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Zero-M-Cordial Labeling of Some Graphs

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ABSTRACT

In this paper we prove that the complete bipartite graph $k_{m,n}$ where m and n are even, join of two cycle graphs c_n and c_m where $n+m \equiv 0 \pmod 4$, split graph of c_n for even "n", $K_n \times P_2$ where n is even are admits a Zero-M-Cordial labeling. Further we prove that $K_n \times P_2$ $B_n = K_{1,n} \times P_2$, of odd n admits a Zero-M-Cordial labeling.

Keywords: Zero-M-Cordial Labeling; Split Graphs; Cartesian Product; H-Cordial

1. Introduction

We begin with finite, connected and undirected graph $G = (V(G), E(G))$. If the vertices of the graph are assigned values subject to certain conditions then it is known as graph labeling. Any graph labeling will have the following three common characteristics. A set of numbers from which Vertex labels are chosen; $v_f(i)$ = number of vertices of G having label i under f
$e_f(i)$ = number of edges of G having label i under f^*.

The concept of cordial labeling was introduced by I. Cahit, who called a graph G is Cordial if there is a vertex labeling $f: v(G) \to \{0,1\}$ such that the induced labeling $f^*: E(G) \to \{0,1\}$, defined by $f^*(xy) = |f(x) - f(y)|$, for all edges $xy \in E(G)$ and with the following inequalities holding and $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

In [1] introduced the concept of H-Cordial labeling. Cahit calls a graph H-Cordial if it is possible to label the edges with the numbers from the set $\{1,-1\}$ in such a way that, for some k, at each vertex v the sum of the labels on the edges incident with v is either k or $-k$ and the inequalities $|v_f(k) - v_f(-k)| \le 1$ and $|e_f(1) - e_f(-1)| \le 1$ are also satisfied where $v(i)$and $e(j)$ are respectively, the number of vertices labeled with i and the number of edges labeled with j. He calls a graph H_n-Cordial if it is possible to label the edges with the numbers from the set $\{\pm 1, \pm 2, \cdots, \pm n\}$ in such a way that, at each vertex v the sum of the labels on the edges incident with v is in the set $\{\pm 1, \pm 2, \cdots, \pm n\}$ and the inequalities $|v_f(i) - v_f(-i)| \le 1$ and $|e_f(i) - e_f(-i)| \le 1$ are also satisfied for each i with $1 \le i \le n$. The concept of Zero-M-Cordial labeling is defined in [2]. A labeling f of a graph G is called Zero-M-Cordial, if for each vertex v, $f(v) = 0$. A graph G is called to be Zero-M-Cordial, if it admits a Zero-M-Cordial labeling. The usefulness of the above definition appears when one tries to find an H-Cordial labeling for a given graph G. If H is a Zero-M-Cordial subgraph of G then H-Cordiality of G\E(H) simply implies H-Cordiality G.

In [1] proved that $k_{n,n}$ is H-Cordial if and only if $n > 2$ and "n" is even; and $k_{m,n}$, $m \neq n$ is H-Cordial if and only if $n \equiv 0$ (mod4), m is even and $m > 2$, $n > 2$.

In [2] proved that k_n is H-Cordial if and only if $n \equiv 0$ or 3 (mod4) and $n \neq 3$. W_n is H-Cordial if and only if n is odd. k_n is not H2-Cordial if $n \equiv 1$ (mod4). Also [2] prove that every wheel has an H_2-Cordial labeling.

In [3] several variations of graph labeling such as graceful, bigraceful, harmonious, cordial, equitable, humming etc. have been introduced by several authors. For definitions and terminologies in graph theory we refer to [4].

In this paper we investigate Zero-M-Cordial labeling on some Cartesian product of graphs, join of two graphs, and bipartite graph.

1.1. *Definition*: The join $G = G_1 + G_2$ of graph G_1 and G_2 with disjoint point sets V_1 and V_2 and edge sets E_1 and E_2 denoted by $G = G_1 + G_2$ is the graph union $G_1 \cup G_2$ together with all the edges joining v_1, v_2. If G_1 is (p_1,q_1) graph and G_2 is (p_2,q_2) graph then $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$.

1.2. *Definition*: Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The Cartesian product of G_1 and G_2 which is denoted by $G_1 \times G_2$ is the graph with vertex set $v = v_1 \times v_2$ consisting of vertices $V = \{u = (u_1, u_2), v = (v_1, v_2) /$ u and v are adjacent in $G_1 \times G_2$ whenever $u_1 = v_1$ and u_2 adjacent to v_2 or u_1 adjacent to v_1 and $u_2 = v_2\}$.

1.3. *Definition*: For a graph G the split graph is obtained by adding to each vertex v, a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G. The resultant graph is denoted by spl (G).

2. Main Results

***Theorem* 2.1:** Every cycle C_n of even order admits a Zero-M-Cordial labeling

Proof: Let $v_1, v_2, \cdots, v_n$ be the vertices of cycle C_n

Define f: $E(G) \to \{1, -1\}$ two cases are to be considered.

Case (i) $n \equiv 2 \pmod 4$

For $1 \le i \le n$

$$f(v_i, v_{i+1}) = \begin{cases} = 1, \text{ if } i \equiv 1 \pmod 2 \\ = -1, \text{ if } i \equiv 0 \pmod 2 \text{ and } 1 \le i \le n \end{cases}$$

In view of the above labeling pattern we give the proof as follows:

When $n \equiv 2 \pmod 4$

The total number of edges labeled with $-1's$ are given by $e_f(-1) = n/2$ and the total number of edges labeled with $1's$ are given by $e_f(1) = n/2$. Therefore the total difference between the edges labeled with $-1's$ and $1's$ are given by $|e_f(1) - e_f(-1)| = 0$. The induced vertex labels are equal to zero. Thus for each vertex v, $f(v) = 0$ and $|e_f(1) - e_f(-1)| \le 1$.

Case (ii) $n \equiv 0 \pmod 4$

The total number of edges labeled with $-1's$ are given by $e_f(-1) = n/2$ and the total number of edges labeled with $1's$ are given by $e_f(1) = n/2$. Therefore the total difference between the edges labeled with $-1's$ and $1's$ are given by $|e_f(1) - e_f(-1)| = 0$. The induced vertex labels are equal to zero. Thus for each vertex v, $f(v) = 0$ and $|e_f(1) - e_f(-1)| \le 1$.

Hence the cycle graph c_n, even n admits a zero-M-cordial labeling.

The vertex and the edge conditions are given in **Table 1**. The illustration is given in **Figures 1** and **2**.

In **Figure 1** illustrates the Zero-M-Cordial labeling for the cycle graph C_6. Among the six edges three edges receive the label +1 and the other three edges receive the label –1. In **Figure 2** illustrates the Zero-M-Cordial labeling for the cycle graph C_8. Among the eight edges four edges receive the label +1 and the other four edges receive the label –1.

***Theorem* 2.2:** The complete bipartite graph $k_{m,n}$ admits a Zero-M-Cordial labeling for all m, n such that $m + n \equiv 0, 2 \pmod 4$.

Proof: Let $v_1, v_2, \cdots, v_m$ and $u_1, u_2, \cdots, u_n$ are the vertex set of the bipartite graph $k_{m,n}$. The number of vertices and the edges of $k_{m,n}$ is $m + n$ and mn respectively.

Define f: $E(G) \to \{1, -1\}$

Table 1. The vertex and the edge conditions of cycle graph C_n.

n	Vertex condition	Edge condition
$n \equiv 2 \pmod 4$	$f(v) = 0$	$e_f(1) = e_f(-1) = n/2$
$n \equiv 2 \pmod 4$	$f(v) = 0$	$e_f(1) = e_f(-1) = n/2$

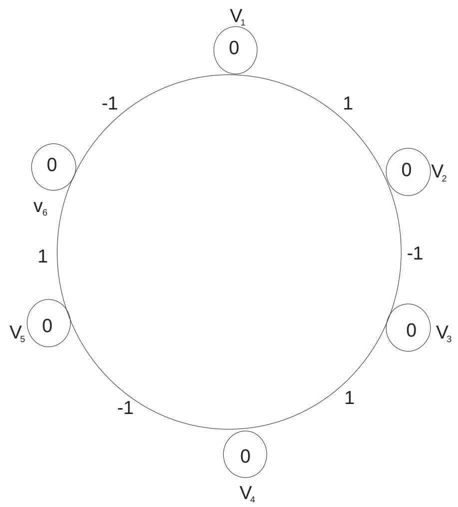

Figure 1. Zero-M-Cordial labeling on C_6.

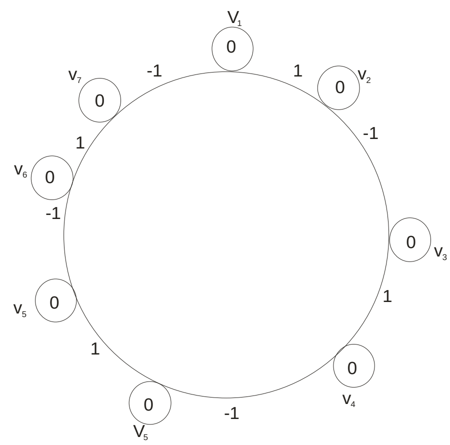

Figure 2. Zero-M-Cordial labeling on C_8.

The edge matrix of $k_{m,n}$ is given in **Table 2**.

In view of the above edge matrix we give the proof as follows.

Case (i) when $m + n \equiv 0 \pmod 4$, $m = n$.

Consider the bipartite graph $k_{4,4}$.

Using **Table 2** the edge label matrix of $k_{4,4}$ is given by

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} \\ \begin{bmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \\ \begin{array}{cccc} 0 & 0 & 0 & 0 \end{array} \end{array} \begin{array}{c} \\ 0 \\ 0 \\ 0 \\ 0 \\ \\ \end{array}$$

Table 2. Edge matrix of $k_{m,n}$.

$$\begin{array}{c} \\ v_1 \\ v_2 \\ : \\ : \\ v_m \\ \\ \end{array}\begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{bmatrix} v_1u_1 & v_1u_2 & \cdots & v_1u_m \\ v_2u_1 & v_2u_2 & \cdots & v_2u_m \\ : & : & & : \\ : & : & & : \\ v_mu_1 & v_mu_2 & \cdots & v_mu_n \end{bmatrix} \\ \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \end{array}\begin{array}{c} \\ v_1 \\ v_2 \\ : \\ : \\ v_m \\ \\ \end{array}$$

With respect to the above labeling the total number of edges labeled with $1'^s$ and $-1'^s$ are given by $e_f(1) = n/2$ and $e_f(-1) = n/2$. Therefore the total difference between the edges labeled with $-1'^s$ and $1'^s$ are given by $|e_f(1) - e_f(-1)| = 0$. The induced vertex labels are equal to zero. Thus for each vertex v, $f(v) = 0$ and $|e_f(1) - e_f(-1)| \le 1$.

Hence the bipartite graph $K_{4,4}$ admits a Zero-M-Cordial labeling. The vertex and the edge conditions are given in **Table 3**. The illustration is given in **Figure 3**.

Figure 3 illustrates the Zero-M-Cordial labeling on $K_{4,4}$. Among the Sixteen edges eight edges receive the label +1 and the other eight edges receive the label –1.

Case (ii) When $m + n \equiv 2 \pmod 4$, $m \ne n$.

Consider the bipartite graph $K_{2,4}$.

Using **Table 2** the edge label matrix is given by

$$\begin{array}{c|cccc|c} & u_1 & u_2 & u_3 & u_4 & \\ \hline v_1 & -1 & -1 & 1 & 1 & 0 \\ v_2 & 1 & 1 & -1 & -1 & 0 \\ \hline & 0 & 0 & 0 & 0 & \end{array}$$

With respect to the above labeling the total number of edges labeled with $1'^s$ and $-1'^s$ are given by $e_f(1) = n/2, e_f(-1) = n/2$. Therefore the total difference between the edges labeled with $-1'^s$ and $1'^s$ are given by $|e_f(1) - e_f(-1)| = 0$. The induced vertex labels are equal to zero. Thus for each vertex v, $f(v) = 0$ and $|e_f(1) - e_f(-1)| \le 1$.

Hence the bipartite graph $k_{2,4}$ admits a zero-M-cordial labeling.

Figure 4 illustrates the Zero-M-Cordial labeling on $k_{2,4}$. Among eight edges four edges receive the label +1 and other four edges receive the label –1.

Case (iii) When $m + n \equiv 0 \pmod 4$ and $m \ne n$.

Consider the bipartite graph $k_{2,6}$.

Using **Table 2** the edge label matrix is given by

$$\begin{array}{c|cccccc|c} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & \\ \hline v_1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 \\ v_2 & -1 & 1 & -1 & 1 & -1 & 1 & 0 \\ \hline & 0 & 0 & 0 & 0 & 0 & 0 & \end{array}$$

With respect to the above labeling the total number of edges labeled with $1'^s$ and $-1'^s$ are given by

Table 3. The vertex and the edge conditions of the bipartite graph $K_{m,n}$.

n	Vertex condition	Edge condition
$m+n \equiv 0 \pmod 4, m = n$	$f(v) = 0$	$e_f(1) = e_f(-1) = n/2$
$m+n \equiv 2 \pmod 4, m \ne n$	$f(v) = 0$	$e_f(1) = e_f(-1) = n/2$
$m+n \equiv 0 \pmod 4$, and $m \ne n$	$f(v) = 0$	$e_f(1) = e_f(-1) = n/2$

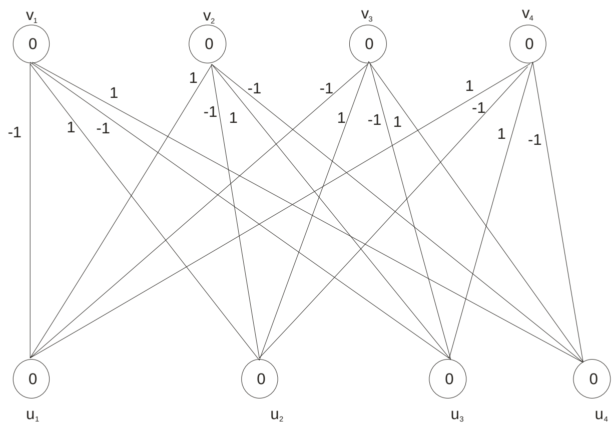

Figure 3. Zero-M-Cordial labeling on $k_{4,4}$.

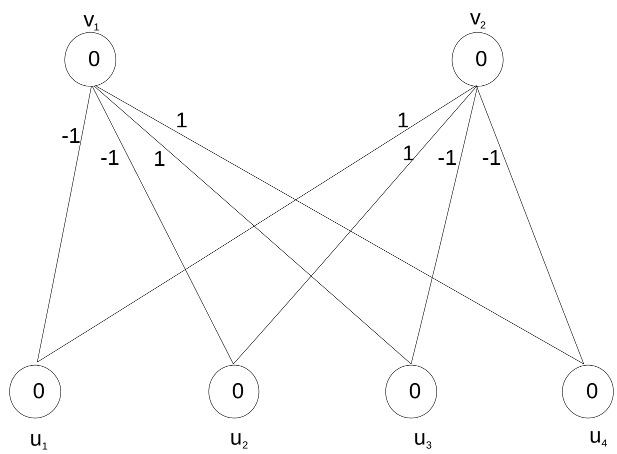

Figure 4. Zero-M-Cordial labeling on $k_{2,4}$.

$e_f(1) = n/2$ and $e_f(-1) = n/2$. Therefore the total difference between the edges labeled with $-1'^s$ and $1'^s$ are given by $|e_f(1) - e_f(-1)| = 0$. The induced vertex labels are equal to zero. Thus for each vertex v, $f(v) = 0$ and $|e_f(1) - e_f(-1)| \le 1$.

Hence the bipartite graph $k_{2,6}$ admits a Zero-M-Cordial labeling. The vertex and the edge conditions are given in **Table 3**.

Figure 5 illustrates the Zero-M-Cordial labeling on $k_{2,6}$. Among the Twelve edges six edges receive the label +1 and the other six edges receive the label –1.

***Theorem* 2.3:** The join of two cycle graphs C_n and C_m

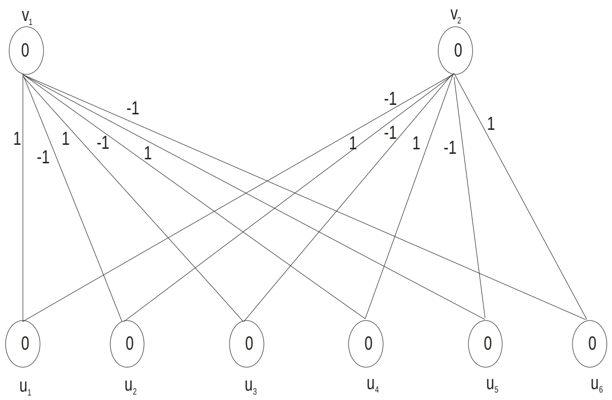

Figure 5. Zero-M-Cordial labeling on $k_{2,6}$.

admits a Zero-M-Cordial labeling

if $n+m \equiv 0 \pmod 4$.

Proof: Let $v_1, v_2, \cdots, v_n$ and $u_1, u_2, \cdots, u_m$ are the vertex set of the cycles C_n and C_m. The edge set E1 and E2 is the graph union of C_n and C_m together with all the edges joining the vertex set $v_1, v_2, \cdots, v_n$ and $u_1, u_2, \cdots, u_m$.

We note that $|V(G)| = p_1 + p_2$ and $|E(G)| = q_1 + q_2 + p_1 p_2$.

Define f: $E(G) \to \{1, -1\}$

The edge matrix of $c_n + c_m$ is given in **Table 4**.

In view of the above labeling pattern we give the proof as follows:

When $n+m \equiv 0 \pmod 4$

Consider the join of two cycle graphs $C_4 + C_4$. Using **Table 4** the edge label matrix of C_4 & C_4 is given by

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ \\ \\ \end{array} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \\ \left[\begin{array}{cccc} -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{array}\right] \\ 1-1 \quad 1-1 \quad -11 \quad -11 \\ 0 \qquad 0 \qquad 0 \qquad 0 \end{array} \begin{array}{cc} & \\ 1-1 & 0 \\ 1-1 & 0 \\ -1\ 1 & 0 \\ 1\ -1 & 0 \end{array}$$

With respect to the above labeling the total number of edges labeled with $1'^s$ and $-1'^s$ are given by $e_f(1) = n/2$ and $e_f(-1) = n/2$. Therefore the total difference between the edges labeled with $-1'^s$ and $1'^s$ are given by $|e_f(1) - e_f(-1)| = 0$. The induced vertex labels are equal to zero. Thus for each vertex v, $f(v) = 0$ and $|e_f(1) - e_f(-1)| \le 1$.

Hence the join of two cycle graphs C_4 and C_4 admits a Zero-M-Cordial labeling. The vertex and the edge conditions are given in **Table 5**.

In **Figure 6** illustrates the zero-M-Cordial labeling on $c_4 + c_4$. Among the twenty four edges twelve edges receive the label +1 and the other twelve edges label –1.

***Theorem* 2.4:** The split graph of C_n, for even n, admits

Table 4. Edge matrix of $C_n + C_m$.

$$\begin{array}{c} \\ v_1 \\ v_2 \\ : \\ : \\ v_n \\ \\ \end{array} \begin{array}{cccc} u_1 & u_2 & \cdots & u_m \\ \left[\begin{array}{cccc} v_1u_1 & v_1u_2 & \cdots & v_1u_m \\ v_2u_1 & v_2u_2 & \cdots & v_2u_m \\ : & : & & : \\ : & : & & : \\ v_nu_1 & v_nu_2 & \cdots & v_nu_m \end{array}\right] \\ u_1u_2, u_1u_m & u_1u_2, u_2u_3 & \cdots & u_1u_m, u_{m-1}u_m \end{array} \begin{array}{c} \\ v_1v_2,\ v_1v_n \\ v_1v_2,\ v_2v_3 \\ : \\ : \\ v_1v_n,\ v_{n-1}v_n \\ \\ \end{array}$$

Table 5. The vertex and the edge conditions of two cycle graphs C_n and C_m.

C_n, C_m	Vertex condition	Edge condition
$m+n \equiv 0 \pmod 4, m=n$	$f(v)=0$	$e_f(1) = e_f(-1) = n/2$

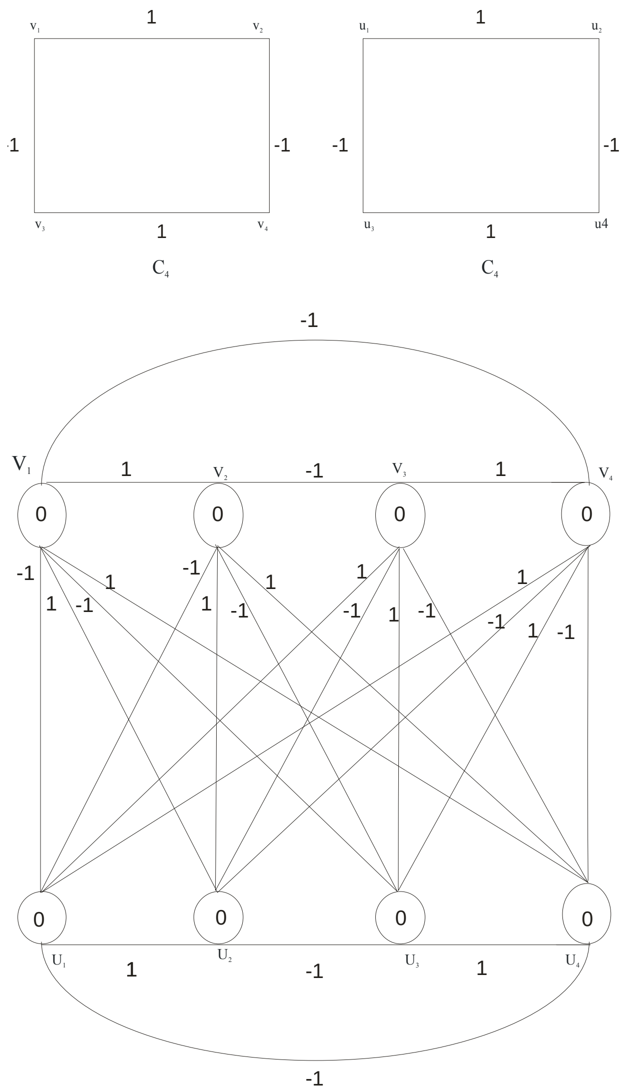

Figure 6. Zero-M-Cordial Labeling on $C_4 + C_4$.

a zero-M-cordial labeling.

Proof:

Let $v_1, v_2, \cdots, v_n$ be the vertices of cycle C_n and $v_1', v_2', \cdots, v_n'$ be the newly added vertices when n is even.

Let G be the split graph of cycle C_n with

$$V(G) = \{v_i, v_i', 1 \le i \le n\},$$

$$E(G) = \{v_i, v_{i+1}', 1 \le i \le n-1, v_n v_1, v_i' v_{i+1}, v_n' v_1, v_i v_{i+1}', 1 \le i \le n-1, v_n v_1'\}$$

we note that $|V(G)| = 2n$ and $|E(G)| = 3n$.

Define $f : E(G) \to \{1, -1\}$ two cases are to be considered.

Case (i) when $n \equiv 0 \pmod 4$

For $1 \le i \le n-1$

$$f(v_i, v_{i+1}) = \begin{cases} 1, & \text{if } i \text{ is odd} \\ -1, & \text{if } i \text{ is even} \end{cases}$$

$$f(v_n, v_1) = -1$$

For $1 \le i \le n-1$

$$f(v_i, v_{i+1}') = -1$$

$$f(v_n, v_1') = -1$$

For $1 \le i \le n-1$

$$f(v_i', v_{i+1}) = 1$$

$$f(v_n', v_1) = 1$$

Case (ii) when $n \equiv 2 \pmod 4$

For $1 \le i \le n-1$

$$f(v_i, v_{i+1}) = \begin{cases} 1, & \text{if } i \equiv 1 \pmod 2 \\ -1, & \text{if } i \equiv 0 \pmod 2 \end{cases}$$

$$f(v_n, v_1) = 1$$

For $1 \le i \le n-1$

$$f(v_i, v_{i+1}') = -1$$

$$f(v_n, v_1') = -1$$

For $1 \le i \le n-1$

$$f(v_i', v_{i+1}) = 1$$

$$f(v_n', v_1) = 1$$

With respect to the above labeling pattern we give the proof as follows.

The total number of edges labeled with $1'^s$ and $-1'^s$ are given by $e_f(1) = n$ and $e_f(-1) = n$. Therefore the total difference between the edges labeled with $-1'^s$ and $1'^s$ are given by $|e_f(1) - e_f(-1)| = |n - n| = 0$, differ by zero. The induced vertex labels are equal to zero. Thus for each vertex v, $f(v) = 0$ and $|e_f(1) - e_f(-1)| \le 1$.

Hence the split graph of C_n for even n admits a Zero-M-Cordial labeling. The vertex and the edge conditions are given in **Table 6**.

In **Figure 7** illustrates the Zero-M-Cordial labeling on split c8. Among the twenty four edges twelve edges receive the label +1 and the other twelve edges receive the label –1.

***Theorem* 2.5:** $K_n \times P_2$ admits a Zero-M-Cordial labeling for even n.

Proof: Let G be the graph $K_n \times P_2$ where n is even and $V(G) = \{V_{ij} \quad i = 1, 2, \cdots, n \text{ and } j = 1, 2\}$ be the vertices of the graph G.

We note that $|V(G)| = 2n$ and $|E(G)| = n^2$ as $|V(k_n)| = n$ and $|E(k_n)| = \frac{n(n-1)}{2}$

Define $f : E(G) \to \{1, -1\}$ as follows

For $1 \le i, k \le n$

$$f(v_{i_1}, v_{k_1}) = 1$$

For $n < i, k \le n+2$

$$f(v_{i_1}, v_{k_1}) = -1$$

For $1 \le i, k \le n$

$$f(v_{i_2}, v_{k_2}) = 1$$

For $n < i, k \le n+2$

$$f(v_{i_2}, v_{k_2}) = -1$$

For $1 \le i \le n$

$$f(v_{i_1}, v_{i_2}) = -1$$

With respect to the above labeling pattern we give the proof as follows.

The total number of edges labeled with $1'^s$ and $-1'^s$ are given by $e_f(1) = n/2$ and $e_f(-1) = n/2$. Therefore the total difference between the edges labeled with $-1'^s$ and $1'^s$ are given by $|e_f(1) - e_f(-1)| = 0$, differ by 0. The induced vertex labels are equal to zero. Thus for each vertex v, $f(v) = 0$ and $|e_f(1) - e_f(-1)| \le 1$.

Hence $K_n \times P_2$ admits a Zero-M-Cordial labeling for even n. The vertex and the edge conditions are given in **Table 7**.

Figure 8 illustrates the Zero-M-Cordial labeling on $k_n \times p_2$. Among the sixteen edges eight edges label +1 and the other eight edges label –1.

***Theorem* 2.6:** $W_n \times P_2$ admits a Zero-M-Cordial labeling for odd n.

Proof: Let G be the graph $W_n \times P_2$ where n is odd and $V(G) = \{V_{ij} \quad i = 1, 2, \cdots, n+1 \text{ and } j = 1, 2\}$ be the vertices of graph G.

We note that $|V(G)| = 2(n+1)$ and $|E(G)| = 5n+1$ as $|V(W_n)| = n+1$ and $|E(W_n)| = 2n$

Table 6. The vertex and the edge conditions of split of C_n.

C_n, C_m	Vertex condition	Edge condition
$n \equiv 0 \pmod 4$	$f(v) = 0$	$e_f(1) = e_f(-1) = n$
$n \equiv 2 \pmod 4$	$f(v) = 0$	$e_f(1) = e_f(-1) = n$

Table 7. Vertex and the edge condition of $k_n \times p_2$.

n	Vertex condition	Edge condition
Even	$f(v) = 0$	$e_f(1) = e_f(-1) = n/2$

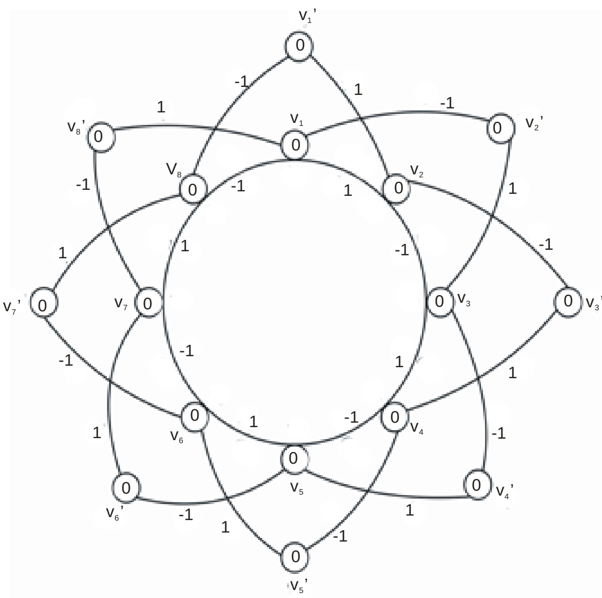

Figure 7. Zero-M-Cordial labeling on split C_8.

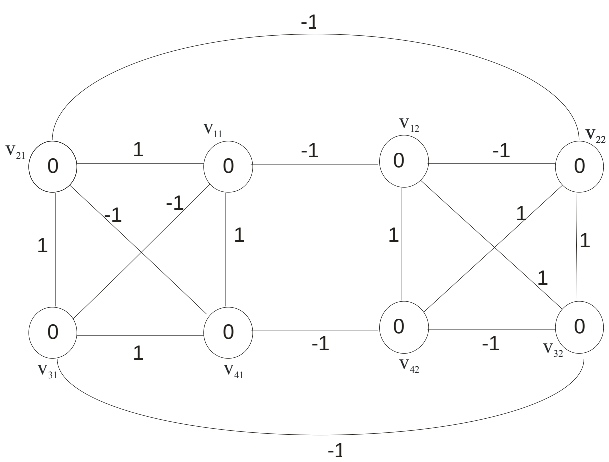

Figure 8. Zero-M-Cordial labeling on $k_4 \times p_2$.

Define $f : E(G) \to \{1, -1\}$ as follows

For $1 \le i, k \le 2n-2$

$f(v_{i_1}, v_{k_1}) = 1$

For $2n-2 < i, k \le 2n$

$f(v_{i_1}, v_{k_1}) = -1$

For $1 \le i, k \le 2n-2$

$f(v_{i_2}, v_{k_2}) = 1$

For $2n-2 < i, k \le 2n$

$f(v_{i_2}, v_{k_2}) = -1$

For $1 \le i \le n+1$

$f(v_{i_1}, v_{i_2}) = -1$

The total number of edges labeled with $1'^s$ and $-1'^s$ are given by $e_f(1) = n/2$ and $e_f(-1) = n/2$. Therefore the total difference between the edges labeled with $-1'^s$ and $1'^s$ are given by $|e_f(1) - e_f(-1)| = 0$, differ by 0. The induced vertex labels are equal to zero. Thus for each vertex v, $f(v) = 0$ and $|e_f(1) - e_f(-1)| \le 1$.

Hence $W_n \times P_2$ admits a Zero-M-Cordial labeling for odd n. The vertex and the edge conditions are given in **Table 8**.

Figure 9 illustrates the Zero-M-Cordial labeling on $W_3 \times P_2$. Among the sixteen edges eight edges receive the label +1 and the other eight edges label –1.

***Theorem* 2.7:** $B_n = k_{1,n} \times P_2$ (also known as book graph) admits a Zero-M-Cordial labeling for odd n.

Proof: Let G be the graph $K_{1,n} \times P_2$ where n is odd and $V(G) = \{V_{ij} \quad i = 1, 2, \cdots, n+1, j = 1, 2\}$ be the vertices of G.

We note that $|V(G)| = 2(n+1)$ and $|E(G)| = 3n+1$.

Define $f : E(G) \to \{1, -1\}$ as follows

For $1 \le i, k \le n-1$

$f(v_{i_1}, v_{k_1}) = 1$

For $n-1 < i, k \le n$

$f(v_{i_1}, v_{k_1}) = -1$

For $1 \le i, k \le n-1$

$f(v_{i_2}, v_{k_2}) = 1$

For $n-1 < i, k \le n$

$f(v_{i_2}, v_{k_2}) = -1$

For $1 \le i \le n$

$f(v_{i_1}, v_{i_2}) = -1$

For $n < i \le n+1$

$f(v_{i_1}, v_{i_2}) = 1$

The total number of edges labeled with $1'^s$ and $-1'^s$ are given by $e_f(1) = n/2$ and $e_f(-1) = n/2$. Therefore the total difference between the edges labeled with $-1'^s$ and $1'^s$ are given by $|e_f(1) - e_f(-1)| = \left|\frac{n}{2} - \frac{n}{2}\right| = 0$, differ by 0. The induced vertex labels are equal to zero. Thus for each vertex v, $f(v) = 0$ and

Table 8. Vertex and the edge condition of $W_n \times P_2$.

n	Vertex condition	Edge condition
Odd	$f(v)=0$	$e_f(1)=e_f(-1)=n/2$

Table 9. Vertex and the edge condition of $B_n = k_{1,n} \times P_2$.

n	Vertex condition	Edge condition
Odd	$f(v)=0$	$e_f(1)=e_f(-1)=n/2$

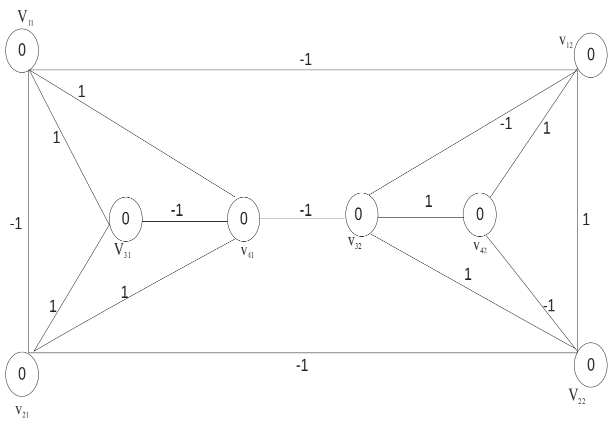

Figure 9. Zero-M-Cordial labeling on $W_3 \times P_2$.

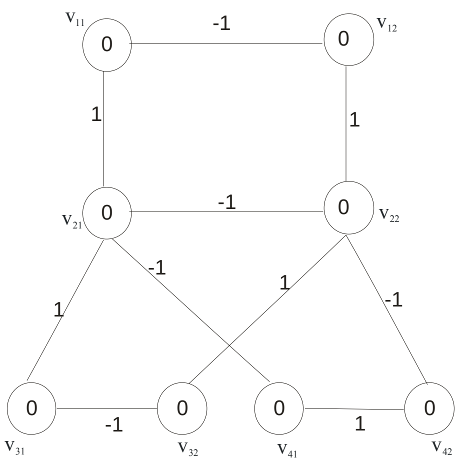

Figure 10. Zero-M-Cordial labeling on $K_{1,3} \times P_2$.

$\left|e_f(1)-e_f(-1)\right| \le 1$.

Hence $B_n = k_{1,n} \times P_2$ (also known as book graph) admits a Zero-M-Cordial labeling for odd *n*. The vertex and the edge conditions are given in **Table 9**.

Figure 10 illustrates Zero-M-Cordial labeling on $K_{1,3} \times P_2$. Among the ten edges five edges receive the label +1 and the other five edges receive the label –1.

3. Concluding Remark

Here we investigate Zero-M-Cordial labeling for Cartesian product of some graphs, join of two cycle graphs, split graphs and bipartite graphs. Similar results can be derived for other graph families and in the context of different graph labeling problem is an open area of research.

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