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The Operator Splitting Method for Black-Scholes Equation

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Abstract

The Operator Splitting method is applied to differential equations occurring as mathematical models in financial models. This paper provides various operator splitting methods to obtain an effective and accurate solution to the Black-Scholes equation with appropriate boundary conditions for a European option pricing problem. Finally brief comparisons of option prices are given by different models.

Keywords: Operator Splitting Method, Black-Scholes Equation, European Option Pricing

1. Introduction

Finance is one of the most rapidly changing and fastest growing areas in the corporate business world. Because of this rapid change, modern financial instruments have become extremely complex. New mathematical models are essential to implement and price these new financial instruments. The world of corporate finance once managed by business students is now controlled by mathematicians and computer scientists. In the early 1970's, Merton [1,2] and Black and Merton [3], made an important breakthrough in the pricing of complex financial instruments by developing what has become known as the Black-Scholes model. Originally, their models are formulated in terms of stochastic differential equations. Under certain restrictive assumptions, these models are written as linear evolutionary partial differential equations with variable coefficients. The Black-Scholes model displayed the importance that mathematics plays in the field of finance. It also led to the growth and success of the new field of mathematical finance or financial engineering [4-10].

In this paper, first, we will give the derivation of the Black-Scholes partial differential equation [4] once more to refresh the minds and ultimately solve the equation for a European call option with the variants of Operator Splitting method.

2. Derivation of the Black-Scholes Equation and Its Similarity Solution

In this section, the price of a derivative security $V(S;t)$ is re-derived [4]. We let the option $V(S;t)$ whose value depends only on S and t, and the option $V(S;t)$ be, at least, twice differentiable in S and differentiable in t. It is not necessary at this stage to specify whether $V(S;t)$ is a call or a put option. In fact, $V(S;t)$ can be the value of a whole portfolio of different options although for simplicity the reader can think of a simple call or put.

From Ito's process we have

$$\mathrm{d}V(S,t)=\frac{\partial V}{\partial S}\mathrm{d}S+\frac{\partial V}{\partial t}\mathrm{d}t+\frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2}\mathrm{d}t \tag{1}$$

This gives the random walk followed by $V(S;t)$.

Now suppose that at time t the asset price is S which obeys to stochastic differential equation

$$\frac{\mathrm{d}S}{S}=\sigma\mathrm{d}X+\mu\mathrm{d}t \tag{2}$$

where σ is a number called volatility and μ is a measure of the average rate of growth of the asset price, also known as the drift.

Plugging (2) into (1) for $\mathrm{d}S$, we have

$$\mathrm{d}V(S,t)=\frac{\partial V}{\partial S}(\mu S\mathrm{d}t+\sigma S\mathrm{d}B)+\frac{\partial V}{\partial t}\mathrm{d}t+\frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2}\mathrm{d}t$$

and this simplifies to

$$\mathrm{d}V(S,t)=\sigma S\mathrm{d}B\frac{\partial V}{\partial S}+\left(\mu S\frac{\partial V}{\partial S}+\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2S^2\frac{\partial^2V}{\partial S^2}\right)\mathrm{d}t \tag{3}$$

Now set up a portfolio long one option, *V*, and short an amount $\frac{\partial V}{\partial S}$ stock.

Note from above that this portfolio is hedged. The value of this portfolio, π, is

$$\pi = V - \frac{\partial V}{\partial S} S \tag{4}$$

The change, $\mathrm{d}\pi$, in the value of this portfolio over a small time interval $\mathrm{d}t$ is given by

$$\mathrm{d}\pi = \mathrm{d}V - \frac{\partial V}{\partial S}\mathrm{d}S \tag{5}$$

Now plugging (3) and stochastic differential equation into (5) for $\mathrm{d}V$ and $\mathrm{d}S$ we get

$$\begin{aligned}\mathrm{d}\pi = \sigma S\mathrm{d}B\frac{\partial V}{\partial S}\mathrm{d}B + \left(\mu S\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)\mathrm{d}t \\ - \frac{\partial V}{\partial S}\left(\mu S\mathrm{d}t + \sigma S\mathrm{d}B\right)\end{aligned} \tag{6}$$

This simplifies to

$$\mathrm{d}\pi = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)\mathrm{d}t \tag{7}$$

It is important to note that this portfolio is completely riskless because it does not contain the random Brownian motion term. Since this portfolio contains no risk it must earn the same as other short-term risk-free securities. If it earned more than this, arbitrageurs could make a profit by shorting the riskfree securities and using the proceeds to buy this portfolio. If the portfolio earned less arbitrageurs could make a riskless profit by shorting the portfolio and buying the risk-free securities. It follows for a riskless portfolio that

$$\mathrm{d}\pi = r\pi\mathrm{d}t \tag{8}$$

where r is the risk free interest rate. Substituting for $\mathrm{d}\pi$ and π from (6) and (3) yields

$$\left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)\mathrm{d}t = r\left(V - S\frac{\partial V}{\partial S}\right)\mathrm{d}t \tag{9}$$

or

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{10}$$

This is the Black-Sholes partial differential equation and is parabolic type equation as in many financial problems. Furthermore, (10) is called backward parabolic equation since the signs of particular derivatives are the same, namely, they appear on the same side of the equation.

With its extensions and variants, it plays the major role in the option pricing theory. By deriving the partial differential equation for a quantity, such as option price, we hope to be able to find an expression for this value by solving this equation. However a partial equation on its own generally has many solutions. The value of an option should be unique (otherwise, arbitrage possibilities would arise) and so, to pin down the solution, we must also impose boundary conditions. A boundary condition specifies the behavior of the required solution at some part of the solution domain. For the moment we restrict our attention to a **European call** with value denoted by $C(S;t)$ with exercise price E and expiry date T.

2.1. Boundary and Final Conditions

Having derived the Black-Sholes equation for the value of an option, we must next consider final and boundary conditions. But, for the moment we restrict our interest to a European call denoted by $C(S;t)$, with exercise price E and expiry date T. The final condition at time $t = T$ can be derived from the definition of a call option. If at expiration $S > E$ the call option will be worth $S - E$ because the buyer of the option can buy the stock for E and immediately sell it for S. If at expiration $S < E$ the option will not be exercised and it will expire worthless. At $t = T$, the value of the option is known for certain to be the payoff

$$C(S;t) = \max(S - E, 0) \tag{11}$$

This is the final condition for our differential equation.

In order to find boundary conditions we consider the value of C when $S = 0$ and as $S \to \infty$: If $S = 0$ then it is easy to see from stochastic differential equation that $\mathrm{d}S = 0$, and therefore, S will never change. If at expiry $S = 0$ then from (10) the payoff must be 0. Consequently, when $S = 0$ we have

$$C(S;t) = 0 \tag{12}$$

Now when $S \to \infty$ it becomes more and more likely the option will be exercised and the payoff will be $S - E$. The exercise price becomes less and less important as $S \to \infty$, so the value of the option is equivalent to

$$C(S;T) \approx S \text{ as } S \to \infty \tag{13}$$

2.2. Similarity Solution

It may occasionally occur that the solution $C(S;t)$ of a partial differential equation, together with its initial and boundary conditions, depends only on one special combination of the two independent variables. In such cases, the problem can be reduced to an ordinary differential equation in which this combination is the independent variable. The solution to this ordinary differential equation is called *a similarity solution* to the original equation. In [7]; Wilmot *et al.* have given the similarity solution of the Black-Scholes equation for a European call option (see pages 97-100). The mathematical reasons for the existence of this reduction are subtle and outside of the scope of this paper, although the numerical calculations of the solution given on the **Table 1** which, we think, necessary for comparison of our numerical results.

3. The Mathematical Foundation of Operator Splitting

In numerous applications in the past revealed that a mixing of the various terms in the equations for the discretization and solver methods made it difficult to solve them together. To overcome this drawback, in 60's or early 70's so called the decomposition methods or splitting methods have been introduced [11]. The main idea of these methods is to decouple a complex equation in various simpler equations and to solve the simpler equations with adapted discretization and solver methods. In general, the simpler parts are collected via the initial conditions the results are coupled together. This decoupling procedure allows us to solve a few simpler systems instead of the whole one [11-14].

In this study, we apply operator splitting and the point in operator splitting is the replacement of the original model with one in which appropriately chosen groups of the sub processes, described by the model, take place successively in time. To illustrate the idea, let S denote some normed space and consider the initial value problem

$$\frac{\mathrm{d}w(t)}{\mathrm{d}t} = Aw(t),\ t \in (0,T], \quad w(0) = w_0, \tag{14}$$

where $w:[0,T] \to S$ is the unknown function, and A is an operator of type $S \to S$. Assume that the operator A can be decomposed into a sum of two simpler operators, for example, as A_1 and A_2. Then defining the splitting step by $\tau = T/n$, where $n \in \mathbb{N}$, is given, we consider the sequence of initial value problems of the form

$$\begin{aligned} &\frac{\mathrm{d}w_k^{(1)}(t)}{\mathrm{d}t} = A_1 w_k^{(1)}(t), \quad t \in ((k-1)\tau, k\tau] \\ &w_k^{(1)}((k-1)\tau) = w_{k-1}^{(2)}((k-1)\tau), \\ &\text{and} \\ &\frac{\mathrm{d}w_k^{(2)}(t)}{\mathrm{d}t} = A_2 w_k^{(2)}(t), \quad t \in ((k-1)\tau, k\tau], \\ &w_k^{(2)}((k-1)\tau) = w_k^{(1)}(k\tau), \end{aligned} \tag{15}$$

for $k = 1, 2, \cdots, n$, where $w_0^{(2)}(0) = w_0$. This procedure is called sequential splitting and can directly be extended to more than two sub operators in a natural way.

Obviously, the alteration of the original problems with the subproblems generally results in some error so called local splitting error. The local splitting error, E_n, of the sequential operator-splitting method can be given as follows:

$$E_n = \frac{1}{\tau_n}\left(\exp(\tau_n(A_1 + A_2)) - \exp(\tau_n A_2)\exp(\tau_n A_1)\right) w_n^1 + O(\tau_n^2) \tag{16}$$

where splitting time step, τ_n, is defined by $\tau_n = n \cdot t - (n-1) \cdot t$

Now, we extend the method to the Black-Scholes equation given in (10).

4. The Operator Splitting of the Black-Scholes Equation and Its Numerical Solution

Splitting methods are important for partial differential equations, because of reducing computational time to solve the equations and accelerating the solver process, see [11]. Based on the splitting launched in Section 3, we have split the Black-Scholes equation given in (10) as follow:

$$-\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} = 0 \tag{17a}$$

$$-\frac{\partial U}{\partial t} - rU = 0 \tag{17b}$$

Next, we discuss our underlying time- and space-discretization approach for our coupled system of Black-Sholes equation given in 17(a-b). Often decoupling methods are applied after discretizing time and space variables. Here, the balance between the time and space discretization methods is important. So, the spatiotemporal schemes can be balanced in implicit-explicit discretization methods. The decoupling in time and space has the advantage of more efficiency and acceleration [11].

4.1. Finite-Difference Approximation

Finite-difference methods are one of the resources of obtaining numerical solutions to partial differential equations and linear complimentary problems. They constitute a very powerful and flexible technique, and, if they applied appropriately, competent of producing accurate numerical solutions to all of the model problems arising in both the physical and financial sciences.

The underlying idea behind finite-difference methods is to replace the partial derivatives occurring in partial differential equations by approximations based on Taylor's series expansions of functions near the point or points of interest. For example, a partial derivative $\partial u/\partial t$ may be defined to be the limiting difference

$$\frac{\partial u}{\partial t}(x,t)=\lim_{\delta t\to 0}\frac{u(x,t+\delta t)-u(x,t)}{\delta t}$$

If instead of taking the limit $\delta t\to 0$, we regard δt as small but nonzero, hence, we obtain the approximation

$$\frac{\partial u}{\partial t}(x,t)\approx\frac{u(x,t+\delta t)-u(x,t)}{\delta t}+O(\delta t)\to 0$$

This is called a finite-difference approximation or a finite difference of $\partial u/\partial t$ because; it involves small but not infinitesimal, differences of the dependent variable u. Furthermore, higher order derivatives can be derived in a similar manner. To continue with the finite difference approximation, we divide the *x*-axis into equally spaced nodes a distance $\delta x=h$ apart, and *t*-axis into equally spaced nodes a distance $\delta t=k$ apart. This divides the (x,t) plane into a mesh, where the mesh points have the form $(i.h, j.k)$. In our case, the grid is made up of the points at asset values $S_i=ih$ and times $t=T-jk$ for the convenience.

Balancing of time and spatial discretization here additional balancing is taken into account, and we proposed the Theta methods.

The following theorem, addresses the delicate situation of time and spatial steps and the fact of reducing the theoretical promised order of the scheme:

4.2. Theta Method

Detaining our attention to values of V at mesh points, and using appropriate finite- difference for the derivatives in (17a) Theta method reads:

$$\begin{aligned}\frac{V_i^{j+1}-V_i^j}{k}=&\frac{1}{2}\sigma^2S_i^2\left(\theta\left(\frac{V_{i-1}^j-2V_i^j+V_{i+1}^j}{h^2}\right)\right.\\&\left.+(1-\theta)\left(\frac{V_{i-1}^{j+1}-2V_i^{j+1}+V_{i+1}^{j+1}}{h^2}\right)\right)\\&+rS_i\left(\theta\left(\frac{V_{i+|1}^j-V_{i-1}^j}{2h}\right)+(1-\theta)\left(\frac{V_{i+1}^{j+1}-V_{i-1}^{j+1}}{2h}\right)\right)\end{aligned}\tag{18}$$

or

$$\begin{aligned}&-(1-\theta)A_iV_{i-1}^{j+1}+(1+(1-\theta)C_i)V_i^{j+1}-(1-\theta)B_iV_{i+1}^{j+1}\\&=\theta A_iV_{i-1}^j+(1-\theta C_i)V_i^j+\theta B_iV_{i+1}^j\end{aligned}\tag{19}$$

where

$$\begin{aligned}A_i&=\left(\frac{1}{2}\sigma^2i^2k-\frac{1}{2}rik\right)\\B_i&=\left(\frac{1}{2}\sigma^2i^2k+\frac{1}{2}rik\right)\\C_i&=\sigma^2i^2k\end{aligned}\tag{20}$$

Detaining our attention to values of U at mesh points, and using appropriate finite- difference for the derivatives in (17b) Theta method reads:

$$\frac{U_i^{j+1}-U_i^j}{k}=-r\left(\theta U_i^j+(1-\theta)U_i^{j+1}\right)\tag{21}$$

or

$$U_i^{j+1}=\frac{(1-rk\theta)}{(1+rk(1-\theta))}U_i^j\tag{22}$$

For readers familiar with Theta method or so called weighted average approximation reduces to:

1) The Explicit Finite Difference Method when we take $\theta=1$.

2) The Implicit Finite Difference Method when we take $\theta=0$.

3) The Crank-Nicolson method when we take $\theta=1/2$.

Now we will make numerical calculations for each method for the cases 1), 2) and 3) to show the applicability and efficiency of each case for Black-Sholes call option model.

1) Explicit Finite Difference Method

For Black-Sholes equation, (18), the explicit method is allocated when $\theta=1$, with $\sigma=0.2$, $r=0.05$ and $E=100$, hence we obtain:

$$\frac{V_i^{j+1}-V_i^j}{k}=\frac{1}{2}\sigma^2S_i^2\left(\frac{V_{i-1}^j-2V_i^j+V_{i+1}^j}{h^2}\right)+rS_i\left(\frac{V_{i+1}^j-V_{i-1}^j}{2h}\right)\tag{23}$$

or

$$V_i^{j+1}=A_i^jV_{i-1}^j+(1-B_i^j)V_i^j+C_i^jV_{i+1}^j\tag{24}$$

where

$$\begin{aligned}A_i^j&=\left(\frac{1}{2}\sigma^2i^2k-\frac{1}{2}rik\right)\\B_i^j&=\sigma^2i^2k\\C_i^j&=\left(\frac{1}{2}\sigma^2i^2k+\frac{1}{2}rik\right)\end{aligned}\tag{25}$$

Equation (24) only holds for $i=1,\cdots,M-1$, *i.e.* for interior points, since V_{-1}^j and V_{M+1}^j are not defined. Thus there are $M - 1$ equations for the $M + 1$ unknowns, the V_i^j. The remaining two equations come from the two boundary conditions on $i = 0$ and $i = M$. These two end points are treated separately.

If we know V_i^j for all i then Equation (24) tells us V_i^{j+1}. Since we know V_i^0, the payoff function, we can easily calculate V_i^1.

The second split equation, (21), reduces to

$$\frac{U_i^{j+1} - U_i^j}{k} = -rU_i^j \tag{26}$$

or

$$U_i^{j+1} = (1 - rk)U_i^j \tag{27}$$

To solve, (27), we take the solution of the first split equation as an initial condition for second split equation, *i.e.*,

$$U_i^0 = V_i^1$$

and calculate U_i^1 easily, which is the option value of one time step before expiry. Using these values we can work step by step back down the grid as far as we required. **Table 2** shows the calculated call option values of Black-Sholes equation. The comparison with exact solutions (see **Table 1**) shows that obtained results are well-matched with exact ones except for lower strike prices, 110 and 120.

2) The Implicit Finite Difference Method

The implicit method for Black-Sholes equation is attained when we take $\theta = 0$ in (18). The calculation may be done in similar manner as in case 1) by using basic numerical linear algebra for the linear systems. The comparison with exact solutions (see **Table 1**) shows that obtained results are well-matched with exact ones except for lower strike prices, 110 and 120 as in the case 1).

3) Crank-Nicolson Method

The Crank-Nicolson method for Black-Sholes equation is attained when we take $\theta = 1/2$ in (18). **Table 3** shows the calculated call option values of Black-Sholes equation with the Crank-Nicolson method. The comparison with exact solutions (see **Table 1**) shows that obtained results are well-matched with exact ones except for lower strike prices, 110 and 120 as in the cases 1) and 2). The calculation may be done in similar manner as in cases 1) and 2) by using the Crank-Nicolson solver frequently used for solving the linear systems.

4.3. Weighted Operator Splitting Method (WOSM)

A more general finite-difference approximation to split Black-Sholes equation, (17(a)-(b)), than those considered is given by

$$\begin{aligned}\frac{V_i^{j+1} - V_i^j}{k} &= \frac{1}{2}w\sigma^2 S_i^2\left(\theta\left(\frac{V_{i-1}^j - 2V_i^j + V_{i+1}^j}{h^2}\right)\right.\\ &\quad\left.+(1-\theta)\left(\frac{V_{i-1}^{j+1} - 2V_i^{j+1} + V_{i+1}^{j+1}}{h^2}\right)\right)\\ &\quad+ wrS_i\left(\theta\left(\frac{V_{i+|1}^j - V_{i-1}^j}{2h}\right) + (1-\theta)\left(\frac{V_{i+1}^{j+1} - V_{i-1}^{j+1}}{2h}\right)\right)\end{aligned} \tag{28}$$

$$U_i^{j+1} = \frac{(1 - r(1-w)k\theta)}{(1 + r(1-w)k(1-\theta))}U_i^j \tag{29}$$

This approximation may be called as Weighted Operator Splitting Method (WOSM) and we think it is useful for practical consideration for unstable equations. Next we will consider this approximation for the Crank-Nicholson case in the following.

Letting $\theta = 1/2$ in Equations (28) and (29) where $w \in (0,1)$ is the weighting factor which is determined by trial and error, gives the results shows in **Table 4**.

To arbitrate the accuracy of our results given in **Tables 2-5**, we have calculated numerical values of the explicit (similarity) solution of the Black-Scholes equation for option call problem for call option for $\sigma = 0.2$, $r = 0.05$ and $E = 100$, given in **Table 1**.

5. Conclusions

In this paper, Black-Scholes equation is solved as a call option problem by variants of splitting method numerically. The comparison of the results obtained by various splitting methods (see **Tables 2-5**) shows that obtained results are well-matched and the diversity among the numerical values are negligible. This may be considered as the splitting method applied to call option problem is consistent. Our calculations, to some extent, for certain values differ from the values obtained by the similarity solution given in **Table 1**. We think that the dissimilarity is practically expected and is due to the fundamental nature of the similarity solution. Because, notice that, the similarity solution contains only one parameter, instead of the four parameters E, σ^2, T and r in the original statement of the problem. The only vital factor controlling the option value is $r\Big/\frac{1}{2}\sigma^2$, which is the only dimensionless parameter in the problem. The effect of all other factors is simply brought in by a straightforward arithmetical calculation. On the other hand, the similarity solution technique is rarely successful in solving a com-

Table 1. Explicit solution for call option for $\sigma = 0.2$, $r = 0.05$ and $E = 100$.

	0	0.111	0.222	0.333	0.444	0.556	0.667	0.778	0.889	1
110	10	10.7514	11.7398	12.7085	13.6288	14.511	15.3455	16.1461	16.9173	17.663
120	20	20.5586	21.1794	21.8672	22.5857	23.3198	24.0453	24.7635	25.4718	26.169
130	30	30.5535	31.1096	31.6848	32.2835	32.9058	33.534	34.1683	34.8046	35.4403
140	40	40.5535	41.1042	41.6554	42.2125	42.7832	43.3571	43.9375	44.5226	45.1106
150	50	50.5535	51.1039	51.6516	52.1984	52.7513	53.3017	53.8552	54.4115	54.9701
160	60	60.5535	61.1039	61.6513	62.196	62.7438	63.2858	63.8278	64.3701	64.913
170	70	70.5535	71.1039	71.6512	72.1956	72.7421	73.2815	73.8191	74.3553	74.8906
180	80	80.5535	81.1039	81.6512	82.1955	82.7418	83.2804	83.8164	84.3502	84.8821
190	90	90.5535	91.1039	91.6512	92.1955	92.7417	93.2801	93.8156	94.3485	94.8789
200	100	100.5535	101.1039	101.6512	102.1955	102.7417	103.28	103.8154	104.3479	104.8777

Table 2. Call option values output by the explicit code. Stock price ranges from 110 to 200, time from 0 (expiration) to 1 with $\sigma = 0.2$, $r = 0.05$ and $E = 100$.

	0	0.111	0.222	0.333	0.444	0.556	0.667	0.778	0.889	1
110	10	10.5531	11.1032	11.6502	12.1941	12.7399	13.2777	13.8126	14.3444	14.8734
120	20	20.5529	21.1028	21.6495	22.1933	22.7388	23.2765	23.8111	24.3428	24.8714
130	30	30.5527	31.1023	31.6489	32.1924	32.7377	33.2752	33.8096	34.3411	34.8695
140	40	40.5525	41.1019	41.6483	42.1916	42.7367	43.2739	43.8081	44.3394	44.8676
150	50	50.5523	51.1015	51.6476	52.1907	52.7356	53.2726	53.8066	54.3377	54.8657
160	60	60.5521	61.1011	61.647	62.1899	62.7346	63.2714	63.8052	64.336	64.8638
170	70	70.5519	71.1006	71.6464	72.189	72.7335	73.2701	73.8037	74.3343	74.8619
180	80	80.5517	81.1002	81.6457	82.1882	82.7324	83.2688	83.8022	84.3326	84.86
190	90	90.5514	91.0998	91.6451	92.1873	92.7314	93.2675	93.8007	94.3309	94.8581
200	100	100.5512	101.0994	101.6445	102.1865	102.7303	103.2663	103.7992	104.3292	104.8562

Table 3. Call option values output by the Crank-Nicolson code. Stock price ranges from 110 to 200, time from 0 (expiration) to 1 with $\sigma = 0.2$, $r = 0.05$ and $E = 100$.

	0	0.111	0.222	0.333	0.444	0.556	0.667	0.778	0.889	1
110	10	10.5533	11.1036	11.6508	12.1949	12.7409	13.2789	13.8139	14.346	14.8751
120	20	20.5531	21.1032	21.6501	22.1941	22.7398	23.2776	23.8125	24.3443	24.8732
130	30	30.5529	31.1027	31.6495	32.1932	32.7387	33.2764	33.811	34.3426	34.8713
140	40	40.5527	41.1023	41.6489	42.1924	42.7377	43.2751	43.8095	44.3409	44.8694
150	50	50.5525	51.1019	51.6482	52.1915	52.7366	53.2738	53.808	54.3392	54.8674
160	60	60.5523	61.1015	61.6476	62.1907	62.7355	63.2725	63.8065	64.3375	64.8655
170	70	70.5521	71.1011	71.647	72.1899	72.7346	73.2713	73.8051	74.3359	74.8638
180	80	80.5519	81.1007	81.6465	82.1892	82.7337	83.2703	83.804	84.3346	84.8623
190	90	90.5518	91.1004	91.646	92.1885	92.7329	93.2693	93.8028	94.3333	94.8608
200	100	100.5516	101.1001	101.6455	102.1879	102.7321	103.2683	103.8016	104.3319	104.8593

Table 4. Call option values output by the Weighted Crank-Nicolson code. Stock price ranges from 110 to 200, time from 0 (expiration) to 1 with $\sigma = 0.2$, $r = 0.05$ and $E = 100$.

	0	0.111	0.222	0.333	0.444	0.556	0.667	0.778	0.889	1
110	10	10.5535	11.104	11.6514	12.1957	12.7419	13.2803	13.8156	14.3479	14.8774
120	20	20.5535	21.1039	21.6512	22.1956	22.7417	23.28	23.8153	24.3476	24.877
130	30	30.5534	31.1038	31.6511	32.1954	32.7415	33.2797	33.815	34.3473	34.8766
140	40	40.5534	41.1037	41.651	42.1952	42.7413	43.2795	43.8147	44.3469	44.8762
150	50	50.5534	51.1036	51.6509	52.1951	52.7411	53.2792	53.8144	54.3466	54.8758
160	60	60.5533	61.1035	61.6507	62.1949	62.7409	63.279	63.8141	64.3463	64.8755
170	70	70.5533	71.1035	71.6506	72.1947	72.7407	73.2788	73.8138	74.3459	74.8751
180	80	80.5532	81.1034	81.6505	82.1946	82.7405	83.2785	83.8136	84.3457	84.8748
190	90	90.5532	91.1033	91.6504	92.1945	92.7403	93.2783	93.8134	94.3454	94.8745
200	100	100.5532	101.1033	101.6503	102.1943	102.7402	103.2781	103.8131	104.3451	104.8742

Table 5. Call option values output by the implicit code. Stock price ranges from 110 to 200, time from 0 (expiration) to 1 with $\sigma = 0.2$, $r = 0.05$ and $E = 100$.

	0	0.111	0.222	0.333	0.444	0.556	0.667	0.778	0.889	1
110	10	10.5538	11.1045	11.6523	12.197	12.7436	13.2823	13.818	14.3509	14.8808
120	20	20.554	21.105	21.6529	22.1978	22.7446	23.2836	23.8195	24.3526	24.8827
130	30	30.5542	31.1054	31.6535	32.1987	32.7457	33.2848	33.821	34.3543	34.8846
140	40	40.5544	41.1058	41.6542	42.1995	42.7467	43.2861	43.8225	44.356	44.8865
150	50	50.5546	51.1062	51.6548	52.2004	52.7478	53.2874	53.824	54.3577	54.8884
160	60	60.5548	61.1067	61.6554	62.2012	62.7489	63.2886	63.8255	64.3594	64.8903
170	70	70.5551	71.1071	71.6561	72.2021	72.7499	73.2899	73.827	74.361	74.8922
180	80	80.5553	81.1075	81.6567	82.2029	82.751	83.2912	83.8284	84.3627	84.8941
190	90	90.5555	91.1079	91.6573	92.2037	92.752	93.2925	93.8299	94.3644	94.896
200	100	100.5557	101.1083	101.658	102.2046	102.7531	103.2937	103.8314	104.3661	104.8979

plete boundary value problems, because it requires such special symmetries in the equation and initial and boundary conditions. Therefore, we can not be confident that the effects we have neglected in making the approximation in both, *i.e.* splitting method and similarity solution are genuinely unimportant.

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