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Extension of Range of MINRES-CN Algorithm

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Abstract

MINRES-CN is an iterative method for solving systems of linear equations with conjugate-normal coefficient matrices whose conspectra are located on algebraic curves of a low degree. This method was proposed in a previous publication of author and KH. D. Ikramov. In this paper, the range of applicability of MINRES-CN is extended in new direction. These are conjugate normal matrices that are low rank perturbations of Symmetric matrices. Examples are given that demonstrate a higher efficiency of MINRES-CN for this class of systems compared to the well-known algorithm GMRES.

Keywords: Conjugate-Normal Matrices, MINRES-CN Algorithm, MINRES-CN2 Algorithm

1. Introduction and Preliminaries

Suppose that one needs to solve the system of linear equations

$$Ax = b \quad (1)$$

with a conjugate-normal $n \times n$-matrix A. In the context of this paper, conjugate-normality means that

$$AA^* = \overline{A^* A} \quad (2)$$

A particular example of conjugate-normal matrices are symmetric matrices.

The method proposed in [1] is a minimum residual algorithm for the subspaces, which are the finite segments of the sequence

$$x, \bar{A}\bar{x}, A^*\bar{x}, \bar{A}Ax, \bar{A}A^T x, A^* A^T x, \bar{A}A\bar{A}\bar{x}, \cdots, \quad (3)$$

Unlike GMRES, this method, called MINRES-CN, is described by a recursion whose (fixed) length depends on the degree m of Γ (the conspectrum (you can see definition of conspectrum in [2]) of A belongs to an algebraic curve Γ of a low degree)). For instance, the length of the recursion is six in the case $m = 2$, which is given the most attention in [1].

2. Extension of Range

In this section, the range of applicability of MINRES-CN is extended in new direction.

We examine the behavior of MINRES-CN for new class of matrices A that can be considered as low rank perturbations of Symmetric matrices.

Let us first recall that any square complex matrix A can be uniquely represented in the form (see [3])

$$A = S + K, S = S^T, K = -K^T \quad (4)$$

We consider the class of conjugate-normal matrices A distinguished by the condition,

$$k = rankK < \frac{k-1}{2} \quad (5)$$

where n is the order of A. The conspectrum of such a matrix belongs to the union of the real axis and (at most) k lines that are parallel to the imaginary axis, *i.e.*, to a degenerate algebraic curve whose degree does not exceed k + 1. Hence, MINRES-CN is applicable to matrices of this type.

3. Numerical Results

Therefore, we can apply MINRES-CN to solving systems with conjugate normal coefficient matrices satisfying conditions (4) and (5).

The efficiency of the method is illustrated by several examples where band systems were solved. The performance of MINRES-CN2 (which is a specialization of MINRES-CN for conjugate normal matrices whose conspectra belong to a second-degree curve) in these examples is compared with that of the Matlab library program implementing GMRES.

In examples, we used the Matlab library function gmres for GMRES and a specially designed Matlab procedure for MINRES-CN2. The same stopping criterion was used for both methods; namely,

$$\|r\|_2 < \epsilon \tag{6}$$

where r is the current residual, while a positive scalar ϵ should be given by the user. For the example under discussion, we set $\epsilon = 10^{-8}$.

In all of our experiments, the order of systems was 2000. The right hand sides were generated as pseudo-random vectors with components distributed uniformly on (0, 1). The calculations were performed on a 2 Duo E630 OEM 1.86 GHz PC with core memory of 1024 Mb.

Example 3.1. Suppose that, $A = (a_{ij})_{n\times n}$, where $a_{ij} \in [10,14]$, for $i = 1,2,\cdots,n-2$, and $a_{n-1,n-1} = a_{n,n} = 0, a_{n-1,n} = -11, a_{n,n-1} = 11$ (where $A = S + K$ and $rankK = 2$), another entries of matrix A are zero. The conspectrum is located on the coordinate axes; *i.e.*, it belong to the second-degree curve,

$$xy = 0 \tag{7}$$

It follows that a system with the matrix A can be processed by MINRES-CN2.

MINRES-CN2 converges faster than GMRES (8 iteration steps and t = 0.02 s against 11 steps and t = 0.09 s).

Example 3.2. Suppose that $A = (a_{ij})_{n\times n}$, where $a_{ij} \in [10,14]$, for $i = 1,2,\cdots,n-2$, and $a_{n-1,n-1} = a_{n,n} = 12, a_{n-1,n} = -11, a_{n,n-1} = 11$ (where $A = S + K$ and $rankK = 2$), another entries of matrix A are zero. The conspectrum is located on the real axis and the line $x = 2$; *i.e.*, it belong to the second-degree curve,

$$(x-12)y = 0.$$

It follows that a system with the matrix A can be processed by MINRES-CN2.

MINRES-CN2 needs 10 steps and t = 0.02 s, while GMRES requires 12 steps and the time 0.09 s.

Example 3.3. Suppose that $A = (a_{ij})_{n\times n}$, where $a_{ij} \in [40,45]$, for $i = 1,2,\cdots,n-4$, and $a_{n-1,n} = -43, a_{n,n-1} = 43, a_{n-3,n-4} = -41, a_{n-4,n-3} = 41$ (where $A = S + K$ and $rankK = 4$), another entries of matrix A are zero. The conspectrum is located on the coordinate axes; *i.e.*, it belongs to the second-degree curve (7). It follows that a system with the matrix A can be processed by MINRES-CN2. 7 iteration steps and t = 0.02 s for MINRES-CN2 against 12 steps and t = 0.08 s for GMRES.

4. References

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