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Adaptive Lag Synchronization of Lorenz Chaotic System with Uncertain Parameters*

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ABSTRACT

The paper discusses lag synchronization of Lorenz chaotic system with three uncertain parameters. Based on adaptive technique, the lag synchronization of Lorenz chaotic system is achieved by designing a novel nonlinear controller. Furthermore, the parameters identification is realized simultaneously. A sufficient condition is given and proved theoretically by Lyapunov stability theory and LaSalle's invariance principle. Finally, the numerical simulations are provided to show the effectiveness and feasibility of the proposed method.

Keywords: Lag Synchronization; Adaptive Technique; Uncertain Parameters; Lorenz Chaotic System

1. Introduction

Since the original work on chaos synchronization by Pecora and Carroll [1] in the drive-response systems, chaos synchronization has attracted much attraction due to its potential applications in many practical engineering fields, such as secure communication [2], information processing [3], image encryption [4], and so on. In the past two decades, many schemes for chaos synchronization have been proposed, including linear and nonlinear feedback approach [5,6], adaptive technique [6], backstepping method [7], impulsive control method [8], etc. At present, the researchers are concentrating on the following types of synchronization phenomena: complete synchronization [9], generalized synchronization [10], phase synchronization [11], lag synchronization [12], dislocated synchronization [13] and so on.

Lag synchronization, where the corresponding state vectors of response system follow the drive system with time delay. Recently, some literatures have been devoted to lag synchronization of chaotic systems. In Reference [14], the lag synchronization of Rössler system and Chua circuit has been investigated via a scalar signal. Li *et al.* [15] applied a nonlinear observer to lag synchronization of hyperchaotic Rössler system and hyperchaotic Matsumoto-Chua-Kobayashi (MCK) circuit. Zhang *et al.* [16] studied the same problem for hyperchaotic Lü system. These design of a controller depends on the considered dynamical system, the method can be used in the system with certain parameters. But in some real physical systems and experimental situations, chaotic systems may have some uncertain parameters, so a systematic design process of lag synchronization of chaotic systems with uncertain parameters is important.

In this paper, we investigate the lag synchronization of Lorenz chaotic system with uncertain parameters. Based on the adaptive technique, a novel controller and parameter adaptive laws are designed such that parameters identification is realized, and lag synchronization of Lorenz chaotic system is achieved simultaneously. Theoretically proof and numerical simulations are given to demonstrate the effectiveness and feasibility of the proposed method.

2. Problem Formulation

The Lorenz chaotic system [17] is proposed in 1963, the nonlinear differential equations for describing it are

$$\begin{cases} \dot{x}_1(t) = a\left(x_2(t) - x_1(t)\right), \\ \dot{x}_2(t) = cx_1(t) - x_1(t)x_3(t) - x_2(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - bx_3(t), \end{cases} \quad (1)$$

having a chaotic attractor when $a = 10$, $b = 8/3$, $c = 28$. The phase portrait is shown in **Figure 1**.

Considering the drive system (1), the response system is controlled Lorenz chaotic system as following

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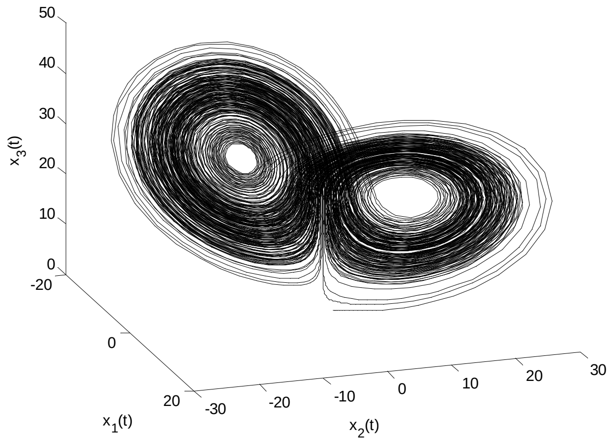

Figure 1. The phase portrait of Lorenz chaotic system (1) with parameter values *a* = 10, *b* = 8/3, *c* = 28.

$$\begin{cases} \dot{y}_1(t) = a_s\left(y_2(t) - y_1(t)\right) + u_1(t), \\ \dot{y}_2(t) = c_s y_1(t) - y_1(t) y_3(t) - y_2(t) + u_2(t), \\ \dot{y}_3(t) = y_1(t) y_2(t) - b_s y_3(t) + u_3(t), \end{cases} \tag{2}$$

where a_s, b_s, c_s of (2) are unknown parameters which need to be identified in the response system,

$$U(t) = \left[u_1(t), u_2(t), u_3(t)\right]^T$$

is the controller which should be designed such that two systems can be lag synchronized.

Let

$$\begin{cases} e_1(t) = y_1(t) - x_1(t-\tau), \\ e_2(t) = y_2(t) - x_3(t-\tau), \\ e_3(t) = y_3(t) - x_2(t-\tau). \end{cases} \tag{3}$$

where $\tau > 0$ is the time delay for the errors dynamical system.

Therefore, the goal of parameters identification and lag synchronization is to find an appropriate controller $U(t)$ and parameter adaptive laws of a_s, b_s, c_s, such that the synchronization errors

$$e_1(t) \to 0, e_2(t) \to 0, e_3(t) \to 0 \quad \text{as} \quad t \to \infty \tag{4}$$

and the unknown parameters

$$\lim_{t\to\infty} a_s = a, \lim_{t\to\infty} b_s = b, \lim_{t\to\infty} c_s = c. \tag{5}$$

Remark 1. When $\tau > 0$, the lag synchronization will appear. When $\tau < 0$, the anticipated synchronization will appear. More in general, complete synchronization will appear when $\tau = 0$.

Remark 2. For the anticipated synchronization and complete synchronization, the discussions are similar to the method given in this paper.

3. Adaptive Lag Synchronization of Lorenz Chaotic System

In this section, based upon the nonlinear adaptive feedback control technique, a systematic design process of parameters identification and lag synchronization of Lorenz chaotic system under the situation of response system with unknown parameters is provided.

According to the systems (1) and (2), we have the errors dynamical system

$$\begin{cases} \dot{e}_1(t) = -ae_1(t) - (a_s - a) y_1(t) - ax_2(t-\tau) \\ \qquad + a_s y_2(t) + u_1(t), \\ \dot{e}_2(t) = -e_2(t) + (c_s - c) y_1(t) - cx_1(t-\tau) + x_1(t-\tau) \\ \qquad \cdot x_3(t-\tau) + cy_1(t) - y_1(t) y_3(t) + u_2(t), \\ \dot{e}_3(t) = -be_3(t) - (b_s - b) y_3(t) - x_1(t-\tau) x_2(t-\tau) \\ \qquad + y_1(t) y_2(t) + u_3(t). \end{cases} \tag{6}$$

Obviously, lag synchronization of systems (1) and (2) appears if the errors dynamical system (6) has an asymptotically stable equilibrium point $e(t) = 0$, where

$$e(t) = \left[e_1(t), e_2(t), e_3(t)\right]^T.$$

Then, we get the following theorem.

Theorem Assuming that the Lorenz chaotic system (1) drives the controlled Lorenz chaotic system (2), take

$$\begin{cases} u_1(t) = ax_2(t-\tau) - a_s y_2(t), \\ u_2(t) = cx_1(t-\tau) - x_1(t-\tau) x_3(t-\tau) \\ \qquad - cy_1(t) + y_1(t) y_3(t), \\ u_3(t) = x_1(t-\tau) x_2(t-\tau) - y_1(t) y_2(t), \end{cases} \tag{7}$$

and parameter adaptive laws

$$\begin{cases} \dot{a}_s = y_1(t) e_1(t), \\ \dot{b}_s = y_3(t) e_3(t), \\ \dot{c}_s = -y_1(t) e_2(t). \end{cases} \tag{8}$$

Systems (1) and (2) can realize lag synchronization and the unknown parameters will be identified, *i.e.*, Equations (4) and (5) will be achieved.

Proof Equation (6) can be converted to the following form under the controller (7)

$$\begin{cases} \dot{e}_1(t) = -ae_1(t) - (a_s - a) y_1(t), \\ \dot{e}_2(t) = -e_2(t) + (c_s - c) y_1(t), \\ \dot{e}_3(t) = -be_3(t) - (b_s - b) y_3(t). \end{cases} \tag{9}$$

Consider a Lyapunov function as

$$V = \frac{1}{2}\Big[e_1^2(t) + e_2^2(t) + e_3^2(t) + (a_s - a)^2 + (b_s - b)^2 + (c_s - c)^2\Big],$$

Obviously, V is a positive definite function. Taking its time derivative along with the trajectories of Equations (8) and (9) leads to

$$\begin{aligned}\dot{V} &= e_1(t)\dot{e}_1(t)+e_2(t)\dot{e}_2(t)+e_3(t)\dot{e}_3(t)+(a_s-a)\dot{a}_s \\ &\quad +(b_s-b)\dot{b}_s+(c_s-c)\dot{c}_s \\ &= -ae_1^2(t)-e_2^2(t)-be_3^2(t)=-e^T Pe\le 0,\end{aligned}$$

where $P = diag\{a,1,b\}$. It is obvious that $\dot{V}=0$ if and only if $e_i(t)=0$, $i=1,2,3$, namely the set

$$M=\{e_1(t)=0, e_2(t)=0, e_3(t)=0, \\ a_s=a, b_s=b, c_s=c\}$$

is the largest invariant set contained in $E=\{\dot{V}=0\}$ for Equation (9). So according to the LaSalle's invariance principle [18], starting with arbitrary initial values of Equation (9), the trajectory converges asymptotically to the set M, *i.e.*, $e_1(t)\to 0$, $e_2(t)\to 0$, $e_3(t)\to 0$, $a_s\to a$, $b_s\to b$ and $c_s\to c$ as $t\to\infty$. This indicates that the lag synchronization of Lorenz chaotic system is achieved and the unknown parameters a_s, b_s, c_s, can be successfully identified by using controller (7) and parameter adaptive laws (8). Now the proof is completed.

Remark 3. Taking our adaptive synchronization method, we can not only achieve synchronization but also identify the system parameters. The values for parameters a, b, c of drive system (1) should be confined to it has a chaotic attractor.

Remark 4. Although this process is focused on the Lorenz chaotic system, the systematic design process could be used for many other complex dynamical systems with uncertain parameters.

4. Numerical Simulations

In order to verify the effectiveness and feasibility of the proposed method, we give some numerical simulations about the lag synchronization and parameters identification between systems (1) and (2). In the numerical simulations, all the differential equations are solved by using the fourth-order Runge-Kutta method.

For this numerical simulations, we assume that the initial states of drive system and response system are $x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$ and $y_1(0)=1$, $y_2(0)=2$, $y_3(0)=3$ and the unknown parameters have zero initial condition, the time delay is chosen as $\tau=1$. The drive signals are from the Lorenz chaotic system (1) with system parameters $a=10$, $b=8/3$, $c=28$ so that it exhibits a chaotic attractor. The simulation results are shown in **Figures 2-4**. **Figures 2** and **3** display the lag synchronization state variables and errors response of systems (1) and (2), respectively. **Figure 4** shows the identification results of unknown parameters a_s, b_s, c_s.

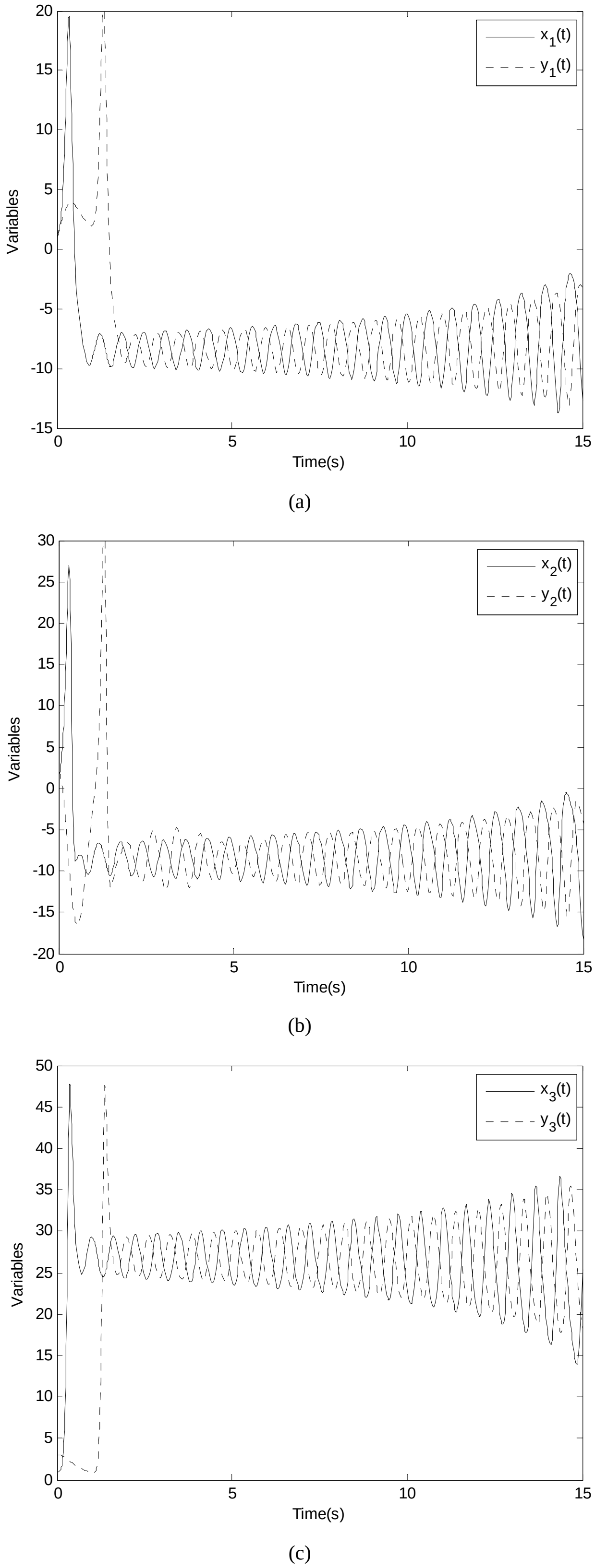

Figure 2. The lag synchronization state variables of systems (1) and (2): (a) Variables $x_1(t)$, $y_1(t)$; (b) Variables $x_2(t)$, $y_2(t)$; (c) Variables $x_3(t)$, $y_3(t)$.

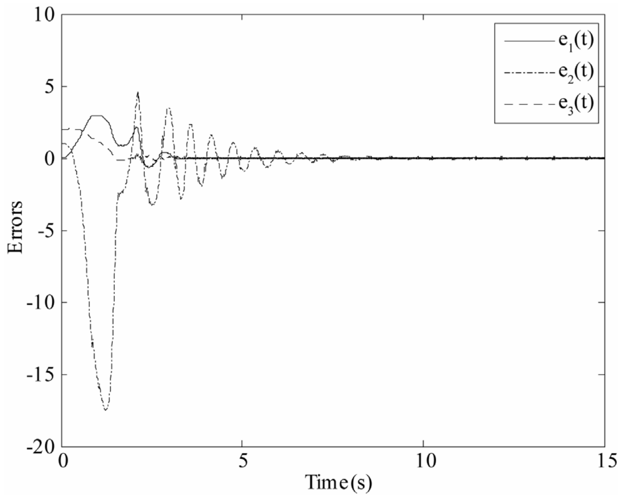

Figure 3. The lag synchronization error evolutions of systems (1) and (2).

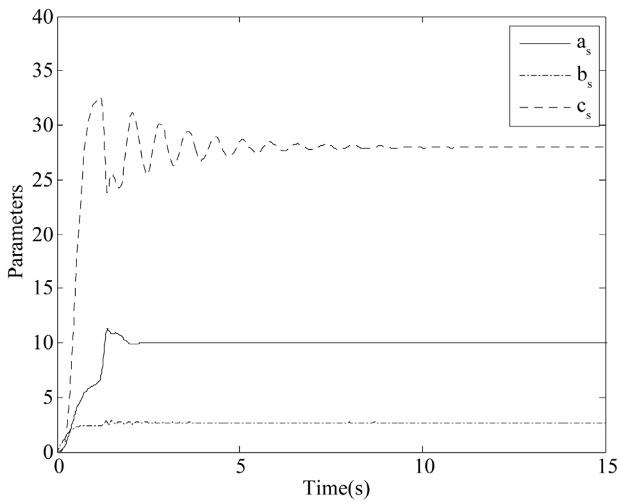

Figure 4. The parameters identification results of response system (2): *a* = 10, *b* = 8/3, *c* = 28.

5. Conclusion

This paper investigates the adaptive lag synchronization for the classical Lorenz chaotic system with the response system parameters unknown. Based on Lyapunov stability theory and LaSalle's invariance principle, the controller and parameter adaptive laws are given to achieve lag synchronization and parameters identification simultaneously. Finally, numerical simulations are provided to demonstrate the effectiveness of the scheme proposed in this work.

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