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Effect of Slip Velocity on Blood Flow through a Catheterized Artery

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Abstract

A mathematical model for pulsatile flow of blood in a catheterized artery in presence of an axisymmetric stenosis with a velocity slip at the constricted wall is proposed. The expressions for the flow characteristics, velocity profiles, the flow resistance, the wall shear stress, the effective viscosity are obtained in the present analysis. The effects of slip velocity on the blood flow characteristics are shown graphically and discussed briefly.

Keywords: Pulsatile, Stenosis, Catheter, Flow Resistance, Wall Shear Stress, Slip Velocity

1. Introduction

Atherosclerosis is the leading cause of death in many countries. There is considerable evidence that vascular fluid dynamics plays an important role in the development and progression of arterial stenosis, which is one of the most widespread diseases in human beings. The fluid mechanical study of blood flow in artery bears some important aspects due to the engineering interest as well as the feasible medical applications. The hemodynamic behavior of the blood flow is influenced by the presence of the arterial stenosis. If the stenosis is present in an artery, normal blood flow is disturbed. The actual causes of stenosis are not well known but its effects on the cardiovascular system can be understood by studying the blood flow in its vicinity [1-4]. Ahmed *et al.* [5] described the effect of stenosis at moderate Reynolds number with a reference to monkey aorta with induced atherosclerosis. Siouffi *et al.* [6] studied experimental analysis of unsteady flows through a stenosis, on the basis of the changes induced by the waveform on post stenostic flow characteristic in a 75% severe stenosis. The study of pulsatile flow through a stenosis is motivated by the need to obtain a better understanding of the impact of flow phenomena on atherosclerosis and stroke. In order to understand the effect of stenosis on blood flow through and beyond the narrowed segment of the artery, many studies have been undertaken experimentally and theoretically. Liu and Yamaguchi [7] find out a systematic study of a pulsatile flow in a stenosed channel to identify how the waveform affects the generation, development and breakdown of the vortex wave. Numerical solutions of pulsatile flow have been reported by several investigators [8,9], which has been done assuming the blood as a Newtonian fluid. A number of researchers have studied the flow of non-Newtonian fluids with the pulsation through arterial stenosis [10-13].

The flow through an annulus with mild constriction at the outer wall can be used as a model for the blood flow through the catheterized stenotic artery. The insertion of a catheter (a long flexible cylindrical tube) into a constricted tube (*i.e.* stenosed artery) results in an annular region between the walls of the catheter and artery. This will alter the flow field, modify the pressure distribution and increase the resistance. Even though the catheter tool devices are used for the measurement of arterial blood pressure or pressure gradient and flow velocity or flow rate, X-ray angiography and intravascular ultrasound diagnosis and coronary balloon angioplasty treatment of various arterial diseases, a little attention has been given in the literature to the flow in catheterized arteries. Roose and Lykoudis [14] studied the fluid mechanics of the ureter with an inserted catheter by considering the peristaltic wave moving along the stationary cylinder. McDonald [15] considered the pulsatile blood flow in a catheterized artery and obtained theoretical estimates for pressure gradient corrections for catheters, which are positioned eccentrically, as well as coaxially with the artery. The effect of catheterization on various flow characteristics in an artery with or without stenosis was studied by Karahalios [16]. Dash *et al.* [17] considered the steady and pulsatile flow of the Casson fluid in a narrow artery when a catheter is inserted into it and estimated the increase in frictional resistance in the artery due to catheterization. In view of the discussions given above the present work is devoted to study the pulsatile flow of blood through a catheterized artery in presence of an axi-symmetric stenosis with a velocity slip at the constricted wall. The theoretical model used here enables one to observe the effects of slip velocity on resistance to flow, the wall shear stress distribution in the stenotic region, and the effective viscosity. To neglect the entrance, end and special wall effects, the artery length is assumed large enough as compared to its radius.

2. Mathematical Formulation

Consider an axially symmetric, laminar, pulsatile and fully developed flow of blood through a catheterized artery with an axisymmetric stenosis as shown in **Figure 1**. The artery is assumed to be a rigid circular tube of radius R_0 and the catheter as a coaxial rigid tube of radius R_c. The artery length is assumed to be large enough as compared to its radius so that the entrance, end and special wall effects can be neglected. The geometry of the stenosis which is assumed to be manifested in the arterial segment is described as

$$\frac{\bar{R}(\bar{z})}{\bar{R}_0} = \begin{cases} 1-\dfrac{\bar{\delta}}{2\bar{R}_0}\left[1+\cos\dfrac{2\pi}{\bar{L}_0}\left(\bar{z}-\bar{d}-\dfrac{\bar{L}_0}{2}\right)\right], & \bar{d}\le\bar{z}\le\bar{d}+\bar{L}_0 \\ 1, & \text{otherwise} \end{cases} \quad (1)$$

where $\bar{R}(\bar{z})$, $\bar{R}_0$ are tube radius with and without stenosis, respectively, $\bar{L}_0$ is the stenosis length and $\bar{d}$ indicate its location, $\bar{\delta}$ is the maximum projection (maximum height) of the stenosis in to the lumen.

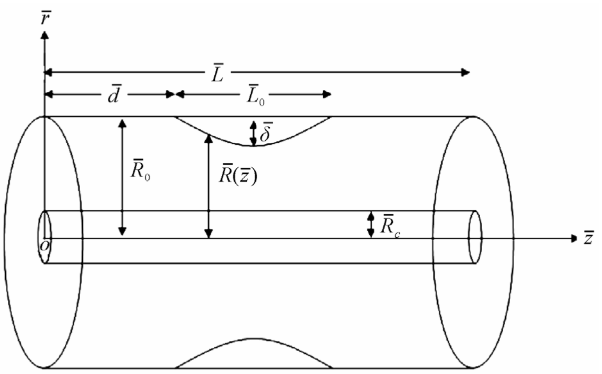

Figure 1. Geometry of an axially symmetrical stenosis with an inserted catheter.

Blood is assumed to be represented by a Newtonian fluid. We have taken here cylindrical coordinate system $(\bar{r},\bar{\theta},\bar{z})$ whose origin is located on the tube axis. It can be shown that the radial velocity is negligibly small in its magnitude and may be neglected for a low mean Reynolds number flow problem with mild stenosis.

The moment equations are

$$\bar{\rho}\frac{\partial\bar{u}}{\partial\bar{t}} = -\frac{\partial\bar{p}}{\partial\bar{z}} - \frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left[\bar{r}\bar{\tau}\right] \quad (2)$$

$$\frac{\partial\bar{p}}{\partial\bar{r}} = 0 \quad (3)$$

$$\frac{\partial\bar{p}}{\partial\bar{\theta}} = 0 \quad (4)$$

where $\bar{u}$ is the fluid velocity in the axial direction, $\bar{\rho}$ is density, $\bar{p}$ is the pressure, $\bar{t}$ is the time, and $\bar{\tau}$ is the shear stress.

For a Newtonian fluid

$$\bar{\tau} = -\bar{\mu}\frac{\partial\bar{u}}{\partial\bar{r}} \quad (5)$$

where $\bar{\mu}$ is the coefficient of viscosity.

The boundary conditions are

$$\bar{u} = \bar{u}_B \text{ at } \bar{r} = \bar{R}(\bar{z}) \quad (6)$$

$$\bar{u} = 0 \text{ at } \bar{r} = \bar{R}_c \quad (7)$$

where $\bar{u}_B$ is the slip velocity at the wall and the radius of the catheter $R_c\,(\ll R_0)$.

The pressure gradient as a function of $\bar{z}$ and $\bar{t}$ can be expressed as

$$-\frac{\partial\bar{p}}{\partial\bar{z}}(\bar{z},\bar{t}) = \bar{q}(\bar{z})\,f(\bar{t}) \quad (8)$$

where $\bar{q}(\bar{z}) = -\frac{\partial\bar{p}}{\partial\bar{z}}(\bar{z},0)$, $f(\bar{t}) = 1 + a\sin\bar{\omega}\bar{t}$, a is the amplitude and $\bar{\omega}$ is the angular frequency of blood flow.

To solve the above system of equations, following non-dimensional variables are introduced.

$$u = \bar{u}\Big/\left(\bar{q}_0\bar{R}_0^2/4\bar{\mu}\right),\quad u_B = \bar{u}_B\Big/\left(\bar{q}_0\bar{R}_0^2/4\bar{\mu}\right),$$

$$\tau = \bar{\tau}\Big/\left(\bar{q}_0\bar{R}_0^2/2\right),\quad r = \frac{\bar{r}}{\bar{R}_0},\quad z = \frac{\bar{z}}{\bar{R}_0},\quad t = \bar{t}\,\bar{\omega},$$

$$d = \frac{\bar{d}}{\bar{R}_0},\quad L_0 = \frac{\bar{L}_o}{\bar{R}_0},\quad \delta = \frac{\bar{\delta}}{\bar{R}_0},\quad \alpha^2 = \frac{\bar{R}_0^2\bar{\omega}}{\left(\bar{\mu}/\bar{\rho}\right)},$$

$$R(z) = \frac{\bar{R}(\bar{z})}{\bar{R}_0},\quad R_C = \frac{\bar{R}_C}{\bar{R}_0}$$

where α is the pulsatile Reynolds numbers for Newtonian fluid and $\bar{q}_0$ is the pressure gradient in a uniform tube without catheter.

Using non-dimensional variables Equations (2)-(5) reduce to

$$\alpha^2 \frac{\partial u}{\partial t} = 4q(z)\, f(t) - \frac{2}{r}\frac{\partial}{\partial r}(r\tau) \tag{9}$$

$$\tau = -\frac{1}{2}\frac{\partial u}{\partial r} \tag{10}$$

An application of Equation (10) in to (9), yields

$$\alpha^2 \frac{\partial u}{\partial t} = 4q(z)\, f(t) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{11}$$

where $f(t) = 1 + a\sin t$, $q(z) = \bar{q}(\bar{z})/\bar{q}_0$, $q(z)\, f(t) > 0$,

The boundary conditions in their non-dimensional form are now expressed as

$$u = u_B \text{ at } r = R(z) \tag{12}$$

$$u = 0 \text{ at } r = R_c \tag{13}$$

The geometry of the stenosis in dimensionless form is given by

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \dfrac{\delta}{2R_0}\left[1 + \cos\dfrac{2\pi}{L_0}\left(z - d - \dfrac{L_0}{2}\right)\right], & d \le z \le d + L_0 \\ 1, & \text{otherwise} \end{cases} \tag{14}$$

The non-dimensional volumetric flow rate is given by

$$Q(t) = 4\int_{R_c}^{R(z)} r u(r,z,t)\,\mathrm{d}r \tag{15}$$

where $Q(t) = \dfrac{\bar{Q}(\bar{t})}{\mathrm{p}\left(\bar{R}_0\right)^4 \bar{q}_0 / 8\bar{\mu}}$ and

$\bar{Q}(\bar{t}) = 2\pi \int_{\bar{R}_c}^{\bar{R}(\bar{z})} \bar{r}\,\bar{u}(\bar{r},\bar{z},\bar{t})$ is the volumetric flow rate.

The effective viscosity μ_e is defined as

$$\bar{\mu}_e = \frac{\pi\left(-\dfrac{\partial\bar{p}}{\partial\bar{z}}\right)\left(\bar{R}(\bar{z})\right)^4}{\bar{Q}(\bar{t})} \tag{16}$$

It can be expressed in dimensionless form as

$$\mu_e = \frac{\left(R(z)\right)^4}{Q(t)} q(z)\, f(t) \tag{17}$$

where $Q(t)$ is defined in Equation (15).

3. Solution

Consider the Womersley parameter to be small. The velocity *u* can be expressed in the following form

$$u(r,z,t) = u_0(r,z,t) + \alpha^2 u_1(r,z,t) + \cdots \tag{18}$$

$$\tau(z,r,t) = \tau_0(z,r,t) + \alpha^2 \tau_1(z,r,t) + \cdots \tag{19}$$

Substituting the expression of *u* from Equation (18) in (11), we get

$$\frac{\partial}{\partial r}\left(r\frac{\partial u_0}{\partial r}\right) = -4rq(z)\, f(t) \tag{20}$$

$$\frac{\partial u_0}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_1}{\partial t}\right) \tag{21}$$

Substituting *u* from Equation (18) into conditions (12) in (13) we get

$$u_0 = u_B,\ u_1 = 0 \text{ at } r = R(z) \tag{22}$$

$$u_0 = 0,\ u_1 = 0 \text{ at } r = R_c \tag{23}$$

Integrating Equations (20) and (21) and using the boundary conditions (22) and (23), we have the expressions for u_0 and u_1 as in Equations (24) and (25).

The expression for velocity u can easily be obtained from Equations (18), (24) and (25).

The wall shear stress τ_w (as a result of Equations (10) and (18)) becomes,

$$\tau_w = -\frac{1}{2}\left(\frac{\partial u_0}{\partial r} + \alpha\frac{\partial u_1}{\partial r}\right)_{r=R(z)} \tag{26}$$

$$u_0 = \left[1 - \frac{\log(r/R)}{\log(R_c/R)}\right]u_B + q(z)\, f(t)\left[\left(R^2 - r^2\right) - \frac{\left(R^2 - R_c^{\,2}\right)}{\log(R_c/R)}\log(r/R)\right] \tag{24}$$

$$\begin{aligned} u_1 = \frac{q(z)\, f'(t)}{16}\Bigg[&\left(4R^2r^2 - r^4 - 3R^4\right) - \frac{\left(R^2 - R_c^{\,2}\right)}{\log(R_c/R)}\left\{4r^2\log(r/R) - 3r^2 + 3R^2\right\} \\ &- \frac{\log(r/R)}{\log(R_c/R)}\left\{4R^2R_c^{\,2} - R_c^{\,4} - 3R^4 - \frac{\left(R^2 - R_1^{\,2}\right)}{\log(R_c/R)}\left(4R_c^{\,2}\log(R_c/R) - 3R^2 + 3R_c^{\,2}\right)\right\}\Bigg] \end{aligned} \tag{25}$$

which is determined, by substituting velocity Equations (24) and (25) into the Equation (26), in the form

$$\begin{aligned}\tau_w = \frac{u_B}{2R\log\left(R_c/R\right)} + q(z)\,f(t)\left\{R + \frac{\left(R^2 - R_c^2\right)}{2R\log\left(R_c/R\right)}\right\} - \frac{\alpha^2}{32}q(z)\,f'(t)\left[4R^3 + 2R\frac{\left(R^2 - R_c^2\right)}{\log\left(R_c/R\right)}\right.\\ \left. - \frac{1}{R\log\left(R_c/R\right)}\left\{\left(4R^2R_c^2 - R_c^4 - 3R^4\right) - \frac{\left(R^2 - R_c^2\right)}{R\log\left(R_c/R\right)}\left(4R_c^2\log\left(\frac{R_c}{R}\right) + 3R^2 - 3R_c^2\right)\right\}\right]\end{aligned} \tag{27}$$

From Equations (15), (24) and (25) the expression for volumetric flow rate is given by

$$\begin{aligned}Q(t) = &\left[2\left(R^2 - R_c^2\right) + \frac{2R_c^2\log\left(R_c/R\right) - \left(R^2 - R_c^2\right)}{\log\left(R_c/R\right)}\right]u_B\\ &+ q(z)\,f(t)\left[\left(R^2 - R_c^2\right)^2 - \frac{\left(R^2 - R_c^2\right)}{\log\left(R_c/R\right)}\left\{R_c^2 - 2R_c^2\log\left(R_c/R\right) - R^2\right\}\right]\\ &+ \frac{\alpha^2}{48}q(z)\,f'(t)\left[\left(18R^2R_c^2 + 2R_c^6 - 12R^2R_c^2 - 8R^6\right) - \frac{\left(R^2 - R_c^2\right)}{\log\left(R_c/R\right)}\left\{6R^4 - 12R_c^4\log\left(R_c/R\right) + 12R_c^4 - 18R^2R_c^2\right\}\right.\\ &\left. + \left\{6R_c^2 + 3\frac{\left(R^2 - R_c^2\right)}{\log\left(R_c/R\right)}\right\}\left\{4R^2R_c^2 - R_c^4 - 3R^4 - \frac{\left(R^2 - R_c^2\right)}{\log\left(R_c/R\right)}\left\{24R_c^2\log\left(R_c/R\right) + 18\left(R^2 - R_c^2\right)\right\}\right\}\right]\end{aligned} \tag{28}$$

The effective viscosity μ_e can be found out with the help of Equations (17) and (28).

If steady flow is considered, then Equation (28) reduces to

$$Q_s = \left[2\left(R^2 - R_c^2\right) + \frac{2R_c^2\log\left(R_c/R\right) - \left(R^2 - R_c^2\right)}{\log\left(R_c/R\right)}\right]u_B + q(z)\left[\left(R^2 - R_c^2\right)^2 - \frac{\left(R^2 - R_c^2\right)}{\log\left(R_c/R\right)}\left\{R_c^2 - 2R_c^2\log\left(R_c/R\right) - R^2\right\}\right] \tag{29}$$

where Q_s is the steady flow rate.

The value of $q(z)$ can be found from Equation (29), taking $Q_s = 1$.

In absence of catheter, (*i.e.* when $R_c = 0$), the Equations (24), (25), (27), (28) reduce to

$$u_0 = u_B + q(z)\,f(t)\left(R^2 - r^2\right) \tag{31}$$

$$u_1 = \frac{q(z)\,f'(t)}{16}\left(4R^2r^2 - r^4 - 3R^4\right) \tag{32}$$

$$\tau_w = q(z)\,f(t)R - \frac{\alpha^2}{8}q(z)\,f'(t)R^3 \tag{33}$$

$$\begin{aligned}Q(t) = \left\{R(z)\right\}^2&\left\{2u_B + q(z)\,f(t)\left\{R(z)\right\}^2\right.\\ &\left. - \frac{\alpha^2}{6}q(z)\,f'(t)\left\{R(z)\right\}^4\right\}\end{aligned} \tag{34}$$

4. Result and Discussions

With a view to examining the applicability of the present mathematical model, a specific numerical illustration has been undertaken with the use of the existing data for the various physical parameters encountered in the analysis. The following data have been made use of in order to carry out the numerical computations:

$$a = 0.5;\ R_c = 0,\ 0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5;$$

$$\alpha = 0.5;\ \delta = 0,\ 0.1,\ 0.2.$$

For the present steady simulation, the profiles of the velocity-field are computed and plotted in **Figures 2** and **3**. **Figure 2** shows the variations of axial velocity, u with radial distance, r for different time periods, t and fixed stenosis height, δ, R_c and α. It is seen that velocity increases rapidly with time, t as t goes from $t = 0^\circ$ to $t = 90^\circ$ and then decreases sharply when t goes from $t = 90^\circ$ to $t = 270^\circ$. It further increases in the time cycle from $t =$

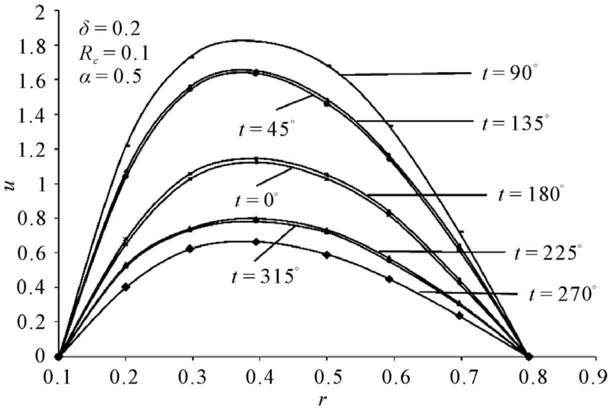

Figure 2. Variation of axial velocity with radial distance.

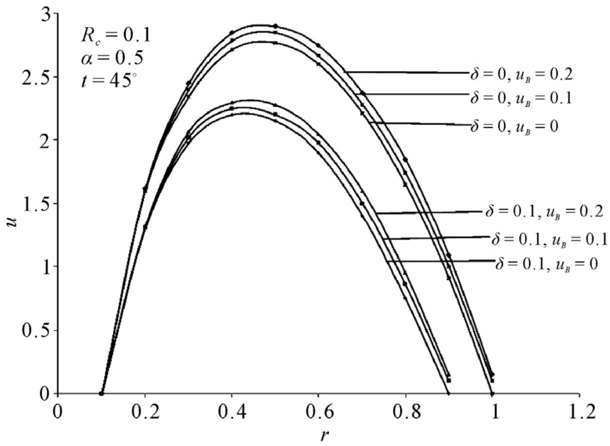

Figure 3. Variation of axial velocity with radial distance.

270° to $t = 360°$.

Axial velocity decreases with increasing stenosis height, δ for different slip velocity, u_B and for fixed values of R_c, α and t (**Figure 3**). It can be clearly observed that the axial velocity assumes higher magnitude in a uniform artery than that in a stenosed artery. Also the axial velocity increases with increasing slip velocity, u_B in both the stenotic and uniform artery. Both these figures also include the corresponding profiles in the absence of stenosis.

The variations of the wall shear stress, τ_w with the axial distance, z for different values of catheter radius, R_c and slip velocity, u_B for fixed α, t and δ are presented in **Figure 4**. The blood flow characteristic, τ_w increases with axial distance, r in the stenotic region in the upstream of the stenosis throat and attains its maximum at the throat and then decreases sharply. The wall shear stress, τ_w decreases with increasing slip velocity for any value of R_c. One notices that the flow characteristic, τ_w assumes higher values in a catheterized artery than that in an uncatheterized artery.

Figure 5 demonstrates the variations of the blood flow characteristic, τ_w with catheter radius for different values of slip velocity, u_B in stenosed and normal artery. It is noticed that increase in catheter radius increases the wall shear stress. On the other hand, increase in slip velocity reduces the wall shear stress in both the normal and the stenosed artery. The variations of effective viscosity, μ_e with the catheter radius, R_c for different values of slip velocity, u_B and fixed stenosis height, δ and time, t are illustrated in **Figure 6**. The effective viscosity, μ_e increases with increasing catheter radius, R_c significantly while it decreases with increasing slip velocity, u_B.

Figure 7 reveals the variations of effective viscosity, μ_e with the catheter radius, R_c for different stenosis heights, δ and t for fixed time. An increase in stenosis height δ increases the effective viscosity μ_e. It is observed that the magnitude of effective viscosity μ_e is less in a normal artery in comparison to that of the stenosed artery.

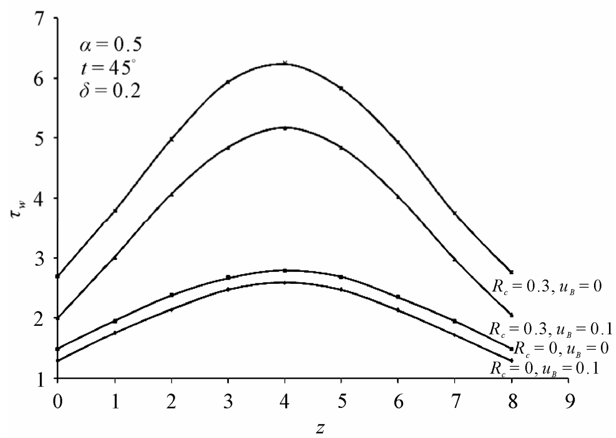

Figure 4. Variation of wall shear stress with axial distance.

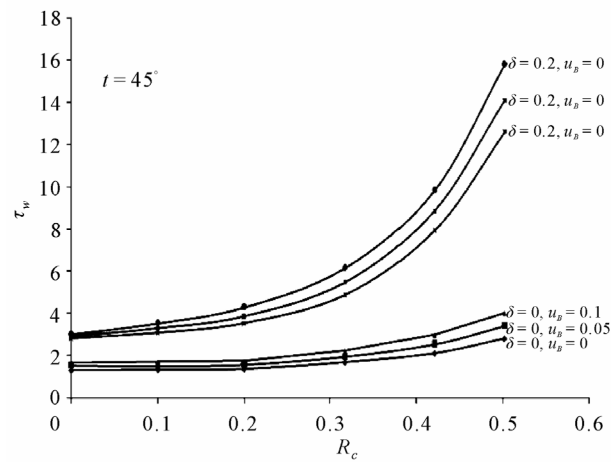

Figure 5. Variation of wall shear stress with catheter radius.

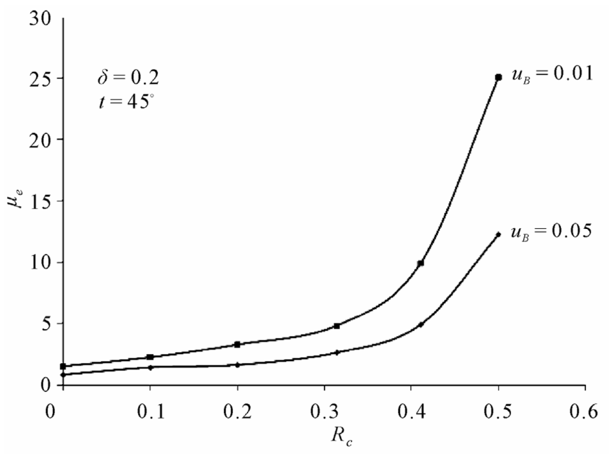

Figure 6. Variation of effective viscosity with catheter radius.

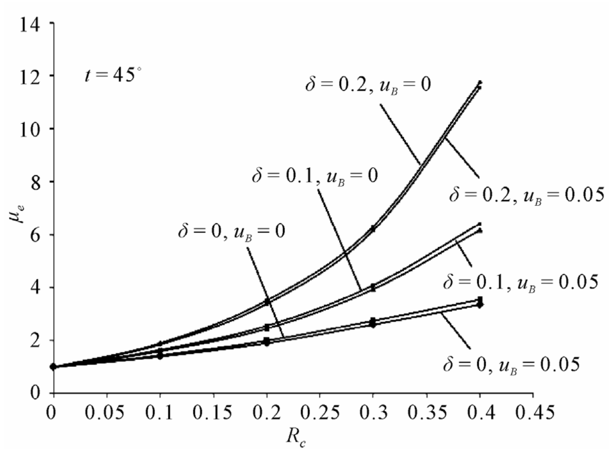

Figure 7. Variation of effective viscosity with catheter radius.

5. Conclusions

To estimate for the increased velocity profiles, wall shear stress and effective viscosity during artery catheterization, pulsatile flow of blood through an axisymmetric stenosis has been analyzed assuming that the flowing blood is represented by a Newtonian fluid. From the analysis it is concluded that the slip velocity plays an important role in reducing wall shear stress and effective viscosity. Elevation of blood viscosity is considered as a risk factor in the cardiovascular disorders, the present model may be used as a tool for reducing the blood viscosity by using slip velocity at the constricted wall. The present study is more useful for the purpose of simulation and validation of different models in different conditions of arteriosclerosis. This study also provides a scope for estimating the influence of the various parameters mentioned above on different flow characteristics and to ascertain which of the parameters has the most dominating role. Further careful investigations are thus suggested to address the problem more realistically and to overcome the restrictions imposed on the present work.

6. References

[1] J. H. Forestor and D. F. Young, "Flow through a Converging Diverging Tube and Its Implications in Occlusive Vascular Disease," *Journal of Biomechanics*, Vol. 3, No. 2, 1970, pp. 297-316. doi:10.1016/0021-9290(70)90031-X

[2] D. A. McDonald, "On Steady Flow through Modeled Vascular Stenosis," *Journal of Biomechanics*, Vol. 12, No. 1, 1979, pp. 13-20.

[3] D. F. Young and F. Y. Tsai, "Flow Characteristics in Model of Arterial Stenosis—Steady Flow," *Journal of Biomechanics*, Vol. 6, No. 4, 1973, pp. 395-410. doi:10.1016/0021-9290(73)90099-7

[4] D. F. Young, "Effects of a Time-Dependent Stenosis of Flow through a Tube," *Journal of Engineering for Industry*, Vol. 90, No. 1, 1968, pp. 248-254.

[5] A. S. Ahmed and D. P. Giddens, "Velocity Measurements in Steady Flow through Axisymmetric Stenosis at Moderate Reynolds Number," *Journal of Biomechanics*, Vol. 16, No. 7, 1983, pp. 505-516. doi:10.1016/0021-9290(83)90065-9

[6] M. Siouffi, V. Depleno and R. Pelissier, "Experimental Analysis of Unsteady Flow through Stenosis," *Journal of Biomechanics*, Vol. 31, No. 1, 1998, pp. 11-19.

[7] H. Liu and T. Yamaguchi, "Waveform Dependence of Pulsatile Flow in a Stenosed Channel," *Journal of Biomechanical Engineering Transactions ASME*, Vol. 123, No. 1, 2001, p. 88.

[8] S. Chakravarty, P. K. Mandal and A. Mandal, "Mathematical Model of Pulsatile Blood Flow in a Distensible Aortic Bifurcation Subject to Body Acceleration," *International Journal of Engineering Sciences*, Vol. 38, No. 2, 2000, pp. 215-238. doi:10.1016/S0020-7225(99)00022-1

[9] J. M. Siegel and C. P. Markou, "A Scaling Law for Wall Shear Rate through an Arterial Stenosis," *Journal of Biomechanical Engineering Transactions ASME*, Vol. 116, No. 1, 1996, pp. 446-451.

[10] S. U. Siddiqui, N. K. Verma and R. S. Gupta, "A Mathematical Model for Pulsatile Flow of Herschel Bulkley Fluid through Stenosed Arteries," *Journal of Science and Technology*, Vol. 5, No. 4, 2010, pp. 49-66.

[11] S. U. Siddiqui, N. K. Verma, S. Mishra and R. S. Gupta, "Mathematical Modelling of Pulsatile Flow of Casson's Fluid in Arterial Stenosis," *Applied Mathematics and Computation*, Vol. 210, No. 1, 2009a, pp. 1-10. doi:10.1016/j.amc.2007.05.070

[12] S. U. Siddiqui, N. K. Verma, S. Mishra and R. S. Gupta, "Mathematical Modelling of Pulsatile Flow of Blood through a Time Dependent Stenotic Blood Vessel," *International Journal of Physical Sciences*, Vol. 21, No.1, 2009b, pp. 241-248.

[13] I. Takuji and L. F. R. Guimaraes, "Effect of Non-Newtonian Property of Blood on Flow through a Stenosed Tube," *Fluid Dynamics Research*, Vol. 22, No. 5, 1998, pp. 251-264. doi:10.1016/S0169-5983(97)00041-5

[14] R. Roos and P. S. Lykoudis, "The Fluid Mechanics of the Ureter with an Inserted Catheter," *Journal of Fluid Mechanics*, Vol. 46, No. 4, 1971, pp. 625-630. doi:10.1017/S0022112071000752

[15] D. A. McDonald, "Pulsatile Flow in a Catheterized Artery," *Journal of Biomechanics*, Vol. 19, No. 3, 1986, pp. 239-249. doi:10.1016/0021-9290(86)90156-9

[16] G. T. Karahalios, "Some Possible Effects of a Catheter on the Arterial Wall," *Medical Physiology*, Vol. 17, No. 5, 1990, pp. 922-925. doi:10.1118/1.596448

[17] R. K. Dash, G. Jayaraman and K. N. Mehta, "Estimation of Increased Flow Resistance in a Narrow Catheterized Artery—A Theoretical Model," *Journal of Biomechanics*, Vol. 29, No. 7, 1996, pp. 917-930. doi:10.1016/0021-9290(95)00153-0