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The Numbers of Thousand Place of Mersenne Primes*

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Abstract

Mersenne primes are a special kind of primes, which are an important content in number theory. The study of Mersenne primes becomes one of hot topics of the nowadays science. Searching for Mersenne primes is very challenging in scientific researches. In this paper, the numbers of thousand place of Mersenne primes are studied, and the conclusion is presented by using the Chinese remainder theorem.

Keywords: Mersenne Primes, The Chinese Remainder Theorem, The Number of Thousand Place

1. Introduction

In 300 BC, ancient Greek mathematician Euclid proved that there are infinitude primes by contradiction, and raised that a small amount of prime numbers could be expressed in numbers of the form $2^p - 1$, where p is a prime. Afterward, many famous mathematicians have researched the prime numbers of this special formulation.

In 1644, French mathematician M. Mersenne stated in the preface to his *Cogitata Physica-Mathematica* that the numbers $2^p - 1$ were primes for p = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127 and 257, and were composite for all other positive integers $p < 257$. Mersenne's (incorrect) conjecture fared only slightly better than Regius', but still got his name attached to these numbers. The formulation of $2^p - 1$ is named as "Mersenne number" and expressed as M_p (so $M_p = 2^p - 1$). If a M_p is a prime number, then it is named as "Mersenne prime".

The Lucas-Lehmer test is a primarily test for Mersenne numbers: For p an odd prime, the Mersenne number $2^p - 1$ is prime if and only if $2^p - 1$ divides S(p – 1), where $S(p+1) = S(p)^2 - 2$, and $S(1) = 4$. The test was originally developed by E. Lucas in 1856, and subsequently improved by Lucas in 1878 and D. Lehmer in the 1930s. The sequence S(p) is computed modulo $2^p - 1$ to save time. The Lucas-Lehmer test is ideal for binary computers because the division of $2^p - 1$ (in binary) can be done by using rotation and addition only.

In 1992, Chinese mathematician and linguist Haizhong Zhou [1] presented the well-known Zhou conjecture on the distribution of Mersenne primes in the natural number system: If $2^{2^n} < p < 2^{2^{n+1}}$ (n = 0, 1, 2, 3,$\cdots$), then there are $2^{n+1} - 1$ Mersenne primes. At the same time he gave the deduction: If $p < 2^{2^{n+1}}$, then there are $2^{n+2} - n - 2$ Mersenne primes.

As the increase of exponent p, searching for Mersenne primes is very challenging. Studying Mersenne primes, not only the advanced theory and practiced skills are needed, but also the arduous calculations are needed to validate whether a Mersenne number is a prime or not [2]. The research of Mersenne primes is very abundant in the contemporary theoretical and practical value. The research of Mersenne primes greatly promotes the development of mathematics, especially number theory. In addition, many well-known mathematics problems, such as Goldbach conjecture, twin primes, Riemann conjecture, etc. are inextricably linked to Mersenne primes. Therefore, the research of Mersenne primes can also speed up the solution to those problems [3]. And the research of Mersenne primes can promote distributed computing and programming arts. It not only requires well-designed distributed architecture, but also improves the numerical calculation methods and algorithm design arts. [4] There are only 47 known Mersenne primes [5].

In [6,7], the last number and the ten place number have been studied, and conclusions have been obtained. In this paper, the numbers of thousand place of Mersenne primes are studied, and the conclusion is presented by using the Chinese remainder theorem.

Theorem If the exponent p of M_p satisfies

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$$\begin{cases} p=8k+1,\ k\equiv 1,12,14,18,22,60, \\ 61,63,66,69,75,103,104 \pmod{125}; \\ p=8k+3,\ k\equiv 14,27,46,54,61,63, \\ 73,76,80,83,87,115 \pmod{125}; \\ p=8k+5,\ k\equiv 3,6,12,41,63,74, \\ 76,84,122,123 \pmod{125}; \\ p=8k+7,\ k\equiv 0,10,13,17,20,24, \\ 52,76,89,108,116,123 \pmod{125} \end{cases}$$

Then, the number of the thousand place of Mersenne primes is 0;

If the exponent p of M_p satisfies

$$\begin{cases} p=8k+1,\ k\equiv 2,6,34,58,71,90,98, \\ 105,107,117,120,124 \pmod{125}; \\ p=8k+3,\ k\equiv 16,17,19,22,25,31, \\ 59,60,82,93,95,99,103 \pmod{125}; \\ p=8k+5,\ k\equiv 8,27,42,44,54,57, \\ 61,64,68,96 \pmod{125}; \\ p=8k+7,\ k\equiv 19,30,32,36,40,78, \\ 79,81,84,87,93,121,122 \pmod{125} \end{cases}$$

Then, the number of the thousand place of Mersenne primes is 1;

If the exponent p of M_p satisfies

$$\begin{cases} p=8k+1,\ k\equiv 10,11,13,16,19,25, \\ 53,54,76,87,89,93,97 \pmod{125}; \\ p=8k+3,\ k\equiv 1,5,8,12,40,64,77, \\ 96,104,111,113,123 \pmod{125}; \\ p=8k+5,\ k\equiv 13,24,26,34, \\ 72,73,78,81,87,116 \pmod{125}; \\ p=8k+7,\ k\equiv 1,14,33,41,48,50, \\ 60,63,67,70,74,102 \pmod{125} \end{cases}$$

Then, the number of the thousand place of Mersenne primes is 2;

If the exponent p of M_p satisfies

$$\begin{cases} p=8k+1, k\equiv 8,21,40,48,55,59, \\ 67,70,74,77,81,109 \pmod{125}; \\ p=8k+3, k\equiv 7,18,20,24,28,66, \\ 67,69,72,75,81,109,110 \pmod{125}; \\ p=8k+5, k\equiv 4,7,11,14,18,46, \\ 83,102,117,119 \pmod{125}; \\ p=8k+7, k\equiv 3,4,6,9,12,18, \\ 46,47,69,80,82,86,90 \pmod{125} \end{cases}$$

Then, the number of the thousand place of Mersenne primes is 3;

If the exponent p of M_p satisfies

$$\begin{cases} p=8k+1, k\equiv 3,4,26,37,39,43,47, \\ 85,86,88,91,94,100 \pmod{125}; \\ p=8k+3,\ k\equiv 2,21,29,36,38,48, \\ 51,55,58,62,90,114 \pmod{125}; \\ p=8k+5,\ k\equiv 22,23,28,31,37, \\ 66,88,99,101,109 \pmod{125}; \\ p=8k+7,\ k\equiv 27,51,64,83,91,98, \\ 100,110,113,117,120,124 \pmod{125} \end{cases}$$

Then, the number of the thousand place of Mersenne primes is 4;

If the exponent p of M_p satisfies

$$\begin{cases} p=8k+1, k\equiv 5,7,17,20,24,27,31, \\ 59,83,96,115,123 \pmod{125}; \\ p=8k+3, k\equiv 0,6,34,35,57,68,70, \\ 74,78,116,117,119,122 \pmod{125}; \\ p=8k+5, k\equiv 33,52,67,69,79, \\ 82,86,89,93,121 \pmod{125}; \\ p=8k+7, k\equiv 5,7,11,15,53,54, \\ 56,59,62,68,96,97,119 \pmod{125} \end{cases}$$

Then, the number of the thousand place of Mersenne primes is 5;

If the exponent p of M_p satisfies

$$\begin{cases} p=8k+1,\ k\equiv 35,36,38,41,44,50,78, \\ 79,101,112,114,118,122 \pmod{125}; \\ p=8k+3,\ k\equiv 15,39,52,71,79,86,88, \\ 98,101,105,108,112 \pmod{125}; \\ p=8k+5,\ k\equiv 16,38,49,51,59,97, \\ 98,103,106,112 \pmod{125}; \\ p=8k+7,\ k\equiv 8,16,23,25,35,38,42, \\ 45,49,77,101,114 \pmod{125} \end{cases}$$

Then, the number of the thousand place of Mersenne primes is 6;

If the exponent p of M_p satisfies

$$\begin{cases} p=8k+1, k\equiv 9,33,46,65,73,80, \\ 82,92,95,99,102,106 \pmod{125}; \\ p=8k+3, k\equiv 3,41,42,44,47,50, \\ 56,84,85,107,118,120,124 \pmod{125}; \\ p=8k+5, k\equiv 2,17,19,29,32,36, \\ 39,43,71,108 \pmod{125}; \\ p=8k+7, k\equiv 21,22,44,55,57,61, \\ 65,103,104,106,109,112,118 \pmod{125} \end{cases}$$

Then, the number of the thousand place of Mersenne primes is 7;

If the exponent p of M_p satisfies

$$\begin{cases} p=8k+1, k\equiv 0,28,29,51,62,64,68, \\ 72,110,111,113,119 (\text{mod}\,125); \\ p=8k+3, k\equiv 4,11,13,23,26,30,33, \\ 37,65,89,102,121 (\text{mod}\,125); \\ p=8k+5, k\equiv 1,9,47,48,53,56, \\ 62,91,113,124 (\text{mod}\,125); \\ p=8k+7, k\equiv 2,26,39,58,66, \\ 73,75,85,88,92,95,99 (\text{mod}\,125) \end{cases}$$

Then, the number of the thousand place of Mersenne primes is 8;

If the exponent p of M_p satisfies

$$\begin{cases} p=8k+1, k\equiv 15,23,30,32,42,45, \\ 49,52,56,84,108,121 (\text{mod}\,125); \\ p=8k+3, k\equiv 9,10,32,43,45,49, \\ 53,91,92,94,97,100,106 (\text{mod}\,125); \\ p=8k+5, k\equiv 21,58,77,92,94,104, \\ 107,111,114,118 (\text{mod}\,125); \\ p=8k+7, k\equiv 28,29,31,34,37,43, \\ 71,72,94,105,107,111,115 (\text{mod}\,125) \end{cases}$$

Then, the number of the thousand place of Mersenne primes is 9.

2. Preliminaries

In order to prove the theorem, we need the following lemma.

Lemma (*The Chinese Remainder Theorem*)

Let $m_1, m_2, \cdots, m_r$ be pair-wise relatively prime positive integers. Then the system of congruencies

$$\begin{cases} x\equiv a_1 (\text{mod}\, m_1) \\ x\equiv a_2 (\text{mod}\, m_2) \\ \quad\vdots \\ x\equiv a_r (\text{mod}\, m_r) \end{cases}$$

has a unique solution modulo $m = m_1 m_2, \cdots, m_r$ [6].

3. Proof of the Theorem

Since the exponent p of M_p is a prime, then $p = 4k + 1$ or $p = 4k + 3$. When $p = 2, 3, 5$, $2^p - 1 < 100$. So, $p = 8k + 1$ or $p = 8k+3$ or $p = 8k+5$ or $p = 8k+7$.

We have five congruence equations as following, when M_p modulo 16,625 separately.

$$\begin{cases} M_p = 2^{8k+1} - 1 = 256^k \times 2 - 1 \equiv -1 (\text{mod}\,16) \\ M_p = 2^{8k+3} - 1 = 256^k \times 2^3 - 1 \equiv -1 (\text{mod}\,16) \\ M_p = 2^{8k+5} - 1 = 256^k \times 2^5 - 1 \equiv -1 (\text{mod}\,16) \\ M_p = 2^{8k+7} - 1 = 256^k \times 2^7 - 1 \equiv -1 (\text{mod}\,16) \end{cases} \quad (1)$$

$$M_p = 2^{8k+1} - 1 \equiv 256^k \times 2 - 1 (\text{mod}\,625) \quad (2)$$

$$M_p = 2^{8k+3} - 1 \equiv 256^k \times 2^3 - 1 (\text{mod}\,625) \quad (3)$$

$$M_p = 2^{8k+5} - 1 \equiv 256^k \times 2^5 - 1 (\text{mod}\,625) \quad (4)$$

$$M_p = 2^{8k+7} - 1 \equiv 256^k \times 2^7 - 1 (\text{mod}\,625) \quad (5)$$

Note that $256^k = 256^{125k_1} \equiv 1 (\text{mod}\,625)$ when $k \equiv 0 (\text{mod}\,125)$, where k is a nonnegative integer. Therefore, we have congruence equations as following, when 256^k modulo 625 separately, where $k \equiv k_i\ (\text{mod}\,125)$ and $k_i = 0,1,2,\cdots,124$.

$256^k \equiv 1$, 256, 536, 341, 421, 276, 31, 436, 366, 571, 551, 431, 336, 391, 96, 201, 206, 236, 416, 246, 476, 606, 136, 441, 396, 126, 381, 36, 466, 546, 401, 156, 561, 491, 71, 51, 556, 461, 516, 221, 326, 331, 361, 541, 371, 601, 106, 261, 566, 521, 251, 506, 161, 591, 46, 526, 281, 61, 616, 196, 176, 56, 586, 16, 346, 451, 456, 486, 41, 496, 101, 231, 386, 66, 21, 376, 6, 286, 91, 171, 26, 406, 186, 116, 321, 301, 181, 86, 141, 471, 576, 581, 611, 166, 621, 226, 356, 511, 191, 146, 501, 131, 411, 216, 296, 151, 531, 311, 241, 446, 426, 306, 211, 266, 596, 76, 81, 111, 291, 121, 351, 481, 11, 316, 271 $(\text{mod}\,625)$.

(6)

Combined congruence Equations (6) and (2), (3), (4), (5) separately, we have

$M_p = 2^{8k+1} - 1 \equiv 1$, 511, 446, 56, 216, 551, 61, 246, 106, 516, 476, 236, 46, 156, 191, 401, 411, 471, 206, 491, 326, 586, 271, 256, 166, 251, 136, 71, 306, 466, 176, 311, 496, 356, 141, 101, 486, 296, 406, 441, 26, 36, 96, 456, 116, 576, 211, 521, 506, 416, 501, 386, 321, 556, 91, 426, 561, 121, 606, 391, 351, 111, 546, 31, 66, 276, 286, 346, 81, 366, 201, 461, 146, 131, 41, 126, 11, 571, 181, 341, 51, 186, 371, 231, 16, 601, 361, 171, 281, 316, 526, 536, 596, 331, 616, 451, 86, 396, 381, 291, 376, 261, 196, 431, 591, 301, 436, 621, 481, 266, 226, 611, 421, 531, 566, 151, 161, 221, 581, 241, 76, 336, 21, 6, 541 $(\text{mod}\,625)$.

(7)

$M_p = 2^{8k+3} - 1 \equiv 7$, 172, 537, 227, 242, 332, 247, 362, 427, 192, 32, 322, 187, 2, 142, 357, 397, 12, 202, 92, 57, 472, 462, 402, 42, 382, 547, 287, 602, 617, 82, 622, 112, 177, 567, 407, 72, 562, 377, 517, 107, 147, 387, 577, 467, 432, 222, 212, 152, 417, 132, 297, 37, 352, 367, 457, 372, 487, 552, 317, 157, 447, 312, 127, 267, 482, 522, 137, 327, 217, 182, 597, 587, 527, 167, 507, 47, 412, 102, 117, 207, 122, 237, 302, 67, 532, 197, 62, 502, 17, 232, 272, 512, 77, 592, 557, 347, 337, 277, 542, 257, 422, 162, 477, 492, 582, 497, 612, 52, 442, 282, 572, 437, 252, 392, 607, 22, 262, 452, 342, 307, 97, 87, 27, 292 $(\text{mod}\,625)$.

(8)

$M_p = 2^{8k+5} - 1 \equiv 31$, 66, 276, 286, 346, 81, 366, 201, 461, 146, 131, 41, 126, 11, 571, 181, 341, 51,186, 371, 231, 16, 601, 361, 171, 281, 316, 526, 536, 596, 331, 616, 451, 86, 396, 381, 291, 376, 261, 196, 431, 591, 301, 436, 621, 481, 266, 226, 611, 421, 531, 566, 151, 161, 221, 581, 241, 76, 336, 21, 6, 541, 1,511, 446, 56, 216, 551, 61, 246, 106, 516, 476, 236, 46, 156, 191, 401, 411, 471, 206, 491, 326, 586, 271, 256, 166, 251, 136, 71, 306, 466, 176, 311, 496, 356, 141, 101, 486, 296, 406, 441, 26, 36, 96, 456, 116, 576, 211, 521, 506, 416, 501, 386, 321, 556, 91, 426, 561, 121, 606, 391, 351, 111, 546 $(\bmod 625)$.

(9)

$M_p = 2^{8k+7} - 1 \equiv 127$,267, 482, 522, 137, 327, 217, 182, 597, 587, 527, 167, 507, 47, 412, 102, 117, 207, 122, 237, 302, 67, 532, 197, 62, 502, 17, 232, 272, 512, 77, 592, 557, 347, 337, 277, 542, 257, 422, 162, 477, 492, 582, 497, 612, 52, 442, 282, 572, 437, 252, 392, 607, 22, 262, 452, 342, 307, 97, 87, 27, 292, 7, 172, 537, 227, 242, 332, 247, 362, 427, 192, 32, 322, 187, 2, 142, 357, 397, 12, 202, 92, 57, 472, 462, 402, 42, 382, 547, 287, 602, 617, 82, 622, 112, 177, 567, 407, 72, 562, 377, 517, 107, 147, 387, 577, 467, 432, 222, 212, 152, 417, 132, 297, 37, 352, 367, 457, 372, 487, 552, 317, 157, 447, 312 $(\bmod 625)$.

(10)

Combined congruence Equations (1) and (7), (8), (9), (10) separately, by using the Chinese Remainder Theorem, we have conclusions as following.

$M_p = 2^{8k+1} - 1 \equiv 8751$, 511, 1071, 4431, 4591, 5551, 1311, 5871, 3231, 7391, 2351, 2111, 671, 2031, 191, 9151, 2911, 5471, 831, 2991, 5951, 3711, 271, 9631, 5791, 2751, 4511, 5071, 8431, 8591, 9551, 5311, 9871, 7231, 1391, 6351, 6111, 4671, 6031, 4191, 3151, 6911, 9471, 4831, 6991, 9951, 7711, 4271, 3631, 9791, 6751, 8511, 9071, 2431, 2591, 3551, 9311, 3871, 1231, 5391, 351, 111, 8671, 31, 8191, 7151, 911, 3471, 8831, 991, 3951, 1711, 8271, 7631, 3791, 751, 2511, 3071, 6431, 6591, 7551, 3311, 7871, 5231, 9391, 4351, 4111, 2671, 4031, 2191, 1151, 4911, 7471, 2831, 4991, 7951, 5711, 2271, 1631, 7791, 4751, 6511, 7071, 431, 591, 1551, 7311, 1871, 9231, 3391, 8351, 8111, 6671, 8031, 6191, 5151, 8911, 1471, 6831, 8991, 1951, 9711, 6271, 5631, 1791$(\bmod 10000)$.

(11)

$M_p = 2^{8k+3} - 1 \equiv 5007$, 2047, 4287, 7727, 8367, 2207, 5247, 3487, 2927, 9567, 9407, 8447, 2687, 8127, 767, 6607, 1647, 1887, 3327, 1967, 3807, 4847, 1087, 8527, 3167, 1007, 8047, 287, 3727, 4367, 8207, 1247, 9487, 8927, 5567, 5407, 4447, 8687, 4127, 6767, 2607, 7647, 7887, 9327, 7967, 9807, 847, 7087, 4527, 9167, 7007, 4047, 6287, 9727, 367, 4207, 7247, 5487, 4927, 1567, 1407, 447, 4687, 127, 2767, 8607, 3647, 3887, 5327, 3967, 5807, 6847, 3087, 527, 5727, 3007, 47, 2287, 5727, 6367, 207, 3247, 1487, 927, 7567, 7407, 6447, 687, 6127, 8767, 4607, 9647, 9887, 1327, 9967, 1807, 2847, 9087, 6527, 1167, 9007, 6047, 8287, 1727, 2367, 6207, 9247, 7487, 6927, 3567, 3407, 2447, 6687, 2127, 4767, 607, 5647, 5887, 7327, 5967, 7807, 8847, 5087, 2527, 7167$(\bmod 10000)$.

(12)

$M_p = 2^{8k+5} - 1 \equiv 31$, 8191, 7151, 911, 3471, 8831, 991, 3951, 1711, 8271, 7631, 3791, 751, 2511, 3071, 6431, 6591, 7551, 3311, 7871, 5231, 9391, 4351, 4111, 2671, 4031, 2191, 1151, 4911, 7471, 2831, 4991, 7951, 5711, 2271, 1631, 7791, 4751, 6511, 7071, 431, 591, 1551, 7311, 1871, 9231, 3391, 8351, 8111, 6671, 8031, 6191, 5151, 8911, 1471, 6831, 8991, 1951, 9711, 6271, 5631, 1791, 8751, 511, 1071, 4431, 4591, 5551, 1311, 5871, 3231, 7391, 2911, 2111, 671, 2031, 191, 9151, 2911, 5471, 831, 2991, 5951, 3711, 271, 9631, 5791, 2751, 4511, 5071, 8431, 8591, 9551, 5311, 9871, 7231, 1391, 6351, 6111, 4671, 6031, 4191, 3151, 6911, 9471, 4831, 6991, 9951, 7711, 4271, 3631, 9791, 6751, 8511, 9071, 2431, 2591, 3551, 9311, 3871, 1231, 5391, 351, 111, 8671$(\bmod 10000)$.

(13)

$M_p = 2^{8k+7} - 1 \equiv 127$, 2767, 8607, 3647, 3887, 5327, 3967, 5807, 6847, 3087, 527, 5167, 3007, 47, 2287, 5727, 6367, 207, 3247, 1487, 927, 7567, 7407, 6447, 687, 6127, 8767, 4607, 9647, 9887, 1327, 9967, 1807, 2847, 9087, 6527, 1167, 9007, 6047, 8287, 1727, 2367, 6207, 9247, 7487, 6927, 3567, 3407, 2447, 6687, 2127, 4767, 607, 5647, 5887, 7327, 5967, 7807, 8847, 5087, 2527, 7167, 5007, 2047, 4287, 7727, 8367, 2207, 5247, 3487, 2927, 9567, 9407, 8447, 2687, 8127, 767, 6607, 1647, 1887, 3327, 1967, 3807, 4847, 1087, 8527, 3167, 1007, 8047, 287, 3727, 4367, 8207, 1247, 9487, 8927, 5567, 5407, 4447, 8687, 4127, 6767, 2607, 7647, 7887, 9327, 7967, 9807, 847, 7087, 4527, 9167, 7007, 4047, 6287, 9727, 367, 4207, 7247, 5487, 4927, 1567, 1407, 447, 4687$(\bmod 10000)$.

(14)

From the congruence Equations (11)-(14), if $k \equiv 0$, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120$(\bmod 125)$, then $p = 8k+5$ are composites, and then these numbers are not considered. So we can draw the conclusion. That completes the proof.

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