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Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Space Using *R*-Weakly Commuting Mappings

Saurabh Manro[1*], Satwinder Singh Bhatia[1], Sanjay Kumar[2]
[1]School of Mathematics and Computer Applications, Thapar University, Patiala, India
[2]Deenbandhu Chhotu Ram, University of Science and Technology, Murthal, India
Email: *sauravmanro@hotmail.com

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ABSTRACT

In this paper, we prove a common fixed point theorem in Intuitionistic fuzzy metric space by using pointwise *R*-weak commutativity and reciprocal continuity of mappings satisfying contractive conditions.

Keywords: Intuitionistic Fuzzy Metric Space; Reciprocal Continuity; *R*-Weakly Commuting Mappings; Common Fixed Point Theorem

1. Introduction

Atanassove [1] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [2] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous *t*-conorms. Recently, in 2006, Alaca *et al.* [3] defined the notion of intuitionistic fuzzy metric space by making use of Intuitionistic fuzzy sets, with the help of continuous *t*-norm and continuous *t*-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [4]. In 2006, Turkoglu [5] *et al.* proved Jungck's [6] common fixed point theorem in the setting of intuitionistic fuzzy metric spaces for commuting mappings. For more details on intuitionistic fuzzy metric space, one can refer to the papers [7-12].

The aim of this paper is to prove a common fixed point theorem in intuitionistic fuzzy metric space by using pointwise *R*-weak commutativity [5] and reciprocal continuity [9] of mappings satisfying contractive conditions.

2. Preliminaries

Definition 2.1 [13]. A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is continuous *t*-norm if * satisfies the following conditions:

1) * is commutative and associative;
2) * is continuous;
3) $a*1=a$ for all $a\in[0,1]$;
4) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Definition 2.2 [13]. A binary operation $\Diamond:[0,1]\times[0,1]\to[0,1]$ is continuous *t*-conorm if ◊ satisfies the following conditions:

1) ◊ is commutative and associative;
2) ◊ is continuous;
3) $a\Diamond 0=a$ for all $a\in[0,1]$;
4) $a\Diamond b\le c\Diamond d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Alaca *et al.* [3] defined the notion of intuitionistic fuzzy metric space as:

Definition 2.3 [3]. A 5-tuple $(X,M,N,*,\Diamond)$ is said to be an intuitionistic fuzzy metric space if *X* is an arbitrary set, * is a continuous *t*-norm, ◊ is a continuous *t*-conorm and M,N are fuzzy sets on $X^2\times[0,\infty)$ satisfying the conditions:

1) $M(x,y,t)+N(x,y,t)\le 1$ for all $x,y\in X$ and $t>0$;
2) $M(x,y,0)=0$ for all $x,y\in X$;
3) $M(x,y,t)=1$ for all $x,y\in X$ and $t>0$ if and only if $x=y$;
4) $M(x,y,t)=M(y,x,t)$ for all $x,y\in X$ and $t>0$;
5) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$ for all $x,y,z\in X$ and $s,t>0$;
6) $M(x,y,.):[0,\infty)\to[0,1]$ is left continuous, for all $x,y\in X$;
7) $\lim_{t\to\infty}M(x,y,t)=1$ for all $x,y\in X$ and $t>0$;
8) $N(x,y,0)=1$ for all $x,y\in X$;
9) $N(x,y,t)=0$ for all $x,y\in X$ and $t>0$ if and only if $x=y$;
10) $N(x,y,t)=N(y,x,t)$ for all $x,y\in X$ and $t>0$;
11) $N(x,y,t)\Diamond N(y,z,s)\ge N(x,z,t+s)$ for all $x,y,z\in X$ and $s,t>0$;
12) $N(x,y,.):[0,\infty)\to[0,1]$ is right continuous, for

*Corresponding author.

all $x, y \in X$;

13) $\lim_{t\to\infty} N(x,y,t) = 0$ for all $x, y \in X$.

The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

***Remark* 2.1 [12].** Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \Diamond)$ such that t-norm * and t-conorm $\Diamond$ are associated as $x \Diamond y = 1 - ((1-x)*(1-y))$ for all $x, y \in X$.

***Remark* 2.2 [12].** In intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$, $M(x, y, *)$ is non-decreasing and $N(x, y, \Diamond)$ is non-increasing for all $x, y \in X$.

Definition 2.4 [3]. Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Then

1) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$$

and

$$\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

2) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$$\lim_{n\to\infty} M(x_n, x, t) = 1$$

and

$$\lim_{n\to\infty} N(x_n, x, t) = 0.$$

Definition 2.5 [3]. An intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

***Example* 2.1 [3].** Let $X = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$ and let * be the continuous t-norm and $\Diamond$ be the continuous t-conorm defined by $a * b = ab$ and $a \Diamond b = \min\{1, a+b\}$ respectively, for all $a, b \in [0,1]$. For each $t \in (0, \infty)$ and $x, y \in X$, define M and N by

$$M(x,y,t) = \begin{cases} \dfrac{t}{t + |x-y|}, & t > 0 \\ 0, & t = 0 \end{cases}$$

and

$$N(x,y,t) = \begin{cases} \dfrac{|x-y|}{t + |x-y|}, & t > 0 \\ 1, & t = 0 \end{cases}$$

Clearly, $(X, M, N, *, \Diamond)$ is complete intuitionistic fuzzy metric space.

Definition 2.6 [3]. A pair of self mappings (A, S) of a intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be commuting if $M(ASx, SAx, t) = 1$ and $N(ASx, SAx, t) = 0$ for all $x \in X$.

Definition 2.7 [3]. A pair of self mappings (A, S) of a intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be weakly commuting if $M(ASx, SAx, t) \ge M(Ax, Sx, t)$ and $N(ASx, SAx, t) \le N(Ax, Sx, t)$ for all $x \in X$ and $t > 0$.

Definition 2.8 [12]. A pair of self mappings (A, S) of a intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be compatible if $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$, and $\lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = u$ for some $u \in X$.

Definition 2.9 [5]. A pair of self mappings (A, S) of a intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be pointwise R-weakly commuting, if given $x \in X$, there exist $R > 0$ such that for all $t > 0$

$$M(ASx, SAx, t) \ge M\left(Ax, Sx, \frac{t}{R}\right),$$

and

$$N(ASx, SAx, t) \le N\left(Ax, Sx, \frac{t}{R}\right)$$

Clearly, every pair of weakly commuting mappings is pointwise R-weakly commuting with $R = 1$.

Definition 2.10 [9]. Two mappings A and S of a Intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ are called reciprocally continuous if $ASu_n \to Az$, $SAu_n \to Sz$, whenever $\{u_n\}$ is a sequence such that $Au_n \to z$, $Su_n \to z$ for some z in X.

If A and S are both continuous, then they are obviously reciprocally continuous but converse is not true.

3. Lemmas

The proof of our result is based upon the following lemmas of which the first two are due to Alaca *et al*. [12]:

Lemma 3.1 [12]. Let $\{u_n\}$ is a sequence in a intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$. If there exists a constant $k \in (0,1)$ such that

$$M(u_n, u_{n+1}, kt) \ge M(u_{n-1}, u_n, t),$$

$$N(u_n, u_{n+1}, kt) \le N(u_{n-1}, u_n, t)$$

for all $n = 0, 1, 2, \cdots$

Then $\{u_n\}$ is a Cauchy sequence in X.

Lemma 3.2 [12]. Let $(X, M, N, *, \Diamond)$ be intuitionistic fuzzy metric space and for all $x, y \in X$, $t > 0$ and if for a number $k \in (0,1)$, $M(x,y,kt) \geq M(x,y,t)$ and $N(x,y,kt) \leq N(x,y,t)$. Then $x = y$.

Lemma 3.3. Let $(X,M,N,*,\Diamond)$ be a complete intuitionistic fuzzy metric space with continuous *t*-norm * and continuous *t*-conorm $\Diamond$ defined by $t*t \geq t$ and $(1-t)\Diamond(1-t) \leq (1-t)$ for all $t \in [0,1]$. Further, let (A,S) and (B,T) be pointwise *R*-weakly commuting pairs of self mappings of *X* satisfying:

(3.1) $A(X) \subseteq T(X), B(X) \subseteq S(X)$,

(3.2) there exists a constant $k \in (0,1)$ such that

$$M(Ax,By,kt) \geq M(Ty,By,t) * M(Sx,Ax,t) * M(Sx,By,\alpha t) * M(Ty,Ax,(2-\alpha)t) * M(Ty,Sx,t)$$

$$N(Ax,By,kt) \leq N(Ty,By,t) \Diamond N(Sx,Ax,t) \Diamond N(Sx,By,\alpha t) \Diamond N(Ty,Ax,(2-\alpha)t) \Diamond N(Ty,Sx,t)$$

for all $x,y \in X$, $t > 0$ and $\alpha \in (0,2)$. Then the continuity of one of the mappings in compatible pair (A,S) or (B,T) on $(X,M,N,*,\Diamond)$ implies their reciprocal continuity.

Proof. First, assume that *A* and *S* are compatible and *S* is continuous. We show that *A* and *S* are reciprocally continuous. Let $\{u_n\}$ be a sequence such that $Au_n \to z$ and $Su_n \to z$ for some $z \in X$ as $n \to \infty$.

Since *S* is continuous, we have $SAu_n \to Sz$ and $SSu_n \to Sz$ as $n \to \infty$ and since (A,S) is compatible, we have

$$\lim_{n\to\infty} M(ASu_n, SAu_n, t) = 1, \lim_{n\to\infty} N(ASu_n, SAu_n, t) = 0$$

$$\Rightarrow \lim_{n\to\infty} M(ASu_n, Sz, t) = 1, \lim_{n\to\infty} N(ASu_n, SAz, t) = 0$$

That is $ASu_n \to Sz$ as $n \to \infty$. By (3.1), for each n, there exists $v_n \in X$ such that $ASu_n = Tv_n$. Thus, we have $SSu_n \to Sz$, $SAu_n \to Sz$, $ASu_n \to Sz$ and $Tv_n \to Sz$ as $n \to \infty$ whenever $ASu_n = Tv_n$.

Now we claim that $Bv_n \to Sz$ as $n \to \infty$.

Suppose not, then taking $\alpha = 1$ in (3.2), we have

$$M(ASu_n, Bv_n, kt) \geq M(Tv_n, Bv_n, t) * M(SSu_n, ASu_n, t) * M(SSu_n, Bv_n, \alpha t) * M(Tv_n, ASu_n, (2-\alpha)t) * M(Tv_n, SSu_n, t)$$

$$N(ASu_n, Bv_n, kt) \leq N(Tv_n, Bv_n, t) \Diamond N(SSu_n, ASu_n, t) \Diamond N(SSu_n, Bv_n, \alpha t) \Diamond N(Tv_n, ASu_n, (2-\alpha)t) \Diamond N(Tv_n, SSu_n, t)$$

Taking $n \to \infty$, we get

$$M(Sz, Bv_n, kt) \geq M(Sz, Bv_n, t) * M(Sz, Sz, t) * M(Sz, Bv_n, t) * M(Sz, Sz, t) * M(Sz, Sz, t)$$

$$N(Sz, Bv_n, kt) \leq N(Sz, Bv_n, t) \Diamond N(Sz, Sz, t) \Diamond N(Sz, Bv_n, t) \Diamond N(Sz, Sz, t) \Diamond N(Sz, Sz, t)$$

That is,

$$M(Sz, Bv_n, kt) \geq M(Sz, Bv_n, t),$$

$$N(Sz, Bv_n, kt) \leq N(Sz, Bv_n, t)$$

by the use of Lemma 3.2, we have $Bv_n \to Sz$ as $n \to \infty$.

Now, we claim that $Az = Sz$. Again take $\alpha = 1$ in (3.2), we have

$$M(Az, Bv_n, kt) \geq M(Tv_n, Bv_n, t) * M(Sz, Az, t) * M(Sz, Bv_n, t) * M(Tv_n, Az, t) * M(Tv_n, Sz, t)$$

$$N(Az, Bv_n, kt) \leq N(Tv_n, Bv_n, t) \Diamond N(Sz, Az, t) \Diamond N(Sz, Bv_n, t) \Diamond N(Tv_n, Az, t) \Diamond N(Tv_n, Sz, t)$$

$n \to \infty$

$$M(Az, Sz, kt) \geq M(Sz, Sz, t) * M(Sz, Az, t) * M(Sz, Sz, t) * M(Sz, Az, t) * M(Sz, Sz, t)$$

$$N(Az, Sz, kt) \leq N(Sz, Sz, t) \Diamond N(Sz, Az, t) \Diamond N(Sz, Sz, t) \Diamond N(Sz, Az, t) \Diamond N(Sz, Sz, t)$$

i.e.

$$M(Az, Sz, kt) \geq M(Sz, Az, t),$$

$$N(Az, Sz, kt) \leq N(Sz, Az, t)$$

therefore, by use of Lemma 3.2, we have $Az = Sz$.

Hence, $SAu_n \to Sz$, $ASu_n \to Sz = Az$ as $n \to \infty$.

This proves that *A* and *S* are reciprocally continuous on *X*. Similarly, it can be proved that *B* and *T* are reciprocally continuous if the pair (B,T) is assumed to be compatible and *T* is continuous.

4. Main Result

The main result of this paper is the following theorem:

Theorem 4.1. Let $(X,M,N,*,\Diamond)$ be a complete intuitionistic fuzzy metric space with continuous *t*-norm * and continuous *t*-conorm $\Diamond$ defined by $t*t \geq t$ and $(1-t)\Diamond(1-t) \leq (1-t)$ for all $t \in [0,1]$.

Further, let (A,S) and (B,T) be pointwise *R*-weakly commuting pairs of self mappings of *X* satisfying (3.1), (3.2). If one of the mappings in compatible pair (A,S) or (B,T) is continuous, then *A*, *B*, *S* and *T* have a unique common fixed point.

Proof. Let $x_0 \in X$. By (3.1), we define the sequences $\{x_n\}$ and $\{y_n\}$ in *X* such that for all $n = 0,1,2,\cdots$ $y_{2n} = Ax_{2n} = Tx_{2n+1}$, $y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$. We show that $\{y_n\}$ is a Cauchy sequence in *X*. By (3.2) take $\alpha = 1-\beta, \beta \in (0,1)$, we have

$$M\left(y_{2n+1},y_{2n+2},kt\right)=M\left(Bx_{2n+1},Ax_{2n+2},kt\right)=M\left(Ax_{2n+2},Bx_{2n+1},kt\right)\geq M\left(Tx_{2n+1},Bx_{2n+1},t\right)$$
$$*M\left(Sx_{2n+2},Ax_{2n+2},t\right)*M\left(Sx_{2n+2},Bx_{2n+1},(1-\beta)t\right)*M\left(Tx_{2n+1},Ax_{2n+2},(1+\beta)t\right)*M\left(Tx_{2n+1},Sx_{2n+2},t\right)$$
$$=M\left(y_{2n},y_{2n+1},t\right)*M\left(y_{2n+1},y_{2n+2},t\right)*M\left(y_{2n+1},y_{2n+1},(1-\beta)t\right)*M\left(y_{2n},y_{2n+2},(1+\beta)t\right)*M\left(y_{2n},y_{2n+1},t\right)$$
$$=M\left(y_{2n},y_{2n+1},t\right)*M\left(y_{2n+1},y_{2n+2},t\right)*1*M\left(y_{2n},y_{2n+1},t\right)*M\left(y_{2n+1},y_{2n+2},\beta t\right)*M\left(y_{2n},y_{2n+1},t\right)$$
$$\geq M\left(y_{2n},y_{2n+1},t\right)*M\left(y_{2n+1},y_{2n+2},t\right)*M\left(y_{2n+1},y_{2n+2},\beta t\right)$$

Now, taking $\beta\to 1$, we have

$$M\left(y_{2n+1},y_{2n+2},kt\right)\geq M\left(y_{2n},y_{2n+1},t\right)*M\left(y_{2n+1},y_{2n+2},t\right)*M\left(y_{2n+1},y_{2n+2},t\right)$$
$$M\left(y_{2n+1},y_{2n+2},kt\right)\geq M\left(y_{2n},y_{2n+1},t\right)*M\left(y_{2n+1},y_{2n+2},t\right)\geq M\left(y_{2n},y_{2n+1},t\right)$$
$$M\left(y_{2n+1},y_{2n+2},kt\right)\geq M\left(y_{2n},y_{2n+1},t\right)$$

Similarly, we can show that

$$M\left(y_{2n+2},y_{2n+3},kt\right)\geq M\left(y_{2n+1},y_{2n+2},t\right)$$

Also,

$$N\left(y_{2n+1},y_{2n+2},kt\right)=N\left(Bx_{2n+1},Ax_{2n+2},kt\right)=N\left(Ax_{2n+2},Bx_{2n+1},kt\right)\leq N\left(Tx_{2n+1},Bx_{2n+1},t\right)\lozenge N\left(Sx_{2n+2},Ax_{2n+2},t\right)$$
$$\lozenge N\left(Sx_{2n+2},Bx_{2n+1},(1-\beta)t\right)\lozenge N\left(Tx_{2n+1},Ax_{2n+2},(1+\beta)t\right)\lozenge N\left(Tx_{2n+1},Sx_{2n+2},t\right)$$
$$=N\left(y_{2n},y_{2n+1},t\right)\lozenge N\left(y_{2n+1},y_{2n+2},t\right)\lozenge N\left(y_{2n+1},y_{2n+1},(1-\beta)t\right)\lozenge N\left(y_{2n},y_{2n+2},(1+\beta)t\right)\lozenge N\left(y_{2n},y_{2n+1},t\right)$$
$$=N\left(y_{2n},y_{2n+1},t\right)\lozenge N\left(y_{2n+1},y_{2n+2},t\right)\lozenge 0\lozenge N\left(y_{2n},y_{2n+1},t\right)\lozenge N\left(y_{2n+1},y_{2n+2},\beta t\right)\lozenge N\left(y_{2n},y_{2n+1},t\right)$$
$$\leq N\left(y_{2n},y_{2n+1},t\right)\lozenge N\left(y_{2n+1},y_{2n+2},t\right)\lozenge N\left(y_{2n+1},y_{2n+2},\beta t\right)$$

Taking $\beta\to 1$, we get

$$N\left(y_{2n+1},y_{2n+2},kt\right)\leq N\left(y_{2n},y_{2n+1},t\right)\lozenge N\left(y_{2n+1},y_{2n+2},t\right)\lozenge N\left(y_{2n+1},y_{2n+2},t\right)$$
$$N\left(y_{2n+1},y_{2n+2},kt\right)\leq N\left(y_{2n},y_{2n+1},t\right)\lozenge N\left(y_{2n+1},y_{2n+2},t\right)\leq N\left(y_{2n},y_{2n+1},t\right)$$
$$N\left(y_{2n+1},y_{2n+2},kt\right)\leq N\left(y_{2n},y_{2n+1},t\right)$$

Similarly, it can be shown that

$$N\left(y_{2n+2},y_{2n+3},kt\right)\leq N\left(y_{2n+1},y_{2n+2},t\right)$$

Therefore, for any *n* and *t*, we have

$$M\left(y_n,y_{n+1},kt\right)\geq M\left(y_{n-1},y_n,t\right),$$
$$N\left(y_n,y_{n+1},kt\right)\leq N\left(y_{n-1},y_n,t\right)$$

Hence, by Lemma 3.1, $\{y_n\}$ is a Cauchy sequence in *X*. Since *X* is complete, so $\{y_n\}$ converges to *z* in *X*. Its subsequences $\{Ax_{2n}\}$, $\{Tx_{2n+1}\}$, $\{Bx_{2n+1}\}$ and $\{Sx_{2n+2}\}$ also converge to *z*.

Now, suppose that (A,S) is a compatible pair and *S* is continuous. Then by Lemma 3.2, *A* and *S* are reciprocally continuous, then $SAx_n\to Sz$, $ASx_n\to Az$ as $n\to\infty$.

As, (A,S) is a compatible pair. This implies

$$\lim_{n\to\infty}M\left(ASx_n,SAx_n,t\right)=1,\lim_{n\to\infty}N\left(ASx_n,SAx_n,t\right)=0;$$

This gives $M\left(Az,Sz,t\right)=1,N\left(Az,Sz,t\right)=0$ as $n\to\infty$.

Hence, $Sz=Az$.

Since $A(X)\subseteq T(X)$, therefore there exists a point $p\in X$ such that $Sz=Az=Tp$.

Now, again by taking $\alpha=1$ in (3.2), we have

$$M\left(Az,Bp,kt\right)\geq M\left(Tp,Bp,t\right)*M\left(Sz,Az,t\right)$$
$$*M\left(Sz,Bp,t\right)*M\left(Tp,Az,t\right)*M\left(Tp,Sz,t\right)$$

$$M\left(Az,Bp,kt\right)\geq M\left(Az,Bp,t\right)*M\left(Az,Az,t\right)$$
$$*M\left(Az,Bp,t\right)*M\left(Az,Az,t\right)*M\left(Az,Az,t\right)$$

and

$$N\left(Az,Bp,kt\right)\leq N\left(Tp,Bp,t\right)\lozenge N\left(Sz,Az,t\right)$$
$$\lozenge N\left(Sz,Bp,t\right)\lozenge N\left(Tp,Az,t\right)\lozenge N\left(Tp,Sz,t\right)$$

$$N\left(Az,Bp,kt\right)\leq N\left(Az,Bp,t\right)\lozenge N\left(Az,Az,t\right)$$
$$\lozenge N\left(Az,Bp,t\right)\lozenge N\left(Az,Az,t\right)\lozenge N\left(Az,Az,t\right)$$

$$M\left(Az,Bp,kt\right)\geq M\left(Az,Bp,t\right),$$

$$N\left(Az,Bp,kt\right)\leq N\left(Az,Bp,t\right)$$

Thus, by Lemma 3.2, we have $Az=Bp$.

Thus, $Az = Bp = Sz = Tp$.

Since, A and S are pointwise R-weakly commuting mappings, therefore there exists $R > 0$, such that

$$M\left(ASz, SAz, t\right) \geq M\left(Az, Sz, \frac{t}{R}\right) = 1$$

and

$$N\left(ASz, SAz, t\right) \leq N\left(Az, Sz, \frac{t}{R}\right) = 0$$

Hence, $ASz = SAz$ and $ASz = SAz = AAz = SSz$.

Similarly, B and T are pointwise R-weakly commuting mappings, we have $BBp = BTp = TBp = TTp$.

Again, by taking $\alpha = 1$ in (3.2),

$$M\left(AAz, Bp, kt\right) \geq M\left(Tp, Bp, t\right) * M\left(SAz, AAz, t\right) * M\left(SAz, Bp, t\right) * M\left(Tp, AAz, t\right) * M\left(Tp, SAz, t\right)$$

$$M\left(AAz, Az, kt\right) \geq M\left(Tp, Tp, t\right) * M\left(AAz, AAz, t\right) * M\left(AAz, Az, t\right) * M\left(Az, AAz, t\right) * M\left(Az, AAz, t\right)$$

and

$$N\left(AAz, Bp, kt\right) \leq N\left(Tp, Bp, t\right) \lozenge N\left(SAz, AAz, t\right) \lozenge N\left(SAz, Bp, t\right) \lozenge N\left(Tp, AAz, t\right) \lozenge N\left(Tp, SAz, t\right)$$

$$N\left(AAz, Az, kt\right) \leq N\left(Tp, Tp, t\right) \lozenge N\left(AAz, AAz, t\right) \lozenge N\left(AAz, Az, t\right) \lozenge N\left(Az, AAz, t\right) \lozenge N\left(Az, AAz, t\right)$$

$$M\left(AAz, Az, kt\right) \geq M\left(AAz, Az, t\right),$$

$$N\left(AAz, Az, kt\right) \leq N\left(AAz, Az, t\right)$$

By Lemma 3.2, we have $SAz = AAz = Az$. Hence Az is common fixed point of A and S. Similarly by (3.2), $Bp = Az$ is a common fixed point of B and T. Hence, Az is a common fixed point of A, B, S and T.

Uniqueness: Suppose that $Ap (\neq Az)$ is another common fixed point of A, B, S and T.

Then by (3.2), take $\alpha = 1$

$$M\left(AAz, BAp, kt\right) \geq M\left(TAp, BAp, t\right) * M\left(SAz, AAz, t\right) * M\left(SAz, BAp, t\right) * M\left(TAp, AAz, t\right) * M\left(TAp, SAz, t\right)$$

$$M\left(Az, Ap, kt\right) \geq M\left(Ap, Ap, t\right) * M\left(Az, Az, t\right) * M\left(Az, Ap, t\right) * M\left(Ap, Az, t\right) * M\left(Ap, Az, t\right),$$ and

$$N\left(AAz, BAp, kt\right) \leq N\left(TAp, BAp, t\right) \lozenge N\left(SAz, AAz, t\right) \lozenge N\left(SAz, BAp, t\right) \lozenge N\left(TAp, AAz, t\right) \lozenge N\left(TAp, SAz, t\right)$$

$$N\left(Az, Ap, kt\right) \leq N\left(Ap, Ap, t\right) \lozenge N\left(Az, Az, t\right) \lozenge N\left(Az, Ap, t\right) \lozenge N\left(Ap, Az, t\right) \lozenge N\left(Ap, Az, t\right)$$

This gives

$M\left(Az, Ap, kt\right) \geq M\left(Az, Ap, t\right)$, and

$N\left(Az, Ap, kt\right) \leq N\left(Az, Ap, t\right)$

By Lemma 3.2, $Ap = Az$.

Thus, uniqueness follows.

Taking $S = T = I_X$ in above theorem, we get following result:

Corollary 4.1. Let $(X, M, N, *, \lozenge)$ be a complete intuitionistic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\lozenge$ defined by $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ for all $t \in [0,1]$. Further, let A and B are reciprocally continuous mappings on X satisfying

$$M\left(Ax, By, kt\right) \geq M\left(y, By, t\right) * M\left(x, Ax, t\right) * M\left(x, By, \alpha t\right) * M\left(y, Ax, (2-\alpha)t\right) * M\left(y, x, t\right)$$

$$N\left(Ax, By, kt\right) \leq N\left(y, By, t\right) \lozenge N\left(x, Ax, t\right) \lozenge N\left(x, By, \alpha t\right) \lozenge N\left(y, Ax, (2-\alpha)t\right) \lozenge N\left(y, x, t\right)$$

for all $u, v \in X$, $t > 0$ and $\alpha \in (0,2)$ then pair A and B has a unique common fixed point.

We give now example to illustrate the above theorem:

Example 4.1. Let $X = [0, \infty)$ and let M and N be defined by $M(u, v, t) = \frac{t}{t + |u - v|}$

and $N(u, v, t) = \frac{|u - v|}{t + |u - v|}$.

Then $(X, M, N, *, \lozenge)$ is complete intuitionistic fuzzy metric space. Let A, B, S and T be self maps on X defined as:

$Ax = Bx = \frac{3x}{4}$ and $Sx = Tx = 2x$ for all $x \in X$.

Clearly,

1) either of pair (A, S) or (B, T) be continuous self-mappings on X;
2) $A(X) \subseteq T(X), B(X) \subseteq S(X)$;
3) {A, S} and {B, T} are R-weakly commuting pairs as both pairs commute at coincidence points;
4) {A, S} and {B, T} satisfies inequality (3.2), for all $x, y \in X$, where $k \in (0,1)$.

Hence, all conditions of Theorem 4.1 are satisfied and $x = 0$ is a unique common fixed point of A, B, S and T.

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