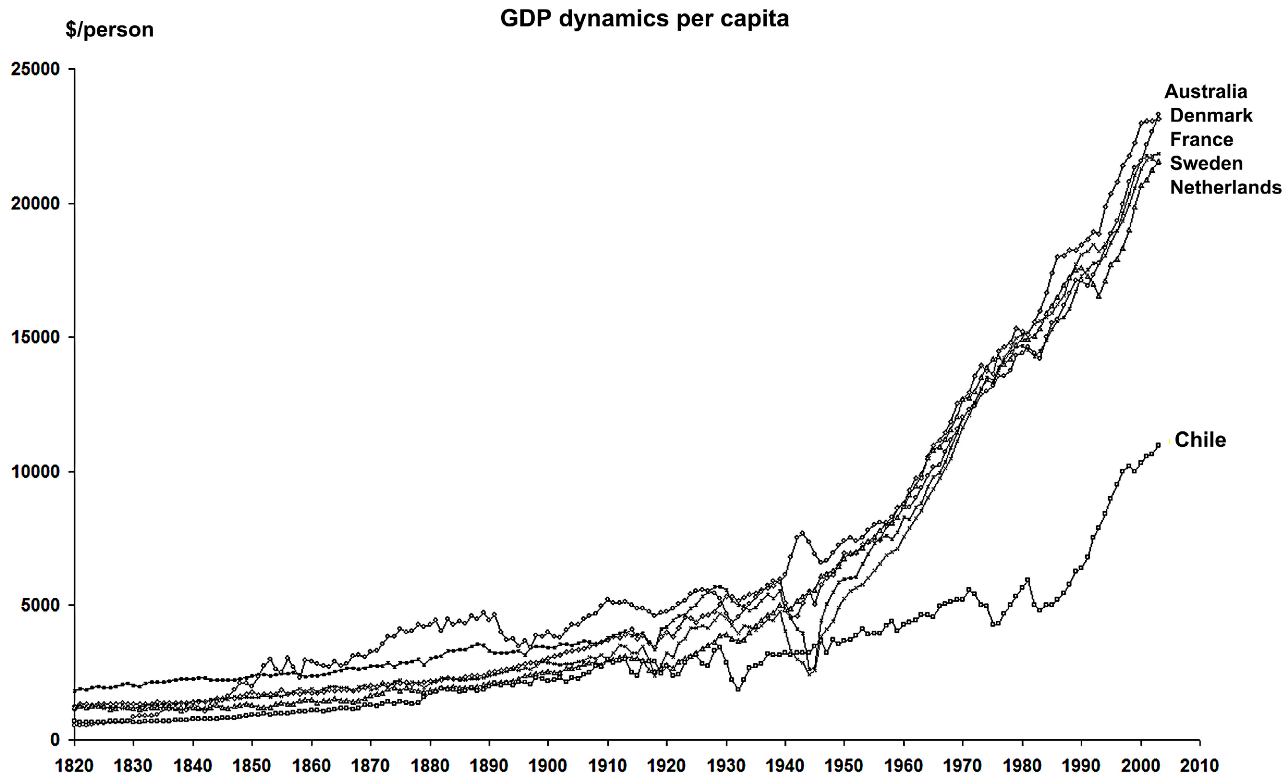

Figure 2. GDP dynamics per capita. Source: http://siteresources.worldbank.org/DATASTATISTICS/Resources/GNI_PPP.pdf.

where it started around 1785 and finished approximately in 1970, which was also determined by the French researcher J. C. Chesnais [4]. Thus, the demographic transition in France lasted for 185 years and was the most continuous one [4]. According to Chesnais, the duration of the demographic transition in Germany constituted 90 years (1876-1965), in India—90 years (1920-2010), China—70 years (1930-2000). He also gave predictive estimate of the duration of the global demographic transition—around 90 years, *i.e.* having started in 1960 it is to end around 2050. [4]. This forecast is supported by Kapitsa's calculations [1] [5]. Thus, as for the majority of Western European and North American countries, the demographic transition started at the beginning-mid 19th century and finished in the second half of the 20th century, in Asia – started at the beginning of the 20th century – finished at the beginning of the 21st century, in Latin America – started in mid-20th century and is finishing now, and in African countries it is beginning just now. Most likely in Africa the demographic transition will be going on the fast track, which will present additional challenges for political, economic and social spheres.

A prominent Russian scholar S. Kapitsa [5] saw the reason of the demographic transition in the crisis of the development of the demographic system: "There are all reasons to believe that demographic crisis is of a fundamental character connected, first and foremost, to achieving speed limit of the system growth. It is accompanied by the deformation of culture, consciousness, disintegration of values" ([5], p. 38). It is no wonder that the passage through the demographic transition escalated by the global interaction is leading to the age marked by the processes of disintegration of the existing pattern of development, fairly called "revolution" by A. Landry. During such periods, as in phase changes in material substances, the "restructuring" occurs, the destruction of old structures and their substitution with the new ones—more advanced structures ([5], p. 36). In the countries that have already gone through the demographic transition, qualitative changes have taken place. There was a change in the fertility behavior: people started to give preference to smaller families, give birth to fewer children, but pay more attention to the quality of their upbringing and education. As a result, the population growth started to slow down already at the end of the 19th century first in West Europe and North America, and its rates fell by more than half in 1870-1950. It happened largely because of reducing a cumulative birth rate. Together with falling birth rate there was the rise of education level. In the developed Western countries general primary education in schools was developed in the mid-19th century, and secondary school education—in the early 20th century. Already in the second half of the 20th century, despite substantial differences in the level of education among the countries, it spread all over the world.

Nowadays there is a global demographic transition which simultaneously covers all humanity as a unique World-System [6]. Examining the stability of the global demographic development, Kapitsa concluded that before the transition the demographic dynamics was unstable and only after it became stable and was going to stay like that ([5], p. 29). Together with it we can see that local development in the world becomes more chaotic, unstable and thus unpredictable. However, these two facts are compatible as, according to the synergetic principle by D. S. Chernavsky [7] chaotic dynamics at the microlevel generates highly determinate systemic pattern of behavior at the macrolevel. The demographic transition should be regarded as a fundamental phenomenon dealing with all spheres of life. As was noted by S. Kapitsa [5], the beginning of the global demographic transition was accompanied by the urban growth, the industrial revolution and extraordinary production growth, development of transport and communication, education and medicine, establishment of the world financial system and incredible development of science and art. This development first and foremost started in Europe and then spread to the rest of the world.

The major part of researchers in the demographic dynamics believes that the world population is going to stabilize in the result of the demographic transition, *i.e.* human development will take place at zero population growth. However, the possibility of sustainable human development with a fixed number raises questions. The fact is that now there is an ongoing transition to a new paradigm of human development based on information and digital technologies, digital economy and a widespread usage of intellectual machines – computers and robots with the elements of artificial intelligence. The transition to the society where intellectual machines will dominate may lead to a rapid change of values. Therefore, it is important to understand what awaits humanity in the long-term perspective and how to manage its development at the global level. Responsible management of the society demands realization of the scale of information and digital revolution, its consequences for humanity and the demographic dynamics, in particular. Europe, many countries of which were the first to come through the demographic transition, joined forces to reform economic, scientific and technological and political space, which points at the processes which may await other countries both at the regional and global levels in the future.

2.1. Stages of Human Development

The history of human development from the point of view of the population growth and its standards of living should be looked at in stages. Following O. Galor and D. N. Weil [8] we may divide it into three stages – Malthusian and Post-Malthusian era, era of the demographic transition and we may add the fourth era—which we are living in—informational and digital one. Indeed, for more than quarter of a century humanity has been going through incredible increase in information and communication technologies (ICT). The network communication is everywhere, and more than 6.5 billion people have cellular phones, and every second uses smartphones which means he or she has Internet access. The number of Internet users has already exceeded 5 billion people. The Internet has become an effective mechanism of collective information network interaction, materialization of collective memory and human intelligence with such effective information retrieval systems as Google and Yandex, as well as technologies of cloud storage and data processing. While networked economy is based on ICT, digital economy is being formed now on the basis of digital technologies and platforms. In the most developed countries, the process of creating Industry 4.0 has already begun, with commercial Internet as its basic infrastructure, and the production will be maintained by intelligent machines only – robots and computers with the elements of artificial intelligence (AI). Information and knowledge have become the key production factors and the main driving force of human development.

2.2. Malthusian Era. The Model of Population Growth According to Malthus

The first known mathematic model of the population growth was exponential model of the demographic dynamics limited by linear growth of the volume of production. The model was developed by the English economist and priest Thomas Malthus [9]. Relying on the data of his time, Malthus supposed that population grows exponentially [9]:

$$N(t) = N_0 \cdot e^{a(t-T_0)}, \tag{1}$$

where N_0 is the number of population at the initial time, $t = T_0$ of the period in question is $t > T_0$; a is a constant coefficient characterising tempos of the population growth. Function (1) is the solution of the simplest linear differential equation:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = aN, \quad N = N_0 \text{ at } t = T_0. \tag{2}$$

This equation suggests that birth rate and death rate change slightly with time, since $a = const$. On the other hand, food production according to the data Malthus had at hand grew linearly, *i.e.* much slower than the number of the population (1). That is why Malthus concluded that “the number of population is inevitably limited by the means of living” ([10], p. 22).

As the number of population (1) in the Malthusian model grows faster than food production level, food shortage soon begins, which leads to the rise in prices and fall in consumption. Malthus argued that a decrease in consumption engenders, in turn, decrease in the population. Then it is time when there is enough food and consumption increases again, provoking population growth, thus, the process is periodic. Indeed, agricultural communities of the past had the periodic dynamics of the population growth [11] [12] described by the Law of Malthus. Yet then already when Malthus wrote his famous work on population, the Industrial Revolution unfolded in England, which guaranteed preemptive growth of means of living in comparison to population size and it lifted resource limitations of the demographic dynamics.

Malthus believed that fall in the population growth rates with decrease in consumption was the law of nature, which was proved for the population size for some species. The law was first described by P.-F. Verhulst with the help of a nonlinear differential equation [13]:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(1 - \frac{N}{K}\right), \quad N = N_0 \text{ at } t = T_0, \tag{3}$$

where $N(t)$ is the current number of population; K is the maximum possible population size with the resources given (capacity of environmental niche, the carrying capacity of the Earth); r is a constant coefficient characterizing the highest possible rate of population growth in favorable conditions. The solution for Equation (3) is a logistic function:

$$N(t) = \frac{K}{1 + \left(\frac{K}{N_0} - 1\right)\exp\left[-r\left(t - T_0\right)\right]}. \tag{4}$$

This logistic function describes very well the Malthusian model with resource limitations to the population growth. It grows exponentially but having passed the flexible crossing point it slows down and with unlimited growth of $t \to \infty$ it asymptotically reaches out for K, *i.e.* $N \to K$ and the population reaches the constant level.

As calculated by A. Maddison [2], average annual per capita income amounted to around 450 international dollars in 1990 in purchasing-power parity in the early our era and remained so until 1000. For comparison, the quantity of the product for a person, minimum essential for a simple reproduction of World-System population, composing minimum cost of living, $m \cong 420$ of international dollars in 1990, according to the estimate given in the work ([14], p. 81). As we can see, in the first millennium A.D. the average annual per capita income slightly exceeded minimum cost of living; that is why the number of population of World-System remained practically the same: if at the beginning of a new era the number of world population was around 230 million people, then it grew up to 261 million only by 1000, and the average growth rate was 0.02% per year ([15], 8.1.1). After 1000 A.D. the world population was increasing with the rate of 0.1% per year and was 438 million people in 1500. Further on it was growing with the average rate of 0.27% per year and increased to 1.04 billion people by 1820 (**Figure 1**).

What about income in this period? From 1000 to 1820 the average annual per capita income increased to 670 dollars in 1990 (average growth rates were 0.05% per year), but world countries already differed significantly in terms of income. For example, in prosperous countries of Western Europe per capita income was 1200 dollars in 1990 ([15], 8.1.1). Thus, the facts show that during periods when per capita income was low and increased slowly, the world population was low as well and increased slowly (**Figure 1**). People for most of the history were occupied with hunting and foraging, and only around 10 thousand years ago agriculture emerged. According to G. Clark [16], hunters and foragers of tens of thousands and millions of years ago consumed as much food per day as people living at the beginning of our era. More than that, food consumption practically did not change until 1800. That is why the period of human history from the emergence of a modern person around 1 billion years ago until 1800 is called Malthusian era.

2.3. Post-Malthusian Era and the Hyperbolic Population Growth. Taapegera's and Kremer's Models

After 1800, there began a notable acceleration of per capita income growth (**Figure 2**) and the world population in particular (**Figure 1**). For example, in 1820-1870 and 1870-2013 average annual per capita income grew at rates of 0.5% and 1.3% per year respectively, *i.e.* ten and twenty-five times higher than in the Malthusian time. The world population growth rate in the periods given also grew and constituted 0.4% per year respectively (1820-1870) and 0.8% per year (1870-2013). Thus, while the Malthusian era was characterized by extremely low population growth rates, post-Malthusian era and vice versa, by increasing population growth rates. However, it did not lead to the decrease in standards of living, as Malthus predicted. On the contrary, during this period there was an accelerated growth of per capita income, which surpassed the population growth. It was unclear how the world managed to turn from the state characteristic of the Malthusian epoch with low income and low population growth to the state with high income and high world population growth. It was like that until the 1960s when the world population growth rates exceeded 2% per year.

It was evident that the Earth's carrying capacity (K) in Equation (3) was not constant, it grew all the time due to numerous technological innovations invented by humanity. For its long history humanity has made a great number of technological innovations and, first and foremost, life-supporting technologies, which enormously increased the ceiling of the Earth's carrying capacity. It was revealed that technological progress rates are proportional to the population size. Surprisingly, the reason for scientific and technological advances and the growth of per capita income in the long term was the population growth. It was first analyzed by the Nobel laureate in economics Simon Kuznets [17], who formulated such assumption: "The more the people, the more the inventors." A little later a prominent Danish researcher E. Boserup [18] independently formulated it, who considered it as anti-Malthusian. In a mathematic form this Kuznets-Boserup assumption was first set down by R. Taagepera [19]:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = cTN^m; \quad c = const, \quad m = const, \tag{5}$$

where T is the level of technological development. Thus, the population growth leads to the increase in technological level in accordance with Equation (5), and the latter, in turn, raises the Earth's carrying capacity and makes it possible for the further population growth. This is the essence of an accelerated population growth in the post-Malthusian era.

Indeed, several millennia ago when the Earth's population was relatively small, the humanity was in the Malthusian-type society. Proceeding from Kusnets-Boserup Equation (5), it becomes evident that with a small number of the population technological progress rates were also extremely low. The slow growth of the technological level meant the slow population growth in a steady state of the Malthusian type. This situation lasted for thousands of years until 1500, when there was a significant increase in scientific and technological progress in Western Europe, as well as the increase of per capita income. Yet economy continued to develop for 300 years in line with the Malthusian mechanism. Only with the beginning of the Industrial Revolution, when technological progress rates surpassed population growth rates, the transition to the process of a sustainable accelerating demographic growth began.

2.3.1. Hyperbolic World Population Growth

As for the regularities in the world population growth in the Malthusian era, it was also specified in 1960 by H. von Foerster, P. Mora, and L. Amiot [20], who convincingly demonstrated that present data for the world population between 1 and 1958 can be accurately described with the help of a quasihyperbolic function:

$$N(t) = \frac{C}{(T_S - t)^{0.99}}, \quad C = const. \tag{6}$$

Point of singularity T_S was estimated by the authors as the year of 2026,87, which corresponded to 13 November 2026. On that day the world population should become infinite if it is to continue grow according to the formula (6). It prompted the authors to give a catchy title to their article "Doomsday: Friday, 13 November, A.D. 2026". However, exactly at the beginning of the 1960s, the accelerated world population growth changed to the process of slowing down and the global demographic transition began. Later S. J. von Hoerner [21] showed that the dynamic of the world population in the period under discussion should be approached with a merely hyperbolic function:

$$N(t)=\frac{C}{T_s-t};\ \ T_s=2025,\ \ C=200\times10^9 \text{ people per year} \tag{7}$$

This function is the solution for the following simplest nonlinear differential equation:

$$\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{N^2}{C}=bN^2,\ \ b=\frac{1}{C}. \tag{8}$$

2.3.2. Taagepera's Model

Taagepera [19] examined quasihyperbolic function to describe the demographic dynamics in the post-Malthusian era:

$$N(t)=\frac{C}{(T_s-t)^M}, \text{ where } C, T_s \text{ and } M \text{ are constants.} \tag{9}$$

He analyzed the period from 400 to 1900, for which he obtained the following values: $M = 0.7$; $T_s = 1980$. To explain the quasihyperbolic logic (9), Taagepera suggested the model of interaction between the population size and the technological level (5):

$$\text{a) } \frac{\mathrm{d}N}{\mathrm{d}t}=kT^nN;\ \text{b) } \frac{\mathrm{d}T}{\mathrm{d}t}=cTN^m, \tag{10}$$

where k, n are constants, and $m=\frac{1}{M}=1.43$. However, it turned out that $M = 1$ was more accurate. It should be noted here that the verification of a hyperbolic regularity (7) with the most precise empirical measurement of the world population made by C. McEvedy and R. Jones [22] for the years of 1000-1500 showed that it could explain 99.6% of the whole demographic dynamics in the period. In a later work [23], Taagepera proposed a three-factor model, supplementing Equation (1) with an equation for the population size, which takes into account the restrictions of the Earth's carrying capacity, which made it possible to consider the slowdown in the population growth during the global demographic transition.

Thus, to explain and justify the hyperbolic growth of the world population (7) it was enough to have two assumptions ([14], p. 30):

1) During the most part of human existence the growth of its size at every moment was restricted by the ceiling of the Earth's carrying capacity, caused by apparent level of life-supporting technologies (Malthusian assumption). The ceiling of the Earth's carrying capacity increased in the result of the growth of the development of life-supporting technologies. Therefore, the rates of the pop-

ulation growth were proportional to the level of the development of life-supporting technologies, which is expressed by Equation (10a) with $n = 1$.

2) Relative growth rates of life-supporting technologies are in direct proportion to the world population (Kuznets assumption), which is expressed by Equation (10b) with $m = 1$.

Hence, during Malthusian and post-Malthusian eras the leading role for the population growth was played by economic and technological factors. Thus, we may assume that at the specified stages of human development economic and technological imperatives dominated the demographic process.

2.3.3. Kremer's Model

The most prominent model based on Malthusian-Kuznets assumption is the model developed by the Nobel laureate M. Kremer [24]. According to the Kremer's model, manufacturing depends on two factors only—the level of technologies T and population size N, and has a form of

$$G = \gamma T N^{\alpha}, \tag{11}$$

where γ is a normalizing factor; α is a constant parameter. The dynamics in Kremer's model is put in the equation for technological growth

$$\frac{\mathrm{d}T}{\mathrm{d}t} = cNT, \quad c = const. \tag{12}$$

$$\bar{N} = \left(\frac{\bar{g}}{T}\right)^{\frac{1}{\alpha-1}}. \tag{13}$$

The dynamic equation is not given for the population as it is believed that this variable is faster than the technological level and thus it instantaneously approaches the equilibrium level $\bar{N}$ defined by the equilibrium level of per capita output $\bar{g}$:

$$\bar{N} = \left(\frac{\bar{g}}{T}\right)^{\frac{1}{\alpha-1}}. \tag{13}$$

If a produced product per capita exceeds $\bar{g}$, then the population increases, if it is less than $\bar{g}$, then the population decreases. With the help of equations (12) and (13), Kremer was able to describe the hyperbolic population growth (7).

To describe the demographic transition and introduce limitations to the population growth in the model, Kremer compiled quite a difficult function which described birth rate depending on the level of income. However, such an approach did not bring any positive results ([14], p. 170). Kremer also tried to reach the goal by modifying equation of technological growth (12) following Ch. I. Jones [25]:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = cN^{\psi} \cdot T^{\varphi}, \tag{12a}$$

where ψ and φ are constant parameters which are not necessarily equal to one. Drawing on this equation, Kremer modified his model and received such values for the parameters given: $\varphi = 2/5$; $\psi = 6/5$. As a result, the modified model considering the population growth (13) and technologies (12a) adequately described the hyperbolic population growth, yet it did not manage to describe the demographic transition even though there was a fall in growth rates since $\varphi < 1$. So, Kremer concluded that $\varphi = 1$ and $\psi = 1$ as well.

2.4. The Epoch of the Demographic Transition. Models by KMKh, Podlazov, Kapitsa, Naydenov and Kozhevnikova

***Korotayev-Malkov-Khalturina Model* (*KMKh*)**

One of the most effective models describing both the hyperbolic growth and the demographic transition is the three-factor model by Korotayev-Malkov-Khalturina (further on abbreviated as KMKh) ([14], chapter 5):

$$\text{a)}\ \frac{\mathrm{d}N}{\mathrm{d}t} = aS(1-L)N\text{; b)}\ \frac{\mathrm{d}S}{\mathrm{d}t} = bNL\text{; c)}\ \frac{\mathrm{d}L}{\mathrm{d}t} = cS(1-L)L\text{,} \qquad (14)$$

where $N(t)$ is the population; $S(t)$ is an excess product produced at a given level of technological development per one person; L is the share of literate population; a, b and c are constant coefficients. An excess product S is the difference between relevantly manufactured products and minimum essential products for a simple reproduction of the population (m = 420$ in 1990). Hence, S is a per capita resource, which can be used for the purpose of human development (healthcare, education, science, culture etc.) apart from preserving life sustaining of an established population size. Since per capita GDP production ($T = G/N$) is the most natural measurable factor corresponding to the definition of "level of technology", then $S = T - m$. Hence, S in equations (14) characterizes the level of technological development.

Considering literacy dynamics (L) of the population is an important characteristic feature of KMKh model (14), which, in turn, allows for considering an empirically determined fact of a negative influence of the literacy level on the birth rate and the population growth rates. Thus, in Equation (14) a tendency to one engenders slowing down of the population growth rate during the demographic transition. Moreover, L is in equation of technological progress (14b), which means the following assumption: a literate population makes more technological innovations than an illiterate one. Hence, the growth of literacy leads to accelerating rates of technological development. During the evolution variable factors demonstrate the following: literacy comes to a permanent state ($L = 1$), then the same happens to the population size, and the technological level grows according to the linear law. KMKh model perfectly corresponds to empirical assessments of the world population size during the whole new era until the 2000s ([14], pp. 90-94). In (14), pp. 79-89) the following generalized model was also considered:

$$\text{a)}\ \frac{\mathrm{d}N}{\mathrm{d}t} = aN^{\varphi_1} \cdot S^{\varphi_2} \cdot (1-L)^{\varphi_3}\text{; b)}\ \frac{\mathrm{d}S}{\mathrm{d}t} = bN^{\varphi_4} \cdot S^{5}\text{;}$$

$$\text{c)}\ \frac{\mathrm{d}L}{\mathrm{d}t} = cL^{\varphi_6} \cdot S^{\varphi_7} \cdot (1-L)^{\varphi_8}\text{,} \qquad (15)$$

where $\varphi_1, \cdots, \varphi_8$ are positive numbers not necessarily equal to one. In the result of the analysis the authors concluded that data available on the world population size was in favor of $\varphi_1 = \varphi_2 = \cdots = \varphi_8 = 1$.

2.4.1. Podlazov's Model and the Technological Imperative

Following Kremer, A. Podlazov [26] [27] tried to mathematically describe the demographic transition and discover objective reasons for the limits of the population growth. Like Kremer, he sees the reason for the hyperbolic growth in a mutual process of the population growth and increase of the technological level, which he describes with equations:

$$\text{a) } \frac{\mathrm{d}N}{\mathrm{d}t} = PN \text{ ; b) } \frac{\mathrm{d}P}{\mathrm{d}t} = NP \text{ ,} \tag{16}$$

where N is the population size; P is the technological level. Yet in contrast to Kremer, for whom technologies are the means of production (11), Podlazov believes that the role of technologies is to prevent human death and prolong his/her life and that is why he calls them "life-saving technologies" (P). While Kremer speaks about the resource niche (13), Podlazov specifies the technological niche, thus implying that the population size would follow the capacity of the technological niche P:

$$N = kP, \quad k = const. \tag{17}$$

In contrast to Kremer, who assigned the main role to the economic and technological factor, thinking of the demographic factor as subordinate, Podlazov gives preference to the technological factor and formulates "the technological imperative": "Population size is defined by the size of the technological niche it has created, *i.e.* that number of people who may be requested by life-saving technologies they have created" ([27], p. 524). This principle is reflected in formula (17).

Thus, Podlazov believed that the birth rate in the growth phase stayed the same and fell during the demographic transition. The exclusion of the birth rate from factors influencing the growth of population leads Podlazov to suggest that it is connected to the fall in death rate, which he believes to be the result of the collective behavior increasing the chances of survival of each individual. Mutual assistance is more effective as there are more people. It is assumed that this mutual assistance is realised via transmission and spread of knowledge and technologies. Podlazov is limited only to life-saving technologies (P), which are used to lessen death rate and raise life expectancy. As for the limiting the population growth, Podlazov sees the reason for it in the fact that it is impossible to nullify death rate coefficient. To sum up all the above mentioned, we may conclude that the population growth for Podlazov (17) is determined by the spread of the human technological niche (P) because of the development of their life-saving technologies. Their measure is the reduction in death rate coefficient achieved through their action. Its approach to natural limits restricts the possibilities of the further population growth.

Thus, Podlazov concludes that there are limits of the technological growth (P_∞) and the demographic growth as well (N_∞), and he estimated them ([27], pp. 525-526):

$$P_\infty \cong (0.05 \pm 0.01)\,\text{year}^{-1};\quad N_\infty = (10 \pm 2)\,\text{billion people} \tag{18}$$

Podlazov underlines that (18) by no means restricts the possibilities of the technological development. However, the life-saving technologies become less effective from the perspective of lessening death rate and prolonging life expectancy, which have biological limitations. As a result, he suggests a phenomenological model describing the demographic transition with the help of the stated technological limit P_∞ ([27], p. 526):

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{PN}{C}\left(1 - \frac{P}{P_\infty}\right),\quad C = const\,. \tag{19}$$

By uniting Equations (16), (17) and (19), we get Podlazov's model for the population growth (N) and life-saving technologies (P), suitable for all the epochs of human development including the epoch of the demographic transition. As is shown in ([27], p. 530), the present model demonstrates the best results from 1500 until now. Podlazov's model is criticized in ([14], pp. 173-174) as for most of the human history birth rate has played more important role than death rate. As we have already seen, in KMKh model it is birth rate which regulates the population, including the epoch of the demographic transition.

2.4.2. The Demographic Imperative and Kapitsa's Model

As we have seen above, the Earth's population growth throughout almost all the human history was subject to a universal pattern of development per hyperbole (7) being completed with the explosive population growth in the 20th century. Then, in 1962-1963 there was a phase transition with a sharp change of the whole human development and, first of all, having reached 2.2% maximum per year the rates of the population growth started to decline, which are nowadays 1.04%, *i.e.* for the past 60 years they have more than doubled. Besides, this global demographic transition is associated with the beginning in the aggravation mode of the limit of relative growth rate of the world's demographic system, since it occurred in the context of further persistent growth in per capita income (see **Figure 2**). It denies the Malthusian population principle, according to which the factor of slowing down the growth is closely related to external resources. Thus, we conclude that the Earth's population growth for tens of thousands of years depended only on the population size itself (7) and (8) and is not connected to any other external factors such as environmental factor, factors of productive forces and resource limitations.

All of this made it possible for S. Kapitsa ([5], p. 12) to formulate the phenomenological principle of the demographic imperative, according to which the quantitative human growth is determined by the world population itself and the development of its consciousness in contrast to the Malthusian population principle when the limit of growth is dependent on external resources—the Earth, energy, food. The demographic imperative means accepting that demographic processes in the history of human development are paramount and self-sufficient, and postulating demographic dependence of many phenomena and processes studied by social sciences. For synergetics [28], this principle is deduced to the assumption that in the variable collection that describe huge historical, social, economic, cultural etc. processes, the population size for significant amount of time is a parameter of order, *i.e.* that leading slow variable to which all the others adjust ([5], p. 20).

To justify the principle of the demographic imperative, Kapitsa thinks of the world population as a unified whole, as a developing interconnected complex dynamic system, in which a common mechanism managing the development of all system by collective interaction. According to Kapitsa, collective interaction takes place with the help of the propagation and multiplication mechanism of summarized information in the human society as a global networked information community ([5], p. 17). It has already been stated above that for several thousand years the dynamic of the world population reflects first of all the dynamic of World-System population, in which by the beginning of the 1st century A.D. lived more than 90% of the world population and which might have been seen as a single system ([14], pp. 117-120). There are enough data to speak about a systemic spread of most important technologic innovations in World-System. There are grounds for saying that creation and spread of innovations was one of the most important mechanisms of World-System integration. At the same time information network system is the most ancient mechanism of World-System integration and it played extremely important role during the whole evolution history of World-System ([14], p. 120). Thus, we see that since the beginning of the hyperbolic population growth, humanity was developing as information society, which gave Kapitsa the basis to propose a cooperative mechanism of development.

The analysis of the hyperbolic humanity growth connecting the size and integral development of humanity allowed suggesting the cooperative mechanism of development, when its measure is the square of population size in contrast to the mere population size (2) characteristic of the animal world. It means that the regularities of the human population growth are of social rather than biological character. That is why S. P. Kapitsa suggested using the quadratic dependence for the population growth rate [5]:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{N^2}{C} = aN^2. \tag{20}$$

The solution for this equation is the hyperbolic function (7). The solutions of such equations as (20) are known as the aggravation modes. The characteristic feature of such equations is that at a certain moment T_S called point of singularity, the solution extends at infinity. In reality, there is a change from the explosive growth to the stabilization mode. In our case, it is the global demographic transition. Here we are dealing with not only the explosive growth of a self-organizing human society, but also with the exhaustion of the possibilities for its further accelerated growth and its replacement by a slowdown mode.

To describe the demographic transition, Kapitsa regularized the quadratic Equation (20), transformed with the help of substitution (7) by introducing the characteristic human lifetime τ, limiting the population growth rate, and found its solution ([5], p. 22):

$$\text{a)}\ \frac{dN}{dt} = \frac{C}{(T_1 - t)^2 + \tau^2},\ \text{б)}\ \ N = K^2 \operatorname{arcctg}\left(\frac{T_1 - t}{\tau}\right),\ \ K^2 = \frac{C}{\tau} \tag{21}$$

Using the data of the world demographic statistics, S. P. Kapitsa calculated the numerical values of the constant parameters in formula (21): $C = 163 \times 10^9$, $K = 60100$, $\tau = 45$ years, $T_1 = 1995$. With these values of the parameters it follows from formula (21b) that the world population asymptotically tends to $N_{\max} = \pi K^2 = 11.36$ billion people. Kapitsa's formula (21b) describes the mode of the world population growth with stabilization and is just only for the case of sustainable human development, assuming an unlimited carrying capacity of the Earth's biosphere.

The graph of the Earth's population growth, calculated according to Kapitsa's formula (21b), is shown in **Figure 1**, which also demonstrates the actual (observed) values of the Earth's population, indicated by dots. It is important to note that the growth rate of the Earth's population ($q_N = \frac{dN}{Ndt}$) has already passed through a maximum (1962-1963, $q_N = 2.2\%$) and began to decline rapidly. Today they are about 1% (according to the World Bank), having more than halved, and in the future, they will only fall, approaching zero. Thus, the global demographic transition is associated with the onset of the limit of the relative growth rate of the world's population in the mode with aggravation. It can be seen from **Figure 1** that Kapitsa's formula (21b) approximates well the demographic dynamics of World-System throughout the new era, especially well during the period of demographic transition. The latter is natural, since the regularized Kapitsa's Equation (21a) is intended to describe precisely the stage of the demographic transition.

Podlazov [27] conducted a comparative numerical analysis of the effectiveness of several models (Kapitsa's, KMKh and the author's) in retrospect with a forecast until 2250 by the example of the world demographic dynamics as at long times (1250 B.C.-2250 A.D.), and at short times (1850-2150). The results of his calculations are given in a graphical form in **Figure 3** ([27], **Figure 7**). As is seen from the examination of the graphs, Kapitsa's model perfectly approximates the real demographic dynamics at the stage of the demographic transition, as well as in the post-Malthusian epoch, yielding in accuracy to KMKh and Podlazov's models at long times in retrospect. It is also clear (see **Figure 3**) that all models show the stabilization of the Earth's population in the 21st-22nd centuries, but at different times and at different levels, and the spread of stationary levels is huge —from 8.5 billion to 11.4 billion people. It should be noted that the population

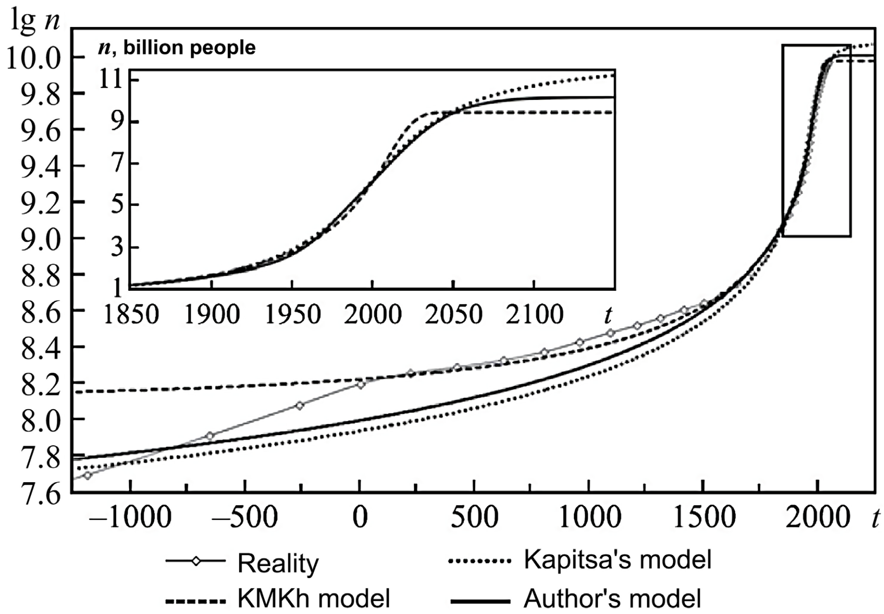

Figure 3. Comparison of the world population dynamics in different models.

in KMKh and Podlazov's models approaches asymptotic limits with a characteristic time that has a finite value, while in Kapitsa's model it is infinite.

Kapitsa's phenomenological model (21) can be successfully used for calculating the demographic dynamics of separate countries that can ensure sustainable development when the population grows according to the stabilization scenario without a noticeable decrease. An indispensable condition is the absence of coercive measures to limit birth rate, as well as a significant impact of migration flows on the social and economic processes in the country. As an example, in **Figure 4** there is a graph illustrating the population growth in little Denmark according to Kapitsa's model (21b). There can be seen a close agreement between the estimated trajectory of demographic development and the actual data in retrospect.

Thus, Kapitsa demonstrated that the Earth's population growth may be described mathematically (20)-(21) without introducing any additional variables other than the population N itself, *i.e.* in fact without any additional factors. This circumstance served as the basis for Kapitsa to formulate demographic imperative according to which global social, historical, economic and cultural processes adjust to the changes in the Earth's population. This value plays the role of a leading slow variable, called the order parameter in synergetics, which subordinates all other variables [28]. Hence, the demographic dynamics plays a primary and decisive role in the history of the development of human society.

The global demographic transition, accompanied by a drastic change in the growth rate, should lead to very significant changes in human development. Kapitsa notes that the global demographic transition will take place in a characteristic time equal to double τ, *i.e.* in 90 years, and will be completed by the middle of the 21st century. It synchronously embraces all humanity. At present, most of

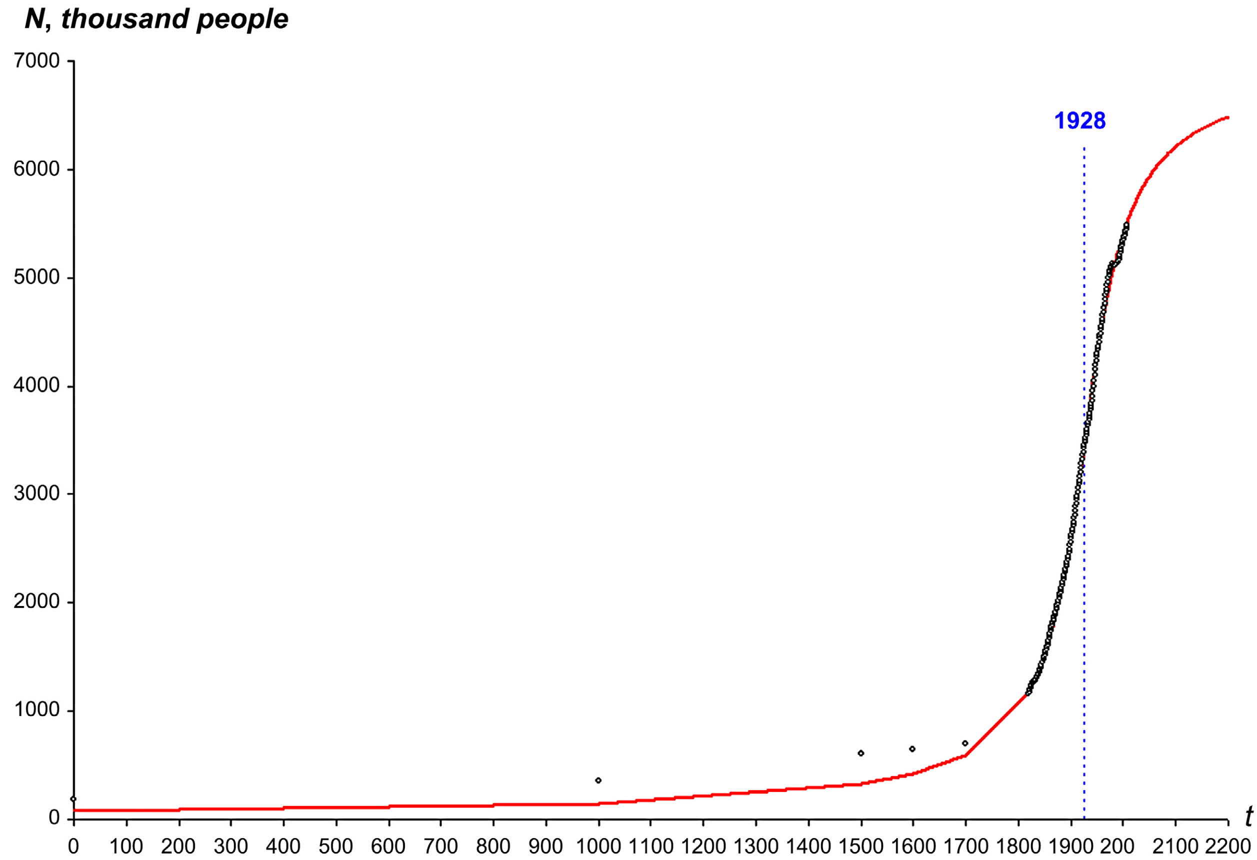

Figure 4. An evolutionary model of the Danish population.

the developed countries have already made this transition and the population of the developed countries has stabilized at one billion. Therefore, here we can see several phenomena that will soon begin to appear in developing countries, which are now just entering the transition stage, and it will be completed only in the second half of the 21st century.

However, the demographic transition in developing countries may transform into a demographic crisis connected to environmental, food and other disasters. Due to its swiftness and inevitability, it triggers great concern: what dangers and challenges do await humanity? It is also supported by growing social and economic inequalities in both developing and developed countries. The modern world, in addition, is embraced by a global political crisis.

It is essential to note that the global demographic transition began when the relative rate of the population growth passed precisely through the maximum (2.2% in 1962-1963) and did not settle at this maximum value. Therefore, as it decreases to zero, the Earth's population will asymptotically level off and stabilize as we saw in Figure 3. It is important that the slowdown in the growth of humanity is connected precisely with reaching the limit of the relative growth rate and not with a shortage of resources. Yet it will be true as long as human interaction with the environment or climate change does not lead to global negative consequences which, in turn, will drastically change the trajectory of human development. Further on we will show it on the example of deforestation.

2.4.3. Naydenov-Kozhevnikova Model

A simple phenomenological model with stabilization proceeding from the demographic imperative was suggested in [29]:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN^2\left(1-\frac{N}{N_C}\right), \quad r = const, \tag{22}$$

where N_C is the limiting stationary population size.

If $N \ll N_C$, this model is deduced to Kapitsa's Equation (20) and therefore describes hyperbolic growth, and then characterizes by a slowdown and entering the asymptotic stage with a gradual approach to the limiting population size N_C, which according to authors' estimates constitutes $N_C = 7.4$ billion people. Moreover, this maximum number was to be reached in the middle of the 21st century. As we can see, the model turned out to be ineffective, as, according to the UN, today the Earth's population has already surpassed 7.8 billion people. We brought this model here because it gave impetus to the development of other interesting models.

3. Models of the Demographic Dynamics with the Return to the Stationary Level

The question of the permissible maximum Earth's population as a stationary level is one of the fundamental issues of our time. There are various evaluating methods of defining the stationary number of the world population N_C, which are covered in [31]. A stable population of 7.7 billion people may be taken as a permissible world population in the resource model of D. Meadows and his associates [32]. Academician V. M. Matrosov, developing the resource model of Meadows' group, defines the permissible population of the Earth at 6.5 billion people [31]. A. Akimov [30] estimates $N_C = 5.2$ billion people. It should be noted that there is an urgent need to develop reliable and reasonably accurate methods for assessing the acceptable stationary population size both for the world as whole and for individual countries. In this connection, one cannot pass over the following historical fact associated with the amazing prediction of Charles Fourier (1772-1837), one of the founders of the theory of utopian socialism. He believed that it would be expedient to "establish a population balance, a proportion between the number of consumers and productive forces", and therefore "reduce the number of inhabitants of the globe to the precise proportion of means and needs to approximately 5 billion people" [33]. Besides, at the beginning of the 19th century, when Charles Fourier made his estimates, the Earth's population was only about 1 billion people. As we can see, the estimates of the permissible stationary Earth's population size are much lower than stationary levels achieved in phenomenological models (8.5 - 11.36 billion people, see **Figure 3**).

In this connection, the most interesting are the modes of the following world population growth, reaching a maximum value and subsequent decline with stabilization around the stationary level, determined by the permissible carrying capacity of the Earth's biosphere. Such modes of the development are called "growth modes with a return". The high probability of such scenarios of human development was first pointed out by A. V. Akimov [30]. He developed an original method for forecasting the world population with the help of an operational description of the demographic transition and a forecast of the modes of demographic development. The disadvantage of Akimov's method is the complexity of its mathematical formalization and the large spread of predictive results with small changes in the mode of demographic development.

3.1. Dolgonosov's Model and Transition to the Information Imperative

B. M. Dolgonosov proposed an informal conception of the world population dynamics [34], supplementing Kapitsa's demographic imperative with the information imperative according to which global demographic processes adjust to changes in the amount of information accumulated by the humanity, which brings information to the level of the driving force of civilization development. Here information is understood as knowledge at the core of life-supporting and life-saving technologies. In the information paradigm the volume of knowledge $q(t)$ takes the role of the order parameter. Moreover, the population size according to Dolgonosov is determined by the rate of information production q:

$$\frac{\mathrm{d}q}{\mathrm{d}t} = \omega N, \tag{23}$$

where ω is the average rate of information processing by a person.

Dolgonosov's mathematical model for the world population size is the development of the Naydenov-Kozhevnikova model and it is as follows:

$$\text{a)}\ \frac{\mathrm{d}N}{\mathrm{d}t} = rN^2\left[1-\frac{N}{K(q)}\right];\ \text{b)}\ K(q) = \frac{N_c}{1-\exp\left(-\alpha\frac{q}{q_c}\right)}, \tag{24}$$

where $r = \frac{\omega}{q_c}$—is a coefficient of the population growth; $K(q)$ is an instantaneous capacity of medium, *i.e.* the largest population size achievable with a given level of knowledge q; $N_c = \frac{q_c}{\omega t_c}$ is the carrying capacity of the Earth's biosphere, which is determined as the stationary human population, at which the biosphere and civilization can firmly coexist; q_c, t_c are characteristic scaled values. It is obvious that $K(q)\to\infty$ if $q\to 0$ and $K(q)\to N_c$ if $q\to\infty$. Thus, the instantaneous capacity of medium (24b) can significantly exceed its stationary value N_c, which overcomes the major disadvantage of Naydenov-Kozhevnikova model (22), which consists in strict restriction of the population growth by the limit of N_c. Parameter α in Equation (24b) characterizes human ability to effectively restore the environment using innovative knowledge and technologies.

Equation (23) for the rate of information production encloses the system. Thus, for $N \ll K(q)$ Equation (24a) reduces to Kapitsa's equation for the hyperbolic growth (20), therefore, $\tau = \frac{1}{N_c t_c} = \frac{1}{C}$ or $N_c = \frac{C}{t_c}$. If we take $t_c = 40$ years, then we get an estimate for $N_c = 5$ billion people.

Dolgonosov's model (23)-(24) may be considered as a universal model that allows, by means of numerical simulation, to analyze various scenarios of human development in the sense of growth scenarios for its population with stabilization, a return and a damped oscillation. All these scenarios for the development of the demographic dynamics are schematically presented in Figure 5. The evolution with damped oscillations proceeds as follows. Even at the stage of the demographic transition the Earth's population is growing at a significant rate and it skips the level corresponding to the Earth's stationary capacity by inertia. The growth continues for some time. Due to overpopulation, the state of the environment is rapidly deteriorating. An ecological or climate crisis sets in, and as a result, the population is reduced to the level lower that the permissible stationary level. During this period the environment is restored and soon after the population increases again and, after several fluctuations, it enters the stabilization phase.

Dolgonosov concludes that the mode of damped oscillations is most suitable for describing the demographic dynamics. However, the corresponding model of information production contains the oscillation frequency β, for which there is no necessary empirical basis. Therefore, to describe the demographic dynamics, Dolgonosov examines only a simple model with an aperiodic return [34]. Besides, the scheme of calculations according to Dolgonosov's model is complicated by the fact that at first it is required to calculate the modes of growth in the information production and only after that—the demographic dynamics.

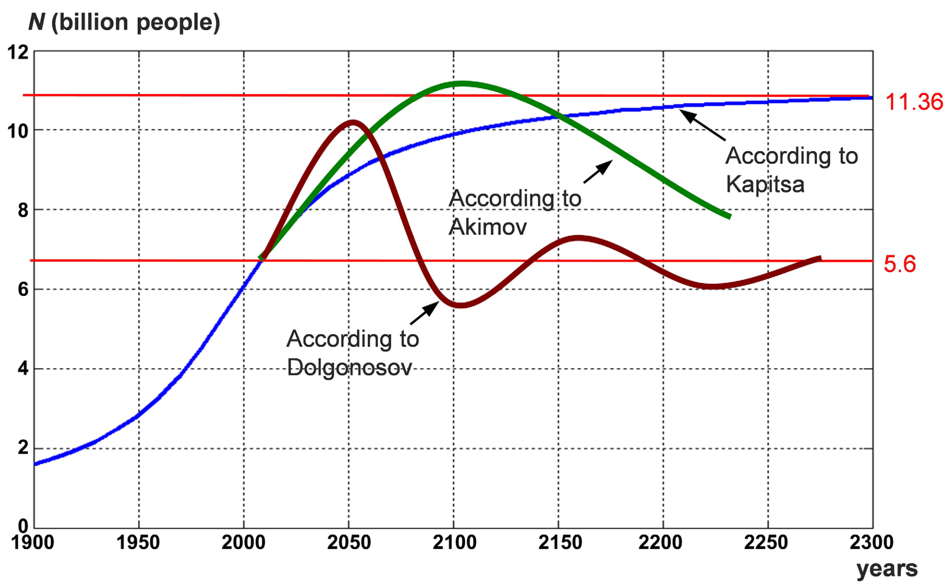

Figure 5. Various scenarios of the world demographic dynamics development.

3.2. Akaev-Sadovnichiy Model

In order to evade the noted difficulties, it is necessary to restrict ourselves to the demographic imperative constructing the function to describe the instantaneous capacity of the environment, *i.e.* $K = K(N)$, which was done in [35], where the following approximate formula was obtained:

$$K = N_c + \gamma N \exp(-kN), \quad \gamma = const. \tag{25}$$

There exist various estimation methods for determining the stationary world population N_c, highlighted in [31]. It is shown that the most probable scientifically substantiated value of the carrying capacity of the Earth's biosphere, which determines the permissible population, lies in the range from 3 to 7 billion people. The stationary world population is estimated at about 5 billion people. The stationary level for the world population in the long-term forecasts of recent years is 5.2 - 5.4 billion people [30] [34]. In the examples below we adhered to the assumption that $N_c = 5.2$ billion people. The stationary population size of a country can approximately be found by dividing the stationary world population by the anthropogenic load index of the country of interest, for which there are special tables [31]. For example, if we take $N_c = 5.2$ billion people for the world as a whole, then for China $N_{ck} = 1.2$ billion people, and for India $N_{cu} = 0.98$ billion people.

V. G. Gorshkov gave an answer to the fundamental question related to bio-consumption: he found out that the biota is able to fully regulate and stabilize the environment, if the value of human consumption of primary biological products does not exceed approximately 1% of total production of the biosphere [36]. He also calculated that this value of permissible bio-consumption corresponded to the permissible Earth's population of about 1 billion people, which was already reached by about the 1820s. According to Gorshkov, nowadays humanity consumes about 22% - 23% of the planet biomass. Therefore, humanity has surpassed the permissible limit of the natural stability of the biosphere by more than 20 times. Thus, practical influence of life-supporting technologies on the instantaneous capacity of the medium started at the beginning of the 19th century, when the Earth's population reached 1 billion people and violated the permissible limit of bio-consumption. To take this circumstance into account, we write (25) in the form:

$$K = N_c + \gamma(N - N_0)\exp[-k(N - N_0)], \quad N_0 = 1 \text{ billion people.} \tag{26}$$

Now we can write equation of the demographic dynamics (24a) as follows:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN^2 \left\{1 - \frac{N}{N_c + \gamma(N - N_0)\exp[-k(N - N_0)]}\right\}. \tag{27}$$

By introducing characteristic time lags, we get:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN^2(t - \tau_1)\left\{1 - \frac{N(t)}{K(N, \tau_2, \tau_3)}\right\}, \tag{28}$$

$$K(N,\tau_2,\tau_3)=N_c+\gamma\left[N(t-\tau_2)-N_0\right]\exp\left\{-k\left[N(t-\tau_3)-N_0\right]\right\},$$

where τ_1 is the average time of the beginning of reproduction ability; τ_2 is diffusion time of basic technologies; τ_3 is the delay in the response of the biosphere to the anthropogenic load. The average time of the beginning of reproductive ability is 25 years. The duration of technology diffusion in our era is 25 - 30 years. The time lag of the biosphere response to the anthropogenic load exceeds 100 years. The introduction of these delays makes it possible to obtain the modes of the demographic dynamics with damped oscillations.

Figure 6 shows the results of predictive calculations of the demographic dynamics for the world under various scenarios of development. As is seen from the figure, model (28) allows to model different scenarios of population development: growth with an aperiodic return to a stationary level (scenario 2); growth and stabilization around a stationary level with the help of damped oscillations (scenario 1). Due to the introduction of time delays τ_1,τ_2 and τ_3, model (28) makes it possible to effectively employ the prehistory of the demographic dynamics for about 100 years and thus excellently coincides with actual data in retrospect. It also follows from the examination of **Figure 6** that the scenario of steady growth with stabilization, described by Kapitsa's equation, can hardly be carried out into practice, since, according to Kapitsa, the stationary level is almost twice the permissible stationary level estimated by a number of authors [30] [31].

Figure 7 represents forecast trajectories of the demographic dynamics for China and India. As is seen from the figure, due to the introduction of a strict birth control mechanism, the demographic dynamics in China is a smooth growth

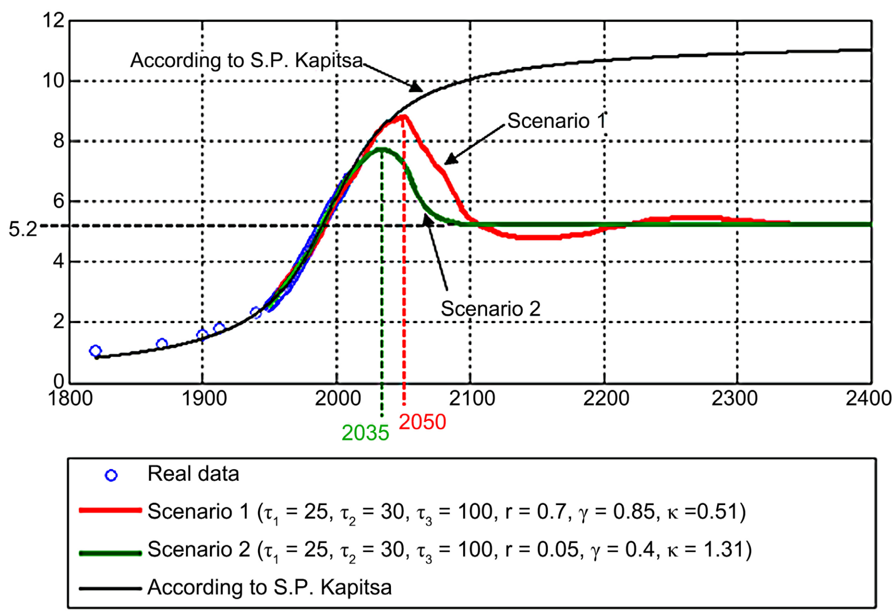

Figure 6. Forecast for the dynamics of the world population size. Circles represent real data. Here and in **Figure 1** $\tau_1 = 25$, $\tau_2 = 30$, $\tau_3 = 100$. Scenario 1: $r = 0.7$, $\gamma = 0.85$, $k = 0.51$; Scenario 2: $r = 0.05$, $\gamma = 0.4$, $k = 1.31$.

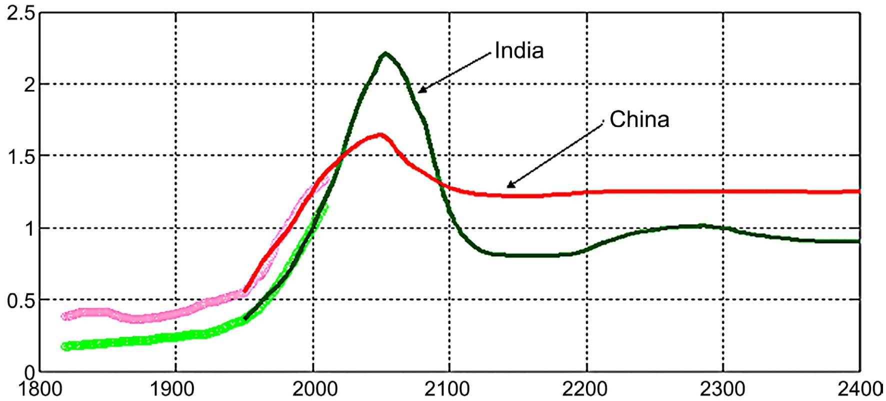

Figure 7. Forecast for the dynamics of the population in China and India. Circles represent real data. India ($N_C = 0.8$, $r = 0.33$, $\gamma = 1.3$, $k = 3$), China ($N_C = 1.2$, $r = 0.15$, $\gamma = 0.50$, $k = 2.77$).

trajectory with an aperiodic return to the stationary level. As for India, if similar measures are not taken there, there will be a large-scale environmental crisis. As a result, a sharp decline in the population will begin, the size of which will subsequently stabilize at great cost in the damped oscillation mode.

3.3. Bologna-Aquino Model. The Influence of Deforestation on the Global Demographic Dynamics

We have already examined models of the population growth with a return to a stationary level, determined by the ceiling of the Earth's carrying capacity. It is possible only if the environment is in satisfactory condition. During the last decades discussions on climate warming have become very acute and acquired global significance. The main reason for the accelerate climate warming was excessive pollution of the World Ocean and the Earth's atmosphere with carbon dioxide (CO_2). This was largely due to intensive deforestation. Humanity during its existence has destroyed 1/3 of the forest cover, reducing it from 60 million sq. km. to 40 million sq. km. Yet trees and forests are the best air filters. The influence forests have on our planet is strong: from carbon absorption and oxygen release to soil fertility preservation and water cycle regulation. Due to the great role that forests play in the Earth's ecosystem, it is hard to imagine that people can survive without them. In [37] the authors calculated that if deforestation continues at the current rate, they will disappear in a hundred to two hundred years at the most. Then, the authors believe, human destiny will be exactly the same as it was with the inhabitants of Easter Island, who disappeared with the disappearance of forests there.

In the mentioned work [37], a model of evolutionary development of humanity is proposed in combination with a deterministic generalized logistic model of interaction between people and the forest:

$$\text{a)}\quad \frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(1-\frac{N}{\beta R}\right);\ \text{b)}\quad \frac{\mathrm{d}R}{\mathrm{d}t} = r'R\left(1-\frac{R}{R_C}\right) - a_0 N\cdot R, \tag{29}$$

where $N(t)$—is the world population; $R(t)$ is the Earth's surface covered with forests; β is a coefficient characterizing the maximum carrying capacity of the planet on the part of the population; r—rates of the population growth in favorable conditions; r' is a parameter representing the ability of forests to recover; R_C is the permissible forest load which used to be 60 million sq. km.; a_0 is a technological parameter measuring rate with which people can use forest resources, given the level of achieved technological development. Calculations performed according to model (29) are shown in a graphical form in **Figure 8**. As is seen from the figure, by the end of the 21st century, the area of forests will be reduced by half, and the Earth's population will be reduced to about 5 billion people. It is important to note that the population steadily declines and there is not any stabilization of the population. In addition, the authors examined an additional stochastic model and showed that if current rates of deforestation continue, then there are several decades only left before the point of no return, after which humanity will enter a collapse trajectory. The last conclusion seems to be too alarmist, but at the same time there is a grain of truth in it.

4. The Forecast of the Demographic Dynamics in the Information and Digital Age

At the end of the 20th-beginning of the 21st centuries information and communication technologies (ICT) have become widespread thus turning into a powerful lever for accelerating technological progress and economic growth. However, their revolutionary influence on all aspects of the life of the society manifested itself in the mid-1990s with the advent of the global computer network Internet. With the development and spread of the Internet network markets and network economy appeared and started to develop rapidly. Electronic networks became the main organization form of the information society, which grew rapidly and soon embraced all countries and continents. The production and consumption of information has become the most important activity of the new society, and information is recognized as the most significant strategic resource. Advances in nanoelectronics have led to the creation of nanochips with unprecedented

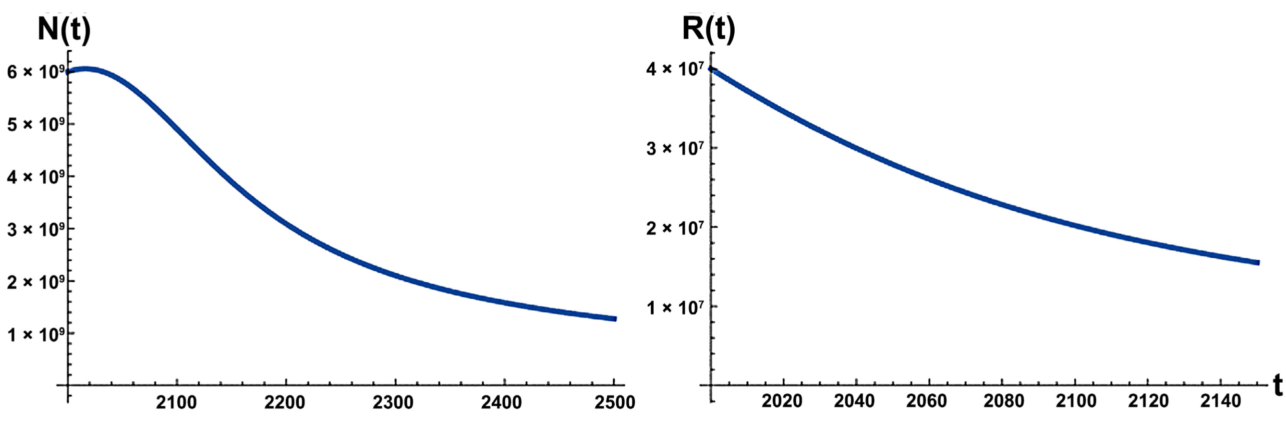

Figure 8. Influence of the Earth's deforestation dynamics on the global demographic dynamics.

processing power, which has revolutionized the sphere of big data processing and machine learning. As a result, highly effective digital technologies and intelligent machines (IM) have appeared – robots and computers with the elements of artificial intelligence. A digital economy has begun its formation, in which digital technologies and platforms based on the Internet, as well as IM, play a key role. Thus, the Internet has become a global driving force for the development of the human society and the whole world economy, and information has become the main resource.

Kapitsa was the first who suggested an information hypothesis to explain the phenomenon of hyperbolic population growth. According to this hypothesis, social interaction of people, their cumulative experience, dissemination and transmission of values qualitatively differ from human evolution and determine the population growth rate [5]. Kapitsa's demographic imperative is based on the information conception, and information is spread through the network structures of the human society. The well-known Metcalfe's law ([38], p. 40) states that the network value for each participant (user) is proportional to the square of the number of other participants (N in our case) of the network, which coincides with the right part of Kapitsa's demographic Equation (20). Following Kapitsa's ideas, Dolgonosov [34] believes human civilization to be an open evolving system endowed with memory and capable of information production and accumulation. Based on this, he formulates the information imperative, in accordance with which global demographic, technological, economic and other processes adjust to changes in the amount of accumulated information, which brings information to the level of the main driving force of human development.

Indeed, as for the most of human history its population has remained proportional to the overall rate of information production (23), it is possible to pass directly from the information to demographic dynamics. Podlazov's technological imperative [27] is also subordinate to information one, since the same knowledge is at the core of life-saving technologies. Moreover, technologies are essentially information systems. Technological evolution is similar to biological one – it is the process of establishing information order in the world of chaos. In the development of technologies one can also observe such properties as autonomy and self-organization, which are characteristic of biological creatures. All this speaks in favor of the leading autonomous role that information plays. In KMKh model (14) the key role is played by the level of literacy (L), which is also determined by the volume and prevalence of knowledge. Thus, it is knowledge (information) that acts as the only driving force of the human development. It goes as follows [34]. Knowledge accumulation contributes to the development of life-supporting and life-saving technologies, which lead to the improvement in the quality of life. As a result, child mortality diminishes, security of all ages increases, and it leads to the increase in average life expectancy and the total population. Increasing life expectancy and improving education lead to the rise of the rate of knowledge production. As a result, we get a closed cycle that guarantees a nonlinear self-accelerating nature of the knowledge production process.

Dolgonosov goes further and formulates the information paradigm in the following strong form: "information is a self-developing entity that can use various types of producers and be based on different media at different stages of its evolution" ([34], p. 51). If we are to proceed from this definition, then it is information that is the driving force of the evolution, choosing the most productive species among those competing and regulating their size. Until recently, humans have been the most productive species. However, now people have a competitor in the form of IM, which are already superior to them in the number or areas related to analysis, processing and production of information [39]. That is why it can be assumed that IM will not only deprive people of middle-skilled jobs, which is much written about [39], but also they will contribute to the reduction of the world population, taking away an increasing part of work on information production and use. Below we shall demonstrate, for the first time on the model, that already in the 21st century the world population will shrink with the development and spread of AI and IM. Several researchers have already noted that AI would affect the reduction in the population, for example, in [40] this phenomenon was examined under the influence of the trend in the development of the energy in virtuality and reality.

On the other hand, the development of society through the information mechanism is basically a non-equilibrium process. The growing non-equilibrium state of the society is manifested in growing social and economic inequality both in developed and developing countries, which causes equally growing social protests. All these destructive processes will continue unless appropriate measures are taken at the global level. It is possible as for all the societies the development was guaranteed by culture and ideology. Life-supporting systems have always ensured human existence but did not determine its development. That is why human society in the nearest future should determine the long-term priorities of human development, including progressive guidelines and goals set by the UN, and immediately begin to implement those mobilizing huge resources that humanity has. First, it is necessary to develop a strategy to overcome depopulation in leading countries and overpopulation in developing countries, where population explosion is about to happen. China can serve as an example, where the main target setting for the development of the country by the middle of the century—by the centenary of the PRC founding—is the construction of the prosperous country with a healthy environment, achievement of comprehensive social justice, formation of a harmonious and prosperous society, in the center of which is the qualitative growth of human capital based on the all-round improvement of the quality of accessible education and healthcare.

A Model for Forecasting the Demographic Dynamics in the Information and Digital Age

As we have seen earlier, Kapitsa's phenomenological model (21) was obtained under the assumption that there was a demographic and information imperative.

It means that Kapitsa's formula (21b)

$$N_K t = K^2 \operatorname{arcctg}\left(\frac{T_1 - t}{\tau}\right) \tag{30}$$

describes the upper limit of the potential maximum population in the 21st-22nd centuries. However, with the beginning of the computer age in the 1950s, part of the studies on the analysis, processing and production of information is transferred to computers, ICT and now IM and it will be further transferred to AI in an ever-growing volume. This led to the fact that the population N_H demanded by the information imperative started to decline, *i.e.* $N_H < N_K$. Indeed, it is observed in **Figure 9**, which shows the trajectory of the N_K demographic curve, determined by Kapitsa's formula (30), normalized according to actual data for the entire industrial period (1800-2000) together with the actual curve of the population growth up to the present day (1800-2020). As is seen from the figure, Kapitsa's demographic curve broke away from the actual one immediately after 1990, *i.e.* with the beginning of the information age.

Thus, the transition from information to its producers can be carried out according to formula (23), where $N(t) = N_K(t)$

$$q(t) = \frac{dQ}{dt} = \omega N_K(t). \tag{31}$$

Here $N_K(t)$ is the number of carriers and producers of information, including people and IM. Besides, in this formula information performs the function of the driving force of evolution, choosing producers of information both among people and IM. On the other hand, according to Kremer [24], the growth of the Earth's population throughout human history has been determined by technological progress (12). It was shown in [14] that the most appropriate value as an

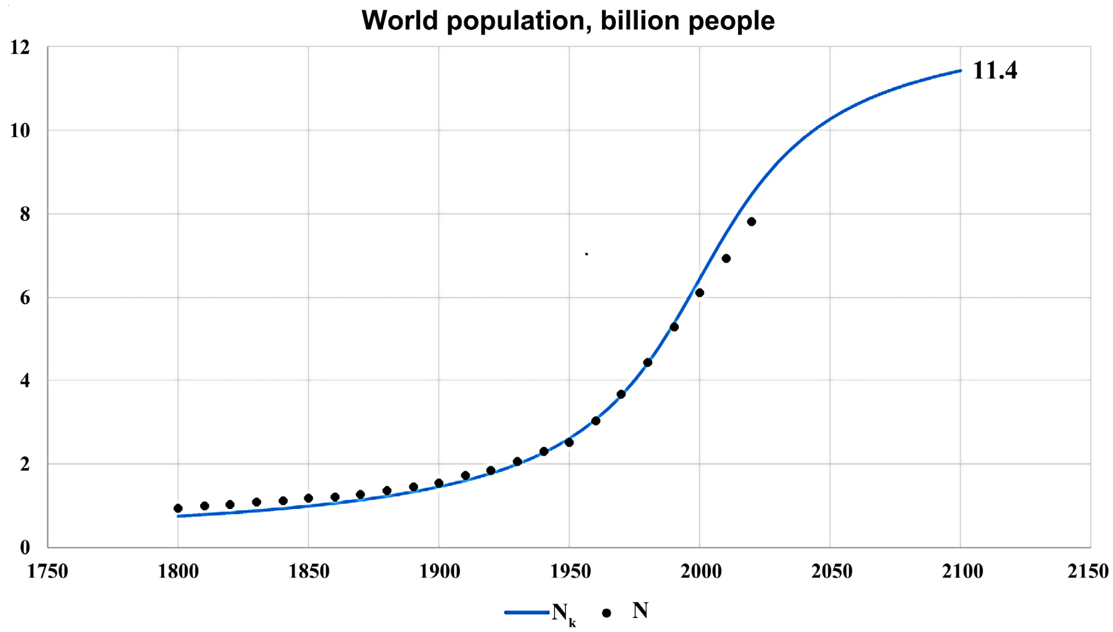

Figure 9. The dynamics of the Earth's population growth in the 19th-21st centuries.

index of technological development is the total number of inventions and discoveries $Q(t)$, made by the point in time of interest t, *i.e.* $T(t) \sim Q(t)$, which simultaneously characterizes the amount of technological information in formula (31). It is important that there are reliable data bases for inventions and discoveries made by people since ancient times, the most complete of which is the one compiled by A. Hellemans and B. Bunch [41]. Therefore, Kuznets-Kremer formula (12) can be written in the following form:

$$q_T(t) = q_Q(t) = \frac{\mathrm{dQ}}{\mathrm{Qdt}} = cN_H(t). \quad (32)$$

Here $N_H(t)$ exclusively characterizes the population dynamics.

We should note that Kuznets-Kremer formula (12) itself was first verified in [14] using the mentioned database by A. Hellemans and B. Bunch. All main inventions and discoveries made by people from ancient times up to the 1980s were collected and systematized in the well-known database by A. Hellemans and B. Bunch [41]. This database was further refined, supplemented, calibrated and brought up to 2005 by L.E. Grinin [42], and later by A.V. Korotaev, S.Yu. Malkov and L.E. Grinin, as well as their colleagues up to 2020. Graphs of the functions $q(t)$ and $Q(t)$, constructed according to the data of the refined and supplemented Hellemans and Bunch base, are shown in **Figure 10** on a logarithmic scale. However, we are mainly interested in speed $q(t)$ (31) and the rate of technological progress $q_Q(t)$ (32), which are shown on a regular scale in **Figure 11** and **Figure 12**.

From Equations (31) and (32) we obtain an identical relation between $N_H(t)$ and N_K:

$$N_H(t) = \frac{\omega}{c}\frac{N_K(t)}{Q(t)} = \gamma\frac{N_K(t)}{Q(t)}, \quad (33)$$

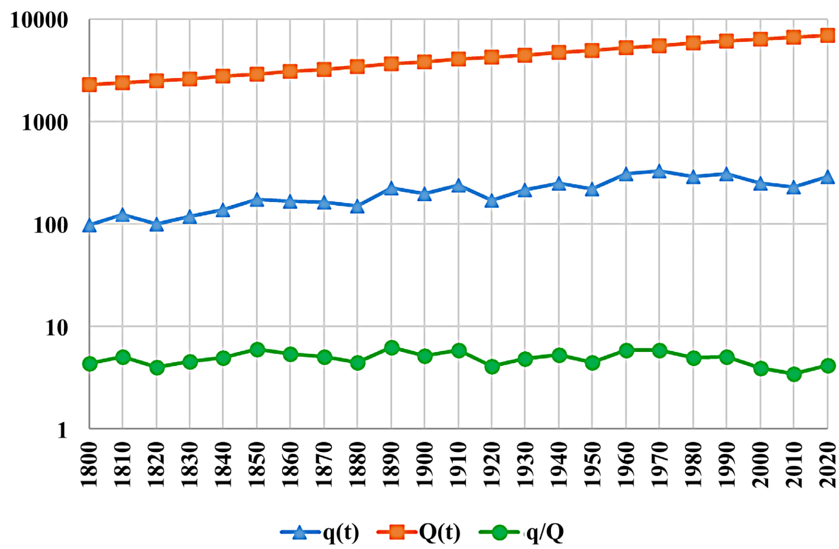

Figure 10. Global dynamics in the number of inventions and discoveries in the industrial (1800-1980) and information (since 1980) ages.

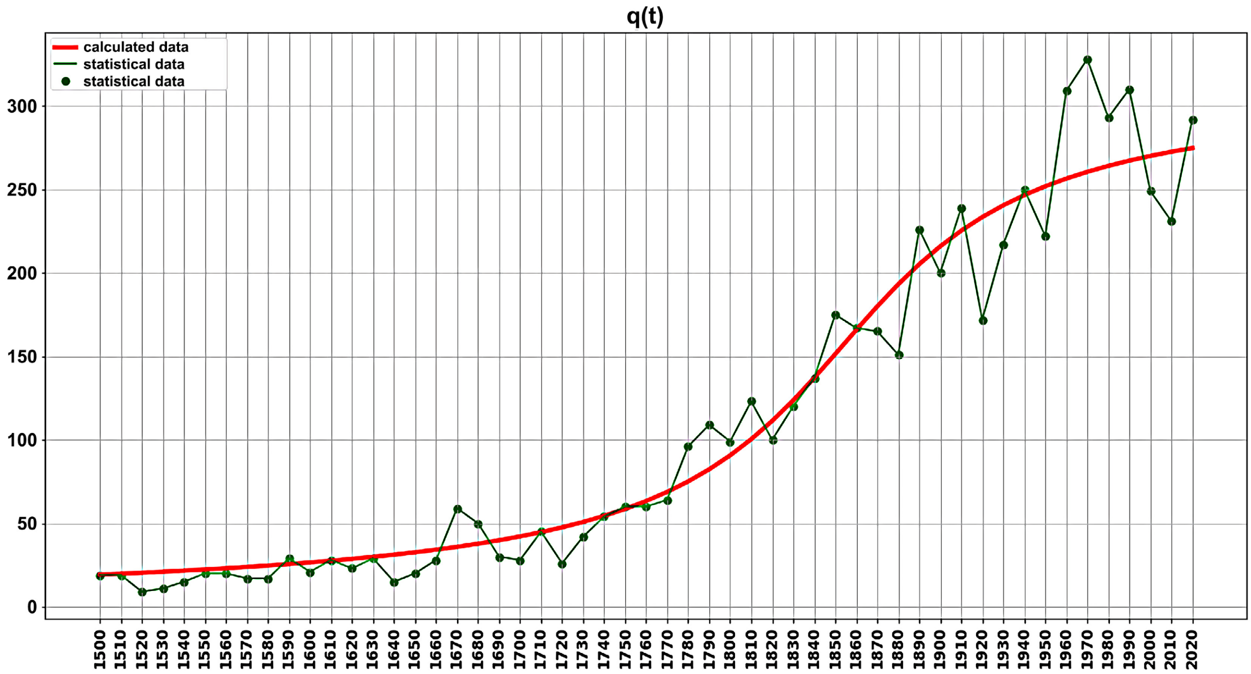

Figure 11. Graphs of the global growth rate movement in the number of inventions and discoveries $q(t)$ is an actual broken curve, $\bar{q}(t)$ is an approximating smooth curve.

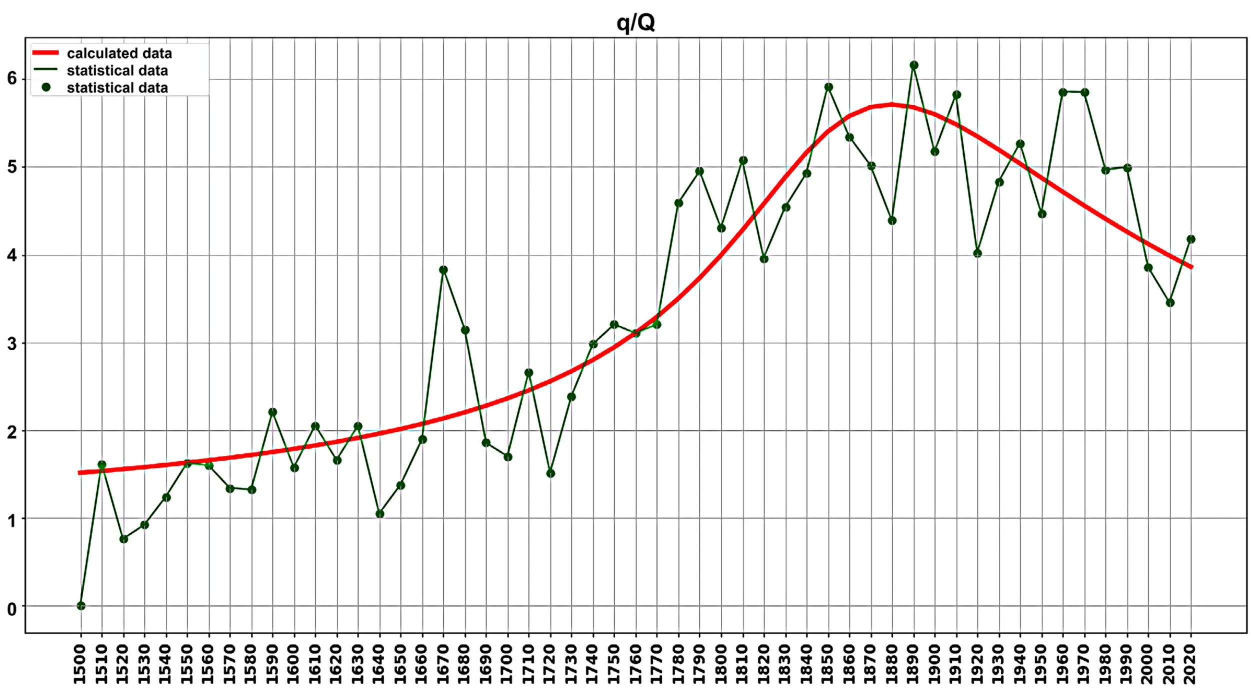

Figure 12. Graphs of the global growth rate movement in the number of inventions and discoveries. $q_Q(t)$ is an actual broken curve, $\bar{q}_Q(t)$ is an approximating smooth curve.

where γ is a normalization coefficient, which is determined by the least squares method from the condition of $N_H(t)$ coincidence at the stage of the information age (1980-2020) with the actual data of the world population, shown in **Figure 9** by discrete dots. As for $N_K(t)$, its trajectory has already been calculated using formula (30) and is presented graphically in **Figure 9**.

Before using Equation (33) to calculate the trajectory of $N_H(t)$, we need to approximate the broken curve, passing through the actual values $q(t)$ (see **Figure 11**), with a smooth continuous function. As is seen from the graph, the discrete function $q(t)$ tended to increase throughout the industrial age (1750-1980) and started to stabilize towards its end in order to reach saturation level in the information and digital age (1980-2050). It is natural to assume that the approximating function belongs to the class of S-shaped functions. Therefore, we used first the simplest logistic function:

$$\bar{q}(t)=\frac{q_m}{1+\mu\cdot\exp\left[-\vartheta\left(t-T_0\right)\right]}, \tag{34}$$

where q_m, μ, ϑ and T_0 are constant parameters. Having determined the best estimates of the values of these parameters with the help of the least square method, we have obtained:

$$q_m=346;\quad \mu=0.68;\quad \vartheta=0.012;\quad T_0=1903; \tag{34a}$$

given that the value of the correlation coefficient $R = 0.98$ and $R^2 = 0.95$. Such a high value of the correlation coefficient indicates the correct choice of the type of approximating function (34).

For comparison, the approximation using Kapitsa formula (30) was also carried out:

$$\bar{q}(t)=\bar{K}^2\operatorname{arcctg}\left(\frac{\bar{T}_K-t}{\bar{\tau}}\right). \tag{35}$$

with the help of the least square method the following estimates for constant parameters were obtained:

$$\bar{K}=10;\quad \bar{\tau}=69;\quad \bar{T}_K=1854;\quad R=0.98;\quad R^2=0.95. \tag{35a}$$

As we can see, both S-shaped functions (34) and (35) are equally good for approximating actual data. However, as it turned out, function (35) perfectly approximates $Q(t)$ and $q_Q(t)$ at the same time, while logistic function (34) does it worse. That is why we have chosen formula (35) as an approximating function, which represents the smooth growth trajectory $\bar{q}(t)$ in **Figure 11**. Let us determine $\bar{Q}(t)$ and $\bar{q}_Q(t)$. Considering (31) and (32) we obtain:

$$\bar{Q}(t)=Q_0+\int_{T_0}^{t}q(t')\mathrm{d}t'=Q_0+\int\bar{K}^2\operatorname{arcctg}\left(\frac{\bar{T}_K-t'}{\bar{\tau}}\right)\mathrm{d}t'$$

$$\text{a)}\quad =Q_0+\bar{K}^2\left\{\left(\bar{T}_K-t\right)\operatorname{arcctg}\left(\frac{\bar{T}_K-t}{\bar{\tau}}\right)+\frac{\bar{\tau}}{2}\ln\left[\bar{\tau}^2+\left(\bar{T}_K-t\right)^2\right]\right. \tag{36}$$

$$\left.-\left(\bar{T}_K-T_0\right)\operatorname{arcctg}\left(\frac{\bar{T}_K-T_0}{\bar{\tau}}\right)-\frac{\bar{\tau}}{2}\ln\left[\bar{\tau}^2+\left(\bar{T}_K-T_0\right)^2\right]\right\}$$

$$\text{b)}\quad \bar{q}_Q(t)=\frac{\bar{q}(t)}{\bar{Q}(t)}=\frac{\bar{K}^2}{\bar{Q}(t)}\operatorname{arcctg}\left(\frac{\bar{T}_K-t}{\bar{\tau}}\right).$$

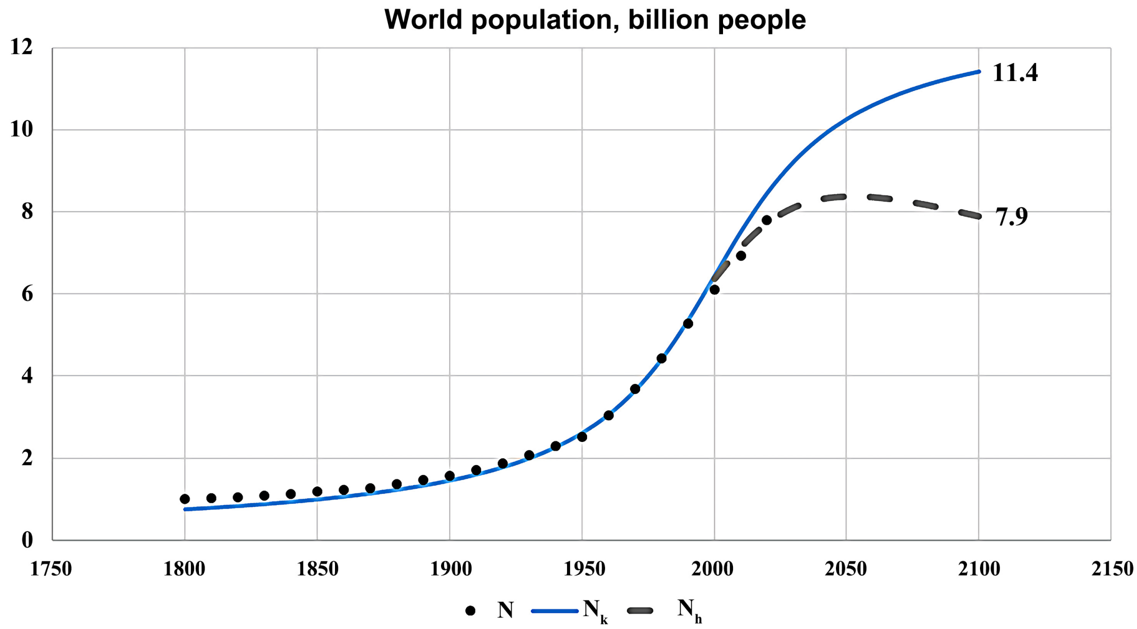

Figure 13. Demographic dynamics of the world in the 21st century influenced by the widespread use of intelligent machines

The latter function (36b) describes a smooth curve $\overline{q}_Q(t)$, which concerns the average trajectory of growth rates of technological progress, shown in Figure 12.

Now we need to calculate the forecast trajectory of the Earth's population $N_H(t)$ in the era of the widespread use of IM, using Equation (33) and formulas (30) and (36a) for $N_K(t)$ and $Q(t)$, respectively. The normalization coefficient γ is determined by the least square method in circumstances where N_H at the stage of information age (1990-2020) is combined with actual data $N(t)$ (see Figure 9). The estimate obtained: $\gamma \cong 6414$. The forecast trajectory of the world population movement in the digital age (2022-2100), calculated by formula (33) is shown in Figure 13. As we can see, the Earth's population, having reached a maximum value of 8.37 billion people in 2050, will then begin to steadily decline, decreasing to 7.9 billion people by 2100. In [35], we proposed a flexible and effective model of the demographic dynamics for calculating the modes of the world (country) population growth with the achievement of a certain peak value and subsequent reduction and further stabilization around a certain stationary value both in aperiodic and oscillatory forms. Yet the question of possibility to stabilize population in the framework of this model remains open.

5. Conclusion

Thus, if we are to proceed from the information imperative [5] [34], according to which global demographic, technological, economic and other processes adjust to changes in the amount of useful information accumulated by people, which brings information to the level of the main driving force, controlling the human development, then we can predict that the Earth's population, having reached a certain maximum ($N_{\max}$), will then begin to decline steadily. It will happen since intelligent machines (IM) will perform most of the main work on information production and usage for the implementation of development goals. Moreover, the information forecast model, proposed by the author, gives an estimate of $N_{\max} \cong 8.52$ billion people, which will be reached presumably in 2050, and then humanity will begin to decline, dropping to 7.9 billion people by 2100. That is why humanity should take decisive measures to counteract the process of information depopulation: first of all, overcoming the current excessive social inequality and guaranteeing the maximum growth of human capital based on the comprehensive improvement of the quality of affordable education and healthcare, as well as labor symbiosis of "human + IM" [43], saving jobs for people and increasing the overall productivity of the symbiosis.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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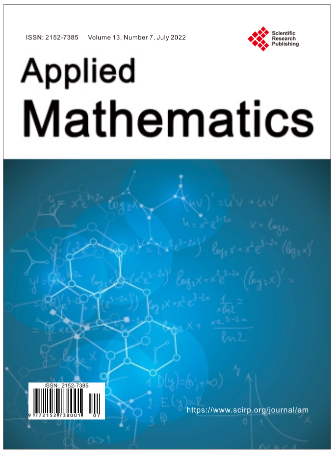

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