Applied Mathematics, **2011, 2, 633-645**
doi:10.4236/am.2011.25084 Published Online May 2011 (http://www.SciRP.org/journal/am)

Uses of the Buys-Ballot Table in Time Series Analysis

Iheanyi S. Iwueze[1], Eleazar C. Nwogu[1], Ohakwe Johnson[2], Jude C. Ajaraogu[3]
[1]*Department of Statistics, Federal University of Technology, Owerri, Nigeria*
[2]*Department of Mathematics and Statistics, Anambra State University, Uli, Nigeria*
[3]*Department of Mathematics and Statistics, Federal Polytechnic, Owerri, Nigeria*
E-mail: isiwueze@yahoo.com
Received December 14, 2010; *revised March* 30, 2011; *accepted April* 3, 2011

Abstract

Uses of the Buys-Ballot table for choice of appropriate transformation (using the Bartlett technique), assessment of trend and seasonal components and choice of model for time series decomposition are discussed in this paper. Uses discussed are illustrated with numerical examples when trend curve is linear, quadratic and exponential.

Keywords: Buys-Ballot Table, Trend Assessment, Assessment of Seasonality, Periodic Averages, Seasonal Averages, Data Transformation, Choice of Model

1. Introduction

A time series is a collection of observations made sequentially in time. Examples occur in a variety of fields, ranging from economics to engineering and methods of analyzing time series constitute an important area of statistics [1]. Time series analysis comprises methods that attempt to understand such time series, often either to understand the underlying context of the data points (Where did they come from? What generated them?), or to make forecasts. Time series forecasting is the use of a model to forecast or predict future events based on known past events.

Methods for time series analyses are often divided into three classes: descriptive methods, time domain methods and frequency domain methods. Frequency domain methods centre on spectral analysis and recently wavelet analysis [2,3], and can be regarded as model-free analyses. Time domain methods [4,5] have a distribution-free subset consisting of the examination of autocorrelation and cross-correlation analysis.

Descriptive methods [1,6] involve the separation of an observed time series into components representing trend (long term direction), the seasonal (systematic, calendar related movements), cyclical (long term oscillations or swings about the trend) and irregular (unsystematic, short term fluctuations) components. The descriptive method is known as time series decomposition. If short period of time are involved, the cyclical component is superimposed into the trend [1] and the observed time series $(X_t, t = 1, 2, \cdots, n)$ can be decomposed into the trend-cycle component (M_t), seasonal component (S_t) and the irregular/residual component (e_t).

Decomposition models are typically additive or multiplicative, but can also take other forms such as pseudo-additive/mixed (combining the elements of both the additive and multiplicative models).

$$\text{Additive Model: } X_t = M_t + S_t + e_t \tag{1}$$

$$\text{Multiplicative Model: } X_t = M_t \times S_t \times e_t \tag{2}$$

$$\text{Pseudo-Additive/Mixed Model; } X_t = M_t \times S_t + e_t \tag{3}$$

The pseudo-additive model is used when the original time series contains very small or zero values. For this reason, this paper will discuss only the additive and multiplicative models.

As far as the traditional method of decomposition is concerned (to be referred to as the Least Squares Method (LSE)), the first step will usually be to estimate and eliminate M_t for each time period from the actual data either by subtraction for Equation (1) or division for Equation (2). The de-trended series is obtained as $X_t - \hat{M}_t$ for Equation (1) or $X_t / \hat{M}_t$ for Equation (2). In the second step, the seasonal effect is obtained by estimating the average of the de-trended series at each season. The de-trended, de-seasonalized series is obtained as $X_t - \hat{M}_t - \hat{S}_t$ for Equation (1) or $X_t / (\hat{M}_t \hat{S}_t)$ for Equation (2). This gives the residual or irregular component. Having fitted a model to a time series, one often wants to see if the residuals are purely random. For detailed discussion of residual analysis, see [4,7].

It is always assumed that the seasonal effect, when it exists, has periods. That is, it repeats after s time periods.

$$S_{t+s} = S_t,\ \text{for all } t \tag{4}$$

For Equation (1), it is convenient to make the further assumption that the sum of the seasonal components over a complete period is zero.

$$\sum_{j=1}^{s} S_{t+j} = 0 \tag{5}$$

Similarly, for Equations (2) and (3), the convenient variant assumption is that the sum of the seasonal components over a complete period is s.

$$\sum_{j=1}^{s} S_{t+j} = s \tag{6}$$

It is also assumed that the irregular component e_t is the Gaussian $N\left(0, \sigma_1^2\right)$ white noise for Equation (1), while for Equation (2), e_t is the Gaussian $N\left(1, \sigma_2^2\right)$ white noise.

This paper discusses the uses of the Buys-Ballot table for 1) choice of appropriate transformations (using the Bartlett technique) 2) assessment of trend and seasonal components and 3) choice of model for time series decomposition. We describe in great detail the Buys-Ballot table in Section 2 for better understanding of the methods used to achieve these objectives.

When any of the assumptions underlying the time series analysis is violated, one of the options available to an analyst is to transform the study series. The choice of appropriate transformation for a study series using Buys-Ballot table is described in Section 3. The presence and nature of trend and seasonal component of a study series can be inferred from the plot and values of the periodic /annual and seasonal averages. Assessment of trend and seasonal component of the actual series from the Buys-Ballot table was discussed in Sections 4 and 5 respectively. A major problem in the use of the descriptive time series analysis is the choice of appropriate model for time series decomposition. This problem was addressed using Buys-Ballot table in Section 6. Numerical examples are also given to illustrate these uses.

2. Buys-Ballot Table

A Buys-Ballot table summarizes data to highlight seasonal variations (**Table 1**). Normally, each line is one period (usually a year) and each column is a season of the period/year (4 quarters, 12 months, etc), A cell, $(i,\ j)$, of this table contains the mean value for all observations made during the period i at the season j. To analyse the data, it is helpful to include the period and seasonal totals $\left(T_{i.} \text{ and } T_{.j}\right)$, period and seasonal averages $\left(\bar{X}_{i.}\right.$ and $\left.\bar{X}_{.j}\right)$, period and seasonal standard deviations $\left(\hat{\sigma}_{i.}\right.$ and $\left.\hat{\sigma}_{.j}\right)$, as part of the Buys-Ballot table. Also included for purposes of analysis are the grand total $\left(T_{..}\right)$, grand mean $\left(\bar{X}_{..}\right)$ and pooled standard deviation $\left(\hat{\sigma}_{..}\right)$. [8] credits these arrangements of the table to [9], hence the table has been called the Buys-Ballot table in the literature.

For easy understanding of **Table 1**, we define the row and column totals, averages and standard deviations as follows:

$$T_{i.} = \sum_{j=1}^{s} X_{(i-1)s+j},\ \ i = 1, 2, \cdots, m,$$

$$T_{.j} = \sum_{i=1}^{m} X_{(i-1)s+j},\ \ j = 1, 2, \cdots, s,$$

Table 1. Buys-Ballot Table.

Period (i)	Season (j)						$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
	1	2	…	j	…	s			
1	X_1	X_2	…	X_j	…	X_s	$T_{1.}$	$\bar{X}_{1.}$	$\hat{\sigma}_{1.}$
2	X_{s+1}	X_{s+2}	…	X_{s+j}	…	X_{2s}	$T_{2.}$	$\bar{X}_{2.}$	$\hat{\sigma}_{2.}$
3	X_{2s+1}	X_{2s+2}	…	X_{2s+j}	…	X_{3s}	$T_{3.}$	$\bar{X}_{3.}$	$\hat{\sigma}_{3.}$
…	…	…	…	…	…	…	…	…	…
i	$X_{(i-1)s+1}$	$X_{(i-1)s+2}$	…	$X_{(i-1)s+j}$	…	$X_{(i-1)s+s}$	$T_{i.}$	$\bar{X}_{i.}$	$\hat{\sigma}_{i.}$
…	…	…	…	…	…	…	…	…	…
m	$X_{(m-1)s+1}$	$X_{(m-1)s+2}$	…	$X_{(m-1)s+j}$	…	X_{ms}	$T_{m.}$	$\bar{X}_{m.}$	$\hat{\sigma}_{m.}$
$T_{.j}$	$T_{.1}$	$T_{.2}$	…	$T_{.j}$	…	$T_{.s}$	$T_{..}$	-	-
$\bar{X}_{.j}$	$\bar{X}_{.1}$	$\bar{X}_{.2}$	…	$\bar{X}_{.j}$	…	$\bar{X}_{.s}$	-	$\bar{X}_{..}$	-
$\hat{\sigma}_{.j}$	$\hat{\sigma}_{.1}$	$\hat{\sigma}_{.2}$	…	$\hat{\sigma}_{.j}$	…	$\hat{\sigma}_{.s}$	-	-	$\hat{\sigma}_{..}$

$$T_{..}=\sum_{i=1}^{m}T_{i.}=\sum_{j=1}^{s}T_{.j},\ \bar{X}_{i.}=\frac{T_{i.}}{s},\ i=1,2,\cdots,m,$$

$$\bar{X}_{.j}=\frac{T_{.j}}{m},\ j=1,2,\cdots,s,\ \bar{X}_{..}=\frac{T_{..}}{ms}=\frac{T_{..}}{ms},\ n=ms,$$

$$\hat{\sigma}_{i.}=\sqrt{\frac{1}{s-1}\sum_{j=1}^{s}\left(X_{(i-1)s+j}-\bar{X}_{i.}\right)^2},\ i=1,2,\cdots,m$$

$$\hat{\sigma}_{.j}=\sqrt{\frac{1}{m-1}\sum_{i=1}^{m}\left(X_{(i-1)s+j}-\bar{X}_{.j}\right)^2},\ j=1,2,\cdots,s$$

$$\hat{\sigma}=\sqrt{\frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left(X_{(i-1)s+j}-\bar{X}_{..}\right)^2}$$

where $X_t, t=1,2,\cdots,n$ is the observed value of the series, m is the number of periods/years, s is the periodicity, and $n=ms$ is the total number of observations /sample size.

Finally, Buys-Ballot table is used to estimate the trend component and seasonal indices from the chosen descriptive time series model. This method, called Buys-Ballot estimation procedure uses the periodic means $\left(\bar{X}_{i.}, i=1,2,\cdots,m\right)$ and the overall mean $\left(\bar{X}_{..}\right)$ to estimate the trend component. Seasonal means $\left(\bar{X}_{.j},\ j=1,2,\cdots,m\right)$ and the overall mean are used to estimate the seasonal indices. The advantages of the Buys-Ballot estimation procedure are that 1) it computes trend easily, 2) gets over the problem of de-trending a series before computing the estimates of the seasonal effects and 3) estimates the error variance without necessarily decomposing the series. For further details on the Buys-Ballot estimation procedure, see [10-14].

3. Choice of Appropriate Transformation

Transformation is a mathematical operation that changes the measurement scale of a variable. Reasons for transformation include stabilizing variance, normalizing, reducing the effect of outliers, making a measurement scale more meaningful, and to linearize a relationship. For further details on reasons for transformation, see [3,14]. Many time series analyst assume no rmality and it is well known that variance stabilization implies normality of the series. The most popular and common are the powers of transformations such as $\log_e X_t$, $\log_e X_t$, $1/X_t$, $1/\sqrt{X_t}$, X_t^2, $1/X_t^2$. Selecting the best transformation can be a complex issue and the usual statistical technique used is to estimate both the transformation and required model for the transformed X_t at the same time [15].

[16] have shown how to apply Bartlett transformation technique [17] to time series data using the Buys-Ballot table and without considering the time series model structure. The relation between variance and mean over several groups is what is needed. If we take random samples from a population, the means and standard deviations of these samples will be independent (and thus uncorrelated) if the population has a normal distribution [18]. Furthermore, if the mean and standard deviation are independent, the distribution is normal.

[16] showed that Bartlett's transformation for time series data is to regress the natural logarithms of the group standard deviations $\left(\hat{\sigma}_{i.}, i=1,2,\cdots,m\right)$ against the natural logarithms of the group means $\left(\bar{X}_{i.},\ i=1,2,\cdots,m\right)$ and determine the slope, β, of the relationship.

$$\log_e \hat{\sigma}_{i.}=\alpha+\beta\log_e \bar{X}_{i.}+\text{error} \quad (7)$$

For non-seasonal data that require transformation, we split the observed time series $X_t,\ t=1,2,\cdots,n$ chronologically into m fairly equal different parts and compute $\left(\bar{X}_{i.},\ i=1,2,\cdots,m\right)$ and $\left(\hat{\sigma}_{i.},\ i=1,2,\cdots,m\right)$ for the parts. For seasonal data with the length of the periodic interval, *s*, the Buys-Ballot table naturally partitions the observed data into m periods or rows for easy application. [16] showed that Bartlett's transformation may also be regarded as the power transformation

$$Y_t=\begin{cases}\log_e X_t,\ \beta=1\\ X_t^{(1-\beta)},\ \beta\neq 1\end{cases} \quad (8)$$

Summary of transformations for various values of β is given in **Table 2**. However, [16] concluded that it is better to use the estimated value of the slope, β, directly in the power transformation (Equation (8)) than to approximate to the known and popular logarithmic, square root, inverse, inverse of the square root, squares and inverse of the squares transformations.

An example that requires logarithmic transformation is the Nigerian Stock Exchange (NSE) All Shares Index (1985 – 2005) that is listed as Appendix A [19]. Summary of the regression analysis of $\log_e\left(\hat{\sigma}_I\right)$ on $\log_e\left(\bar{X}_i\right)$, $i=1,2,\cdots,21$ is given in **Table 3**.

Table 2. Bartlett's transformation for some values of *β*.

S/No	1	2	3	4	5	6	7
β	0	1/2	1	3/2	2	3	–1
Transformation	No transformation	$\sqrt{X_t}$	$\log_e X_t$	$1/\sqrt{X_t}$	$1/X_t$	$1/X_t^2$	X_t^2

Table 3. Regression analysis of $\log_e \hat{\sigma}_{i.}$ on $\log_e \bar{X}_{i.}$, $i = 1,2,\text{L},21$ for various transformations of the Nigerian Stock Exchange (NSE) All Shares Index (1985-2005).

Transformations	Regression equation	R^2	t – test for $\hat{\beta} = 1.0$	
			df	t – value
X_t : Original	$\log_e \hat{\sigma}_i = -2.5797 + 1.0260 \log_e \bar{X}_i$	0.94	19	0.44
$Y_t = \log_e X_t$	$\log_e \hat{\sigma}_i = -2.8857 + 0.2490 \log_e \bar{Y}_i$	0.02	19	*N/A*
$W_t = X_t^{(1-\hat{\beta})}, \hat{\beta} = 1.0260$	$\log_e \hat{\sigma}_i = -6.2186 + 0.0794 \log_e \bar{W}_i$	0.00	19	*N/A*

It is clear from **Table 3** that we can approximate the value of $\hat{\beta} = 1.0260$ to $\hat{\beta} \approx 1.0$ and the suitable transformation using **Table 2** is $Y_t = \log_e X_t$. Regression analysis summaries for $Y_t = \log_e X_t$ and $W_t = X_t^{(1-\hat{\beta})}$, $\hat{\beta} = 1.0260$ are also given in **Table 3**. The transformation $W_t = X_t^{(1-\hat{\beta})}, \hat{\beta} = 1.0260$ surely removes the relationships between the standard deviations and the means for the row/yearly groupings.

4. Assessment of Trend

The time plot of a time series, according to [1] reveals the nature of the trend which can describe the pattern in the series. Among other features, the time plot of the periodic means follows the same pattern as the plot of the entire series with respect to the trend. Therefore, instead of looking at the plot of the entire series, one may look at only the plot of the period/annual means in order to choose the appropriate trend. We use the following examples to illustrate this.

4.1. Linear Trend

The data of **Table 4** (linear trend and additive model) and **Figure 1** is a simulation of 100 values from the additive model

$$X_t = a + bt + S_t + e_t \tag{9}$$

with $a = 5.0$, $b = 0.2$, $S_1 = -1.5$, $S_2 = 2.5$, $S_3 = 3.5$, $S_4 = -4.5$ and e_t being Gaussian $N(0,1)$ white noise. The data of **Table 4** (linear trend and multiplicative model) and **Figure 2** is a simulation of 100 values from the multiplicative model

$$X_t = (a + bt) \times S_t \times e_t \tag{10}$$

with $a = 5.0$, $b = 0.2$, $S_1 = 0.6$, $S_2 = 1.1$, $S_3 = 0.9$, $S_4 = 1.4$ and e_t being Gaussian $N(1.0, 0.01)$ white noise. Listed in **Table 4** are the periodic/row means $(\bar{X}_{i.}, i = 1,2,\cdots,m)$ of the simulated data, while the time plots of the actual series and periodic/row means are shown in **Figures 1** and **2**, respectively. It is clear that the time plot of $\bar{X}_{i.}$ for data with linear trend curve mimics the time plot of the entire series and the row averages $(\bar{X}_{i.})$ can therefore, be used to estimate trend.

The estimate of the parameters of the trend of the entire series can be determined from the estimate of the trend of the periodic means by recognizing that periodic averages are centred at the midpoints of the periodic intervals. Thus, while successive values in the actual series are one unit of time apart starting from $t = 1$, successive values in periodic average series $(\bar{X}_{i.})$ are s units of time apart starting from $t = (s+1)/2$. Thus, periodic averages are derived from the original series by translation of the original series by a factor $((s+1)/2)$ and dilation by a factor s. That is, periodic averages, $\bar{X}_{i.}$, $i = 1,2,\cdots,m$ may be looked at as the values of the original series at times t_i, $i = 1,2,\cdots,m$. That is,

$$\bar{X}_{i.} = X_{t_i} = a + bt_{i.} \tag{11}$$

where $t_i = \frac{s+1}{2}, \frac{3s+1}{2}, \frac{5s+1}{2}, \cdots, \frac{(2m-1)s+1}{2}$, for $i = 1,2,3,\cdots,m$, Hence,

$$\begin{aligned} \bar{X}_{i.} = X_{t_i} &= a + b\left(\frac{(2i-1)s+1}{2}\right) \\ &= a - b\left(\frac{s-1}{2}\right) + (bs)i \\ &= a' + b'i \end{aligned} \tag{12}$$

where $a' = a - b\left(\frac{s-1}{2}\right)$, $b' = bs$ or $b = b'/s$, $a = a' + b\left(\frac{s-1}{2}\right)$.

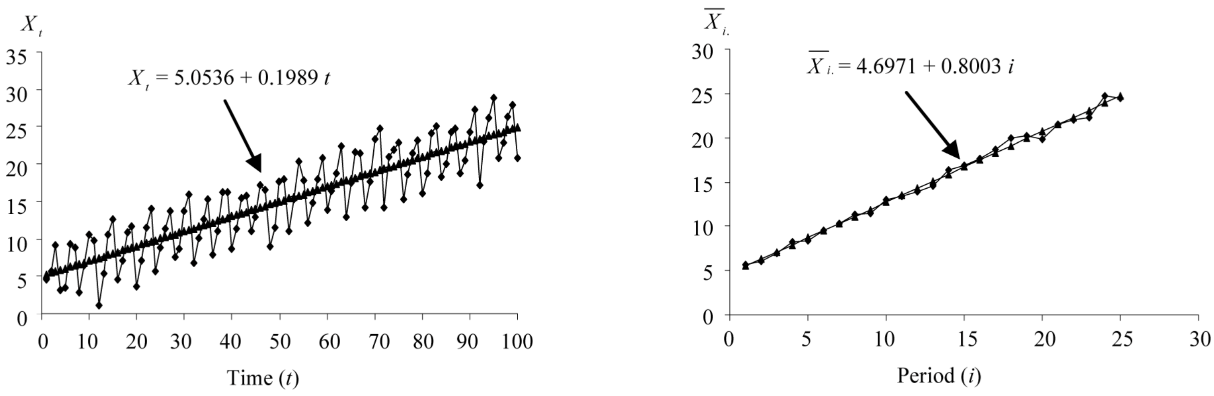
As an illustration, we observe from **Figure 1** that $s = 4$, $b' = 0.8003$, $a' = 4.6971$. Hence, $b = b'/s = 0.8003/4 = 0.2001$, $a = a' + b((s-1)/2) = 4.6971 + 0.2001((4-1)/2) = 4.9972$.

The estimate of the parameters of the trend of the entire series can be determined from the estimate of the trend of the periodic means in quadratic [12] and exponential [13] trend curves as shown below.

Table 4. Periodic means of the simulated series.

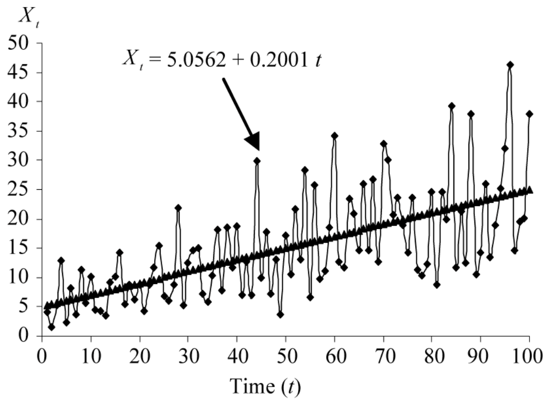
Period/Year (i)	Linear Equations (9) and (10)		Quadratic Equations (13) and (14)		Exponential Equations (16) and (17)	
	Add. $(\bar{X}_{i.})$	Mult. $(\bar{X}_{i.})$	Add. $(\bar{X}_{i.})$	Mult. $(\bar{X}_{i.})$	Add. $(\bar{X}_{i.})$	Mult. $(\bar{X}_{i.})$
1	5.631	5.950	56.900	8.180	10.647	11.260
2	6.106	6.396	67.800	19.830	11.197	11.580
3	6.947	6.147	90.200	33.760	12.187	10.710
4	8.260	9.288	124.300	90.800	13.728	15.680
5	8.320	7.340	168.400	101.000	14.101	12.210
6	9.568	10.060	224.800	190.000	15.755	16.580
7	10.359	10.911	292.000	264.000	17.053	18.020
8	11.267	11.881	370.500	347.200	18.576	19.670
9	11.519	10.417	459.600	365.000	19.562	17.450
10	13.019	14.206	561.100	577.000	21.922	24.230
11	13.396	14.125	672.600	667.000	23.299	24.700
12	13.937	12.031	795.600	629.000	24.989	21.380
13	14.520	13.255	929.800	785.000	26.883	24.280
14	16.377	18.476	1076.400	1203.000	30.227	34.800
15	16.880	18.418	1232.900	1319.000	32.408	35.700
16	17.583	17.168	1400.800	1324.000	34.995	34.100
17	18.712	20.478	1580.400	1720.000	38.232	42.460
18	19.977	24.094	1771.200	2168.000	41.850	51.580
19	20.258	20.132	1972.300	1940.000	44.740	44.800
20	19.843	14.640	2183.900	1517.000	47.222	34.110
21	21.522	23.094	2408.800	2558.000	52.110	56.400
22	21.978	20.834	2643.600	2448.000	56.098	53.200
23	22.310	16.033	2889.600	1958.000	60.330	42.130
24	24.741	30.636	3148.800	3983.000	67.052	85.100
25	24.467	23.045	3416.500	3156.000	71.492	67.200
$\bar{X}_{..}$	15.100	15.162	1221.600	1174.900	32.266	32.373

Note: Add = Additive; Mult = Multiplicative

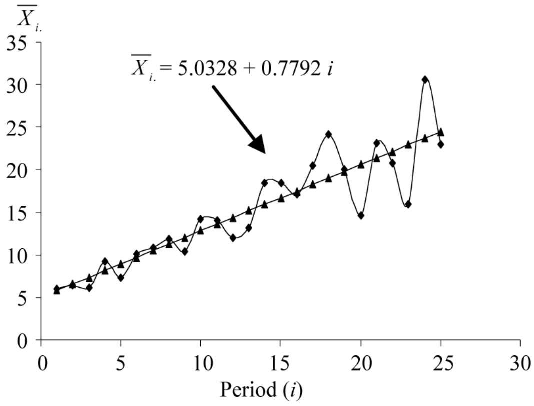


(a) Actual series (b) Periodic means

Figure 1. Time plot of actual series and periodic means of simulated series using Equation (9).



(a) Actual series



(b) Periodic means

Figure 2. Time plot of actual series and periodic means of simulated series using Equation (10).

4.2. Quadratic Trend

The data of **Table 4** (quadratic trend and additive model) and **Figure 3** is a simulation of 100 values from the additive model

$$X_t = \left(a + bt + ct^2\right) + S_t + e_t \quad (13)$$

with $s = 4$, $a = 5.0$, $b = -0.35$, $c = 0.35$, $S_1 = -50$, $S_2 = 30$, $S_3 = 80$, $S_4 = -60$ and $e_t \sim N(0,\ 1)$. The data of **Table 4** (quadratic trend and multiplicative model) and **Figure 4** is a simulation of 100 values from the multiplicative model

$$X_t = \left(a + bt + ct^2\right) \times S_t \times e_t \quad (14)$$

with $s = 4$, $a = 5.0$, $b = -0.35$, $c = 0.35$, $S_1 = 0.6$, $S_2 = 1.1$, $S_3 = 0.9$, $S_4 = 1.4$ and $e_t \sim N(1, 0.333^2)$ white noise. Listed in Table 4 are the periodic/row means $\left(\overline{X}_{i.},\ i = 1, 2, \cdots, m\right)$ of the simulated data, while the time plots of the actual series and periodic/row means are shown in **Figure 3** and **Figure 4**, respectively.

Figures 3 and **4** show that the graphs of the periodic means follow the same pattern as the plot of the actual series with respect to quadratic trend in both the additive and multiplicative models. Thus, as in the linear trend, one may look at only the plot of the periodic means in order to choose the appropriate trend. As in linear trend also, the estimate of the parameters of the trend of the entire series can be determined from the estimate of the trend of the periodic means by recognizing that periodic averages are centred at the midpoints of the periodic intervals. That is, periodic averages, $\overline{X}_{i.},\ i = 1, 2, \cdots, m$ may be expressed in terms of the original series at times $t_i, = 1, 2, \cdots, m$ as

$$\overline{X}_{i.} = X_{t_i} = a + bt_i + ct_1^2$$

$$\begin{aligned} &= a + b\left(\frac{(2i-1)s+1}{2}\right) + c\left(\frac{(2i-1)s+1}{2}\right)^2 \\ &= a - b\left(\frac{s-1}{2}\right) + c\left(\frac{s-1}{2}\right)^2 \\ &\quad + \left(bs - cs(s-1)\right)i + c(s)^2 i^2 \\ &= a' + b'i + c'i^2 \end{aligned} \quad (15)$$

where $a' = a - b\left(\frac{s-1}{2}\right) + c\left(\frac{s-1}{2}\right)^2$, $b' = \left(bs - cs\left(s-1\right)\right)$ and $c' = c\left(s\right)^2$

As an illustration, from **Figure 4**, the length of the periodic interval s = 4, the parameters of the trend of the periodic averages are $a' = 6.6541$, $b' = -5.5799$ and $c' = 5.5992$. Hence, the parameters of the actual series are:

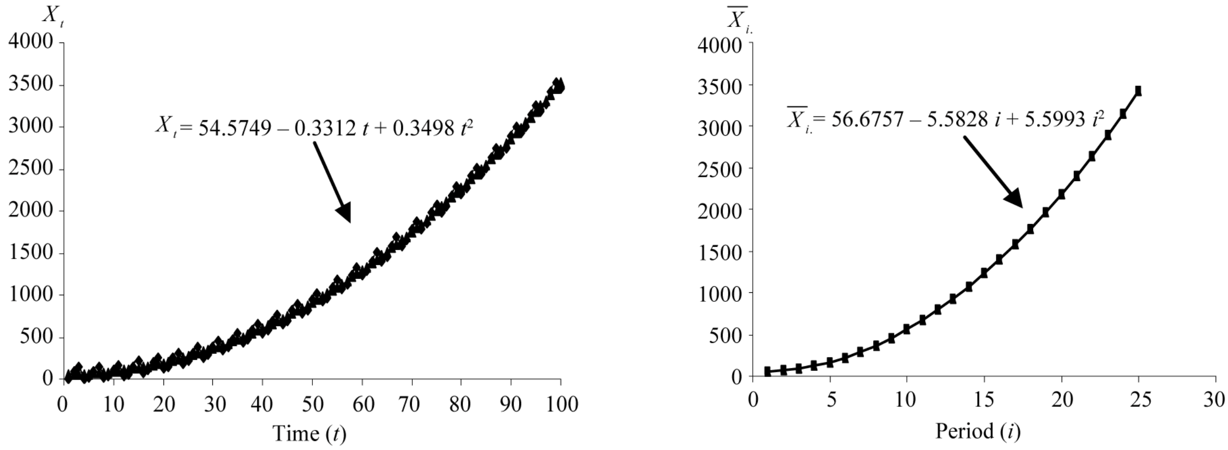
$$c = \frac{c'}{s^2} = \frac{5.5992}{4^2} = 0.3500,$$

$$\begin{aligned} b &= \frac{b' + cs\left(s-1\right)}{s} \\ &= \frac{-5.5799 + \left(0.3500\right)\left(4\right)\left(4-1\right)}{4} = -0.3451 \end{aligned}$$

$$\begin{aligned} a &= a' + b\left(\frac{s-1}{2}\right) - c\left(\frac{s-1}{2}\right)^2 \\ &= 6.6541 - 0.3451\left(\frac{4-1}{2}\right) - 0.3500\left(\frac{4-1}{2}\right)^2 \\ &= 5.3490 \end{aligned}$$

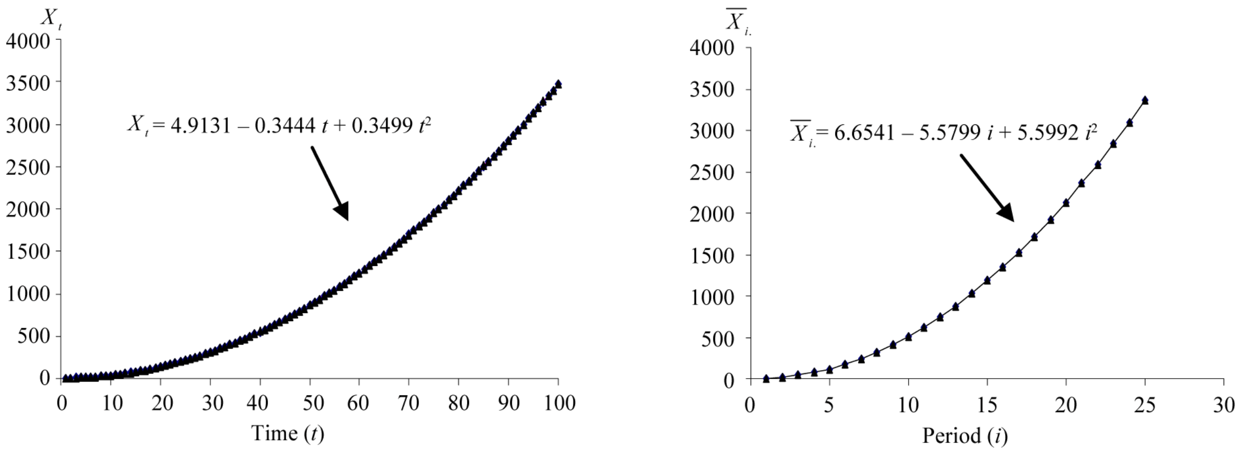
4.3. Exponential Trend

The data of **Table 4** (exponential trend and additive model) and **Figure 5** is a simulation of 100 values from the additive model



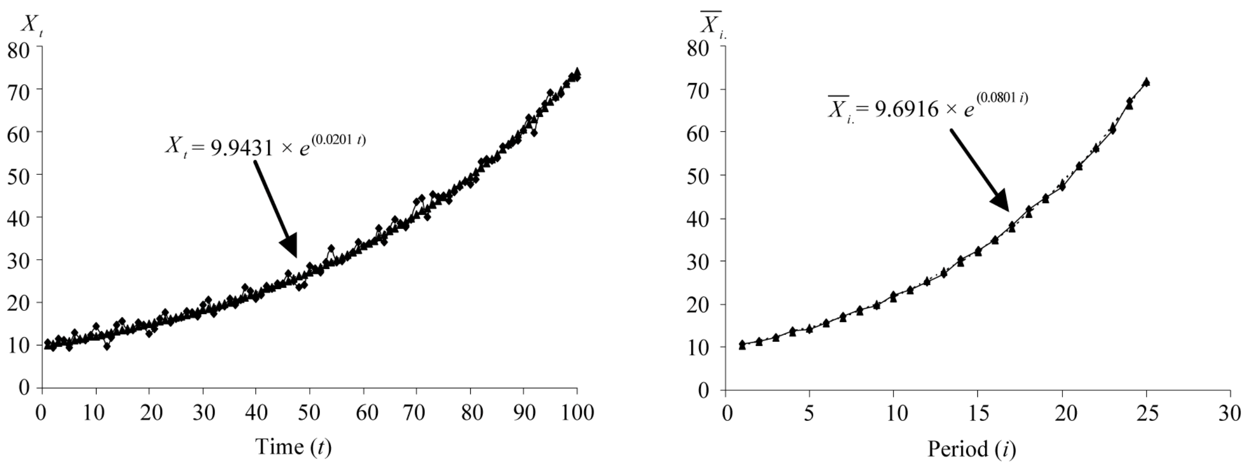
(a) Actual series (b) Periodic means

Figure 3. Plot of actual series and periodic means of simulated series using Equation (13).



(a) Actual series (b) Periodic means

Figure 4. Plot of actual series and periodic means of simulated series using Equation (14).



(a) Actual series (b) Periodic means

Figure 5. Actual series and periodic means of simulated series from Equation (16).

$$X_t = be^{ct} + S_t + e_t \tag{16}$$

with $s = 4$, $b = 10$, $c = 0.02$, $S_1 = -1.5$, $S_2 = 2.5$, $S_3 = 3.5$, $S_4 = -4.5$ and $e_t \sim N(0, 1)$. The data of **Table 4** (exponential trend and multiplicative model) and **Figure 6** is a simulation of 100 values from the multiplicative model

$$X_t = be^{ct} \times S_t \times e_t \tag{17}$$

with $s = 4$, $b = 10$, $c = 0.02$, $S_1 = 0.6$, $S_2 = 1.1$, $S_3 = 0.9$, $S_4 = 1.4$ and $e_t \sim N(1, 0.333^2)$ white noise. Listed in **Table 4** are the periodic/row means $(\bar{X}_{i.}, i = 1, 2, \cdots, m)$ of the simulated data, while the time plots of the actual series and periodic/row means are shown in **Figures 5** and **6**, respectively.

The corresponding graphs of the actual series and the periodic means, shown in **Figures 5** and **6**, indicate clearly that the pattern in the plot of the periodic means is similar to that of the actual series in both the additive and multiplicative models.

As in linear and quadratic trend curves, the estimate of the parameters of the exponential trend of the entire series can be determined from the estimate of the trend of the periodic means. That is, periodic averages, $\bar{X}_{i.}$, $i = 1, 2, \cdots, m$ may be expressed in terms of the original series at times t_i, $i = 1, 2, \cdots, m$ as

$$\begin{aligned} \bar{X}_{i.} = X_{t_i} &= be^{ct_i} = be^{c\left[\frac{(2i-1)s+1}{2}\right]} \\ &= \left(be^{-c\left(\frac{s-1}{2}\right)}\right)e^{(cs)i} = b'e^{c'i} \end{aligned} \tag{18}$$

where $b' = be^{-c\left(\frac{s-1}{2}\right)}$ and $c' = cs$.

As an illustration, from **Figure 6** the length of the periodic interval s = 4, the parameters of the trend of the periodic averages are $b' = 9.9323$, and $c' = 0.0780$. Hence, the parameters of the actual series are:

$$c = \frac{c'}{s} = \frac{0.0780}{4} = 0.0195,$$

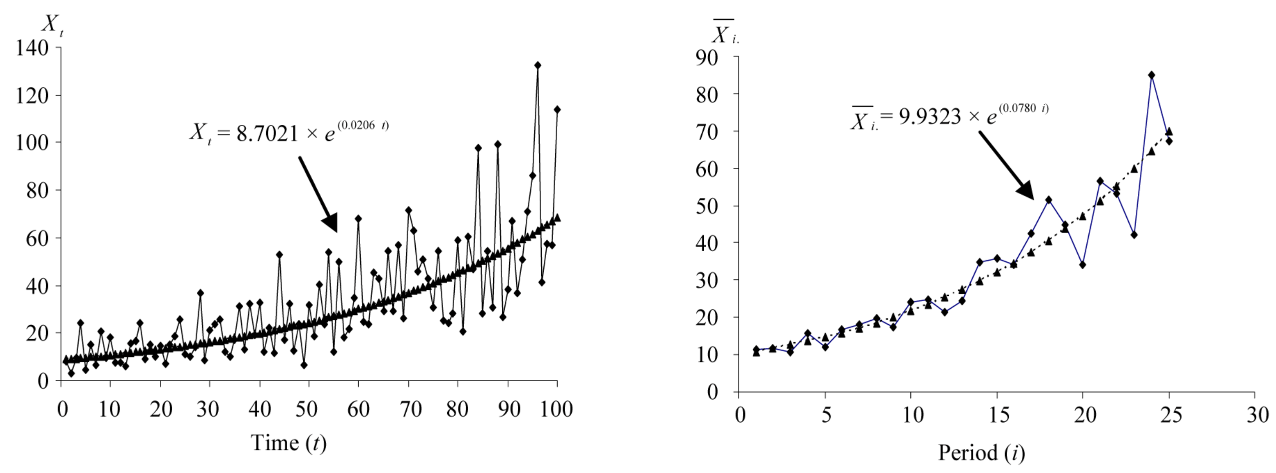
$$b = b'e^{c\left(\frac{s-1}{2}\right)} = 9.9323 \times e^{0.0195\left(\frac{4-1}{2}\right)} = 10.2273$$

5. Assessment of Seasonal Component

The seasonal component consists of effects that are reasonably stable with respect to timing, direction and magnitude. Seasonality in a time series can be identified from the time plot of the entire series by regularly spaced peaks and troughs which have a consistent direction and approximately the same magnitude every period/year, relative to the trend.

For time series which contain a seasonal effect, the overall average $(\bar{X}_{..})$ and the seasonal average $(\bar{X}_{.j}, j = 1, 2, \cdots, s)$ of the Buys-Ballot table are used to assess the effects either as a difference $(\bar{X}_{.j} - \bar{X}_{..})$ or as a ratio $(\bar{X}_{.j} / \bar{X}_{..})$. That is, the deviations of the differences seasonal averages and the overall average (additive model) from zero or the ratios of the seasonal averages to the overall average from unity (multiplicative model) is used to assess the presence of seasonal effect. The wider the deviations, the greater the seasonal effect.

This is illustrated below with stimulated time series data for the additive and multiplicative models when trend-cycle components are assumed 1) linear (Equations (9) and (10), respectively) 2) quadratic (Equations (13) and (14), respectively) and 3) exponential (Equations (16) and (17), respectively). The assessed values of the sea-



(a) Actual series (b) Periodic means

Figure 6. Actual series and periodic means of simulated series from Equation (17).

sonal effects from these series are given in **Table 5** while their corresponding graphs are given in **Figures 7**, **8** and **9** respectively. As **Table 5** and **Figures 7**, **8** and **9** show, the patterns of the deviations $\bar{X}_{.j} - \bar{X}_{..}$ (additive model) of the seasonal averages ($\bar{X}_{.j}$) from the overall average ($\bar{X}_{..}$) and ratios $\bar{X}_{.j}/\bar{X}_{..}$ (multiplicative model) mimics/follow those of the actual seasonal indices S_j used in the simulation in all series. Thus, an analyst interested in studying the seasonal effect in any study series only needs to look at either $\bar{X}_{.j} - \bar{X}_{..}$ or $\bar{X}_{.j}/\bar{X}_{..}$ to determine if there is seasonal effect or not.

However, this should not be used as a conclusive test for the presence of or otherwise of seasonal effect in a study series. The use of seasonal averages to measure seasonal effect is most appropriate in a series with no trend. When trend dominates other components in any series, the true seasonal effect may be visible from $\bar{X}_{.j} - \bar{X}_{..}$ or $\bar{X}_{.j}/\bar{X}_{..}$. Therefore, this assessment procedure should be used with great caution.

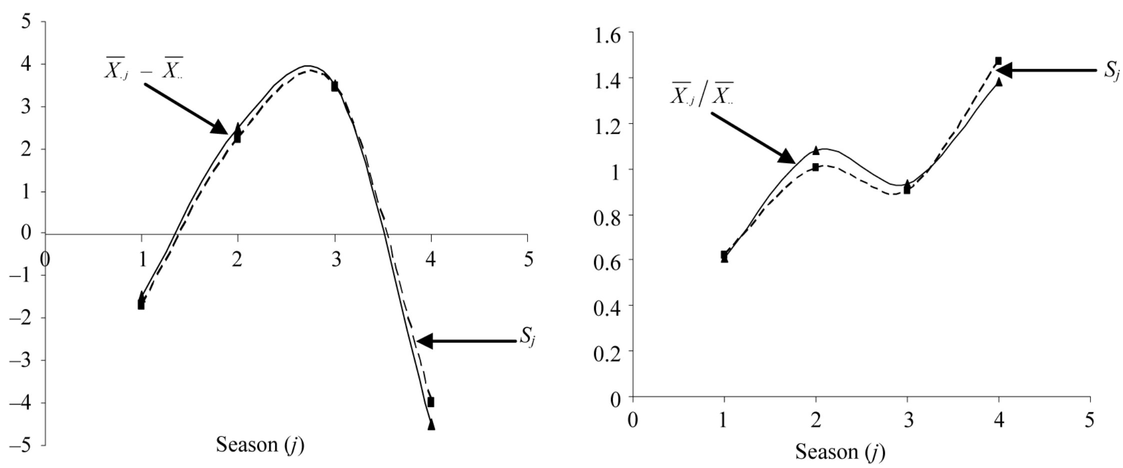
6. Choice of Appropriate Model

Traditionally, the time plot of the entire series is used to make the appropriate choice between the additive and multiplicative models. In some time series, the amplitude of both the seasonal and irregular variations do not change as the level of the trend rises or falls. In such cases, an additive model is appropriate. In many time series, the amplitude of both the seasonal and irregular variations increases as the level of the trend rises. In this situation, a multiplicative model is usually appropriate. The multiplicative model cannot be used when the original time series contains very small or zero values. This is because it is not possible to divide a number by zero. In these cases, a pseudo-additive model combining the elements of both the additive and multiplicative models is used.

How can an appropriate model be obtained from the parameters of the Buys-Ballot table? The relationship between the seasonal means $\left(\bar{X}_{.j},\ j = 1, 2, \cdots, s\right)$ and the

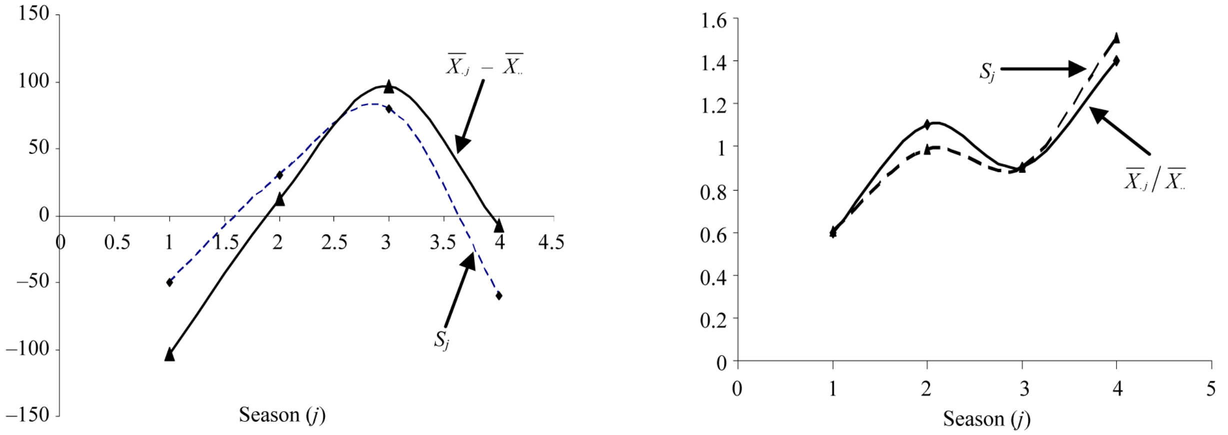
Table 5. Actual values of seasonal effects $\left(S_j\right)$, deviations of seasonal averages from the overall averages $\left(\bar{X}_{.j} - \bar{X}_{..}\right)$ and ratios of seasonal averages to the overall averages $\left(\bar{X}_{.j}/\bar{X}_{..}\right)$ for the simulated series.

Season j	Linear				Quadratic				Exponential			
	Additive Equation (9)		Multiplicative Equation (10)		Additive Equation (13)		Multiplicative Equation (14)		Additive Equation (16)		Multiplicative Equation (17)	
	S_j	$\bar{X}_{.j} - \bar{X}_{..}$	S_j	$\bar{X}_{.j} - \bar{X}_{..}$	S_j	$\bar{X}_{.j} - \bar{X}_{..}$	S_j	$\bar{X}_{.j} - \bar{X}_{..}$	S_j	$\bar{X}_{.j} - \bar{X}_{..}$	S_j	$\bar{X}_{.j} - \bar{X}_{..}$
1	–1.5	–1.71	0.61	0.62	–50	–103.0	0.6	0.60	–0.6	–1.47	0.6	0.61
2	2.5	2.24	1.08	1.01	30	12.00	1.1	0.99	1.1	0.62	1.1	1.01
3	3.5	3.47	0.93	0.90	80	97.00	0.9	0.90	0.9	1.09	0.9	0.89
4	–4.5	–4.01	1.38	1.47	–60	–7.00	1.4	1.51	–1.4	–0.24	1.4	1.50



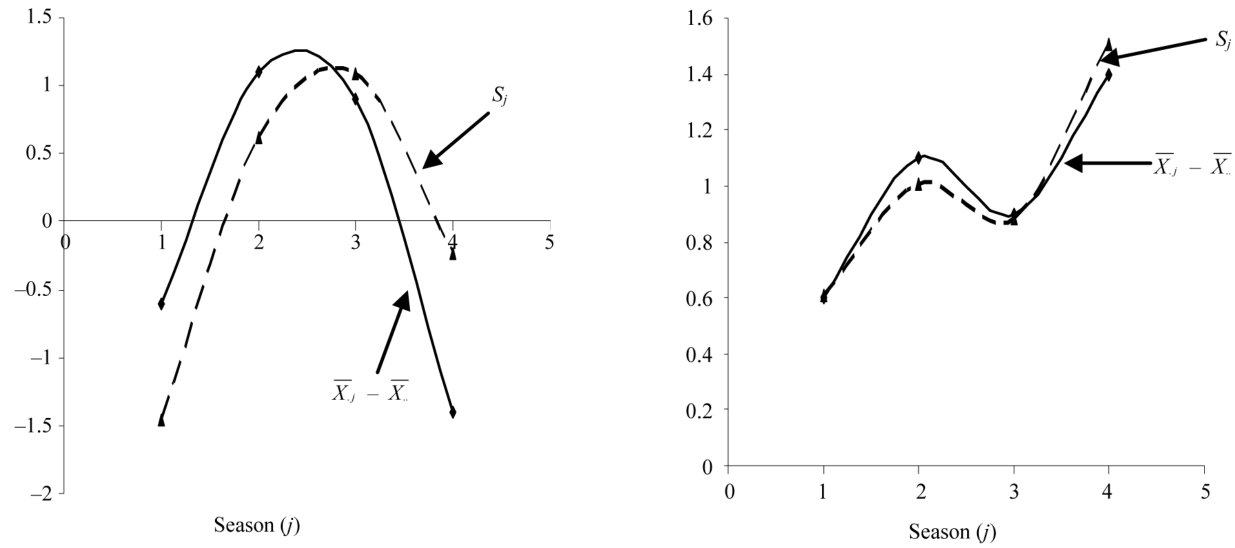
(a) Additive model (b) Multiplicative model

Figure 7. Assessment of seasonal effects when trend is linear.



(a) Additive model (b) Multiplicative model

Figure 8. Assessment of seasonal effect when trend is quadratic.



(a) Additive model (b) Multiplicative model

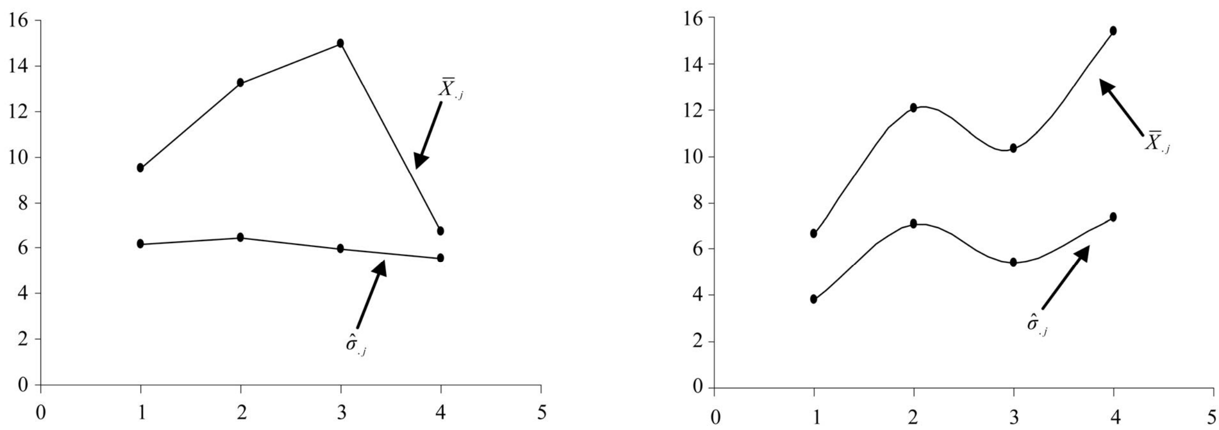
Figure 9. Assessment of seasonal effect when trend is exponential.

seasonal standard deviations $\left(\hat{\sigma}_{.j},\ j = 1, 2, \cdots, s\right)$ gives an indication of the desired model. An additive model is appropriate when the seasonal standard deviations show no appreciable increase/decrease relative to any increase or decrease in the seasonal means. On the other hand, a multiplicative model is usually appropriate when the seasonal standard deviations show appreciable increase/decrease relative to any increase/decrease in the seasonal means. This is vividly demonstrated in **Table 6** for additive and multiplicative models. Time plots of $\overline{X}_{.j}$ and $\hat{\sigma}_{.j}$, $(j = 1, 2, \cdots, s)$ are given in **Figures 10** through **12** for values of **Table 6**.

As **Tables 6** and **Figures 10** through **12** show, the observations are true for all trending curves and for both additive and multiplicative models. For the multiplicative model, the plot of standard deviation, ($\hat{\sigma}_{.j}$) clearly show appreciable increase or decrease as the seasonal averages ($\overline{X}_{.j}$) change. For the additive model, the plot of ($\hat{\sigma}_{.j}$) show no appreciable change relative to the plot of seasonal averages ($\overline{X}_{.j}$) in all the series. However, as noted earlier, when trend dominates other components, the seasonal standard deviations may not follow the observed pattern and therefore, may not be used effectively for choice of appropriate model for decomposition. Therefore, the use of the plot of the seasonal averages and standard deviations as basis for the choice of appropriate model should be done with great care.

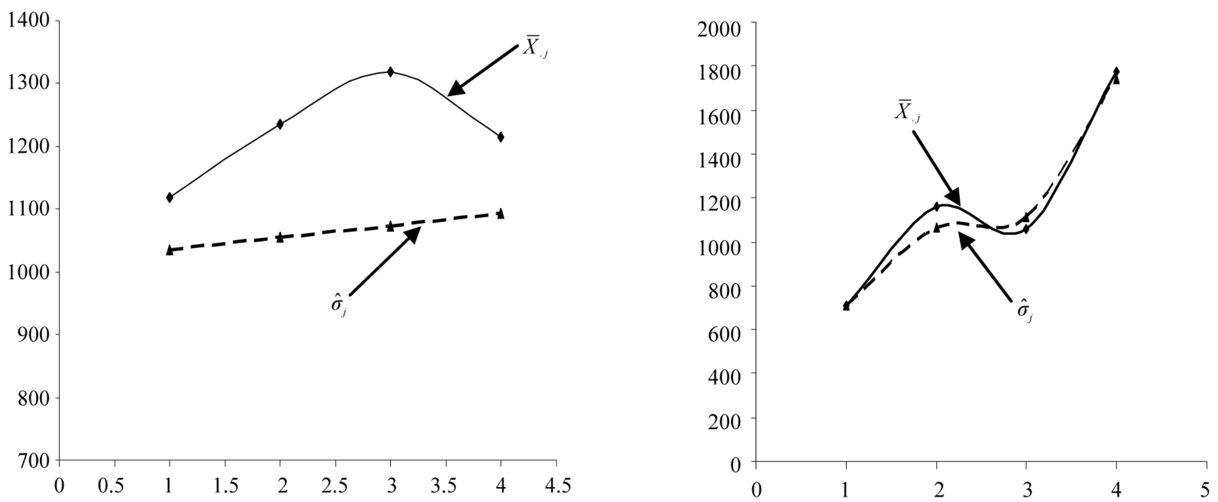
Table 6. Seasonal means and standard deviations for the simulated series.

Season j	Linear				Quadratic				Exponential			
	Additive Equation (9)		Multiplicative Equation (10)		Additive Equation (13)		Multiplicative Equation (14)		Additive Equation (16)		Multiplicative Equation (17)	
	$\overline{X}_{.j}$	$\hat{\sigma}_j$	$\overline{X}_{.j}$	$\hat{\sigma}_j$	$\overline{X}_{.j}$	$\hat{\sigma}_j$	$\overline{X}_{.j}$	$\hat{\sigma}_j$	$\overline{X}_{.j}$	$\hat{\sigma}_j$	$\overline{X}_{.j}$	$\hat{\sigma}_j$
1	13.40	6.05	9.41	5.13	1119	1034	708	707	30.8	17.95	19.61	13.28
2	17.34	5.94	15.25	7.89	1234	1054	1161	1068	32.88	18.14	32.56	20.08
3	18.57	6.07	13.7	7.83	1319	1074	1058	1113	c	18.72	28.68	21.11
4	11.09	5.84	22.29	10.55	1215	1094	1772	1740	32.03	18.88	48.64	31.8



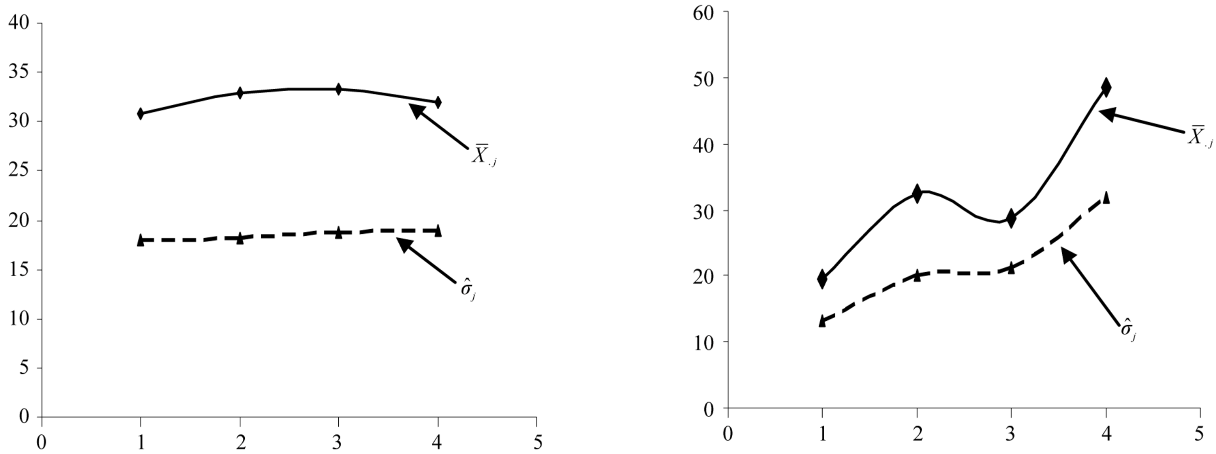
(a) Additive model (b) Multiplicative model

Figure 10. Line plot of $\overline{X}_{.j}$ and $\hat{\sigma}_{.j}$ for simulated data when trend is linear (Equations 9 and 10).



(a) Additive model (b) Multiplicative model

Figure 11. Seasonal means $\left(\overline{X}_{j}\right)$ and standard deviations $\left(\hat{\sigma}_{j}\right)$ of simulated series from quadratic trend (Equations 13 and 14).



(a) Additive model (b) Multiplicative model

Figure 12. Seasonal means $\left(\bar{X}_{.j}\right)$ and standard deviations $\left(\hat{\sigma}_j\right)$ of simulated series from exponential trend.

7. Conclusions

This paper has examined four uses of the Buys-Ballot table. Uses examined in detail include 1) data transformation 2) assessment of trend 3) assessment of seasonality and 4) choice of model for decomposition. Use of Buys-Ballot table for the estimation of trend and computation of seasonal indices was not discussed in details. For data transformation, the relationship between period /annual averages and standard deviations was used. Assessment of trend is based on the period/annual averages while the assessment of the seasonal effect is based on the seasonal and overall averages. The choice of appropriate model for decomposition is based on the seasonal averages and standard deviations.

8. References

[1] C. Chatfield, "The Analysis of Time Series: An Introduction," Chapman and Hall/CRC Press, Boca Raton, 2004.

[2] D. B. Percival and A. T. Walden, "Wavelet Methods for Time Series Analysis," Cambridge University Press, Cambridge, 2000.

[3] M. B. Priestley, "Spectral Analysis and Time Series Analysis," Academic Press, London, Vols. 1-2, 1981.

[4] G. E. P. Box, G. M. Jenkins and G. C. Reinsel, "Time Series Analysis, Forecasting and Control," 3rd Edition, Prentice-Hall, Englewood Cliffs, 1994.

[5] W. W. S. Wei, "Time Series Analysis: Univariate and Multivariate Methods," Addison-Wesley, Redwood City, 1989.

[6] M. G. Kendal and J. K. Ord, "Time Series," 3rd Edition, Charles Griffin, London, 1990.

[7] G. M. Ljung and G. E. P. Box, "On a Measure of Lack of Fit in Time Series Models," *Biometrika*, Vol. 65, No. 2, 1978, pp. 297-303. doi:10.1093/biomet/65.2.297

[8] H. Wold, "A Study in the Analysis of Stationary Time Series," 2nd Edition, Almqrist and Witsett, Stockholm, 1938.

[9] C. H. D. Buys-Ballot, "Leo Claemert Periodiques de Temperature," Kemint et Fills, Utrecht, 1847.

[10] I. S. Iwueze and A. C. Akpanta, "Effect of the Logarithmic Transformation on the Trend-Cycle Component," *Journal of Applied Science*, Vol. 7, No. 17, 2007, pp. 2414-2422.

[11] I. S. Iwueze and E. C. Nwogu, "Buys-Ballot Estimates for Time Series Decomposition," *Global Journal of Mathematics*, Vol. 3, No. 2, 2004, pp. 83-98.

[12] I. S. Iwueze and J. Ohakwe, "Buys-Ballot Estimates When Stochastic Trend is Quadratic," *Journal of the Nigerian Association of Mathematical Physics*, Vol. 8, 2004, pp. 311-318.

[13] I. S. Iwueze and E. C. Nwogu "Buys-Ballot Estimates for Exponential and S-Shaped Curves, for Time Series," *Journal of the Nigerian Association of Mathematical Physics*, Vol. 9, 2005, pp 357-366.

[14] I. S. Iwueze, E. C. Nwogu and J. C. Ajaraogu, "Properties of the Buys-Ballot Estimates When Trend-Cycle Component of a Time Series is Linear: Additive Case," *International Journal of Methematics and Computation*, Vol. 8, No. S10, 2010, pp. 18-27.

[15] G. E. P. Box and D. R. Cox, "An Analysis of Transformations," *Journal of the Royal Statistical Society*, Series B, Vol. 26, No. 2, 1964, pp. 211-243.

[16] A. C. Akpanta and I. S. Iwueze, "On Applying the Bartlett Transformation Method to Time Series Data," *Journal of Mathematical Sciences*, Vol. 20, No. 3, 2009, pp. 227-243.

[17] M. S. Bartlett, "The Use of Transformations," *Biometrika*, Vol. 3, 1947, pp. 39-52.

[18] R. V. Hogg and A. T. Craig, "Introduction to Mathematical Statistics," 4th Edition, MacMillan Publishing Company, New York, 1978.

[19] Central Bank of Nigeria, The Statistical Bulletin, Vol. 18, 2007.

Appendix

All Shares Index of the Nigerian Stock Exchange (1985-2005)

Year	Month												$\overline{X}_{i.}$	$\hat{\sigma}_{i.}$
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.		
1985	111.3	112.2	113.4	115.6	116.5	116.3	117.2	117.0	116.9	119.1	124.6	127.3	117.3	4.7
1986	134.6	139.7	140.8	146.2	144.2	147.4	150.9	151.0	155.0	160.9	163.3	163.8	149.8	9.5
1987	166.9	166.2	161.7	157.5	154.2	196.1	193.4	193.0	194.9	154.8	193.4	190.9	176.9	17.9
1988	190.8	191.4	195.5	200.1	199.2	206.0	211.5	217.6	224.1	228.5	231.4	233.6	210.8	15.9
1989	239.7	251.0	256.9	257.5	257.1	259.2	269.2	281.0	279.9	298.4	311.2	325.3	273.9	26.1
1990	343.0	349.3	356.0	362.0	382.3	417.4	445.4	463.6	468.2	480.3	502.6	513.8	423.7	63.1
1991	528.7	557.0	601.0	625.0	649.0	651.8	688.0	712.1	737.3	757.5	769.0	783.0	671.6	83.7
1992	794.0	810.7	839.1	844.0	860.5	870.8	879.7	969.3	1022.0	1076.5	1098.0	1107.6	931.0	117.0
1993	1113.4	1119.9	1130.5	1147.3	1186.9	1187.5	1180.8	1195.5	1217.3	1310.9	1414.5	1543.8	1229.0	131.1
1994	1666.3	1715.3	1792.8	1845.6	1875.5	1919.1	1926.3	1914.1	1956.0	2023.4	2119.3	2205.0	1913.2	154.5
1995	2285.3	2379.8	2551.1	2785.5	3100.8	3586.5	4314.3	4664.6	4858.1	5068.0	5095.2	5092.2	3815.0	1149.0
1996	5135.1	5180.4	5266.2	5412.4	5704.1	5798.7	5919.4	6141.0	6501.9	6634.8	6775.6	6992.1	5955.0	652.0
1997	7268.3	7699.3	8561.4	8729.8	8592.3	8459.3	8148.8	7682.0	7130.8	6554.8	6395.8	6440.5	7639.0	876.0
1998	6435.6	6426.2	6298.5	6113.9	6033.9	5892.1	5817.0	5795.7	5697.7	5671.0	5688.2	5672.7	5961.9	293.6
1999	5494.8	5376.5	5456.2	5315.7	5315.7	5977.9	4964.4	4946.2	4890.8	5032.5	5133.2	5266.4	5264.2	304.3
2000	5752.9	5955.7	5966.2	5892.8	6095.4	6466.7	6900.7	7394.1	7298.9	7415.3	7164.4	8111.0	6701.0	778.0
2001	8794.2	9180.5	9159.8	9591.6	10153.8	10937.3	10576.4	10329.0	10274.2	11091.4	11169.6	10963.1	10185.0	825.0
2002	10650.0	10581.9	11214.4	11399.1	11486.7	12440.7	12458.2	12327.9	11811.6	11451.5	11622.7	12137.7	11632.0	637.0
2003	13298.8	13668.8	13531.1	13488.0	14086.3	14565.5	13962.0	15426.0	16500.5	18743.5	19319.3	20128.9	15560.0	2502.0
2004	22712.9	24797.4	22896.4	25793.0	27730.8	28887.4	27062.1	23774.3	22739.7	23354.8	23270.5	23844.5	24739.0	2131.0
2005	23078.3	21953.5	20682.4	21961.7	21482.1	21564.8	21911.0	22935.4	24635.9	25873.8	24635.9	24085.8	22877.0	1563.0
$\overline{X}_{.j}$	5533.1	5648.2	5579.6	5818.3	5981.3	6216.6	6099.8	6077.6	6129.1	6357.2	6329.4	6472.8	6020.3	
$\hat{\sigma}_{.j}$	6955.2	7116.6	6743.2	7281.1	7539.4	7782.9	7503.3	7246.9	7369.4	7761.4	7609.5	7723.7		7235.3

Source: Statistical Bulletin of the Central Bank of Nigeria (CBN, 2007)

