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Image Mathematics

—Mathematical Intervening Principle Based on "Yin Yang Wu Xing" Theory in Traditional Chinese Mathematics (I)

Yingshan Zhang[1], Weilan Shao[2]

[1]School of Finance and Statistics, East China Normal University, Shanghai, China
[2]The College English Teaching and Researching Department, Xinxiang University, Xinxiang, China
Email: ysh_zhang@163.com, yszhang@stat.ecnu.edu.cn

ABSTRACT

By using mathematical reasoning, this paper demonstrates the mathematical intervening principle: "Virtual disease is to fill his mother but real disease is to rush down his son" (虚则补其母，实则泄其子) and "Strong inhibition of the same time, support the weak" (抑强扶弱) based on "Yin Yang Wu Xing" Theory in image mathematics of Traditional Chinese Mathematics (TCMath). We defined generalized relations and generalized reasoning, introduced the concept of steady multilateral systems with two non-compatibility relations, and discussed its energy properties. Later based on the intervention principle in image mathematics of TCMath and treated the research object of the image mathematics as a steady multilateral system, it has been proved that the mathematical intervening principle is true. The kernel of this paper is the existence and reasoning of the non-compatibility relations in steady multilateral systems, and it accords with the oriental thinking model.

Keywords: Image Mathematics; "Yin Yang Wu Xing" Theory; Steady Multilateral Systems; Opposite
Non-Compatibility Relations; Intervention Rule; Self-Protection Rule; Mathematical Side Effects;
Mathematical Intervention Resistance Problem

1. Main Differences between Traditional Chinese Mathematics and Western Mathematics

In Western Mathematics (mathematics; Greek: $\mu\,\alpha\,\theta\,\eta\,\mu\,\alpha\,\tau\,\iota\,\kappa$), the word comes from the ancient Greek in the west of the $\mu\,\theta\,\eta\,\mu\,\alpha$ (mathēma), its have learning, studying, science, and another relatively narrow meaning and technical sense-"mathematics study", even in its neck comes in. The adjective $\mu\,\alpha\,\theta\,\eta\,\mu\,\alpha\,\tau\,\iota\,\kappa$ (mathēmatikos), meaning and learning about or for the hard, also can be used to index of learning. In English on the function of the plural form of, and in French surface mathematiques plural form, can be back to Latin neutral plural mathematica, due to the Greek plural $\tau\,\alpha\,\mu\,\alpha\,\theta\,\eta\,\mu\,\alpha\,\tau\,\iota\,\kappa$ (tamathēmatika), the Greek by Aristotle brought refers to "all things several" concept. (Latin: Mathemetica) original intention is number and count of the technology.

According to the understanding of the now in Western mathematics, mathematics is the real world number relationship and the form of the space science. Say simply, is the study of the form and number of science. The start of the study is always from the Axiomatic system. Any Axiomatic system of form and number comes from the observations. Because of the demand on life and labor, and even the most original nationality, also know simple count, and the fingers or physical count development to use digital count. Basic mathematics knowledge and use always individual and community essential to life. The basic concept of the refining is as early as in ancient Egypt, Mesopotamia, and in ancient India of ancient mathematical text and see in considerable. Since then, its development will continue to have a modest progress, and the 16th century until the Renaissance, because of the new scientific and found interactions and the generated mathematical innovation leads to the knowledge of the acceleration, until today.

Mathematics is used in different areas of the world, including science, engineering, medicine and economics, etc. The application of mathematics in these fields are usually called applied mathematics, sometimes also stir up new mathematical discovery, and led to the development of new subject. Mathematician also study didn't any application value of pure mathematics, even if the application is found in often after.

The Boolean school, founded in the 1930s in France by Boolean, thinks: mathematics, at least pure mathematics, is the study of the theory of abstract structure.

Structure, it is the initial concept and the deduction system of Axiom. Boolean school also thinks that, there are three basic kinds of abstract structure: algebraic structure (group, ring, the domain...), sequence structure (partial order, all the sequence...), the topological structure (neighborhood, limit, connectivity, dimension...). Mathematics is a kind of transformation, an abstract model, and a sign system, the real world converted into mathematical model, using mathematical language describe them later, after operation, the results can back, explained in the real world specific scientific.

There is a basic logic that the human is everything, they can be observed to establish an Axiom system, make nature operations according to the Axiom system by human assumption. But in the traditional Chinese philosophy, always think that humans are the small, they can not establish a set of rules, let nature operation in accordance with the running rules of the human assumption. In front to nature, human's only doing things is its behavior request, with its development with nature.

In other words, the western mathematics thinks the mathematical object of study is a simple system, which can be observed to establish an Axiom system, and then logical analysis. It is because a simple system can be assumed. But the image mathematics of the traditional Chinese mathematics thinks the mathematical object of study is a complex system, human can't do specific research object hypothesis, humans can only be clear, for general object of study (model-free), what kind of logic structure analysis can reach the humans to understand the research object of certain relations. It is because a complex system cannot be assumed.

Simple said: the western mathematics deals directly with the research object through the directly observed, but the image mathematics of the traditional Chinese mathematics researches object through the relationship between indirect processing analysis.

In fact, Western mathematics late nineteenth century was introduced into China, initially, "mathematics" to be directly translated as "arithmetic (算术)", and then said "arithmetic learn (算学)", and then they changed to "mathematics (数学)" words. But the ancient Chinese for this concept, in 3000 years ago, has been officially use the word "Gua (卦)" as the form, the "Xiang (象)" as the number. In the Yi-Jing ("易经"), this "mathematics (数学)" concept is defined as "Image Mathematics (象数学)". It is part of the Traditional Chinese Mathematics (TCMath). This article mainly concerns image mathematical content in TCMath, so also said the image mathematics as TCMath. Image mathematics generally contains hexagrams ("Gua (卦)") and images ("Xiang (象)") two content. The hexagrams ("Gua (卦)") is only the hexagrams mathematical symbols, which there is not the size since the size of number is about the definition of

human beings. In general, the research object in the complex system is independent of human definition. The image ("Xiang (象)") is the study way or the calculation method for some mathematical indexes in order to know the objective existence of the fixed a state. The way or the calculation method for some mathematical indexes is independent of the complex system and only is Human's some methods of operation in order to study the relationship of the complex system.

The ancients speak of "mathematics" in Chinese is a word as the way for running (intervening and controlling) a complex system through the analysis of the use of hexagrams ("Gua (卦)") and images ("Xiang (象)"). This and what we now understand the mathematical completely different things. In other words, in the image mathematics of Traditional Chinese Mathematics (TCMath), both intervening and controlling of an engineering are believed to as a complex system. It is because to run an engineering is difficult and complex in which there are the loving relation, the killing relation and the equivalent relation among many Axiom systems. The loving and killing relations are non-compatibility relations, which can compose the whole energy of the system greater than or less than the sum of each part energy of the system, respectively, rarely equal conditions. Mathematics means managing or controlling or intervening for the complex system through the analysis of the use of hexagrams ("Gua (卦)") and images ("Xiang (象)"), and so on. Pursue the goal is the harmonious sustainable of the complex system in order to compose the complex system not outward expansion development. Generally speaking, the assumption involved the behavior of people is not needed since the system is complex.

But, in Western mathematics, mathematics means first through the observation to establish one Axiom system, then performing mathematical inference from the Axiom system. Both obtaining and inference of reasoning are believed to as a simple system, because all the conclusions and definition of are compatible with the Axiom system. Compared with Axiom system speaking, there are true and false. Major mathematical analysis method is to judge true or false from simple assumptions or simple model. It is because to obtain the true and false relationship of one Axiom system is easy and simple in which there is only a compatibility relation or a generalized equivalent relation under one Axiom system assumption. The compatibility relation or generalized equivalent relation can compose the whole energy of the system equal to the sum of each part energy of the system. Thus to obtain or to analyze under one Axiom system assumption can compose the simple system outward expansion development. Therefore, pursue the goal is for obtaining or analyzing in order to compose the simple system outward expansion development. Generally speaking, the various

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hypothetical models involves the behavior of people. This phenomenon in the image mathematics of TCMath is not allowed since a complex system cannot be supposed. Both true and false cannot be judged if the Axiom system has not been assumed.

Western mathematics using simple assumptions or simple models treats directly mathematical complex system from Microscopic point of view, always destroy the original mathematical complex system's balance, and has none beneficial to mathematical complex system's immunity. Western mathematical intervention method can produce imbalance of mathematical complex system, having strong side effects. Excessively using methods of mathematical intervention for a complex system can easily paralysis the mathematical complex system's immunity, which the debate of mathematical schools under different from Axiom systems is a product of Western mathematics since there are a number of Axiom systems in nature which are different to the people of faith. Using the method of mathematical intervention for a complex system too little can easily produce the mathematical intervention resistance problem.

The image mathematics of TCMath studies the world from the Macroscopic point of view, and its target is in order to maintain the original balance of mathematical complex system and in order to enhance the mathematical complex system's immunity. The image mathematics believes that each mathematical intervention has one-third of badness. She never encourage government to use mathematical intervention in long term. The ideal way is *Wu Wei Er Wu Bu Wu* (无为而无不为)—by doing nothing, everything is done. The image mathematics has over 5000-year history. It has almost none side effects or mathematical intervention resistance problem.

After long period of practicing, our ancient mathematical scientists use "Yin Yang Wu Xing" Theory extensively in the image mathematics to explain the origin of mathematical complex system, the law of mathematical complex system, mathematical changes, mathematical diagnosis, mathematical prevention, mathematical self-protection and mathematical intervening. It has become an important part of the image mathematics. "Yin Yang Wu Xing" Theory has a strong influence to the formation and development of traditional Chinese mathematical theory. As is known to all, China in recent decades, economy and related mathematical work have made great strides in development. Its reason is difficult to say the introduction of western mathematics, the fact that the Chinese traditional culture is in all kinds of mathematical decision plays a role. Her many mathematical intervening methods come from the traditional Chinese medicine since both human body and mathematical research objects of image mathematics are all complex systems. But, many Chinese and foreign schol-

ars still have some questions on the reasoning of image mathematics, such as Traditional Chinese Medicine which is due to image mathematics. In this article, we will start to the western world for presentation of image mathematics introduced some mathematical and logic analysis concept.

Zhang's theories, multilateral matrix theory [1] and multilateral system theory [2-19], have given a new and strong mathematical reasoning method from macro (Global) analysis to micro (Local) analysis. He and his colleagues have made some mathematical models and methods of reasoning [20-35], which make the mathematical reasoning of image mathematics possible based on "Yin Yang Wu Xing" Theory [36-38]. This paper will use steady multilateral systems to demonstrate the intervening principle of image mathematics: "Real (mathematical) disease is to rush down his son but virtual (mathematical) disease is to fill his mother" and "Strong inhibition of the same time, support the weak".

The article proceeds as follows. Section 2 contains basic concepts and main theorems of steady multilateral systems while the intervening principle of image mathematics is demonstrated in Section 3. Some discussions in image mathematics are given in Section 4 and conclusions are drawn in Section 5.

2. Basic Concept of Steady Multilateral Systems

In the real world, we are enlightened from some concepts and phenomena such as "biosphere", "food chain", "ecological balance" etc. With research and practice, by using the theory of multilateral matrices [1] and analyzing the conditions of symmetry [20-24] and orthogonality [25-35] what a stable complex system must satisfy, in particular, with analyzing the basic conditions what a stable working procedure of good product quality must satisfy [10,29], we are inspired and find some rules and methods, then present the logic model of analyzing stability of complex systems-steady multilateral systems [2-19]. There are a number of essential reasoning methods based on the stable logic analysis model, such as "transition reasoning", "atavism reasoning", "genetic reasoning" etc. We start and still use concepts and notations in papers [3-6].

2.1. Generalized Relations and Reasoning

Let V be a non-empty set and $V \times V = \{(x, y) : x \in V, y \in V\}$. The non-empty subset $R \subset V \times V$ is called a relation of V. Image mathematics mainly researches general relation rules for general V rather than for special V. So the general V cannot be supposed. We can only do matter is to research the structure of the set of relations $\Re = \{R_0, \cdots, R_{m-1}\}$.

For any relation set $\Re = \{R_0, \cdots, R_{m-1}\}$, we can define both an inverse relationship of $R_i \in \Re$ and a relation multiplication between $R_i \in \Re$ and $R_j \in \Re$ as follows:

$$R_i^{-1} = \{(x,y) : (y,x) \in R_i\}$$

and $R_i * R_j = \{(x,y) :$ there is at least a $u \in V$ such that $(x,u) \in R_i$ and $(u,y) \in R_j\}$.

The relation $R_i \in \Re$ is called reasonable if $R_i^{-1} \in \Re$. A generalized reasoning of general V is defined as for $R_i * R_j \neq \varnothing$ there is a relation $R_k \in \Re$ such that $R_i * R_j \subset R_k$.

The generalized reasoning satisfies the associative law of reasoning, i.e., $(R_i * R_j) * R_k = R_i * (R_j * R_k)$. This is the basic requirement of reasoning in TCMath. But there are a lot of reasoning forms which do not satisfy the associative law of reasoning in Western Science. For example, in true and false binary of proposition logic, the associative law does not hold on its reasoning because

$$(false * false) * false = true * false = false$$
$$\neq true = false * true = false * (false * false).$$

2.2. Equivalence Relations

Let V be a non empty set and R_0 be its a relation. We call it **an equivalence relation**, denoted by ~, if the following three conditions are all true:

1) *Reflexive*: $(x,x) \in R_0$ for all $x \in V$, i.e., $x \sim x$;

2) *Symmetric*: if $(x,y) \in R_0$, then $(y,x) \in R_0$, i.e., if $x \sim y$, then $y \sim x$;

3) *Conveyable (Transitivity)*: if $(x,y) \in R_0$, $(y,z) \in R_0$, then $(x,z) \in R_0$, i.e., if $x \sim y$, $y \sim z$, then $x \sim z$.

Furthermore, the relation R is called a compatibility relation if there is a non-empty subset $R_1 \subset R$ such that R_1 satisfies at least one of the conditions above. And the relation R is called a non-compatibility relation if there doesn't exist any non-empty subset $R_1 \subset R$ such that R_1 satisfies any one of the conditions above. Any one of compatibility relations can be expanded into an equivalent relation to some extent [2].

Western Science only considers the reasoning under one Axiom system such that only compatibility relation reasoning is researched. However there are many Axiom systems in Nature. Traditional Chinese Science mainly researches the reasoning among many Axiom systems in Nature. Of course, she also considers the reasoning under one Axiom system but she only expands the reasoning as the equivalence relation reasoning.

2.3. Two Kinds of Opposite Non-Compatibility Relations

Equivalence relations, even compatibility relations, can not portray the structure of the complex systems clearly. In the following, we consider two non-compatibility relations.

In image mathematics, any Axiom system is not considered, but should first consider to use a logic system. Believe that the rules of Heaven and the behavior of Human can follow the same logic system (天人合一). This logic system is equivalent to a group of computation. The method is to abide by the selected logic system to the research object classification, without considering the specific content of the research object, namely classification taking images (比类取象). Analysis of the relationship between research object, make relationships with computational reasoning comply with the selected logic system operation. And then in considering the research object of the specific content of the conditions, according to the logic of the selected system operation to solve specific problems. In mathematics, the method of classification taking images is explained in the following Definition 2.1.

Definition 2.1. *Suppose that there exists a finite group* $G^m = \{g_0, \cdots, g_{m-1}\}$ *of order m where g_0 is identity. Let V be a none empty set satisfying that* $V = V_{g_0} + \cdots + V_{g_{m-1}}$ *where the notation means that* $V = V_{g_0} \cup \cdots \cup V_{g_{m-1}}$, $V_{g_i} \cap V_{g_j} = \varnothing, \forall i \neq j$ *(the following the same).*

In image mathematics, the V_{g_j} *is first called a factor image of group element* g_j *for any j, and* $V = V_{g_0} + \cdots + V_{g_{m-1}}$ *is called a factor space (all "Gua (卦)"). We do not consider the factor size (class variables) and only consider it as mathematical symbols ("Gua (卦)"), such as, 0 or 1, because the size is defined by a human behavior for V, but we have no assumption of V.*

A mathematical index of the unknown multivariate function $f(x_{g_0}, \cdots, x_{g_{m-1}})$, $\forall x_{g_j} \in V_{g_j}$ $j = 0, \cdots, m-1$ *is called a function image of V. All mathematical indexes of the unknown multivariate function f compose of the formation of a new set, namely image space*

$$F(V) = F_{\omega_0}(V) + \cdots + F_{\omega_{g-1}}(V)$$

where $G^g = \{\omega_0, \cdots, \omega_{g-1}\}$ *is also a finite group of order g. The* $F_{\omega_j}(V)$ *is also called an Axiom system for any j if* $F_{\omega_j}(V) \neq \varnothing$ *because any Axiom system is the assumption of*

$$F(V) = F_{\omega_0}(V) + \cdots + F_{\omega_{g-1}}(V)$$

(or, equivalent, V) in which there is only the compatibility relations, i.e., pursuing the same mathematics index $I(F_{\omega_j}(V))$. *We do not consider the special multivariate function f (i.e., special function image) and only consider the calculation way of the general mathematical indexes of f from the factor space* $V = V_{g_0} + \cdots + V_{g_{m-1}}$ *in order to know some causal relations, because we have no assumption of f. But the size of the data image should be*

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considered if we study specific issues by the general rules of data images.

Say simply, a study of the hexagrams ("Gua (卦)") in image mathematics is to learn the generalized properties of the inputs $x_{g_0}, \cdots, x_{g_{m-1}}$ of any multivariate function f for the given factor space $V = V_{g_0} + \cdots + V_{g_{m-1}}$, such as there are non-size, non-order relation, orthogonal relations and symmetrical relations, and so on. A study of the image ("Xiang (象)") in image mathematics is to learn the generalized properties of all outputs f for the image space $F(V) = F_{\omega_0}(V) + \cdots + F_{\omega_{g-1}}(V)$, such as there are size specific meaning, a sequence of relationship, killing relations, loving relations, equivalent relations, and so on.

Without loss of generality, we put the function image space $F(V) = F_{\omega_0}(V) + \cdots + F_{\omega_{g-1}}(V)$ and the factor image space $V = V_{g_0} + \cdots + V_{g_{m-1}}$, still keep for V because of no assumption of V. In order to study the generalized relations and generalized reasoning, image mathematics researches the following relations.

Denoted $V_{g_i} \times V_{g_j} = \left\{ (x, y) : x \in V_{g_i}, y \in V_{g_j} \right\}$, where the note $\times$ is the usual Cartesian product or cross join. Define relations

$$R_{g_r} = \sum_{g \in G^m} V_g \times V_{gg_r}, \quad r = 0, \cdots, m-1,$$

where $R_{g_0} = R_{g_0}^{-1} = R_{g_0^{-1}}$ is called an equivalence relation of V if g_0 is identity; denoted by $\sim$; $R_{g_s} = R_{g_s}^{-1} = R_{g_s^{-1}}$ is called a symmetrical relation of V if $g_s = g_s^{-1}$, $s \neq 0$; denoted by $\xleftarrow{R_s}$ or $\leftrightarrow$; $R_{g_1} = R_{g_1^{-1}}^{-1}$ is called a neighboring relation of V if $g_1 \neq g_1^{-1}$; denoted by $\xrightarrow{R_1}$ or $\to$; $R_{g_a} = R_{g_a^{-1}}^{-1} \neq R_{g_a^{-1}}, R_{g_1}, R_{g_1^{-1}}$ is called an alternate (or atavism) relation of V if $g_a \neq g_a^{-1}$, g_1, g_1^{-1}, $a \geq 2$; denoted by $\xrightarrow{R_a}{\Rightarrow}$ or $\Rightarrow$. #

In this case, the equivalence relations and symmetrical relations are compatibility relations but both neighboring relations and alternate relations are non-compatibility relations. For the given relation set $\Re = \left\{ R_{g_0}, \cdots, R_{g_{m-1}} \right\}$, these relations R_{g_i} are all reasoning relations since the relation $R_{g_i}^{-1} = R_{g_i^{-1}} \in \Re$ if $R_{g_i} \in \Re$.

The equivalence relation R_{g_0}, symmetrical relations R_{g_s}, neighboring relation R_{g_1} and alternate relations R_{g_a} are all the possible relations for the method of classification taking images. In this paper, we mainly consider the equivalence relation R_{g_0}, neighboring relation R_{g_1} and alternate relations R_{g_a}.

There is an unique generalized reasoning between the

two kinds of opposite non-compatibility relations for case $m = 5$. For example, let V be a none empty set, there are two kinds of opposite relations: the neighboring relation R_1, denoted $\to$ and the alternate (or atavism) relation R_2, denoted $\Rightarrow$. The logic reasoning architecture [2-19] of "Yin Yang Wu Xing" Theory in Ancient China is equivalent to the following reasoning:

1) If $x \to y$, $y \to z$, then $x \Rightarrow z$; i.e., if $(x, y) \in R_1$, $(y, z) \in R_1$, then $(x, z) \in R_2$; or, $R_1 * R_1 \subset R_2$; $\Leftrightarrow$ if $x \to y$, $x \Rightarrow z$, then $y \to z$; i.e., if $(x, y) \in R_1$, $(x, z) \in R_2$, then $(y, z) \in R_1$; or, $R_1^{-1} * R_2 \subset R_1$; $\Leftrightarrow$ if $x \Rightarrow z$, $y \to z$, then $x \to y$; i.e., if $(x, z) \in R_2$, $(y, z) \in R_1$, then $(x, y) \in R_1$; or, $R_2 * R_1^{-1} \subset R_1$.

2) If $x \Rightarrow y$, $y \Rightarrow z$ then $z \to x$; i.e., if $(x, y) \in R_2$, $(y, z) \in R_2$, then $(z, x) \in R_1$; or, $R_2 * R_2 \subset R_1^{-1}$; $\Leftrightarrow$ if $z \to x$, $x \Rightarrow y$, then $y \Rightarrow z$; i.e., if $(z, x) \in R_1$, $(x, y) \in R_2$ then $(y, z) \in R_2$; or, $R_1 * R_2 \subset R_2^{-1}$; $\Leftrightarrow$ if $y \Rightarrow z$, $z \to x$ then $y \Rightarrow z$; i.e., if $(y, z) \in R_2$, $(z, x) \in R_1$, then $(y, z) \in R_2$; or, $R_2 * R_1 \subset R_2^{-1}$.

Let $R_3 = R_2^{-1}$ and $R_4 = R_1^{-1}$. Then above reasoning is equivalent to the calculating as follows:

$$R_i * R_j \subset R_{\mathrm{mod}(i+j,5)}, \forall i, j \in \{1, 2, 3, 4\}$$

where the $\mathrm{mod}(i+j, 5)$ is the addition of module 5.

Two kinds of opposite relations can not be exist separately. Such reasoning can be expressed in **Figure 1**. The first triangle reasoning is known as a jumping-transition reasoning, while the second triangle reasoning is known as an atavism reasoning. Reasoning method is a triangle on both sides decided to any third side. Both neighboring relations and alternate relations are not compatibility relations, of course, none equivalence relations, called non-compatibility relations.

2.4. Genetic Reasoning

Let V be a none empty set with the equivalent relation R_0, the neighboring relation R_1 and the alternate relations $R_a \neq R_1^{-1}$, $a > 1$. Then a genetic reasoning is defined as follows:

1) If $x \sim y$, $y \to z$, then $x \to z$; i.e., if $(x, y) \in R_0$, $(y, z) \in R_1$, then $(x, z) \in R_1$; or, $R_0 * R_1 \subset R_1$;

2) If $x \sim y$, $y \Rightarrow z$, then $x \Rightarrow z$; i.e., if $(x, y) \in R_0$, $(y, z) \in R_2$, then $(x, z) \in R_2$; or, $R_0 * R_2 \subset R_2$;

3) If $x \to y$, $y \sim z$, then $x \to z$; i.e., if $(x, y) \in R_1$, $(y, z) \in R_0$, then $(x, z) \in R_1$; or, $R_1 * R_0 \subset R_1$;

4) If $x \Rightarrow y$, $y \sim z$, then $x \Rightarrow z$; i.e., if $(x, y) \in R_2$, $(y, z) \in R_0$, then $(x, z) \in R_2$; or, $R_2 * R_0 \subset R_2$.

The genetic reasoning is equivalent to that

$$
\begin{array}{ccc}
x & & x \\
\Downarrow \searrow & & \Downarrow \nwarrow \\
z \leftarrow y & & y \Rightarrow z
\end{array}
$$
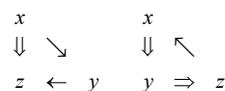
Figure 1. Triangle reasoning.

$$R_0 * R_j = R_j * R_0 = R_j, \forall j \in \{0, 1, \cdots, m-1\} = G_0^m$$

since $R_0 * R_j \supset R_j$, $R_j * R_0 \supset R_j$. The genetic reasoning is equivalent to that there is a group $G_0^m = \{0, \cdots, m-1\}$ with the operation * such that V can be cut into $V = V_0 + \cdots + V_{m-1}$ where V_i may be an empty set and the corresponding relations of reasoning can be written as the forms as follows

$$R_r = \sum_{i=0}^{m-1} V_i \times V_{i*r}, r = 0, \cdots, m-1,$$

satisfying $R_i * R_j \subset R_{i*j}, \forall i, j \in G_0^m$.

2.5. Steady Multilateral Systems

For a none empty set V and its a relation set $\Re = \{R_0, \cdots, R_{m-1}\}$, the form $(V, \Re)$ (or simply, V) is called a multilateral system [2-19], if $(V, \Re)$ satisfies the following properties:

1) $R_0 + \cdots + R_{m-1} \subset V \times V$, i.e. $R_i \cap R_j = \varnothing, \forall i \neq j$.
2) $R_0 * R_j = R_j * R_0 = R_j, \forall j \in \{0, 1, \cdots, m-1\} = G_0^m$
3) The relation $R_i^{-1} \in \Re$ if $R_i \in \Re$.
4) For $R_i * R_j \neq \varnothing$, there is a relation $R_k \in \Re$ such that $R_i * R_j \subset R_k$.

The 4) is called the reasoning, the 1) the uniqueness of reasoning, the 2) the hereditary of reasoning (or genetic reasoning) and the 3) the equivalent property of reasoning of both relations $R_i \in \Re$ and $R_i^{-1} \in \Re$, i.e., the reasoning of $R_i \in \Re$ is equivalent to the reasoning of $R_i^{-1} \in \Re$. In this case, the two-relation set $\{R_i, R_i^{-1}\}$ is a lateral relation of $(V, \Re)$. The R_0 is called an equivalence relation. The multilateral system $(V, \Re)$ can be written as $(V_0 + \cdots + V_{n-1}, \{R_0, \cdots, R_{m-1}\})$. Furthermore, the V and $\Re$ are called the state space and relation set considered of $(V, \Re)$, respectively. For a multilateral system $(V, \Re)$, it is called complete (or, perfect) if "$\subset$" changes into "$=$". And it is called complex if there exists at least a non-compatibility relation $R_i \in \Re$. In this case, the multilateral system is also called a logic analysis model of complex systems.

Let R_1 be a non-compatibility relation. A complex multilateral system

$$(V, \Re) = (V_0 + \cdots + V_{n-1}, \{R_0, \cdots, R_{m-1}\})$$

is said as a steady multilateral system (or, a stable multilateral system) if there exists a number n such that $R_1^{*n} = R_0$ where $R_1^{*n} = R_1 \overset{n}{* * *} R_1$. The condition is equivalent to there is a the chain $x_1, \cdots, x_n \in V$ such that $(x_1, x_2) \in R_1, \cdots, (x_n, x_1) \in R_1$, or $x_1 \to x_2 \to \cdots \to x_n \to x_1$. The steady multilateral system is equivalent to the complete multilateral system. The stability definition given above, for a relatively stable system, is most essential. If there is not the chain or circle, then there will be some

elements without causes or some elements without results in a system. Thus, this system is to be in the state of finding its results or causes, i.e., this system will fall into an unstable state, and there is not any stability to say.

Theorem 2.1. The system $(V, \Re)$ is a multilateral system if and only if there exists a finite group $G^m = \{g_0, \cdots, g_{m-1}\}$ of order m where g_0 is identity such that the relation set $\Re = \{R_{g_0}, \cdots, R_{g_{m-1}}\}$ satisfying $R_{g_i} * R_{g_j} \subset R_{g_i g_j}, \forall i, j \in \{0, 1, \cdots, m-1\}$.#

In this case, the multilateral system $(V, \Re)$ can be written as $(V_{g_0} + \cdots + V_{g_{m-1}}, \{R_{g_0}, \cdots, R_{g_{m-1}}\})$ satisfying $R_{g_r} = \sum_{g \in G^m} V_g \times V_{g g_r}, r = 0, \cdots, m-1$, where V_{g_i} may be an empty set.

Theorem 2.2. If the multilateral system $(V, \Re) = (V_0 + \cdots + V_{n-1}, \{R_0, \cdots, R_{m-1}\})$ is a steady multilateral system, then $n = m$ and $\Re = \{R_0, \cdots, R_{m-1}\}$ is a finite group of order m about the relation multiplication $R_i * R_j = R_k$ where V_i must be a non empty set. #

Definition 2.2. Suppose that a multilateral system $(V, \Re)$ can be written as $(V_{g_0} + \cdots + V_{g_{m-1}}, \{R_{g_0}, \cdots, R_{g_{m-1}}\})$ satisfying $R_{g_r} = \sum_{g \in G^m} V_g \times V_{g g_r}, r = 0, \cdots, m-1$ and

$$R_{g_i} * R_{g_j} \subset R_{g_i g_j}, \forall i, j \in \{0, 1, \cdots, m-1\}.$$

The group $G^m = \{g_0, \cdots, g_{m-1}\}$ of order m where g_0 is identity is called the representation group of the multilateral system $(V, \Re)$. The representing function of R_{g_r} is defined as follows

$$I(R_{g_r}) = \{(x, y) : x^{-1} y = g_r, x, y \in G^m\}, r = 0, \cdots, m-1.$$

Let multilateral systems $(V^i, \Re^i), i = 1, 2$ be with two representation groups $G_i, i = 1, 2$, respectively. Both multilateral systems $(V^i, \Re^i), i = 1, 2$ are called isomorphic if the two representation groups $G_i, i = 1, 2$ are isomorphic. #

Theorems 2.1 and 2.2 and Definitions 2.1 and 2.2 are key concepts in multilateral system theory because they show the classification taking images as the basic method. In the following, introduce two basic models to illustrate the method.

Theorem 2.3 Suppose that $G_0^2 = \{0, 1\}$ with multiplication table

*	0	1
0	0	1
1	1	0

i.e., the multiplication of G_0^2 is the addition of module 2. In other words, $i * j = \mod(i + j, 2)$.

And assume that $(V,\Re) = (V_0 + V_1,\{R_0,R_1\})$ *satisfying*

$$R_r = \sum_{i=0}^{1} V_i \times V_{\text{mod}(i+r,2)}, r = 0,1,$$

$$R_i * R_j \subset R_{\text{mod}(i+j,2)}, \forall i,j \in G_0^2.$$

Then $(V,\Re)$ *is a steady multilateral system with one equivalent relation* R_0 *and one symmetrical relation* R_1 *which is a simple system since there is not any non-compatibility relation. In other words, the relations* R_i*'s are the simple forms as follows*:

$$I(R_0) = \{(0,0),(1,1)\}, \quad I(R_1) = \{(0,1),(1,0)\}$$

where (i,j) *is corresponding to* $V_i \times V_j$. #

It will be proved that the steady multilateral system in Theorem 2.3 is the reasoning model of "*Tao*" (道) in TCMath if there are two energy functions $\varphi(V_0)$ and $\varphi(V_1)$ satisfying $\varphi(V_0) > \varphi(V_1)$, called Dao model, denoted by V^2.

Theorem 2.4. *For each element x in a steady multilateral system V with two non-compatibility relations, there exist five equivalence classes below*:

$$X = \{y \in V | y \sim x\}, \quad X_S = \{y \in V | x \to y\},$$

$$X_K = \{y \in V | x \Rightarrow y\}, \quad K_X = \{y \in V | y \Rightarrow x\},$$

$$S_X = \{y \in V | y \to x\}$$

which the five equivalence classes have relations in **Figure 2**. #

It can be proved that the steady multilateral system in Theorem 2.4 is the reasoning model of "Yin Yang Wu Xing" in TCMath if there are five energy functions (Defined in Section 3) $\varphi(X)$, $\varphi(X_S)$, $\varphi(X_K)$, $\varphi(K_X)$ and $\varphi(S_X)$ satisfying

$$\varphi(X_K) > \varphi(X_S) > \varphi(X) > \varphi(K_X) > \varphi(S_X)$$

called Wu-Xing model, denoted by V^5.

By Definition 2.2, the Wu-Xing model V^5 can be written as follows:
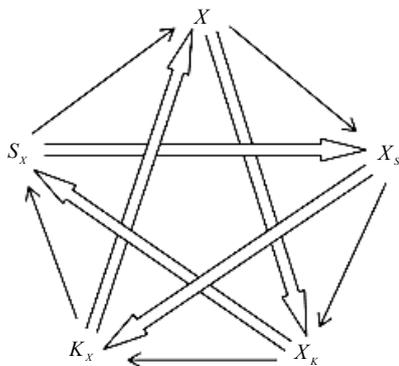
Figure 2. The method of finding Wu-Xing.

Define $V_0 = X$, $V_1 = X_S$, $V_2 = X_K$, $V_3 = K_X$, $V_4 = S_X$, corresponding to wood, fire, soil, metal, water, and assume $V = V_0 + V_1 + \cdots + V_4$ and $\Re = \{R_0, R_1, \cdots, R_4\}$, satisfying

$$R_r = \sum_{i=0}^{4} V_i \times V_{\text{mod}(i+r,5)}, r \in G_0^5,$$

$$R_i * R_j \subset R_{\text{mod}(i+j,5)}, \forall i,j \in G_0^5$$

i.e., the relation multiplication of V^5 is isomorphic to the addition of module 5. Then V^5 is a steady multilateral system with one equivalent relation R_0 and two non-compatibility relations $R_1 = R_4^{-1}$ and $R_2 = R_3^{-1}$.

These Theorems can been found in [1-6,11-16]. Figure in Theorem 2.4 is the Figure of "Yin Yang Wu Xing" Theory in Ancient China. The steady multilateral system V with two non-compatibility relations is equivalent to the logic architecture of reasoning model of "Yin Yang Wu Xing" Theory in Ancient China. What describes the general method of complex systems can be used in mathematical complex system.

3. Relationship Analysis of Steady Multilateral Systems

3.1. Energy of a Multilateral System

Energy concept is an important concept in Physics. Now, we introduce this concept to the multilateral systems (or image mathematics) and use these concepts to deal with the multilateral system diseases (mathematical index too bad or too good).In mathematics, a multilateral system is said to have Energy (or Dynamic) if there is a none negative function $\varphi(*)$ which makes every subsystem meaningful of the multilateral system.

For two subsystems V_i and V_j of multilateral system V, denote $V_i \to V_j$ (or $V_i \Rightarrow V_j$, or $V_i \sim V_j$, or $V_i \leftrightarrow V_j$) means $x_i \to x_j$, $\forall x_i \in V_i$, $x_j \in V_j$ (or $x_i \Rightarrow x_j$, $\forall x_i \in V_i$, $x_j \in V_j$, or $x_i \sim x_j$, $\forall x_i \in V_i$, $x_j \in V_j$, or $x_i \leftrightarrow x_j$, $\forall x_i \in V_i$, $x_j \in V_j$).

For subsystems V_i and V_j where $V_i \cap V_j = \varnothing$, $\forall i \neq j$. Let $\varphi(V_i)$, $\varphi(V_j)$ and $\varphi(V_i,V_j)$ be the energy function of V_i, the energy function of V_j and the total energy of both V_i and V_j, respectively.

For an equivalence relation $V_i \sim V_j$, if

$$\varphi(V_i,V_j) = \varphi(V_i) + \varphi(V_j)$$

(the normal state of the energy of $V_i \sim V_j$), then the neighboring relation $V_i \sim V_j$ is called that V_i likes V_j which means that V_i is similar to V_j. In this case, the V_i is also called the brother of V_j while the V_j is also called the brother of V_i. In the causal model, the V_i is called the similar family member of V_j while the V_j is also called the similar family member of V_i. There are not any causal relation considered between V_i and V_j.

For a symmetrical relation $V_i \leftrightarrow V_j$, if

$$\varphi(V_i, V_j) = \varphi(V_j, V_i) = \varphi(V_i) + \varphi(V_j) + \tau(V_i, V_j)$$

(the normal state of the energy of $V_i \leftrightarrow V_j$) where the $\tau(V_i, V_j)$ is an interaction of V_i and V_j satisfying $\tau(V_i, V_j) = \tau(V_j, V_i)$, then the symmetrical relation $V_i \leftrightarrow V_j$ is called that V_i is corresponding to V_j which means that V_i is positively (or non-negatively) corresponding to V_j if $\tau(V_i, V_j) > 0$ (or $\tau(V_i, V_j) \geq 0$) and that V_i is negatively corresponding to V_j if $\tau(V_i, V_j) < 0$. In this case, the V_i is also called the counterpart of V_j while the V_j is also called the counterpart of V_i. In the causal model, the V_i is called the reciprocal causation of V_j while the V_j is also called the reciprocal causation of V_i. There is a reciprocal causation relation considered between V_i and V_j.

For an neighboring relation $V_i \to V_j$, if

$$\varphi(V_i, V_j) > \varphi(V_j, V_i) > \varphi(V_i) + \varphi(V_j)$$

(the normal state of the energy of $V_i \to V_j$), then the neighboring relation $V_i \to V_j$ is called that V_i bears (or loves) V_j [or that V_j is born by (or is loved by) V_j] which means that V_i is beneficial on V_j each other. In this case, the V_i is called the mother of V_j while the V_j is called the son of V_i. In the causal model, the V_i is called the beneficial cause of V_j while the V_j is called the beneficial effect of V_i.

For an alternate relation $V_i \Rightarrow V_j$, if

$$\varphi(V_i, V_j) < \varphi(V_j, V_i) < \varphi(V_i) + \varphi(V_j)$$

(the normal state of the energy of $V_i \Rightarrow V_j$), then the alternate relation $V_i \Rightarrow V_j$ is called as that V_i kills (or hates) V_j [or that V_j is killed by (or is hated by) V_i] which means that V_i is harmful on V_j each other. In this case, the V_i is called the bane of V_j while the V_j is called the prisoner of V_i. In the causal model, the V_i is called the harmful cause of V_j while the V_j is called the harmful effect of V_i.

In the future, if not otherwise stated, any equivalence relation is the liking relation, any symmetrical relation is the reciprocal causation relation, any neighboring relation is the born relation (or the loving relation), and any alternate relation is the killing relation.

Suppose V is a steady multilateral system having energy, then V in the multilateral system during normal operation, its energy function for any subsystem of the multilateral system has an average (or expected value in Statistics), this state is called normal when the energy function is nearly to the average. Normal state is the better state.

That a subsystem of a multilateral system is not running properly (or, abnormal), is that the energy deviation from the average of the subsystems is too large (or too big), the high (mathematical real disease or economic overheating or real disease) or the low (mathematical virtual disease or economic downturn or virtual disease). Both mathematical real disease and mathematical virtual disease are all diseases of mathematical complex systems.

In a subsystem of a multilateral system being not running properly, if this sub-system through the energy of external forces increase or decrease, making them return to the average (or expected value), this method is called intervention (or making a mathematical treatment) to the multilateral system.

The purpose of intervention is to make the multilateral system return to normal state. The method of intervention is to increase or decrease the energy of a subsystem.

What kind of intervening should follow the principle to treat it? For example, Western economics emphasizes direct intervening, but the indirect intervening of oriental economics is required. In mathematics, which is more reasonable?

Based on this idea, many issues are worth further discussion. For example, if an economic intervening has been done to an economic society, what situation will happen?

3.2. Intervention Rule of a Multilateral System

For a steady multilateral system V with two non-compatibility relations, suppose that there is an external force (or an intervening force) on the subsystem X of V which makes it the energy $\varphi(X)$ change increment $\Delta\varphi(X)$, then the energies $\varphi(X_S)$, $\varphi(X_K)$, $\varphi(K_X)$, $\varphi(S_X)$ of other subsystems X_S, X_K, K_X, S_X (defined in Theorem 2.4) of V will be changed by the increments $\Delta\varphi(X_S)$, $\Delta\varphi(X_K)$, $\Delta\varphi(K_X)$ and $\Delta\varphi(S_X)$, respectively.

It is said that the multilateral system has the capability of intervention reaction if the multilateral system has capability to response the intervention force.

If a subsystem X of multilateral system is intervened, then the energies of the subsystems X_S and S_X which have neighboring relations to X will change in the same direction of the force outside on X. We call them beneficiaries. But the energies of the subsystems X_K and K_X which have alternate relations to X will change in the opposite direction of the force outside on X. We call them victims.

In general, there is an essential principle of intervenetion: any beneficial subsystem of X changes in the same direction of X, and any harmful subsystem of X changes in the opposite direction of X. The size of the energy changed is equal, but the direction opposite.

Intervention Rule: In the case of virtual disease or economic downturn, the intervening method of intervention is to increase the energy. If the intervening has been done on X, the energy increment (or, increase degree)

$\left|\Delta\varphi\left(X_S\right)\right|$ of the son X_S of X is greater than the energy increment $\left|\Delta\varphi\left(S_X\right)\right|$ of the mother S_X of X, *i.e.*, the best beneficiary is the son X_S of X. But the energy decrease degree $\left|\Delta\varphi\left(X_K\right)\right|$ of the prisoner X_K of X is greater than the energy decrease degree $\left|\Delta\varphi\left(K_X\right)\right|$ of the bane of X, *i.e.*, the worst victim is the prisoner X_K of X.

In the case of real disease or economic overheating, the intervening method of intervention is to decrease the energy. If the intervening has been done on X, the energy decrease degree $\left|\Delta\varphi\left(S_X\right)\right|$ of the mother S_X of X is greater than the energy decrease degree $\left|\Delta\varphi\left(X_S\right)\right|$ of the son of X, *i.e.*, the best beneficiary is the mother S_X of X. But the energy increment $\left|\Delta\varphi\left(K_X\right)\right|$ of the bane K_X of X is greater than the energy increment $\left|\Delta\varphi\left(X_K\right)\right|$ of the prisoner X_K of X, *i.e.*, the worst victim is the bane K_X of X.

In mathematics, the changing laws are as follows.

1) If $\Delta\varphi\left(X\right)=\Delta>0$, then $\Delta\varphi\left(X_S\right)=\rho_1\Delta$, $\Delta\varphi\left(X_K\right)=-\rho_1\Delta$, $\Delta\varphi\left(K_X\right)=-\rho_2\Delta$, $\Delta\varphi\left(S_X\right)=\rho_2\Delta$;

2) If $\Delta\varphi\left(X\right)=-\Delta<0$, then $\Delta\varphi\left(X_S\right)=-\rho_2\Delta$, $\Delta\varphi\left(X_K\right)=\rho_2\Delta$, $\Delta\varphi\left(K_X\right)=\rho_1\Delta$, $\Delta\varphi\left(S_X\right)=-\rho_1\Delta$;

where $1\geq\rho_1\geq\rho_2\geq0$. Both ρ_1 and ρ_2 are called intervention reaction coefficients, which are used to represent the capability of intervention reaction. The larger ρ_1 and ρ_2, the better the capability of intervention reaction. The state $\rho_1=\rho_2=1$ is the best state but the state $\rho_1=\rho_2=0$ is the worst state.

This intervention rule is similar to force and reaction in Physics. In other words, if a subsystem of multilateral system V has been intervened, then the energy of subsystem which has neighboring relation changes in the same direction of the force, and the energy of subsystem which has alternate relation changes in the opposite direction of the force. The size of the energy changed is equal, but the direction opposite.

In general, ρ_1 and ρ_2 are decreasing functions of the intervention force Δ since the intervention force $\Delta\varphi\left(X\right)$ is easily to transfer all if Δ is small but the intervention force is not easily to transfer all if Δ is large. The energy function of complex system, the stronger the more you use. In order to magnify ρ_1 and ρ_2, should set up a mathematical complex system of the intervention reaction system, and often use it.

Mathematical intervening resistance problem is that such a question, beginning more appropriate mathematical intervening method, but is no longer valid after a period. It is because the capability of intervention reaction is bad, *i.e.*, the intervention reaction coefficients ρ_1 and ρ_2 is too small. In the state $\rho_1=\rho_2=1$, any mathematical intervening resistance problem is non-existence but in the state $\rho_1=\rho_2=0$, mathematical intervening resistance problem is always existence. At this point, the paper advocates the essential principle of intervening to avoid mathematical intervening resistance problems.

3.3. Self-Protection Rule of a Multilateral System

If there is an intervening force on the subsystem X of a steady multilateral system V which makes the energy $\varphi\left(X\right)$ changed by increment $\Delta\varphi\left(X\right)$ such that the energies $\varphi\left(X_S\right)$, $\varphi\left(X_K\right)$, $\varphi\left(K_X\right)$, $\varphi\left(S_X\right)$ of other subsystems X_S, X_K, K_X, S_X (defined in Theorem 2.4) of V will be changed by the increments $\Delta\varphi\left(X_S\right)$, $\Delta\varphi\left(X_K\right)$, $\Delta\varphi\left(K_X\right)$, $\Delta\varphi\left(S_X\right)$, respectively, then can the multilateral system V has capability to protect the worst victim to restore?

It is said that the steady multilateral system has the capability of self-protection if the multilateral system has capability to protect the worst victim to restore. The capability of self-protection of the steady multilateral system is said to be better if the multilateral system has capability to protect all victims to restore.

In general, there is an essential principle of self-protection: any harmful subsystem of X should be protected by using the same intervention force but any beneficial subsystem of X should not.

Self-protection Rule: In the case of virtual disease or economic downturn, the intervening method of intervention is to increase the energy. If the intervening has been done on X by the increment $\Delta\varphi\left(X\right)=\Delta>0$, the worst victim is the prisoner X_K of X which has the increment $\left(-\rho_1\Delta\right)$. Thus the intervening principle of self-protection is to restore the prisoner X_K of X and the restoring method of self-protection is to increase the energy $\varphi\left(X_K\right)$ of the prisoner X_K of X by using the intervention force on X according to the intervention rule. In general, the increase degree is $\left(\rho_3\Delta\right)$ where $\rho_3\leq\rho_1$.

In the case of real disease or economic overheating, the intervening method of intervention is to decrease the energy. If the intervening has been done on X by the increment $\Delta\varphi\left(X\right)=-\Delta<0$, the worst victim is the bane K_X of X which has the increment $\left(\rho_1\Delta\right)$. Thus the intervening principle of self-protection is to restore the bane K_X of X and the restoring method of self-protection is to decrease the energy $\varphi\left(K_X\right)$ of the bane K_X of X by using the same intervention force on X according to the intervention rule. In general, the decrease degree is $\left(-\rho_3\Delta\right)$ where $\rho_3\leq\rho_1$.

In mathematics, the following self-protection laws hold.

1) *If $\Delta\varphi\left(X\right)=\Delta>0$, then the energy of subsystem X_K will decrease the increment $\left(-\rho_1\Delta\right)$, which is the worst victim. So the capability of self-protection increases the energy of subsystem X_K by increment $\left(\rho_3\Delta\right)$ where $\rho_3\leq\rho_1$, in order to restore the worst victim by according to the intervention rule.*

2) *If $\Delta\varphi\left(X\right)=-\Delta<0$, then the energy of subsystem K_X will increase the increment $\left(\rho_1\Delta\right)$, which is the worst victim. So the capability of self-protection decreases the energy of subsystem K_X by increment $\left(-\rho_3\Delta\right)$ where*

$\rho_3 \le \rho_1$, *in order to restore the worst victim by according to the intervention rule.*

In general, $0 \le \rho_3 \le \rho_1 \le 1$. The ρ_1 is the intervention reaction coefficient. The ρ_3 is called an self-protection coefficient, which is used to represent the capability of self-protection. The larger ρ_3, the better the capability of self-protection. The state $\rho_3 = \rho_1 = 1$ is the best state but the state $\rho_3 = 0$ is the worst state of self-protection. According to the general economy of the protection principle, ρ_3 should be not greater than ρ_1 since the purpose of protection is to restore the victims and not reward the victims.

The self-protection rule can be explained as: the general principle of self-protection subsystem is that the most affected is protected firstly, the protection method and intervention force are in the same way.

In general, ρ_3 is also a decreasing function of the intervention force Δ since the worst victim is easily to restore all if Δ is small but the worst victim is not easily to restore all if Δ is large. The energy function of complex system, the stronger the more you use. In order to magnify ρ_3, should set up an economic society of the self-protection system, and often use it.

Theorem 3.1. *Suppose that a steady multilateral system V which has energy function $\varphi(*)$ and capabilities of intervention reaction and self-protection is with intervention reaction coefficients $\rho_1 = \rho_1(\Delta)$ and $\rho_2 = \rho_2(\Delta)$, and with self-protection coefficient $\rho_3 = \rho_3(\Delta)$. If the capability of self-protection wants to restore both subsystems X_K and K_X, then the following statements are true.*

1) In the case of virtual disease, the treatment method is to increase the energy. If an intervention force on the subsystem X of steady multilateral system V is implemented such that its energy $\varphi(X)$ has been changed by increment $\Delta\varphi(X) = \Delta > 0$, then all five subsystems will be changed finally by the increments as follows:

$$\Delta\varphi(X)_2 = \Delta\varphi(X) + \Delta\varphi(X)_1 = (1 - \rho_2\rho_3)\Delta > 0,$$
$$\Delta\varphi(X_S)_2 = \Delta\varphi(X_S) + \Delta\varphi(X_S)_1 = (\rho_1 + \rho_2\rho_3)\Delta > 0,$$
$$\Delta\varphi(X_K)_2 = \Delta\varphi(X_K) + \Delta\varphi(X_K)_1 = -(\rho_1 - \rho_3)\Delta \le 0,$$
$$\Delta\varphi(K_X)_2 = \Delta\varphi(K_X) + \Delta\varphi(K_X)_1 = -(\rho_2 - \rho_1\rho_3)\Delta \le 0,$$
$$\Delta\varphi(S_X)_2 = \Delta\varphi(S_X) + \Delta\varphi(S_X)_1 = (\rho_2 - \rho_1\rho_3)\Delta \ge 0$$
$$\forall \Delta\varphi(X) = \Delta > 0.$$
$$(1)$$

2) In the case of real disease, the treatment method is to increase the energy. If an intervention force on the subsystem X of steady multilateral system V is implemented such that its energy $\varphi(X)'$ has been changed by increment $\Delta\varphi(X)' = -\Delta < 0$, then all five subsystems will be changed by the increments as follows:

$$\Delta\varphi(X)'_2 = \Delta\varphi(X) + \Delta\varphi(X)'_1 = -(1 - \rho_2\rho_3)\Delta < 0,$$
$$\Delta\varphi(X_S)'_2 = \Delta\varphi(X_S) + \Delta\varphi(X_S)'_1 = -(\rho_2 - \rho_1\rho_3)\Delta \le 0,$$
$$\Delta\varphi(X_K)'_2 = \Delta\varphi(X_K) + \Delta\varphi(X_K)'_1 = (\rho_2 - \rho_1\rho_3)\Delta \ge 0,$$
$$\Delta\varphi(K_X)'_2 = \Delta\varphi(K_X) + \Delta\varphi(K_X)'_1 = (\rho_1 - \rho_3)\Delta \ge 0,$$
$$\Delta\varphi(S_X)'_2 = \Delta\varphi(S_X) + \Delta\varphi(S_X)'_1 = -(\rho_1 + \rho_2\rho_1)\Delta < 0$$
$$\forall \Delta\varphi(X)' = -\Delta < 0.$$
$$(2)$$

where the $\Delta\varphi(*)_1$'s and $\Delta\varphi(*)'_1$'s are the increments under the capability of self-protection. #

Corollary 3.1. *Suppose that a steady multilateral system V which has energy function $\varphi(*)$ and capabilities of intervention reaction and self-protection is with intervention reaction coefficients $\rho_1 = \rho_1(\Delta)$ and $\rho_2 = \rho_2(\Delta)$, and with self-protection coefficient $\rho_3 = \rho_3(\Delta)$. Then the capability of self-protection can make both subsystems X_K and K_X to be restored at the same time, i.e., the capability of self-protection is better, if and only if $\rho_2 = \rho_1\rho_3$ and $\rho_3 = \rho_1$.* #

Side effects of mathematical intervening problems were the question: in the mathematical intervening process, destroyed the normal balance of non-fall ill subsystem or non-intervention subsystem. By Theorem 3.1 and Corollary 3.1, it can be seen that if the capability of self-protection of the steady multilateral system is better, i.e., the multilateral system has capability to protect all the victims to restore, then a necessary and sufficient condition is $\rho_2 = \rho_1\rho_3$ and $\rho_3 = \rho_1$. General for a complex system of mathematical complex system, the condition $\rho_2 = \rho_1\rho_3$ is easy to meet since it can restore two subsystems by Theorem 3.1, the condition $\rho_3 = \rho_1$ is difficult to meet it only can restore one subsystem by Theorem 3.1. At this point, the paper advocates the principle to avoid any side effects of intervening.

3.4. Mathematical Reasoning of Intervening Principle by Using the Neighboring Relations of Steady Multilateral Systems

Intervening principle by using the neighboring relations of steady multilateral systems is "Virtual disease or mathematical virtual disease is to fill his mother but mathematical real disease is to rush down his son". In order to show the rationality of the intervening principle, it is needed to prove the following theorems.

Theorem 3.2. *Suppose that a steady multilateral system V which has energy function and capabilities of intervention reaction and self-protection is with intervention reaction coefficients $\rho_1 = \rho_1(\Delta)$ and $\rho_2 = \rho_2(\Delta)$, and with self-protection coefficient $\rho_3 = \rho_3(\Delta)$ satisfying*

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$\rho_2 = \rho_1\rho_3$ and $\rho_3 = \rho_1$. *Then the following statements are true.*

In the case of virtual disease, if an intervention force on the subsystem X of steady multilateral system V is implemented such that its energy $\varphi(X)$ increases the increment $\Delta\varphi(X) = \Delta > 0$, then the subsystems S_X, X_K and K_X can be restored at the same time, but the subsystems X and X_S will increase their energies by the increments

$$\Delta\varphi(X)_2 = (1 - \rho_2\rho_3)\Delta\varphi(X)$$
$$= (1 - \rho_2\rho_3)\Delta = (1 - \rho_1^3)\Delta > 0$$

and

$$\Delta\varphi(X_S)_2 = (\rho_1 + \rho_2\rho_3)\Delta\varphi(X)$$
$$= (\rho_1 + \rho_2\rho_3)\Delta = (\rho_1 + \rho_1^3)\Delta > 0,$$

respectively.

On the other hand, in the case of real disease, if an intervention force on the subsystem X of steady multilateral system V is implemented such that its energy $\varphi(X)'$ decreases, i.e., by the increment $\Delta\varphi(X)' = -\Delta < 0$, the subsystems X_S, X_K and K_X can also be restored at the same time, and the subsystems X and X_S will decrease their energies, i.e., by the increments

$$\Delta\varphi(X)'_2 = (1 - \rho_2\rho_3)\Delta\varphi(X)'$$
$$= -(1 - \rho_2\rho_3)\Delta = -(1 - \rho_1^3)\Delta < 0$$

and

$$\Delta\varphi(S_X)'_2 = (\rho_1 + \rho_2\rho_3)\Delta\varphi(X)'$$
$$= -(\rho_1 + \rho_2\rho_3)\Delta = -(\rho_1 + \rho_1^3)\Delta < 0,$$

respectively. #

Theorem 3.3. *For a steady multilateral system V which has energy function $\varphi(*)$ and capabilities of intervening reaction and self-protection, assume intervention reaction coefficients are ρ_1 and ρ_2, and let the self-protection coefficientbe ρ_3, which satisfy $\rho_2 = \rho_1\rho_3$, $\rho_3 = \rho_1$ and $\rho_1 \geq \rho_0$ where $\rho_0 \approx (<)0.5897545123$ (the following the same) is the solution of $2\rho_1^3 + \rho_1 = 1$. Then the following statements are true.*

1) *If an intervention force on the subsystem X of steady multilateral system V is implemented such that its energy $\varphi(X)$ has been changed by increment $\Delta\varphi(X) = \Delta > 0$, then the final increment $(\rho_1 + \rho_2\rho_3)\Delta$ of the energy $\varphi(X_S)$ of the subsystem X_S changed is greater than or equal to the final increment $(1 - \rho_2\rho_3)\Delta$ of the energy $\varphi(X)$ of the subsystem X changed based on the capability of self-protection.*

2) *If an intervention force on the subsystem X of steady multilateral system V is implemented such that its energy $\varphi(X)$ has been changed by increment $\Delta\varphi(X) = -\Delta < 0$, then the final increment $-(\rho_1 + \rho_2\rho_3)\Delta$ of the energy $\varphi(S_X)$ of the subsystem S_X changed is less than or*

equal to the final increment $-(1 - \rho_2\rho_3)\Delta$ of the energy $\varphi(X)$ of the subsystem X changed based on the capability of self-protection. #

Corollary 3.2. *For a steady multilateral system V which has energy function $\varphi(*)$ and capabilities of intervening reaction and self-protection, assume intervention reaction coefficients are ρ_1 and ρ_2, and let the self-protection coefficient be ρ_3, which satisfy $\rho_2 = \rho_1\rho_3$, $\rho_3 = \rho_1$ and $\rho_1 < \rho_0$. Then the following statements are true.*

1) *In the case of virtual disease, if an intervention force on the subsystem X of steady multilateral system V is implemented such that its energy $\varphi(X)$ has been changed by increment $\Delta\varphi(X) = \Delta > 0$, then the final increment $(\rho_1 + \rho_2\rho_3)\Delta$ of the energy $\varphi(X_S)$ of the subsystem X_S changed is less than the final increment $(1 - \rho_2\rho_3)\Delta$ of the energy $\varphi(X)$ of the subsystem X changed based on the capability of self-protection.*

2) *In the case of real disease, if an intervention force on the subsystem X of steady multilateral system V is implemented such that its energy $\varphi(X)$ has been changed by increment $\Delta\varphi(X) = -\Delta < 0$, then the final increment $-(\rho_1 + \rho_2\rho_3)\Delta$ of the energy $\varphi(S_X)$ of the subsystem S_X changed is greater than the final increment $-(1 - \rho_2\rho_3)\Delta$ of the energy $\varphi(X)$ of the subsystem X changed based on the capability of self-protection.* #

By Theorems 3.2 and 3.3 and Corollary 3.2, the intervention method of "Virtual disease or economic downturn is to fill his mother but real disease or economic overheating is to rush down his son" should be often used in case: $\rho_2 = \rho_1\rho_3$, $\rho_3 = \rho_1$ and $\rho_1 \geq \rho_0$ since in this time, $(\rho_1 + \rho_2\rho_3)\Delta \geq (1 - \rho_2\rho_3)\Delta$.

3.5. Mathematical Reasoning of Intervening Principle by Using the Alternate Relations of Steady Multilateral Systems

Intervening principle by using the alternate relations of steady multilateral systems is "Strong inhibition of the same time, support the weak". In order to show the rationality of the intervening Principle, it is needed to prove the following theorems.

Theorem 3.4. *Suppose that a steady multilateral system V which has energy function $\varphi(*)$ and capabilities of intervention reaction and self-protection is with intervention reaction coefficients $\rho_1 = \rho_1(\Delta)$ and $\rho_2 = \rho_2(\Delta)$, and with self-protection coefficient $\rho_3 = \rho_3(\Delta)$. Then the following statements are true.*

Assume there are two subsystems X and X_K of V with an alternate relation such that X encounters virtual disease, and at the same time, X_K befalls real disease. If an intervention force on the subsystem X of steady multilateral system V is implemented such that its energy $\varphi(X)$ has been changed by increment $\Delta\varphi(X) = \Delta > 0$, and at

the same time, another intervention force on the subsystem X_K of steady multilateral system V is also implemented such that its energy $\varphi(X_K)$ has been changed by increment $\Delta\varphi(X_K)' = -\Delta < 0$, then all other subsystems: S_X, K_X and X_S can be restored at the same time, and the subsystems X and X_K will increase and decrease their energies by the same size but the direction opposite, i.e., by the increments

$$\Delta\varphi(X)_3 = (1-\rho_2\rho_3)\Delta\varphi(X)$$
$$= (1-\rho_2\rho_3)\Delta = (1-\rho_1^3)\Delta > 0$$

and

$$\Delta\varphi(X_K)_3 = (1-\rho_2\rho_3)\Delta\varphi(X_K)$$
$$= -(1-\rho_2\rho_3)\Delta = -(1-\rho_1^3)\Delta < 0$$

respectively.

Assume there are two subsystems X and K_X of V with an alternate relation such that X encounters real disease, and at the same time, K_X befalls virtual disease. If an intervention force on the subsystem X of steady multilateral system V is implemented such that its energy $\varphi(X)$ has been changed by increment $\Delta\varphi(X) = -\Delta < 0$, and at the same time, another intervention force on the subsystem K_X of steady multilateral system V is also implemented such that its energy $\varphi(K_X)$ has been changed by increment $\varphi(K_X) = \Delta > 0$, then all other subsystems: S_X, X_K and X_S can be restored at the same time, and the subsystems X and K_X will decrease and increase their energies by the same size but the direction opposite, i.e., by the increments

$$\Delta\varphi(X)_3 = (1-\rho_2\rho_3)\Delta\varphi(X)$$
$$= -(1-\rho_2\rho_3)\Delta = -(1-\rho_1^3)\Delta < 0$$

and

$$\Delta\varphi(K_X)_3 = (1-\rho_2\rho_3)\Delta\varphi(K_X)$$
$$= (1-\rho_2\rho_3)\Delta = (1-\rho_1^3)\Delta > 0$$

respectively. #

By Theorems 3.3 and 3.4 and Corollary 3.2, the method of "Strong inhibition of the same time, support the weak" should be used in case: $\rho_2 = \rho_1\rho_3$, $\rho_3 = \rho_1$ and $\rho_1 < \rho_0$ since $(\rho_1 + \rho_2\rho_3)\Delta < (1-\rho_2\rho_3)\Delta$.

4. Rationality of Intervening Principle of Traditional Chinese Mathematics and "Yin Yang Wu Xing" Theory

4.1. Chinese Traditional Mathematics and "Yin Yang Wu Xing" Theory

Ancient Chinese "Yin Yang Wu Xing" [38] Theory has been surviving for several thousands of years without

dying out, proving it reasonable to some extent. If we regard $\sim$ as the same category, the neighboring relation $\rightarrow$ as beneficial, harmony, obedient, loving, etc. and the alternate relation $\Rightarrow$ as harmful, conflict, ruinous, killing, etc., then the above defined stable logic analysis model is similar to the logic architecture of reasoning of "Yin Yang Wu Xing". Both "Yin" and "Yang" mean that there are two opposite relations in the world: harmony or loving $\rightarrow$ and conflict or killing $\Rightarrow$, as well as a general equivalent category $\sim$. There is only one of three relations $\sim$, $\rightarrow$ and $\Rightarrow$ between every two objects. Everything X makes something ($X_S \neq \varnothing$), and is made by something ($S_X \neq \varnothing$); Everything restrains something ($X_K \neq \varnothing$), and is restrained by something ($K_X \neq \varnothing$); i.e., one thing overcomes another thing and one thing is overcome by another thing. The ever changing world V, following the relations: $\sim$, $\rightarrow$ and $\Rightarrow$, must be divided into five categories by the equivalent relation $\sim$, being called "Wu Xing": wood (X), fire (X_S), soil (X_K), metal (K_X), water (S_X). The "Wu Xing" is to be "neighbor is friend": wood (X) $\rightarrow$ fire (X_S) $\rightarrow$ soil (X_K) $\rightarrow$ metal (K_X) $\rightarrow$ water (S_X) $\rightarrow$ wood (X), and "alternate is foe": wood (X) $\Rightarrow$ soil (X_K) $\Rightarrow$ water (S_X) $\Rightarrow$ fire (X_S) $\Rightarrow$ metal (K_X) $\Rightarrow$ wood (X). In other words, the ever changing world must be divided into five categories:

$$V = X + X_S + X_K + K_X + S_X$$

satisfying

$$X \rightarrow X_S \rightarrow X_K \rightarrow K_X \rightarrow S_X \rightarrow X$$

and

$$X \Rightarrow X_K \Rightarrow S_X \Rightarrow X_S \Rightarrow K_X \Rightarrow X$$

where elements in the same category are equivalent to one another. We can see, from this, the ancient Chinese "Yin Yang Wu Xing" theory is a reasonable logic analysis model to identify the stability and relationship of complex mathematical systems.

Image mathematics firstly uses the verifying relationship method of "Yin Yang Wu Xing" Theory to explain the relationship between mathematical complex system and environment. Secondly, based on "Yin Yang Wu Xing" Theory, the relations of development processes of mathematical complex system can be shown by the neighboring relation and alternate relation of five subsets. Then a normal mathematical complex system can be shown as a steady multilateral system in which there are the loving relation and the killing relation and the liking relation. The loving relation in image mathematics can be explained as the neighboring relation, called "Sheng (生)". The killing relation in image mathematics can be explained as the alternate relation, called "Ke (克)". The liking relation can be explained as the equivalent relation, called "Tong-Lei (同类)". Constraints and conversion

between five subsets are equivalent to the two kinds of triangle reasoning. So a normal mathematical complex system can be classified into five equivalence classes corresponding five mathematical indexes, respectively.

For example, in image mathematics, a mathematical complex system is similar to a human body. A mathematical index system of normal complex system following the "Yin Yang Wu Xing" Theory was classified into five equivalence classes as follows [29]:

Consider a complex system, its input $x_1, \cdots, x_m, \omega$ and output y can be written as

$$y = f(x_1, \cdots, x_m, \omega) = g(x_1, \cdots, x_m) + \varepsilon_g$$
$$g(x_1, \cdots, x_m) = E(f(x_1, \cdots, x_m, \omega) | x_1, \cdots, x_m), \quad (2)$$
$$\varepsilon_g = f(x_1, \cdots, x_m, \omega) - E(f(x_1, \cdots, x_m, \omega) | x_1, \cdots, x_m),$$

where not only all output functions
$y = f = f(x_1, \cdots, x_m, \omega)$, $g(x_1, \cdots, x_m)$ and ε_g are not known, but also the input variables $x_1, \cdots, x_m, \omega$ are not known. The problem is called model-free.

The inputs $x_1, \cdots, x_m$ are called controllable if they are observed and controlled by human. So

$$g(x_1, \cdots, x_m) = E(f(x_1, \cdots, x_m, \omega) | x_1, \cdots, x_m)$$

can be observed if the controllable inputs $x_1, \cdots, x_m$ can be choose.

The input ω is called uncontrolled if it is not observed or controlled by human. So the freedom model error

$$\varepsilon_g = f(x_1, \cdots, x_m, \omega) - E(f(x_1, \cdots, x_m, \omega) | x_1, \cdots, x_m)$$

can not be assumed. But we can show the following properties:

$$E(\varepsilon_g | x_1, \cdots, x_m) = E(f(x_1, \cdots, x_m, \omega) | x_1, \cdots, x_m)$$
$$- E[E(f(x_1, \cdots, x_m, \omega) | x_1, \cdots, x_m) | x_1, \cdots, x_m] = 0,$$

$$Var(\varepsilon_g^2 | x_1, \cdots, x_m) =: \sigma_{x_1, \cdots, x_m}^2 \geq 0,$$

$$Cov((g(x_1, \cdots, x_m), \varepsilon_{g(x_1, \cdots, x_m)}) | x_1, \cdots, x_m)$$
$$= E((g - Eg)(\varepsilon_g - E\varepsilon_g) | x_1, \cdots, x_m) = 0.$$

The condition is not hypothesis since they can be obtained if f makes the calculation meaningful, such as, if f is continuous.

In general, we can consider the inputs $x_1, \cdots, x_m, \omega$ as independent random variables with continuous distributions $F_1(x_1), \cdots, F_m(x_m), F_\omega(\omega)$, respectively, since the inputs $x_1, \cdots, x_m$ are controllable which can be selected independently by human under some similar conditions. If not independent, by factor analysis can select or-

thogonal factor. This operation of experiment under some similar conditions is equivalent to the uncontrolled input ω is independent of the controllable input $x_1, \cdots, x_m$ since $Cov(g(x_1, \cdots, x_m), \varepsilon_g) = 0$. This operation of experiment that the inputs $x_1, \cdots, x_m$ can be selected independently by human is equivalent to that the inputs $x_1, \cdots, x_m$ are independent random variables one another. For example, take $x_1, \cdots, x_m$ based on an orthogonal array since the orthogonality is equivalent to independence for discrete random variables and continuous random variables can be in a discrete random variable approximation [8].

In this case, it is well known that the inputs

$$u_1 = F_1(x_1), \cdots, u_m = F_m(x_m), u_\omega = F_\omega(\omega)$$

are independent random variables with the same continuous distribution $U(0,1)$ Assume

$$v_1 = a_1 + u_1(b_1 - a_1), \cdots, v_m = a_m + u_m(b_m - a_m),$$
$$v_\omega = a_\omega + u_\omega(b_\omega - a_\omega),$$

where $b_j > a_j, j = 1, \cdots, m, b_\omega > a_\omega$. Then the inputs $v_1, \cdots, v_m, v_\omega$ are independent random variables with continuous distributions $U(a_1, b_1), \cdots, U(a_m, b_m), U(a_\omega, b_\omega)$. In this case, the function

$$h(v_1, \cdots, v_m, v_\omega) = f\left(F_1^{-1}\left(\frac{v_1 - a_1}{b_1 - a_1}\right)\right), \cdots,$$
$$F_m^{-1}\left(\frac{v_m - a_m}{b_m - a_m}\right), F_\omega^{-1}\left(\frac{v_\omega - a_\omega}{b_\omega - a_\omega}\right)\right)$$

can replace f as a new system function since each of both h and f are all not known. Therefore, without loss of generality, we always consider $x_1, \cdots, x_m, \omega$ as independent random variables with continuous distributions

$$U(a_1, b_1), \cdots, U(a_m, b_m), U(a_\omega, b_\omega).$$

On the other hand, the function f is considered as continuous, in order to ensure that condition expectations and partial derivative of existence and make the conventional mathematics method has significance.

To the complex system f, we need to decide an energy goal t, make y more close to target, the greater the function of the system. In general, the Target t is the maximum energy of y.

Image mathematics in TCMath considers the complex system stability problem, because the core problem of any complex system is stability. The stability can only through the fixed program to observe to do a test or experiment since the function f is not known. In general, the human wants to find a testing or experimental center $x^0 = (x_1^0, \cdots, x_m^0)$ and testing or experimental tolerance $\Delta x = (\Delta x_1, \cdots, \Delta x_m)$, $\Delta_{\max} = \max(\Delta x_1, \cdots, \Delta x_m)$, for the observed function

$$g(x_1, \cdots, x_m) = E\left(f(x_1, \cdots, x_m, \omega) \big| x_1, \cdots, x_m\right)$$

under some similar conditions, such that

1) $\left(Eg(x_1, \cdots, x_m) - t\right)^2$

   $= \left(Eg(x_1^0, \cdots, x_m^0) - t\right)^2 + O(\Delta_{max}) \to \min;$

2) $E(y-t)^2 \to \min;$

where the controllable inputs $x_1, \cdots, x_m$ are independent random variables and

$$x_j \sim U(a_j, b_j) = U(x_j^0 - \Delta x_j, x_j^0 + \Delta x_j), j = 1, \cdots, m.$$

In order to solve the stability problem, we get easily the following theorem [29]:

Theorem 4.1. *Suppose that f is continuous and*

$$g = g(x_1, \cdots, x_m) = E\left(f(x_1, \cdots, x_m, \omega) \big| x_1, \cdots, x_m\right).$$

Then

$$E(y-t)^2 = (Eg-t)^2 + Var(g) + Var(\varepsilon_g),$$

where $Var(\varepsilon_g) = E\sigma_{x_1, \ldots, x_m}^2.$ #

1) In image mathematics, the index $U_g^2 = Var(g(x_1, \cdots, x_m))$ is called Distortion Degree. It belongs to the wood (X) subsystem of the complex system f since it cognizes the structure function g of the complex system f which is the beginning or birth stage of all things, just like in the Spring of a year. In mathematics,

$$U_g^2 = Var(g(x_1, \cdots, x_m))$$

$$= \sum_{j=1}^m \left(\frac{\partial g(x^0)}{\partial x_j}\right)^2 Var(x_j) + o(\Delta_{max}) = U_g^2(\Delta_{max})$$

where $\Delta_{max} = \max(\Delta x_1, \cdots, \Delta x_m)$. It is thought as the wood (X) image of generally complex system f since it is an objective constant independent of human observations, expressing birth, although the maximum and controllable experimental tolerance Δ_{max} can be choose by human. The index $\varphi(X) = Var(g(x_1, \cdots, x_m))^{-1} = U_g^{-2}$ can be taken as the energy function of the subsystem wood (X) since the smaller the distortion degree U_g^2, the better the stability of the complex system f.

Although the distortion degree U_g^2 is unknown for freedom model speaking, it can be easily estimated by using orthogonal arrays $L_n(p_1, \cdots, p_m) = (a_{ij})_{n \times m}$, $0 \le a_{ij} \le p_j - 1, j = 1, \cdots, m$ (see [29]). For example, take

$$x_{ij} = x_j(a_{ij}) = x_j^0 + C_j(a_{ij})\Delta x_j$$

where

$$C_j(a_{ij}) = -1 + 2/(p_j - 1),$$

then $Var(x_j) = W(p_j)\Delta x_j^2$, where

$$W(p_j) = \frac{1}{p_j} \sum_{k=0}^{p_j-1} C_j(k)^2.$$

And for the experiment data $y_i = f(x_{i1}, x_{im}, \omega_{i\omega})$, $i = 1, \cdots, n$, we have

$$\frac{\widehat{\partial g(x^0)}}{\partial x_j} = \frac{\frac{1}{p_j} \sum_{k=0}^{p_j-1} C_j(k)\hat{\mu}_{jk}}{W(p_j)\Delta x_j},$$

$$\hat{\mu}_{jk} \triangleq \frac{1}{r_{jk}} \sum_{s \in H_{jk}} y_s, \quad r_{jk} \triangleq |H_{jk}|,$$

where $H_{jk} = \{s : a_{sj} = k, s = 1, \cdots, n\}$. Thus, we can obtained the estimation of the distortion degree $U_g^2 = Var(g(x_1, \cdots, x_m))$ as

$$\widehat{U_g^2} = \widehat{Var(g)} = \sum_{j=1}^m \frac{\left(\frac{1}{p_j} \sum_{k=0}^{p_j-1} C_j(k)\hat{\mu}_{jk}\right)^2}{W(p_j)}.$$

The $\Delta_{max} = \max(\Delta x_1, \cdots, \Delta x_m)$ smaller, more exact estimate.

2) The index $\gamma_f^2 = Var(f) = Var(g) + Var(\varepsilon_g)$ is called Disturb Degree. It belongs to the fire (X_S) subsystem of the complex system f since it controls the fluctuations of the complex system f which is the development and growth stage of all things, just like in the Summer of a year. In mathematics,

$$\gamma_f^2 = Var(f(x_1, \cdots, x_m))$$

$$= \sum_{j=1}^m \left(\frac{\partial g(x^0)}{\partial x_j}\right)^2 Var(x_j) + E\sigma_{x_1, \ldots, x_m}^2 + o(\Delta_{max})$$

$$= \gamma_g^2(\Delta_{max})$$

where $\Delta_{max} = \max(\Delta x_1, \cdots, \Delta x_m)$. It is thought as the fire (X_S) image of generally complex system f since it is also an objective constant independent of human observations, expressing growth, although the maximum and controllable experimental tolerance Δ_{max} can be choose by human. The index $\varphi(X_S) = \gamma_f^2$ can be taken as the energy function of the subsystem fire (X_S) since the smaller the disturb degree γ_f^2, the better the stability of the complex system f.

Although the disturb degree γ_f^2 is unknown for freedom model speaking, it can be easily estimated by using experiment data $y_1, \cdots, y_n$ of orthogonal arrays (see [29]). For example, a good estimation of $\gamma_f^2 = Var(f)$ is the data standard variance

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$$\widehat{\gamma_f^2} = \widehat{Var(f)} = S_n^2 = \frac{1}{n-1}\sum_{s=1}^{n}(y_s - \bar{y})^2,$$

where $\bar{y} = \frac{1}{n}\sum_{s=1}^{n}y_s$.

3) The index $\eta_f = (Ef)^2 / Var(f)$ is called Information Decomposition Ratio (or Signal to Noise Ratio). It belongs to the soil (X_K) subsystem of the complex system f since it makes the coordination of the center and fluctuation in the complex system f which is the continuous development and combined stage of all things, just like in the Long-Summer of a year. In mathematics,

$$\eta_f = (Ef)^2 / Var(f(x_1,\cdots,x_m))$$
$$= \frac{(Eg)^2}{\sum_{j=1}^{m}\left(\frac{\partial g(x^0)}{\partial x_j}\right)^2 Var(x_j) + E\sigma_{x_1,\ldots,x_m}^2 + o(\Delta_{max})}$$
$$= \eta_g(\Delta_{max})$$

where $\Delta_{max} = \max(\Delta x_1,\cdots,\Delta x_m)$. It is thought as the soil (X_K) image of generally complex system f since it is an objective constant independent of human observations, expressing combined, although the maximum and controllable experimental tolerance Δ_{max} can be choose by human. The index $\varphi(X_K) = \eta_f$ can be taken as the energy function of the subsystem soil (X_K) since the bigger the information decomposition ratio η_f, the better the stability of the complex system f.

Although the information decomposition ratio η_f is unknown for freedom model speaking, it can be easily estimated by using the experimental data $y_1,\cdots,y_n$ of orthogonal arrays (see [29]). For example, a good estimation of η_f is

$$\widehat{\eta_f} = \frac{n\bar{y}^2}{S_n^2}, S_n^2 = \frac{1}{n-1}\sum_{s=1}^{n}(y_s - \bar{y})^2, \bar{y} = \frac{1}{n}\sum_{s=1}^{n}y_s.$$

Note: In data analysis situation, often taking $\eta_f = nE\bar{y}^2 / \gamma_g^2$ instead of $\eta_f = (Ef)^2 / Var(f)$. Limited to data observation point of view, they are in the statistical meaning equivalent, but the former is more advantageous to the statistical analysis.

4) The index $\rho_f^2 = (Ef - t)^2$ is called Deviation Degree. It belongs to the metal (K_X) subsystem of the complex system f since it makes function characteristics (or data center and the expected goal deviation) in the complex system f which is the getting-results and accepted stage of all things, just like in the Autumn of a year. In mathematics,

$$\rho_f^2 = (Ef - t)^2 = (Eg - t)^2$$
$$= (g(x^0) - t)^2 + o(\Delta_{max}) = \rho_g^2(\Delta_{max})$$

where $Ey = Ef = Eg = g(x^0) + o(\Delta_{max})$. It is thought

as the metal (K_X) image of a generally complex system f since it is an objective constant independent of human observations expressing accepted, although the controllable inputs $x_1,\cdots,x_m$ of the observed function

$$g(x_1,\cdots,x_m) = E(f(x_1,\cdots,x_m,\omega)|x_1,\cdots,x_m)$$

can be choose by human. The index $\varphi(K_X) = \rho_f^{-2}$ can be taken as the energy function of the subsystem metal (K_X) since the smaller the deviation degree ρ_f^2, the better the stability of the complex system f.

Although the deviation degree ρ_f^2 is unknown for freedom model speaking, it can be easily estimated by using the experimental data $y_1,\cdots,y_n$ of orthogonal arrays (see [29]). For example, a good estimation of ρ_f^2 is

$$\widehat{\rho_f^2} = (\bar{y} - t)^2, \bar{y} = \frac{1}{n}\sum_{s=1}^{n}y_s.$$

5) The index $R_f^2 = E(y - t)^2$ is called Risk Function (or Risk, or Loss Function). It belongs to the water (S_X) subsystem of the complex system f since it makes the expected value of loss $L(y,t) = (y - t)^2$ between each data y and the goal t in the complex system f which is the risk and hiding stage of all things, just like in the winter of a year. The best condition is $R_f^2 = E(y - t)^2 = 0$, but generally this is very hard to achieve, because right now all the data are equal to the target t and the system f is a simple system $f = t$. In mathematics,

$$R_f^2 = E(y - t)^2$$
$$= \rho_g^2(\Delta_{max}) + U_g^2(\Delta_{max}) + E\sigma_{x_1,\ldots,x_m}^2$$
$$= R_g^2(\Delta_{max}).$$

where $E\sigma_{x_1,\ldots,x_m}^2 = E(\varepsilon_g^2|x_1,\cdots,x_m)$. It is thought as the water (S_X) image of generally complex system f since it is an objective constant independent of human observations, expressing hiding, although the observed function

$$g(x_1,\cdots,x_m) = E(f(x_1,\cdots,x_m,\omega)|x_1,\cdots,x_m)$$

can be choose by human. The index $\varphi(S_X) = R_f^{-2}$ can be taken as the energy function of the subsystem water (S_X) since the smaller the risk R_f^2, the better the stability of the complex system f.

Although the risk R_f^2 is unknown for freedom model speaking, it can be easily estimated by using the experimental data $y_1,\cdots,y_n$ of orthogonal arrays (see [29]). For example, a good estimation of R_f^2 is

$$\widehat{R_f^2} = \frac{1}{n}\sum_{s=1}^{n}(y_s - t)^2.$$

In image mathematics, each of the rows of orthogonal arrays is called one gua (卦) which is independent of the

unknown system function f (model-free). The state space V of a multilateral system $(V, \Re)$ based the unknown system function f is the set of mathematical indexes of the unknown system function f. By Theorem 2.4, the set V can be divided into five categories. Corresponding to every kind of five categories, each of mathematical indexes of the unknown system function f is called an image. Each of images must shows the complex system certain characteristics, such as, wood, fire, soil, metal, water.

For given each of the gua (卦) or each the rows of orthogonal arrays and for any unknown continuous system function f, it can be proved easily that each of true image mathematical indexes above can be obtained if the experiments or observations can be repeated (law of averages of great numbers). The way or calculation method of the image indexes is also independent of the unknown system function f (model-free).

In image mathematics, each of mathematical indexes represents an "Axiom system", called a class. For each of classes, all theories and methods are in order to increase the energy of class or to make the corresponding mathematical index becomes to better. There are the loving and hating (or killing) relations among all images or mathematical indexes of classes. Generally speaking, close is love, alternate is hate.

In every category of internal, think that they are equivalent relationship, between each two of their elements there is a force of similar material accumulation of each other. It is because their pursuit of the goal is the same, $i.e.$, follows the same "Axiom system". It can increase the energy of the class if they accumulate together. All of nature material activity follows the principle of maximizing so energy. In general, the force of similar material accumulation of each other is smaller than the loving force or the killing force in a stable complex system. The stability of any complex system first needs to maintain the equilibrium of the killing force and the loving force. For a stable complex system, if the killing force is large, $i.e.$, ρ_1 becomes larger, then the loving force is large and the force of similar material accumulation of each other is also large. They can make the complex system more stable. If the killing force is small, $i.e.$, ρ_1 becomes smaller, then the loving force is small and the force of similar material accumulation of each other is also small. They can make the complex system becoming unstable.

It has been shown in Theorems 2.1 - 2.4 that the classification of five subsets is quite possible based on the mathematical logic. As for the characteristics of the five subsets is rational or not, it is need more research work. It has been also shown in Theorems 3.1 - 3.4 that the logical basis of image mathematics is a steady multilateral system.

The vigor energy (or, Chi, Qi) of image mathematics means the energy function in a steady multilateral system.

There are two kinds of mathematical diseases in image mathematics: Mathematical real disease and mathematical virtual disease. They generally means the subsystem is abnormal, its energy is too high for mathematical real disease or too low for mathematical virtual disease.

The intervening method of image mathematics is to "xie Chi" which means to rush down the energy if a mathematical real disease is treated, or to "bu Chi" which means to fill the energy if a mathematical virtual disease is treated. Like intervening the subsystem, decrease when the energy is too high, increase when the energy is too low.

Both the capability of intervention reaction and the capability of self-protection of the multilateral system are equivalent to the Immunization of image mathematics. This capability is really existence for a mathematical complex system. Its target is to protect other mathematical subsystem while treating one mathematical subsystem. It is because if the capability is not existence, then $\rho_1 = \rho_2 = \rho_3 = 0$. In this time, the energy of the system will be the sum of energy of each part. Thus the mathematical system will be a simple mathematical system which is not what we consider range.

4.2. Intervening Principle if Only One Subsystem of the Mathematical Complex System Falls Ill

If we always intervene the abnormal subsystem of the mathematical complex system directly, the intervention method always destroy the balance of the mathematical complex system because it is having strong side effects to the mother or the son of the subsystem which may be non-disease of mathematical subsystem or non-intervened subsystem by using Theorem 3.2. The intervening method also decrease the capability of intervention reaction because the method which don't use the capability of intervention reaction makes the ρ_1 and ρ_2 near to 0. The state $\rho_1 = \rho_2 = 0$ is the worst state of the mathematical complex system, namely mathematical intervenetion failure. On the way, the mathematical intervening resistance problem will be occurred since any mathematical intervening method is possible too little for some small ρ_1 and ρ_2.

But, by Corollary 3.2, it will even be better if we intervene subsystem X itself directly $\rho_2 = \rho_1 \rho_3$, $\rho_3 = \rho_1$ and $\rho_1 < \rho_0$. In this case: $\rho_1 + \rho_2 < 0.9375648971$. It can be explained that if a multilateral system which has a poor capability of intervention reaction, then it is better to intervene the subsystem itself directly than indirectly. But similar to above, the intervention method is always

to destroy the balance of multilateral systems such that there is at least one of side effects occurred. And the intervention method also have harmful to the capability of intervention reaction making the mathematical intervening resistance problem also occurred by Theorem 3.2. Therefore the intervention method directly can be used in case $\rho_1 < \rho_0$ but should be used as little as possible.

If we always intervene the abnormal subsystem of the mathematic complex system indirectly, the intervention method can be to maintain the balance of the mathematical complex system because it has not any side effects to all other subsystems which are not both the mathematical disease subsystem and the mathematical intervened subsystem by using Theorem 3.2. The intervening method also increase the capability of intervention reaction because the method of using the intervention reaction makes the ρ_1 and ρ_2 near to 1. The state $\rho_1 = \rho_2 = 1$ is the best state of the mathematical complex system. On the way, it is almost none mathematical intervening resistance problem since any mathematical intervening method is possible good for some large ρ_1 and ρ_2.

For example, in China, many mathematical engineers generally only care about risk R_f^2 (or, income, water (S_X)) and interfere γ_f^2 (or, disturb, fire (X_S)) as two mathematical indexes which have the killing relations for a complicated engineering problem based on the freedom model (3). They frequently used method is: If only the risk R_f^2 (i.e., water (S_X) is not normal (or, the bigger, virtual disease), then they reduce deviation degree ρ_f^2 (i.e., increase the energy $\varphi(K_X) = \rho_f^{-2}$ of metal (K_X)) with linear optimization method since metal (K_X) is the mother of water (S_X); If only the interfere degree γ_f^2 (i.e., fire (X_S)) is not normal (or, the bigger, virtual disease), then they reduce distortion degree U_f^2 (i.e., increase the energy $\varphi(X) = U_f^{-2}$ of wood (X)) of system function f with the method of adjusting stable center x^0 of the system function since wood (X) is the mother of fire (X_S) (see [29]). The idea is precisely "Virtual disease is to fill his mother" if one subsystem of mathematical complex system falls virtual ill.

All in all, the mathematical complex system satisfies the intervention rule and the self-protection rule. It is said a healthy mathematical complex system when the intervention reaction coefficient ρ_1 satisfies $\rho_1 \geq \rho_0$. In logic and practice, it's reasonable $\rho_1 + \rho_2$ near to 1 if the input and output in a complex system is balanced, since a mathematical output subsystem is absolutely necessary social other subsystems of all consumption. In case: $\rho_1 + \rho_2 = 1$, all the energy for intervening mathematical complex subsystem can transmit to other mathematical complex subsystems which have neighboring relations or alternate relations with the intervening mathematical complex subsystem. The condition $\rho_1 \geq \rho_0$ can be satisfied when $\rho_2 = \rho_1\rho_3$ and $\rho_3 = \rho_1$ for

a mathematical complex system since $\rho_1 + \rho_2 = 1$ implies $\rho_1 = (\sqrt{5}-1)/2 \approx 0.618 > \rho_0$ and $\rho_2 = 1 - (\sqrt{5}-1)/2 \approx 0.382$. If this assumptions is set up, then the intervening principle: "Real disease is to rush down his son and virtual disease is to fill his mother" based on "Yin Yang Wu Xing" Theory in image mathematics, is quite reasonable. But, in general, the ability of self-protection often is insufficient for an usual mathematical complex system, i.e., ρ_3 is small. A common standard is $\rho_3 = \dfrac{1-\rho_1}{2\rho_2} \approx \dfrac{1}{2}$, i.e., there is a principle which all losses are bear in mathematical complex system. Thus the general condition often is $\rho_1 \approx 0.618 \geq \rho_3 \approx 0.5 \geq \rho_2 \approx 0.382$. Interestingly, they near to the golden numbers.

On the other hand, in image mathematics, mathematical real disease and mathematical virtual disease have their reasons. Mathematical real disease is caused by the born subsystem and mathematical virtual disease is caused by the bear subsystem. Although the reason can not be proved easily in mathematics or experiments, the intervening method under the assumption is quite equal to the intervening method in the intervention indirectly. It has also proved that the mathematical intervening principle is true from the other side.

4.3. Intervening Principle If Only Two Subsystems with the Loving Relation of the Mathematical Complex System Encounter Sick

Suppose that the two subsystems X and X_S of the mathematic complex system are abnormal (mathematical virtual disease or mathematical real disease). In the mathematical complex system of relations between two non-compatible with the constraints, by Theorem 3.2, only two situations may occur:

1) X encounters mathematical virtual disease, and at the same time, X_S befalls mathematical virtual disease, i.e., the energy of X is too low and the energy of X_S is also too low. It is because X bears X_S. The mathematical virtual disease causal is X.

2) X encounters mathematical real disease, and at the same time, X_S befalls mathematical real disease, i.e., the energy of X is too high and the energy of X_S is also too high. It is because X_S is born by X. The mathematical virtual disease causal is X_S.

It can be shown by Theorem 3.3 that when intervenetion reaction and self-protection coefficients satisfy $\rho_2 = \rho_1\rho_3$, $\rho_3 = \rho_1$ and $\rho_1 \geq \rho_0$, if one wants to treat the abnormal subsystems X and X_S, then

1) For mathematical virtual disease, the one should in-

tervene subsystem X directly by increasing its energy. It means "mathematical virtual disease is to fill his mother" because the mathematical virtual disease causal is X;

2) For mathematical real disease, the one should intervene subsystem X_S directly by decreasing its energy. It means "mathematical real disease is to rush down his son" because the mathematical real disease causal is X_S.

For example, in China, many factories and enterprises generally only care about cost R_f^2 (or, risk, water (S_X)) and quality ρ_f^2 (or, deviation degree, metal (K_X)) as two mathematical indexes which have the loving relation for a complicated economical problem based on the freedom model (3). They frequently used method is: If both cost and quality are all not normal (or, virtual diseases, *i.e.*, high cost and poor quality), then they reduce deviation degree ρ_f^2 (*i.e.*, improve quality, increase the energy $\varphi(K_X) = \rho_f^{-2}$ of metal (K_X)) with linear optimization method since metal (K_X) is the mother of water (S_X) (see [29]). The idea is precisely "Virtual disease is to fill his mother" if one subsystem of mathematical complex system falls virtual ill.

The intervention method can be to maintain the balance of the mathematical complex system because only two mathematical virtual disease subsystems are treated, by using Theorem 3.2, such that there is not any side effect for all other subsystems. And the intervention method can also be to enhance the capability of intervention reaction because the method of using intervention reaction makes the ρ_1 and ρ_2 greater and near to 1. The state $\rho_1 = \rho_2 = 1$ is the best state of the mathematical complex system. On the way, it almost have none mathematical intervening resistance problem since any mathematical intervening method is possible good for some large ρ_1 and ρ_2.

4.4. Intervening Principle If Only Two Subsystems with the Killing Relation of the Mathematical Complex System Encounter Sick

Suppose that the subsystems X and X_K of a mathematical complex system are abnormal (real disease or virtual disease). In the mathematical complex system of relations between two non-compatible with the constraints, only a situation may occur: X encounters mathematical virtual disease, and at the same time, X_K befalls mathematical real disease, *i.e.*, the energy of X is too low and the energy of X_K is too high. The disease is serious because the X_K has harmed the X by using the method of incest, *i.e.* damaged the king relation of X and X_K.

It can be shown by Theorems 3.3 and 3.4 that when intervention reaction and self-protection coefficients satisfy $\rho_2 = \rho_1\rho_3$, $\rho_3 = \rho_1$ and $\rho_1 < \rho_0$, if one wants to treat the abnormal subsystems X and X_K, the one should

intervene subsystem X directly by increasing its energy, and at the same, intervene subsystem X_K directly by decreasing its energy. It means that "Strong inhibition of the same time, support the weak".

For example, in China, many sociologists generally only care about social and economic benefits $\varphi(S_X) = R_f^{-2}$ (or social benefits, water (S_X)) and social equity extent $\varphi(X_K) = \eta_f$ (or, information decomposition ratio, soil (X_K)) with two mathematical killing indexes for a complicated social system based on the freedom model (3). If both social benefit $\varphi(S_X) = R_f^{-2}$ and social equity extent $\varphi(X_K) = \eta_f$ are not normal, often appear serious problem is that social benefit $\varphi(S_X) = R_f^{-2}$ is too good (or, R_f^2 too low, real disease) but social equity extent $\varphi(X_K) = \eta_f$ is too bad (or, η_f too low, *i.e.*, virtual disease).The disease is serious because the X_K has been harmed by the S_X with the method of incest such that the soil (X_K) cannot kill water (S_X). For the serious disease, many mathematical sociologists of China now frequently used method is: to increase social equity extent η_f (or, to increase the information decomposition ratio η_f which is the energy of the soil (X_K)) but to decrease social and economic benefits $\varphi(S_X) = R_f^{-2}$ (*i.e.*, to decrease the energy of water (S_X)) at the same time since soil (X_K) is the bane of water (S_X). The idea is "Strong inhibition of the same time, support the weak" if X_K falls virtual disease and at the same time, S_X befalls real disease.

For another example, thirty years ago, China's social coordination function $\varphi(X_K) = \eta_f$ is very good, the complex system is rife with average socialist, but both social benefit $\varphi(S_X) = R_f^{-2}$ and social structure $\varphi(X) = U_f^{-2}$ are all very poor. In addition, the mathematical complex system is not rich. In other words, both S_X and X fall virtual diseases and at the same time, X_K befalls real disease based on the freedom model (3). The disease is very serious because not only the wood (X) has been harmed by the soil (X_K) with the method of incest such that wood (X) cannot kill soil (X_K), but also there are 3 subsystems falling ill. Generally speaking, there are three or more than three of disease of the subsystem of a complex system, it is very difficult to cure. In order to cure the very serious disease, Deng Xiao-Ping's taking method is to break the "iron bowl" and to develop the economy (to fill up the energies $\varphi(X) = U_f^{-2}$ and $\varphi(S_X) = R_f^{-2}$ of both wood (X) and water (S_X) at the same time, *i.e.*, strengthen social structure and social benefit), and to allow a few people to get rich (to rush down the energy $\varphi(X_K) = \eta_f$ of soil (X_K), abate the coordinated ability). The idea is, at the same time, to use both "Strong inhibition of the same time, support the weak" and "Virtual disease is to fill his mother", if both S_X and X fall virtual diseases and at the same time, X_K befalls real disease.

The intervention method can be to maintain the bal-

ance of mathematical complex system because only two mathematical virtual disease subsystems are treated, by using Theorems 3.2 and 3.4, such that there is none side effects for all other subsystems. And the intervention method can also be to enhance the capability of intervention reaction and self-protection because the method of using intervention reaction and self-protection makes the ρ_3 and ρ_1 greater and near to 1. The state $\rho_3 = \rho_1 = 1$ is the best state of the steady multilateral system. On the way, it almost have none mathematical intervening resistance problem since any mathematical intervening method is possible good for some large ρ_3 and ρ_1.

5. Conclusions

This work shows how to treat the mathematical diseases (real or virtual) of a mathematical complex system in image mathematics and three methods are presented.

If only one subsystem falls ill, mainly the intervening method should be to intervene it indirectly for case: $\rho_2 = \rho_1\rho_3$, $\rho_3 = \rho_1$ and $\rho_1 \geq \rho_0$, according to the intervening principle of "Real disease is to rush down his son but virtual disease is to fill his mother". The intervention method directly can be used in case $\rho_2 = \rho_1\rho_3$, $\rho_3 = \rho_1$ and $\rho_1 < \rho_0$ but should be used as little as possible.

If two subsystems with the loving relation encounter sick, the intervening method should be intervene them directly according to the intervening principle of "Real disease is to rush down his son but virtual disease is to fill his mother".

If two subsystems with the killing relation encounter sick, the intervening method should be intervene them directly also according to the intervening principle of "Strong inhibition of the same time, support the weak".

Other properties, such as balanced, orderly nature of Wu-Xing, mathematical forecast, and so on, will be discussed in the next articles.

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