

# Propagation of Waves in a Two-Temperature Rotating Thermoelastic Solid Half-Space without Energy Dissipation

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## ABSTRACT

The present paper is concerned with the propagation of plane waves in an isotropic two-temperature generalized thermoelastic solid half-space in context of Green and Naghdi theory of type II (without energy dissipation). The governing equations in  $x$ - $z$  plane are solved to show the existence of three coupled plane waves. The reflection of plane waves from a thermally insulated free surface is considered to obtain the relations between the reflection coefficients. A particular example of the half-space is chosen for numerical computations of the speeds and reflection coefficients of plane waves. Effects of two-temperature and rotation parameters on the speeds and the reflection coefficients of plane waves are shown graphically.

**Keywords:** Two-Temperature; Generalized Thermoelasticity; Reflection; Reflection Coefficients; Energy Dissipation; Rotation

## 1. Introduction

Lord and Shulman [1] and Green and Lindsay [2] extended the classical dynamical coupled theory of thermoelasticity to generalized thermoelasticity theories. These theories treat heat propagation as a wave phenomenon rather than a diffusion phenomenon and predict a finite speed of heat propagation. Ignaczak and Ostoja-Starzewski [3] explained in detail, the above theories in their book on "Thermoelasticity with Finite Wave Speeds". The theory of thermoelasticity without energy dissipation is another generalized theory, which was formulated by Green and Naghdi [4]. It includes the isothermal displacement gradients among its independent constitutive variables and differs from the previous theories in that it does not accommodate dissipation of thermal energy. The representative theories in the range of generalized thermoelasticity are reviewed by Hetnarski and Ignaczak [5]. Wave propagation in thermoelasticity has many applications in various engineering fields. Some problems on wave propagation in coupled or generalized thermoelasticity are studied by various researchers, for example, Deresiewicz [6], Sinha and Sinha [7], Sinha and Elsibai [8,9], Sharma, *et al.* [10], Othman and Song [11], Singh [12,13], and many more.

Gurtin and Williams [14,15] suggested the second law

of thermodynamics for continuous bodies in which the entropy due to heat conduction was governed by one temperature, that of the heat supply by another temperature. Based on this suggestion, Chen and Gurtin [16] and Chen *et al.* [17,18] formulated a theory of thermoelasticity which depends on two distinct temperatures, the conductive temperature  $\Phi$  and the thermodynamic temperature  $T$ . The two-temperature theory involves a material parameter  $a^* > 0$ . The limit  $a^* \rightarrow 0$  implies that  $\Phi \rightarrow T$  and the classical theory can be recovered from two-temperature theory. The two-temperature model has been widely used to predict the electron and phonon temperature distributions in ultrashort laser processing of metals. Warren and Chen [19] stated that these two temperatures can be equal in time-dependent problems under certain conditions, whereas  $\Phi$  and  $T$  are generally different in particular problems involving wave propagation. Following Boley and Tolins [20], they studied the wave propagation in the two-temperature theory of coupled thermoelasticity. They showed that the two temperatures  $T$  and  $\Phi$ , and the strain are represented in the form of a travelling wave plus a response, which occurs instantaneously throughout the body. Puri and Jordan [21] discussed the propagation of harmonic plane waves in two temperature theory. Quintanilla and Jordan [22] presented exact solutions of two initial-boundary

value problems in the two temperature theory with dual-phase-lag delay. Youssef [23] formulated a theory of two-temperature generalized thermoelasticity. Kumar and Mukhopadhyay [24] extended the work of Puri and Jordan [21] in the context of the linear theory of two-temperature generalized thermoelasticity formulated by Youssef [23]. Magana and Quintanilla [25] studied the uniqueness and growth of solutions in two-temperature generalized thermoelastic theories. Recently, Youssef [26] presented a theory of two-temperature thermoelasticity without energy dissipation.

In the present paper, we have applied Youssef [26] theory to study the wave propagation in an isotropic two-temperature thermoelastic solid. The governing equations are solved to obtain the cubic velocity equation. The required boundary conditions at thermally insulated stress free surface are satisfied by the appropriate solutions in an isotropic thermoelastic solid half-space and we obtain three relations between the reflection coefficients for an incident plane wave. The speeds and reflection coefficients of plane waves are also computed numerically for a particular model of the half-space to capture the effect of the two-temperature and rotation parameters.

### 2. Basic Equations

We consider a two-temperature thermoelastic medium, which is rotating uniformly with an angular velocity  $\Omega = \Omega n$ , where  $n$  is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame of reference has two additional terms: Centripetal acceleration,  $\Omega \times (\Omega \times u)$  due to time-varying motion only and the Corioli's acceleration,  $2\Omega \times \dot{u}$  where  $u$  is the dynamic displacement vector. These terms do not appear in non-rotating media. Following Youssef [26], the governing equations for a rotating two-temperature generalized thermoelastic half-space without energy dissipation are taken in the following form:

(i) The heat conduction equation

$$K^* \Phi_{,ii} = \rho c_E \ddot{\theta} + \gamma T_0 \ddot{e}_{kk}, \tag{1}$$

(ii) The displacement-strain relation

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \tag{2}$$

(iii) The equation of motion

$$\rho [\ddot{u}_i + \Omega \times (\Omega \times u) + 2\Omega \times \dot{u}] = (\lambda + \mu) u_{j,ij} + \mu u_{i,ij} - \gamma \theta_{,i}, \tag{3}$$

(iv) The constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \gamma \theta) \delta_{ij}, \tag{4}$$

where  $\gamma = (3\lambda + 2\mu)\alpha_i$ , is a coupling parameter and

$\alpha_i$ , is the thermal expansion coefficient.  $\lambda$  and  $\mu$  are called Lamé's elastic constants,  $\delta_{ij}$  is the Kronecker delta,  $K^*$  is material characteristic constant,  $T$  is the mechanical temperature,  $\Phi_0 = T_0$  is the reference temperature,  $\theta = T - T_0$  with  $|\theta/T_0| \ll 1$ ,  $\sigma_{ij}$  is the stress tensor,  $e_{ij}$  is the strain tensor,  $\rho$  is the mass density,  $c_E$  is the specific heat at constant strain,  $u_i$  are the components of the displacement vector,  $\Phi$  is the conductive temperature and satisfies the relation

$$\Phi - \theta = a^* \Phi_{,ii}, \tag{5}$$

where  $a^* > 0$  is the two-temperature parameter.

### 3. Analytical 2D Solution

We consider a homogeneous and isotropic thermoelastic medium of an infinite extent with Cartesian coordinates system  $(x, y, z)$ , which is previously at uniform temperature  $T_0$ . The origin is taken on the plane surface  $z = 0$  and the  $z$ -axis is taken normally into the medium  $z \geq 0$ . The surface  $z = 0$  is assumed stress-free and thermally insulated. The present study is restricted to the plane strain parallel to  $x$ - $z$  plane, with the displacement vector  $u = (u_1, 0, u_3)$  and rotational vector  $\Omega = (0, \Omega, 0)$ . Now, the Equation (3) has the following two components in  $x$ - $z$  plane

$$\begin{aligned} & (\lambda + 2\mu)u_{1,11} + (\lambda + \mu)u_{3,13} + \mu u_{1,33} - \gamma \theta_{,1} \\ & = \rho [\ddot{u}_1 - \Omega^2 u_1 + 2\Omega \dot{u}_3], \end{aligned} \tag{6}$$

$$\begin{aligned} & (\lambda + 2\mu)u_{3,33} + (\lambda + \mu)u_{1,13} + \mu u_{3,11} - \gamma \theta_{,3} \\ & = \rho [\ddot{u}_3 - \Omega^2 u_3 - 2\Omega \dot{u}_1], \end{aligned} \tag{7}$$

The heat conduction Equation (1) is written in  $x$ - $z$  plane as

$$K^* (\Phi_{,11} + \Phi_{,33}) = \rho c_E \frac{\partial^2 \theta}{\partial t^2} + \gamma T_0 \frac{\partial^2}{\partial t^2} (u_{1,1} + u_{3,3}), \tag{8}$$

and, the Equation (5) becomes,

$$\Phi - \theta = a^* (\Phi_{,11} + \Phi_{,33}). \tag{9}$$

The displacement components  $u_1$  and  $u_3$  are written in terms of potentials  $q$  and  $\psi$  as

$$u_1 = \frac{\partial q}{\partial x} - \frac{\partial \psi}{\partial z}, u_3 = \frac{\partial q}{\partial z} + \frac{\partial \psi}{\partial x}. \tag{10}$$

Using Equations (9)-(10) in Equations (6)-(8), we obtain

$$\mu \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} \right) = \rho (\ddot{\psi} - \Omega^2 \psi - 2\Omega \dot{q}) \tag{11}$$

$$\begin{aligned} & (\lambda + 2\mu) \left( \frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right) - \gamma \left[ \Phi - a^* \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \right] \\ & = \rho (\ddot{q} - \Omega^2 q + 2\Omega \dot{\psi}), \end{aligned} \tag{12}$$

$$K^* \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) = \rho c_E \frac{\partial^2 \Phi}{\partial t^2} - a^* \rho c_E \frac{\partial^2}{\partial t^2} \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) + \gamma T_0 \frac{\partial^2}{\partial t^2} \left( \frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right), \tag{13}$$

Solutions of Equations (11)-(13) are now sought in the form of harmonic travelling wave

$$(q, \Phi, \psi) = (A, B, C) \exp[ik(\sin \theta x + \cos \theta z - Vt)], \tag{14}$$

in which  $V$  is the phase speed,  $k$  is the wave number and  $(\sin \theta, \cos \theta)$  denotes the projection of wave normal onto  $x$ - $z$  plane. Making use of Equation (14) into the Equations (11)-(13), we obtain a homogenous system of equations in  $A, B$  and  $C$ , which admits the non-trivial solution if

$$V^6 + AV^4 + BV^2 + C = 0, \tag{15}$$

where

$$A = - \left[ K_a^* + \frac{\Omega^* (c_1^2 + c_1^2 + \varepsilon)}{\left( \Omega^{*2} - 4 \frac{\Omega^2}{\omega^2} \right)} \right]$$

$$B = \left[ K_a^* (c_1^2 + c_2^2) \Omega^* + c_2^2 (c_1^2 + \varepsilon) \right] / \left( \Omega^{*2} - 4 \frac{\Omega^2}{\omega^2} \right)$$

$$C = -K_a^* c_1^2 c_2^2 / \left( \Omega^{*2} - 4 \frac{\Omega^2}{\omega^2} \right),$$

and

$$K_a^* = \frac{K^*}{\rho c_E (1 + a^* k^2)}, \quad \varepsilon = \frac{\gamma^2 T_0}{\rho^2 c_E}$$

$$c_1^2 = \frac{(\lambda + 2\mu)}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}, \quad \Omega^* = 1 + \frac{\Omega^2}{\omega^2}.$$

The three roots of the cubic Equation (15) are complex. Using the relation  $V_j^{-1} = v_j^{-1} - i\omega^{-1}q_j, (j=1,2,3)$ , we obtain three real values  $v_1, v_2$  and  $v_3$  of the speeds of three plane waves, namely,  $P_1, P_2$  and  $P_3$  waves, respectively.

### 4. Limiting Cases

#### 4.1. In Absence of Rotation Parameters

In absence of rotation parameters, we have  $\frac{\Omega}{\omega} = 0$  and the velocity Equation (15) reduces to

$$(V^2 - c_2^2) [V^4 - (K_a^* + c_1^2 + \varepsilon)V^2 + K_a^* c_1^2] = 0, \tag{16}$$

which gives the speeds of  $P$ , thermal and  $SV$  waves in an isotropic two-temperature thermoelastic medium without

energy dissipation.

#### 4.2. In Absence of Rotation and Thermal Parameters

In absence of rotation and thermal parameters, we have  $K_a^* \rightarrow 0, \varepsilon \rightarrow 0$  and the Equation (15) reduces to

$$(V^2 - c_1^2)(V^2 - c_2^2) = 0, \tag{17}$$

which gives the speeds of  $P$  and  $SV$  waves in an isotropic elastic media.

### 5. Boundary Conditions

We consider the incidence of  $P_1$  wave. The boundary conditions at the stress-free thermally insulated surface  $z = 0$  are satisfied, if the incident  $P_1$  wave gives rise to a reflected  $P_1, P_2$  and  $P_3$  waves. The required boundary conditions at free surface  $z = 0$  are as

(i) Vanishing of the normal stress component

$$\sigma_{zz} = 0, \tag{18}$$

(ii) Vanishing of the tangential stress component

$$\sigma_{zx} = 0, \tag{19}$$

(iii) Vanishing of the normal heat flux component

$$\frac{\partial \Phi}{\partial z} = 0, \tag{20}$$

where

$$\sigma_{zz} = \lambda \left( \frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right) + 2\mu \left( \frac{\partial^2 \psi}{\partial x \partial z} \right) + 2\mu \frac{\partial^2 q}{\partial z^2} - \gamma \left[ \Phi - a^* \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \right], \tag{21}$$

$$\sigma_{zx} = \mu \left[ 2 \frac{\partial^2 q}{\partial x \partial z} - \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} \right]. \tag{22}$$

The appropriate displacement and temperature potentials  $q, \Phi, \psi$  are taken in the following form

$$q = A_0 \exp(ik_1(x \sin \theta_0 + z \cos \theta_0 - V_1 t)) + A_1 \exp(ik_1(x \sin \theta_1 - z \cos \theta_1 - V_1 t)) + A_2 \exp(ik_2(x \sin \theta_2 - z \cos \theta_2 - V_2 t)) + A_3 \exp(ik_3(x \sin \theta_3 - z \cos \theta_3 - V_3 t)), \tag{23}$$

$$\Phi = \eta_1 A_0 \exp(ik_1(x \sin \theta_0 + z \cos \theta_0 - V_1 t)) + \eta_1 A_1 \exp(ik_1(x \sin \theta_1 - z \cos \theta_1 - V_1 t)) + \eta_2 A_2 \exp(ik_2(x \sin \theta_2 - z \cos \theta_2 - V_2 t)) + \eta_3 A_3 \exp(ik_3(x \sin \theta_3 - z \cos \theta_3 - V_3 t)), \tag{24}$$

$$\begin{aligned} \psi = & \zeta_1 A_0 \exp(ik_1(x \sin \theta_0 + z \cos \theta_0 - V_1 t)) \\ & + \zeta_1 A_1 \exp(ik_1(x \sin \theta_1 - z \cos \theta_1 - V_1 t)) \\ & + \zeta_2 A_2 \exp(ik_2(x \sin \theta_2 - z \cos \theta_2 - V_2 t)) \\ & + \zeta_3 A_3 \exp(ik_3(x \sin \theta_3 - z \cos \theta_3 - V_3 t)), \end{aligned} \tag{25}$$

where the wave normal to the incident  $P_1$  wave makes angle  $\theta_0$  with the positive direction of  $z$ -axis and those of reflected  $P_1, P_2$  and  $P_3$  waves make angles  $\theta_1, \theta_2$  and  $\theta_3$ , respectively with the same direction, and

$$\frac{\eta_j}{k_j^2} = \frac{\Omega^* V_j^2 - c_1^2 + \Omega_j}{\bar{\gamma}(1 + a^* k_j^2)}, \zeta_j = -\frac{2i \frac{\Omega}{\omega} V_j^2}{c_2^2 - \Omega^* V_j^2}, (j = 1, 2, 3),$$

where

$$\bar{\gamma} = \frac{\gamma}{\rho}, \Omega_j = \frac{4 \frac{\Omega^2}{\omega^2} V_j^4}{c_2^2 - \Omega^* V_j^2}, (j = 1, 2, 3).$$

### 6. Reflection Coefficients

The ratios of the amplitudes of the reflected waves to the amplitude of incident  $P_1$  wave, namely  $\frac{A_1}{A_0}, \frac{A_2}{A_0}$  and

$\frac{A_3}{A_0}$  are the reflection coefficients (amplitude ratios) of reflected  $P_1, P_2$  and  $P_3$  wave, respectively. The wave numbers  $k_1, k_2, k_3$  and the angles  $\theta_0, \theta_1, \theta_2, \theta_3$  are connected by the relation

$$k_1 \sin \theta_0 = k_1 \sin \theta_1 = k_2 \sin \theta_2 = k_3 \sin \theta_3, \tag{26}$$

at surface  $z = 0$ . In order to satisfy the boundary conditions (18)-(20), the relation (26) is also written as

$$\frac{\sin \theta_0}{v_1} = \frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2} = \frac{\sin \theta_3}{v_3} \tag{27}$$

with the help of the potentials given by Equations (23)-(25) and the Snell's law Equations (26) and (27), the boundary conditions (18)-(20) results into a system of following three non-homogeneous equations

$$\sum_{j=1}^3 a_{ij} Z_j = b_i, (i = 1, 2, 3), \tag{28}$$

where  $Z_j = \frac{A_j}{A_0}, (j = 1, 2, 3)$  are the reflection coefficients of reflected  $P_1, P_2$  and  $P_3$  waves, and

$$\begin{aligned} a_{11} = & -\lambda + \mu \zeta_1 \sin 2\theta_0 - 2\mu \cos^2 \theta_0 \\ & - \rho V_1^2 + (\lambda + 2\mu) - \left( \frac{\Omega^2}{\omega^2} + 2i \zeta_1 \frac{\Omega}{\omega} \right) \rho V_1^2 \end{aligned}$$

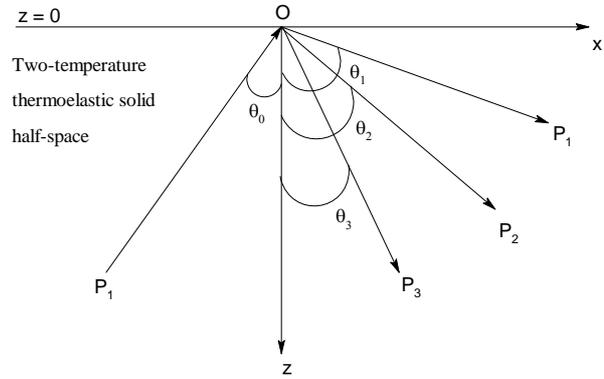


Figure 1. Geometry of the problem.

$$\begin{aligned} a_{1j} = & \left[ -\lambda + 2\mu \zeta_j (V_j/V_1) \sin \theta_0 \sqrt{1 - \frac{V_j^2}{V_1^2} \sin^2 \theta_0} \right. \\ & \left. - 2\mu \left( 1 - \frac{V_j^2}{V_1^2} \sin^2 \theta_0 \right) - \rho V_j^2 + (\lambda + 2\mu) \right. \\ & \left. - \left( \frac{\Omega^2}{\omega^2} + 2i \zeta_j \frac{\Omega}{\omega} \right) \rho V_j^2 \right] (V_1^2/V_j^2), (j = 2, 3) \end{aligned}$$

$$a_{21} = \sin 2\theta_0 + \zeta_1 \cos 2\theta_0$$

$$\begin{aligned} a_{2j} = & \left[ 2(V_j/V_1) \sin \theta_0 \sqrt{1 - \frac{V_j^2}{V_1^2} \sin^2 \theta_0} \right. \\ & \left. + \zeta_j \left( 1 - 2 \frac{V_j^2}{V_1^2} \sin^2 \theta_0 \right) \right] (V_1^2/V_j^2), (j = 2, 3) \end{aligned}$$

$$a_{31} = \eta_1 \cos \theta_0, a_{3j} = \frac{V_1}{V_3} \left[ \eta_j \sqrt{1 - \frac{V_j^2}{V_1^2} \sin^2 \theta_0} \right], (j = 2, 3)$$

$$\begin{aligned} b_1 = & \lambda + \mu \zeta_1 \sin 2\theta_0 + 2\mu \cos^2 \theta_0 + \rho V_1^2 - (\lambda + 2\mu) \\ & + \left( \frac{\Omega^2}{\omega^2} + 2i \zeta_1 \frac{\Omega}{\omega} \right) \rho V_1^2 \end{aligned}$$

$$b_2 = \sin 2\theta_0 - \zeta_1 \cos 2\theta_0, b_3 = \eta_1 \cos \theta_0.$$

### 7. Numerical Results and Discussion

To study the effects of two-temperature and rotation parameters on the speeds of propagation and reflection coefficients of plane waves, we consider the following physical constants of aluminium as an isotropic thermoelastic solid half space

$$\begin{aligned} \rho = & 2.7 \times 10^3 \text{ kg/m}^3, \lambda = 7.59 \times 10^{10} \text{ N/m}^2, \\ \mu = & 1.89 \times 10^{10} \text{ N/m}^2 \\ K = & 237 \text{ W/m} \cdot \text{deg}, C_e = 24.2 \text{ J/kg} \cdot \text{deg}, \\ T_0 = & 296 \text{ K}. \end{aligned}$$

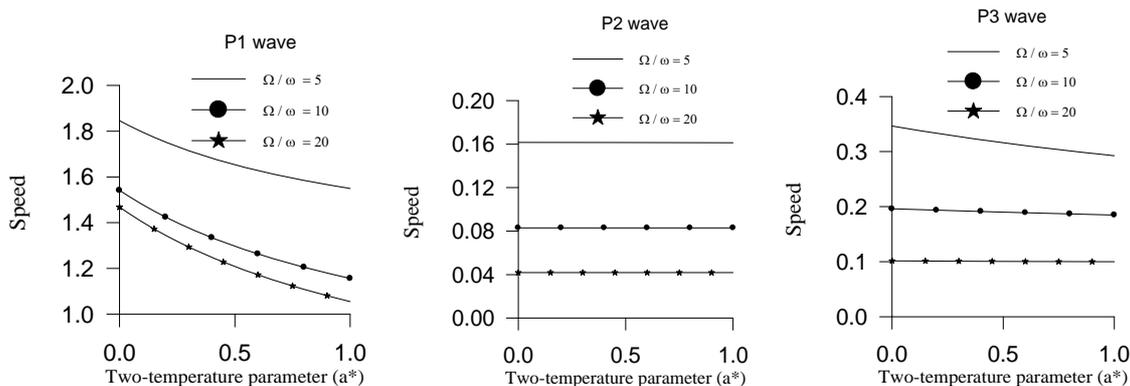
Using the relation  $V_j^{-1} = v_j^{-1} - i\omega^{-1} q_j, (j = 1, \dots, 3)$  in

Equation (15), the real values of the propagation speeds of  $P_1, P_2$  and  $P_3$  waves are computed for the range  $0 \leq a^* \leq 1$  of two-temperature parameter, when  $\Omega/\omega = 5, 10$  and  $20$ . The speeds of  $P_1, P_2$  and  $P_3$  waves are shown graphically versus the two-temperature parameter  $a^*$  in **Figure 2**. The speed of  $P_1$  wave decreases with an increase in two-temperature parameter, whereas the speeds of  $P_2$  and  $P_3$  wave are affected less due to the change in two-temperature parameter. It is also observed from **Figure 2** that the speed of each plane wave decreases with the increase in value of rotation parameter.

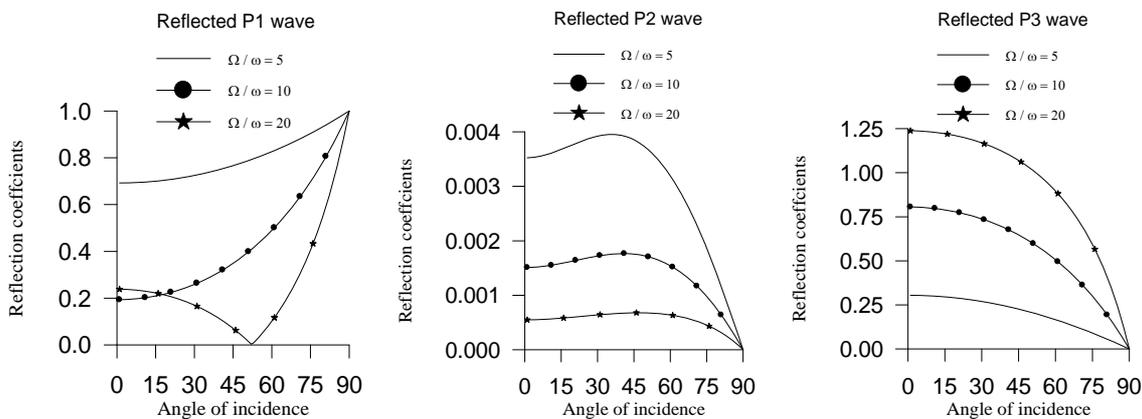
With the help of Equation (28), the reflection coefficients of reflected  $P_1, P_2$  and  $P_3$  waves are computed for the incidence of  $P_1$  wave. For the range  $0^\circ < \theta_0 \leq 90^\circ$  of the angle of incidence of  $P_1$  wave, the reflection coefficients of the  $P_1, P_2$  and  $P_3$  waves are shown graphically in **Figure 3**, when the rotation parameter  $\Omega/\omega = 5, 10, 20$  and two-temperature parameter  $a^* = 0.5$ . For  $\Omega/\omega = 5$  and  $10$ , the reflection coefficient of  $P_1$  wave increases from its minimum value at  $\theta_0 = 1^\circ$  to its maximum value one at  $\theta_0 = 90^\circ$  and for  $\Omega/\omega = 20$ , its reflection coefficient first decreases to its minimum value zero at  $\theta_0 = 52^\circ$  and then increases to its maximum

value one at  $\theta_0 = 90^\circ$ . For each value of  $\Omega/\omega$ , the reflection coefficient of  $P_2$  wave first increases slightly and then decreases to its minimum value zero at  $\theta_0 = 90^\circ$ . For all value of  $\Omega/\omega$ , the reflection coefficient of  $P_3$  wave decreases from its maximum value at  $\theta_0 = 1^\circ$  to its minimum value zero at  $\theta_0 = 90^\circ$ . From **Figure 3**, it is also observed that the effect of rotation parameter  $\Omega/\omega$  on reflection coefficients of  $P_1$  is maximum near  $\theta_0 = 52^\circ$ , whereas it is maximum at  $\theta_0 = 42^\circ$  for  $P_2$  wave. There is no effect of rotation parameter on these reflected waves at grazing incidence. The reflection coefficients of  $P_1$  and  $P_2$  waves decrease with the increase in value of rotation parameter at each angle of incidence except the grazing incidence, whereas the reflection coefficient of  $P_3$  wave increases.

For the range  $0^\circ < \theta_0 \leq 90^\circ$  of the angle of incidence of  $P_1$  wave, the reflection coefficients of the  $P_1, P_2$  and  $P_3$  waves are shown graphically in **Figure 4**, when two-temperature parameter  $a^* = 0, 0.5, 1$  and rotation parameter  $\Omega/\omega = 10$ . For all values of  $a^*$ , the reflection coefficient of  $P_1$  wave increases from its minimum value at  $\theta_0 = 1^\circ$  to its maximum value one at  $\theta_0 = 90^\circ$ . For all values of  $a^*$ , the reflection coefficient of  $P_2$  wave first increases and then decreases to its minimum value zero



**Figure 2.** Variations of the speeds of plane waves versus two-temperature parameter  $a^*$ .



**Figure 3.** Variations of the reflection coefficients versus angle of incidence when two-temperature parameter  $a^* = 0.5$ .

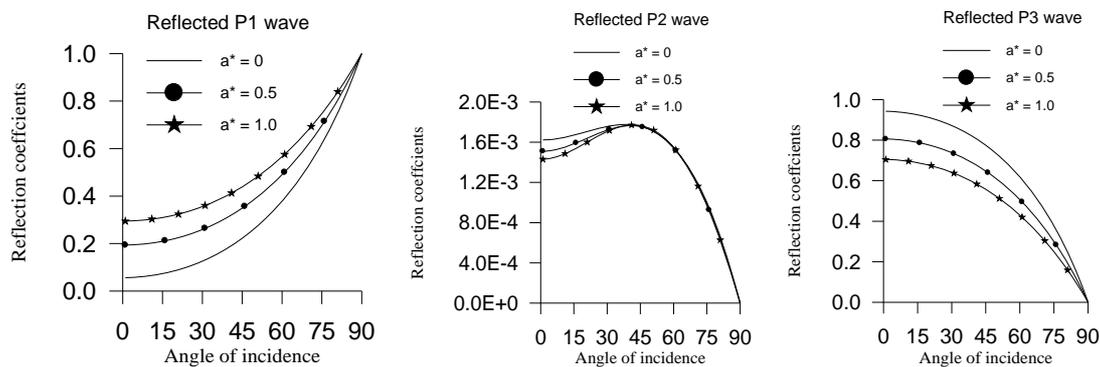


Figure 4. Variations of the reflection coefficients versus angle of incidence when rotation parameter  $\Omega/\omega = 10$ .

at  $\theta_0 = 90^\circ$ . The reflection coefficient of  $P_3$  wave decreases from its maximum value at  $\theta_0 = 1^\circ$  to its minimum value zero at  $\theta_0 = 90^\circ$ . From Figure 4, it is also observed that the effect of two-temperature parameter  $a^*$  on all reflected waves is maximum near normal incidence. For grazing incidence, there is no effect of two-temperature parameter on all the reflected waves. The reflection coefficients of  $P_1$  wave increases with the increase in value of two-temperature parameter at each angle of incidence except grazing incidence, whereas the reflection coefficient of  $P_3$  wave decreases. For the range  $0^\circ < \theta_0 \leq 42^\circ$  of the angle of incidence of  $P_1$  wave, the reflection coefficients of the  $P_2$  decreases with an increase in two-temperature parameter. Beyond  $\theta_0 \geq 42^\circ$ , there is little effect of two-temperature parameter on the reflection coefficients of the  $P_2$  wave.

## 8. Conclusion

Two-dimensional solution of the governing equations of an isotropic two-temperature thermoelastic medium without energy dissipation indicates the existence of three plane waves, namely,  $P_1, P_2$  and  $P_3$  waves. The appropriate solutions in the half-space satisfy the required boundary conditions at thermally insulated free surface and the relations between reflection coefficients of reflected  $P_1, P_2$  and  $P_3$  waves are obtained for the incidence of  $P_1$  wave. The speeds and reflection coefficients of plane waves are computed for a particular material representing the model. From theory and numerical results, it is observed that the speeds and reflection coefficients of plane waves are significantly affected by the two-temperature and rotation parameters.

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