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Uniqueness of Meromorphic Functions with Their Nonlinear Differential Polynomials Share a Small Function

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Abstract

In this paper we deal with the uniqueness of meromorphic functions when two nonlinear differential polynomials generated by two meromorphic functions share a small function. We consider the case for some general differential polynomials [$f^nP(f)\ f'$] where *P*(*f*) is a polynomial which generalize some result due to Abhijit Banerjee and Sonali Mukherjee [1].

Keywords: Entire Functions, Meromorphic Functions, Nonlinear Differential Polynomials, Uniqueness

1. Introduction

In this paper, we use the standard notations and terms in the value distribution theory [2]. For any nonconstant meromorphic function $f(z)$ on the complex plane C, we denote by $S(r,f)$ any quantity satisfying $S(r,f) = o\{T(r,f)\}$, as $r\to+\infty$, except possibly for a set of r of finite linear measures. A meromorphic function $a(z)$ is called a small function with respect to *f*(*z*) if $T(r,a)=S(r,f)$. Let $S(f)$ be the set of meromorphic function in the complex plane C which are small functions with respect to *f*. Set $E(a(z),f)=\{z: f(z)-a(z)=0\}, a(z)\in S(f)$, where a zero point with multiplicity m is counted m times in the set. If these zero points are only counted once, then we denote the set by $\bar{E}(a,f)$. Let k be $a(z)$ a positive integer. Set

$$E_{k)}(a(z),f)=$$
$$\left\{z: f(z)-a(z)=0, \exists i, 1\le i\le k, s.t., f^{(i)}(z)-a^{(i)}(z)\neq 0\right\}$$

where a zero point with multiplicity m is counted *m* times in the set.

Let *f*(*z*) and *g*(*z*) be two transcendental meromorphic functions, $a(z)\in S(f)\cap S(g)$. If $E(a(z),f)=E(a(z),g)$, then we say that $f(z)$ and $g(z)$ share the value $a(z)$ CM, especially, we say that *f*(*z*) and *g*(*z*) have the same fixed points when $a(z)=z$. If $\bar{E}(a,f)=\bar{E}(a,g)$, then we say that *f*(*z*) and *g*(*z*) share the $a(z)$ IM. If $E_{k)}(a(z),f)=E_{k)}(a(z),g)$, we say that $f(z)-a$ and $g(z)-a$ have same zeros with the same multiplicities $\le k$.

Moreover, we also use the following notations.

We denote by $N_{k)}(r,f)$ the counting function for poles of *f*(*z*) with multiplicities $\le k$, and by $\bar{N}_{k)}(r,f)$ the corresponding one for which the multiplicity is not counted. Let $N_{(k}(r,f)$ be the counting function for poles of *f*(*z*) with multiplicities $\ge k$, and let $\bar{N}_{(k}(r,f)$ be the corresponding one for which the multiplicity is not counted. Set

$$N_k(r,f)=\bar{N}(r,f)+\bar{N}_{(2}(r,f)+\ldots+\bar{N}_{(k}(r,f).$$

Similarly, we have the notations

$$N_{k)}\left(r,\frac{1}{f}\right),\bar{N}_{k)}\left(r,\frac{1}{f}\right),N_{(k}\left(r,\frac{1}{f}\right),\bar{N}_{(k}\left(r,\frac{1}{f}\right),N_k\left(r,\frac{1}{f}\right).$$

Let *f*(*z*) and *g*(*z*) be two nonconstant meromorphic functions and $\bar{E}(1,f)=\bar{E}(1,g)$. We denote by $\bar{N}_L\left(r,\frac{1}{f-1}\right)$ the counting function for 1-points of both $f(z)$ and $g(z)$ about which $f(z)$ has larger multiplicity than $g(z)$, with multiplicity is not being counted, and denote by $N_{1]}\left(r,\frac{1}{f-1}\right)$ the counting function for common simple 1-points of both *f*(*z*) and *g*(*z*) where multiplicity is not counted. Similarly, we have the notation $\bar{N}_L\left(r,\frac{1}{(g-1)}\right)$.

In 2002 Fang and Fang [3] and in 2004 Lin-Yi [4] independently proved the following result.

Theorem A ([3,4]). Let f and g be two nonconstant meromorphic functions and $n(\geq 13)$ be an integer. If $f^n(f-1)^2 f'$ and $g^n(g-1)^2 g'$ share 1 CM, then $f \equiv g$.

In 2004 Lin-Yi [5] improved Theorem A by generalizing it in view of fixed point. Lin-Yi [5] proved the following result.

Theorem B ([5]). Let f and g be two transcendental meromorphic functions and $n(\geq 13)$ be an integer. If $f^n(f-1)^2 f'$ and $g^n(g-1)^2 g'$ share z CM, then $f \equiv g$.

With the notion of weighted sharing of value recently the first author [6] improved Theorem A as follows.

Theorem C ([6]). Let f and g be two nonconstant meromorphic functions and

$$n > \left[12 - 2\Theta(\infty; f) - 2\Theta(\infty; g) - \min\{\Theta(\infty; f), \Theta(\infty; g)\}\right],$$

is an integer. If $f^n(f-1)^2 f'$ and $g^n(g-1)^2 g'$ share "$(1,2)$" then $f \equiv g$.

In the mean time Lahiri and Sarkar [7] also studied the uniqueness of meromorphic functions corresponding to nonlinear differential polynomials which are different from that of previously mentioned and proved the following.

Theorem D ([7]). Let f and g be two nonconstant meromorphic functions such that $f^n(f-1)^2 f'$ and $g^n(g-1)^2 g'$ share "$(1,2)$", where $n(\geq 13)$ is an integer then either $f \equiv g$ or $f \equiv -g$. If n is an even integer then the possibility of $f \equiv -g$ does not arise.

In 2008, Banerjee and Murkherjee [1] proved the following theorem.

Theorem E ([1]). Let f and g be two transcendental meromorphic functions such that $f^n(af^2 + bf + c)f'$ and $g^n(ag^2 + bg + c)g'$ where $a \neq 0$ and $|b| + |c| \neq 0$ share "$(\alpha, 2)$". Then the following holds:

1) If $b \neq 0, c = 0$ and

$$n > \max\left[12 - 2\Theta(\infty; f) - 2\Theta(\infty; g) - \min\{\Theta(\infty; f), \Theta(\infty; g)\}, \frac{4}{\Theta(\infty; f) + \Theta(\infty; g)} - 2\right]$$

be an integer, where $\Theta(\infty; f) + \Theta(\infty; g) > 0$, then $f \equiv g$.

2) If $b \neq 0, c \neq 0$ and

$$n > \max\left[12 - 2\Theta(\infty; f) - 2\Theta(\infty; g) - \min\{\Theta(\infty; f), \Theta(\infty; g)\}\right],$$

the roots of the equation $az^2 + bz + c = 0$ are distinct and one of f and g is nonentire meromorphic function having only multiple poles, then $f \equiv g$.

3) If $b \neq 0, c \neq 0$ and

$$n > \max\left[12 - 2\Theta(\infty; f) - 2\Theta(\infty; g) - \min\{\Theta(\infty; f), \Theta(\infty; g)\}\right]$$

and the roots of the equation $az^2 + bz + c = 0$ coincides, then $f \equiv g$.

4) If $b = 0$, $c \neq 0$ and

$$n > \max\left[12 - 2\Theta(\infty; f) - 2\Theta(\infty; g) - \min\{\Theta(\infty; f), \Theta(\infty; g)\}\right],$$

then either $f \equiv g$ or $f \equiv -g$. If n is an even integer then the possibility $f \equiv -g$ does not arise.

Here, we obtain unicity theorem when [$f^n P(f) f'$] and [$g^n P(g) g'$] share a small function.

Theorem 1. Let f and g be two transcendental meromorphic functions. Let $P(f) = a_m f^m + a_{m-1} f^{m-1} + \ldots + a_1 f + a_0, (a_m \neq 0)$, and $a_i (i = 0, 1, \cdots, m)$ is the first nonzero coefficient from the right, and n, m, k be a positive integer with

$$n > \left[m + 10 - 2\Theta(\infty; f) - 2\Theta(\infty; g) - \min\{\Theta(\infty; f); \Theta(\infty; g)\}\right].$$

If [$f^n P(f) f'$] and [$g^n P(g) g'$] share "$(\alpha, 2)$" then $f \equiv g$.

2. Lemmas

In this section we present some lemmas which will be needed in the sequel. Let f, g, F_1, G_1 be four nonconstant meromorphic functions. Henceforth we shall denote by h and H the following two functions.

$$h = \left(\frac{f''}{f'} - \frac{2f'}{f-1}\right) - \left(\frac{g''}{g'} - \frac{2g'}{g-1}\right)$$

$$\text{and } H = \left(\frac{F_1''}{F_1'} - \frac{2F_1'}{F_1 - 1}\right) - \left(\frac{G_1''}{G_1'} - \frac{2G_1'}{G_1 - 1}\right).$$

Lemma 2.1. ([1]) If for a positive integer k, $N_k(r, 0; f' | f \neq 0)$ denotes the counting function of those zeros of f' which are not the zeros of f, where a zero of f' with multiplicity m is counted m times if $m \leq k$ and k times if $m > k$ then

$$N_k(r, 0; f' | f \neq 0) \leq \bar{N}(r, 0; f) + \bar{N}(r, \infty; f) - \sum_{p=k+1}^{\infty} \bar{N}\left(r, 0; \frac{f'}{f} | \geq p\right) + S(r, f).$$

Lemma 2.2. ([1]) Let f, g be share "$(1,2)$" and $h \neq 0$. Then

$$T(r, f) \leq N_2(r, 0; f) + N_2(r, \infty; f) + N_2(r, 0; g) + N_2(r, \infty; g) - \sum_3^{\infty} \bar{N}\left(r, 0; \frac{g'}{g} | \geq p\right) + S(r, f) + S(r, g).$$

Lemma 2.3. ([8]) Let f be a nonconstant meromorphic function and $P(f)=a_0+a_1f+\cdots+a_nf^n$, where $a_0,a_1,\cdots,a_n$ are constants and $a_n\neq 0$. Then

$$T(r,P(f))=nT(r,f)+S(r,f).$$

Lemma 2.4. Let $F_1=\dfrac{f^nP(f)f'}{\alpha}$ and $G_1=\dfrac{g^nP(g)g'}{\alpha}$, where $\alpha(\neq 0,\infty)$ is a small function of f and g. Then $S(r,F_1)=S(r,f)$ and $S(r,G_1)=S(r,g)$.

Proof. Using Lemma 2.3 we see that

$$\begin{aligned}T(r,F_1)&\le(n+m)T(r,f)+T(r,f')+S(r,f)\\&=(n+m+2)T(r,f)+S(r,f).\end{aligned}$$

And

$$\begin{aligned}(n+m)T(r,f)&=T\left(r,f^nP(f)\right)+O(1)\\&\le T(r,F_1)+T(r,f')+S(r,f),\end{aligned}$$

that is, $T(r,F_1)\ge(n+m-2)T(r,f)+S(r,f)$. Hence $S(r,F_1)=S(r,f)$.

In the same way we can prove $S(r,G_1)=S(r,g)$. This proves the Lemma.

Lemma 2.5. ([9]) If $h\equiv 0$ and

$$\limsup_{r\to\infty}\frac{\bar N(r,0;f)+\bar N(r,\infty;f)+\bar N(r,0;g)+\bar N(r,\infty;g)}{T(r)}<1,$$

$r\in I$ then $f\equiv g$ or $fg\equiv 1$.

Lemma 2.6. Let f,g be two nonconstant meromorphic functions. Then

$$f^nP(f)f'g^nP(g)g'\neq\alpha^2,$$

where $n+m(\ge 6)$ is an integer.

Proof. Let

$$f^nP(f)f'g^nP(g)g'\equiv\alpha^2. \tag{2.1}$$

Let z_0 be a 1-point of f with multiplicity $p(\ge 1)$. Then z_0 is a pole of g with multiplicity $q(\ge 1)$ such that $np+p-1=nq+q+mq+1$, *i.e.*,

$$mq+2=(n+1)(p-q) \tag{2.2}$$

From (2.2) we get $q\ge\dfrac{n-1}{m}$ and again from (2.2) we obtain

$$p\ge\frac{1}{n+1}\left[\frac{(n+m+1)(n-1)}{m}+2\right]=\frac{n+m-1}{m}.$$

Let z_1 be a zero of $P(f)$ with multiplicity $p_1(\ge 1)$. Then z_1 is a pole of g with multiplicity $q_1(\ge 1)$, say. So from (2.1) we get

$$2p_1-1=(n+m+1)q+1\ge(n+m+2)$$

i.e., $p_1\ge\dfrac{(n+m+3)}{2}$.

Since a pole of f is either a zero of $g^nP(g)$ or a zero of g', we have

$$\begin{aligned}\bar N(r,\infty;f)&\le\bar N(r,0;g)+\bar N\left(r,0;g^m\right)+\bar N_0(r,0;g')\\&\quad+S(r,f)+S(r,g)\\ \bar N(r,\infty;f)&\le\frac{m}{n+m-1}N(r,0;g)+\frac{2}{n+m+3}N\left(r,0;g^m\right)\\&\quad+\bar N_0(r,0;g')+S(r,f)+S(r,g)\\&\le\left(\frac{m}{n+m-1}+\frac{2m}{n+m+3}\right)T(r,g)+\bar N_0(r,0;g')\\&\quad+S(r,f)+S(r,g),\end{aligned}$$

where $\bar N_0(r,0;g')$ denotes the reduced counting function of those zeros of g' which are not the zeros of $gP(g)$.

As $P(f)=a_mf^m+a_{m-1}f^{m-1}+\cdots+a_1f+a_0$ where $a_m,a_{m-1},\cdots,a_0$ are m distinct complex numbers. Then by second fundamental theorem of Nevanlinna we get

$$\begin{aligned}mT(r,f)&\le\bar N(r,\infty;f)+\bar N(r,0;f)-\bar N_0(r,0;f')\\&\quad+\sum_{j=1}^{m}\bar N\left(r,a_j;f\right)+S(r,f)\\&\le\bar N(r,0;f)+\bar N(r,\infty;f)+\bar N\left(r,a;f^m\right)\\&\quad-\bar N_0(r,0;f')+S(r,f)\qquad(2.3)\\&\le\left(\frac{m}{n+m-1}+\frac{2m}{n+m+3}\right)\left\{T(r,g)+T(r,f)\right\}\\&\quad+\bar N(r,0;g')-\bar N_0(r,0;f')\\&\quad+S(r,f)+S(r,g).\end{aligned}$$

Similarly, we have

$$\begin{aligned}mT(r,g)&\le\left(\frac{m}{n+m-1}+\frac{2m}{n+m+3}\right)\left\{T(r,g)+T(r,f)\right\}\\&\quad+\bar N_0(r,0;f')-\bar N_0(r,0;g')+S(r,f)+S(r,g).\end{aligned}\tag{2.4}$$

Adding (2.3) and (2.4) we obtain

$$\begin{aligned}&\left(1-\frac{2}{n+m-1}-\frac{4}{n+m+3}\right)\left\{T(r,g)+T(r,f)\right\}\\&\le S(r,f)+S(r,g)\end{aligned}$$

which is a contradiction. This proves the Lemma.

Lemma 2.7. Let f and g be two transcendental meromorphic function and

$$F=f^{n+1}\left[\frac{a_m}{m+n+1}f^m+\frac{a_{m-1}}{m+n}f^{m-1}+\cdots+\frac{a_0}{n+1}\right]$$

$$G=g^{n+1}\left[\frac{a_m}{m+n+1}g^m+\frac{a_{m-1}}{m+n}g^{m-1}+\cdots+\frac{a_0}{n+1}\right]$$

where $n(>m+2)$ is an integer. Then $F' \equiv G'$ implies that $F \equiv G$.

Proof. Let $F' \equiv G'$, then $F \equiv G + c$ where c is a constant. Let $c \neq 0$. Then by second fundamental theorem we get

$$T(r,F) \le \bar{N}(r,\infty;F) + \bar{N}(r,0;F) + \bar{N}(r,c;F) + S(r,F)$$
$$\le \bar{N}(r,\infty;f) + \bar{N}(r,0;f) + \bar{N}\left(r,\frac{a_m}{m+n+1};f^m\right)$$
$$+\bar{N}(r,0;g) + \bar{N}\left(r,\frac{a_m}{m+n+1};g^m\right) + S(r,f)$$
$$\le 2T(r,f) + mT(r,f) + T(r,g) + mT(r,g) + S(r,f).$$

Hence we get

$$(m+n+1)T(r,f) \le (2+m)T(r,f) + (m+1)T(r,g) + S(r,f). \quad (2.5)$$

Similarly, we have

$$(m+n+1)T(r,g) \le (2+m)T(r,g) + (m+1)T(r,f) + S(r,g). \quad (2.6)$$

Adding (2.5) and (2.6) we obtain

$$(m+n+1)\{T(r,f)+T(r,g)\} \le (3+2m)T(r,f) + (3+2m)T(r,g) + S(r,f) + S(r,g)$$

i.e., $(n-m-2)\{T(r,f)+T(r,g)\} \le S(r,f)+S(r,g)$.

which is a contradiction. So $c=0$ and the Lemma is proved.

Lemma 2.8. ([10]) Let f be a nonconstant meromorphic function. Then

$$N(r,0;f^k) \le k\bar{N}(r,\infty;f) + N(r,0;f) + S(r,f).$$

Lemma 2.9. Let F and G be given as in Lemma 2.7 and F_1, G_1 be given by Lemma 2.4. Then

1) $T(r,F) \le T(r,F_1) + N(r,0;f) + N(r,b_1;f) + \cdots + N(r,b_m;f) - N(r,c_1;f) - \cdots - N(r,c_m;f) - N(r,0;f') + S(r,f)$

2) $T(r,G) \le T(r,G_1) + N(r,0;g) + N(r,b_1;g) + \cdots + N(r,b_m;g) - N(r,c_1;g) - \cdots - N(r,c_m;g) - N(r,0;g') + S(r,g)$

where $b_1, b_2, \cdots, b_m$ are roots of the algebraic equation

$$\frac{a_m}{m+n+1}z^m + \frac{a_{m-1}}{m+n}z^{m-1} + \cdots + \frac{a_0}{n+1} = 0$$

and $c_1, c_2, \cdots, c_m$ are roots of the algebraic equation

$$a_m z^m + a_{m-1}z^{m-1} + \cdots + a_0 = 0.$$

Proof. By the Nevanlinna's first fundamental theorem and Lemmas 2.3 we obtain

$$T(r,F) = T\left(r,\frac{1}{F}\right) + O(1)$$
$$= N(r,0;F) + m\left(r,\frac{1}{F}\right) + O(1)$$
$$\le N(r,0;F) + m\left(r,\frac{F'}{F}\right) + m(r,0;F') + O(1)$$
$$= T(r,F') + N(r,0;F) - N(r,0;F') + S(r,F)$$
$$\le T(r,F_1) + N(r,0;f) + N(r,b_1;f) + \cdots + N(r,b_m;f)$$
$$-N(r,c_1;f) - \cdots - N(r,c_m;f) - N(r,0;f') + S(r,f).$$

Similarly, we have

$$T(r,G) \le T(r,G_1) + N(r,0;g) + N(r,b_1;g) + \cdots$$
$$+N(r,b_m;g) - N(r,c_1;g) - N(r,c_2;g) - \cdots$$
$$-N(r,c_m;g) - N(r,0;g') + S(r,g)$$

where $b_1, b_2, \cdots, b_m$ are roots of the algebraic equation

$$\frac{a_m}{m+n+1}z^m + \frac{a_{m-1}}{m+n}z^{m-1} + \cdots + \frac{a_0}{n+1} = 0$$

and $c_1, c_2, \cdots, c_m$ are roots of the algebraic equation

$$a_m z^m + a_{m-1}z^{m-1} + \cdots + a_0 = 0.$$

This proves the Lemma.

3. Proofs of the Theorems

Proof of Theorem 1. Let F, G be defined as in Lemma 2.7 and F_1 and G_1 be defined as in Lemma 2.4. Then it follows that F' and G' share "$(\alpha,2)$" and hence F_1 and G_1 share "$(\alpha,2)$". Suppose $H \neq 0$. Then by Lemma 2.2, 2.4 and (2.3) we get

$$T(r,F_1) \le N_2(r,0;F_1) + N_2(r,\infty;F_1) + N_2(r,0;G_1)$$
$$+N_2(r,\infty;G_1) + S(r,f) + S(r,g)$$
$$\le 2\bar{N}(r,0;f) + 2\bar{N}(r,\infty;f) + N(r,c_1;f) + \cdots$$
$$+N(r,c_m;f) + N(r,0;f') + 2\bar{N}(r,0;g) + 2\bar{N}(r,\infty;g)$$
$$+N(r,c_1;g) + \cdots + N(r,c_m;g) + N(r,0;g')$$
$$+S(r,f) + S(r,g). \quad (3.1)$$

Now from Lemma 2.3, 2.8 and 2.9 we can obtain from (3.1) for $\varepsilon(>0)$

$$(m+n+1)T(r,f)$$
$$\le 2\bar{N}(r,0;f) + 2\bar{N}(r,\infty;f) + mT(r,f)$$
$$+N(r,0;f) + 2\bar{N}(r,0;g) + 2\bar{N}(r,\infty;g)$$
$$+mT(r,g) + N(r,0;g') \quad (3.2)$$
$$\le (m+3)\left[T(r,f) + T(r,g)\right] + 2\bar{N}(r,\infty;f)$$
$$+3\bar{N}(r,0;g) + S(r,f) + S(r,g).$$

$$(m+n+1)T(r,f)$$
$$\le (2m+11-3\Theta(\infty;g)-2\Theta(\infty;f)+2\varepsilon)T(r)+S(r).$$

In a similar manner we can obtain

$$(m+n+1)T(r,g)$$
$$\le (2m+11-3\Theta(\infty;f)-2\Theta(\infty;g)+2\varepsilon)T(r)+S(r). \tag{3.3}$$

From (3.2) and (3.3) we get

$$\begin{aligned}&[n-m-10+2\Theta(\infty;f)+2\Theta(\infty;g)\\ &+\min\{\Theta(\infty;f);\Theta(\infty;g)\}-2\varepsilon]T(r)\\ &\le S(r).\end{aligned} \tag{3.4}$$

Since $\varepsilon(>0)$ is arbitrary, (3.4) implies a contradiction. Hence $H\equiv 0$. Since for $\varepsilon(>0)$ we have

$$\begin{aligned}&\bar{N}(r,0;f')\le T(r,f')-m(r,1/f')\\ &\le m(r,f)+N(r,\infty;f)+\bar{N}(r,\infty;f)-m(r,1/f')\\ &+S(r,f)\\ &\le (2-\Theta(\infty;f)+\varepsilon)T(r,f)-m(r,1/f')+S(r,f).\end{aligned}$$

We note that

$$\begin{aligned}&\bar{N}(r,\infty;F_1)+\bar{N}(r,0;F_1)+\bar{N}(r,\infty;G_1)+\bar{N}(r,0;G_1)\\ &\le \bar{N}(r,0;f)+\bar{N}(r,c_1;f)+\cdots+\bar{N}(r,c_m;f)\\ &+\bar{N}(r,\infty;f)+\bar{N}(r,0;f')+\bar{N}(r,0;g)\\ &+\bar{N}(r,c_1;g)+\cdots+\bar{N}(r,c_m;g)+\bar{N}(r,\infty;g)+\bar{N}(r,0;g').\\ &\le (2m+8-2\Theta(\infty;f)-2\Theta(\infty;g)+2\varepsilon)T(r)\\ &-m(r,0;f')-m(r,0;g')+S(r).\end{aligned} \tag{3.5}$$

Also using Lemma 2.3 we get

$$\begin{aligned}&T(r,F')+m(r,1/f')=m(r,f^nP(f)f')+m(r,1/f')\\ &+N(r,\infty;f^nP(f)f')\\ &\ge m(r,f^nP(f))+N(r,\infty;f^nP(f))\\ &=T(r,f^nP(f))=(n+m)T(r,f)+O(1).\end{aligned} \tag{3.6}$$

Similarly

$$T(r,G')+m(r,1/g')\ge (n+m)T(r,g)+O(1). \tag{3.7}$$

From (3.6) and (3.7) we get

$$\max\{T(r,F_1),T(r,G_1)\}\ge (n+m)T(r)-m(r,1/f'). \tag{3.8}$$

By (3.5) and (3.8) applying Lemma 2.5 we get either $F_1\equiv G_1$ or $F_1G_1\equiv 1$.

Now from Lemma 2.6 it follows that $F_1G_1\ne 1$. Again $F_1\equiv G_1$ implies $F'\equiv G'$. So from Lemma 2.7 the theorem follows.

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