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G-Type Seismic Wave in Magnetoelastic Monoclinic Layer

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Abstract

This paper deals with the study of propagation of G type waves along the plane surface at the interface of two different types of media. The upper medium is taken as monoclinic magnetoelastic layer whereas the lower half-space is inhomogeneous isotropic. Dispersion equation and condition for maximum energy flow near the surface are obtained in compact form. The dispersion equation is in assertion with the classical Love-type wave equation for the isotropic case. Effect of magnetic field and inhomogeneity on phase velocity and variation of group velocity with scaled wave number has been depicted by means of graphs. It is observed that inhomogenetity decreases phase velocity and the magnetic field has the favouring effect. A comparative study for the case of isotropic layer and monoclinic layer over the same isotropic inhomogeneous half space has been made through graphs.

Keywords: G-Waves, Magnetoelastic, Dispersion Equation, Monoclinic, Transform Technique

1. Introduction

Horizontally polarized surface wave of shear type is known as G-type wave after B. Gutenberg [1-2], who established the existence of a low velocity layer in the earth mantle. It has been studied by researchers that such waves propagate with a group velocity of 4.4 km/s [3-5] and since the group velocity of Love waves over the period range from about 40 to 300 s is same, so on other hand Love waves with long periods (60 to 300 s) may also be recognized as G-waves. These waves are followed by dispersed Love waves, especially for continental paths and they exhibit a transient pulse-like character in record. A sequence of G-waves may be observed after a large earthquake. An outstanding case of well developed G-type waves was provided by the earthquake in Peru (January, 1960). Bath and Arroyo [6] presented the result obtained from this earthquake, especially with regard to absorption and velocity dispersion of G-waves. Aki [7] discussed the generation and propagation of G-waves from the Niigata earthquake of June 16, 1964. Some other notable works in this field are done by Jeffreys [8], Bhattacharya [9], Chattopadhyay [10], Haskell [11] and others. Possibility of generation of G-waves in different medium has been investigated by different authors. Mal [12] studied the generation of G-waves taking the medium to be isotropic. The wave propagation in crystalline media plays a very interesting role in geophysics and also in ultrasonic and signal processing. Chattopadhyay and Keshri [13] presented the low velocity layer by assuming the law of variation in the lower semi-infinite anisotropic medium of two different media of monoclinic symmetry. Recently, Chattopadhyay *et al.* [14] discussed the dispersion of G-type waves in low velocity layer. The variation for the half-space in elastic constants and density reduces the equation of motion into Hill's equation with periodic coefficients which is solved by the method given by Valeev [15]. Valeev considered a certain class of system of linear differential equations with periodic coefficients which have the property that, by means of Laplace transformation, they may be converted to a system of linear difference equations, which in turn may be solved by the method of infinite determinants. This method of solving Hill's differential equation has been successfully employed by Bhattacharya [9], Mal [12] and Chattopadhyay [10].

Theoretical and experimental studies regarding to better understanding of the real earth has led a need for more realistic representation of the medium through which seismic waves propagate. The propagation of seismic waves is affected by the elastic properties of the layered materials. Moreover, the materials of the layer might be magnetoelastic in nature and the interplay of electromagnetic field with the motion of a deformable solid has its importance in various fields of science and technology.

In the present paper we study of the propagation of G type waves along the plane surface at the interface of two different types of media. The upper medium is taken as monoclinic magnetoelastic layer whereas the lower half-space is inhomogeneous isotropic. Keeping terms up to first order, the Laplace transform of the displacement is obtained. Dispersion equation and condition for maximum energy flow near the surface are obtained in compact form. The dispersion equation is in assertion with the classical Love-type wave equation for the isotropic case. Effect of magnetic field and inhomogeneity on phase velocity and variation of group velocity with scaled wave number has been depicted by means of graphs. It is observed that inhomogenetity decreases phase velocity and the magnetic field has the favouring effect. A comparative study for the case of isotropic layer and monoclinic layer over the same isotropic inhomogeneous half space has been made through graphs.

The formulation part and solution part of the problem has been dealt in Section 2. In the same section dispersion equation, the condition for maximum energy flow near the surface and the expression for group velocity have been obtained. Special cases for the dispersion equation obtained in Section 2 are considered in Section 3. Section 4 deals with the numerical calculation and graphical illustration for the problem. Finally, Section 5 concludes the study.

2. Formulation and Solution of the Problem

We consider a monoclinic magnetoelastic layer of thickness *H* lying over an inhomogeneous isotropic half-space. The variation for half-space is taken in following manner

$$\left.\begin{aligned}\mu_2 &= \mu_0\left(1-\varepsilon\cos sy\right)\\ \rho_2 &= \rho_0\left(1-\varepsilon\cos sy\right)\end{aligned}\right\} \quad (1)$$

where ε is small positive constant and s is real depth parameter. The axes Z and Y are taken horizontally and vertically downwards respectively (**Figure 1**).

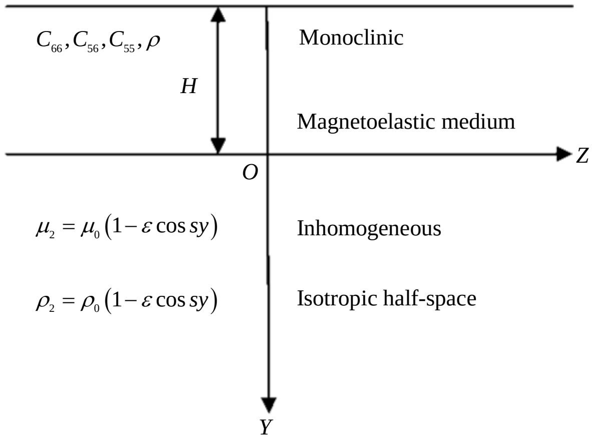

Figure 1. Geometry of the problem.

At first, we deduce the equation governing the propagation of shear wave in monoclinic magnetoelastic crustal layer.

The strain-displacement relations for monoclinic medium are

$$S_1 = \frac{\partial u}{\partial x},\ S_2 = \frac{\partial v}{\partial y},\ S_3 = \frac{\partial w}{\partial z},\ S_4 = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z},\\ S_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x},\ S_6 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad (2)$$

where u, v, w are displacement components in the direction x, y, z respectively, and $S_i\left(i = 1, 2, \cdots, 6\right)$ are the strain components.

Also, the stress-strain relation for a rotated y-cut quartz plate, which exhibits monoclinic symmetry with x being the diagonal axis are

$$\begin{aligned}T_1 &= C_{11}S_1 + C_{12}S_2 + C_{13}S_3 + C_{14}S_4,\\ T_2 &= C_{12}S_1 + C_{22}S_2 + C_{23}S_3 + C_{24}S_4,\\ T_3 &= C_{13}S_1 + C_{23}S_2 + C_{33}S_3 + C_{34}S_4,\\ T_4 &= C_{14}S_1 + C_{24}S_2 + C_{34}S_3 + C_{44}S_4,\\ T_5 &= C_{55}S_5 + C_{56}S_6,\\ T_6 &= C_{56}S_5 + C_{66}S_6\end{aligned} \quad (3)$$

where $T_i\left(i = 1, 2, \cdots, 6\right)$ are the stress components and $C_{ij} = C_{ji}\left(i = 1, 2, \cdots, 6\right)$ are the elastic constants.

Equations governing the propagation of small elastic disturbances in a perfectly conducting monoclinic medium having electromagnetic force $\mathbf{J}\times\mathbf{B}$ (the Lorentz force, $\mathbf{J}$ being the electric current density and $\mathbf{B}$ being the magnetic induction vector) as the only body forces are

$$\begin{aligned}\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} + \left(J\times B\right)_x &= \rho\frac{\partial^2 u}{\partial t^2},\\ \frac{\partial T_6}{\partial x} + \frac{\partial T_2}{\partial y} + \frac{\partial T_4}{\partial z} + \left(J\times B\right)_y &= \rho\frac{\partial^2 v}{\partial t^2},\\ \frac{\partial T_5}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_3}{\partial z} + \left(J\times B\right)_z &= \rho\frac{\partial^2 w}{\partial t^2}\end{aligned} \quad (4)$$

where ρ is the density of the layer.

For SH wave propagating in the z-direction and causing displacement in the x-direction only, we shall assume that

$$u = u\left(y, z, t\right),\ v = w = 0 \text{ and } \frac{\partial}{\partial x} \equiv 0. \quad (5)$$

Using Equations (2) and (5), the stress-strain relation (3) becomes

$$T_1 = T_2 = T_3 = T_4 = 0,$$
$$T_5 = C_{55}\frac{\partial u}{\partial z} + C_{56}\frac{\partial u}{\partial y}, \quad (6)$$
$$T_6 = C_{56}\frac{\partial u}{\partial z} + C_{66}\frac{\partial u}{\partial y}.$$

Using Equation (6) in Equation (4), the only non-vanishing equation we have

$$C_{66}\frac{\partial^2 u}{\partial y^2} + 2C_{56}\frac{\partial^2 u}{\partial y \partial z} + C_{55}\frac{\partial^2 u}{\partial z^2} + (J \times B)_x = \rho\frac{\partial^2 u}{\partial t^2}. \quad (7)$$

The well known Maxwell's equations governing the electromagnetic field are

$$\nabla \cdot \mathbf{B} = 0, \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \nabla \times \mathbf{H} = \mathbf{J}$$
$$\text{with } \mathbf{B} = \mu_e \mathbf{H}, \mathbf{J} = \sigma\left(\mathbf{E} + \frac{\partial \mathbf{u}}{\partial t} \times \mathbf{B}\right) \quad (8)$$

where **E** is the induced electric field, **J** is the current density vector and magnetic field **H** includes both primary and induced magnetic fields. μ_e and σ are the induced permeability and conduction coefficient respectively.

The linearized Maxwell's stress tensor $\left(\tau_{ij}^0\right)^{M_x}$ due to the magnetic field is given by

$$\left(\tau_{ij}^0\right)^{M_x} = \mu_e\left(H_i h_j + H_j h_i - H_k h_k \delta_{ij}\right).$$

Let $\mathbf{H} = (H_1, H_2, H_3)$, $\mathbf{u} = (u, v, w)$ and $h_i = (h_1, h_2, h_3)$ where h_i is the change in the magnetic field. In writing the above equations, we have neglected the displacement current. From Equation (8), we get

$$\nabla^2 \mathbf{H} = \mu_e \sigma\left\{\frac{\partial \mathbf{H}}{\partial t} + \nabla \times \left(\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H}\right)\right\}. \quad (9)$$

In component form, Equation (9) can be written as

$$\frac{\partial H_1}{\partial t} = \frac{1}{\mu_e \sigma}\nabla^2 H_1 + \frac{\partial\left(H_2 \frac{\partial u}{\partial t}\right)}{\partial y} + \frac{\partial\left(H_3 \frac{\partial u}{\partial t}\right)}{\partial z},$$
$$\frac{\partial H_2}{\partial t} = \frac{1}{\mu_e \sigma}\nabla^2 H_2, \quad (10)$$
$$\frac{\partial H_3}{\partial t} = \frac{1}{\mu_e \sigma}\nabla^2 H_3.$$

For perfectly conducting medium *i.e.* $\sigma \to \infty$, the Equation (10) becomes

$$\frac{\partial H_2}{\partial t} = \frac{\partial H_3}{\partial t} = 0, \quad (11)$$

and

$$\frac{\partial H_1}{\partial t} = \frac{\partial\left(H_2 \frac{\partial u}{\partial t}\right)}{\partial y} + \frac{\partial\left(H_3 \frac{\partial u}{\partial t}\right)}{\partial z}. \quad (12)$$

It is clear from Equation (11) that there is no perturbation in H_2 and H_3, however from Equation (12) there may be perturbation in H_1. Therefore, taking small perturbation, say h_1 in H_1, we have $H_1 = H_{01} + h_1$, $H_2 = H_{02}$ and $H_3 = H_{03}$, where (H_{01}, H_{02}, H_{03}) are components of the initial magnetic field $\mathbf{H}_0$.

We can write $\mathbf{H}_0 = (0, H_0 \sin\phi, H_0 \cos\phi)$, where $H_0 = |\mathbf{H}_0|$ and ϕ is the angle at which the wave crosses the magnetic field. Thus we have

$$\mathbf{H} = (h_1, H_0 \sin\phi, H_0 \cos\phi) \quad (13)$$

We shall take initial value of h_1 as $h_1 = 0$. Using Equation (13) in Equation (12), we get

$$\frac{\partial h_1}{\partial t} = \frac{\partial\left(H_0 \sin\phi \frac{\partial u}{\partial t}\right)}{\partial y} + \frac{\partial\left(H_0 \cos\phi \frac{\partial u}{\partial t}\right)}{\partial z}. \quad (14)$$

Integrating with respect to t, we get

$$h_1 = H_0 \sin\phi \frac{\partial u}{\partial y} + H_0 \cos\phi \frac{\partial u}{\partial z}. \quad (15)$$

Considering $\nabla\left(\frac{H^2}{2}\right) = -(\nabla \times \mathbf{H}) \times \mathbf{H} + (\mathbf{H} \cdot \nabla)\mathbf{H}$ and Equation (8), we get

$$\mathbf{J} \times \mathbf{B} = \mu_e\left\{-\nabla\left(\frac{H^2}{2}\right) + (\mathbf{H} \cdot \nabla)\mathbf{H}\right\}. \quad (16)$$

In the component form Equation (16) can be written as

$$(\mathbf{J} \times \mathbf{B})_y = (\mathbf{J} \times \mathbf{B})_z = 0 \quad \text{and}$$
$$(\mathbf{J} \times \mathbf{B})_x = \mu_e H_0{}^2\left(\sin^2\phi \frac{\partial^2 u}{\partial y^2} + \sin 2\phi \frac{\partial^2 u}{\partial y \partial z} + \cos^2\phi \frac{\partial^2 u}{\partial z^2}\right). \quad (17)$$

Using Equations (7) and (17), we find the equation of motion for the magnetoelastic monoclinic medium in the form

$$M_{66}\frac{\partial^2 u}{\partial y^2} + 2M_{56}\frac{\partial^2 u}{\partial y \partial z} + M_{55}\frac{\partial^2 u}{\partial z^2} = \rho\frac{\partial^2 u}{\partial t^2}. \quad (18)$$

where

$$M_{66} = C_{66}\left(1 + m_H \sin^2\phi\right),$$
$$M_{55} = C_{66}\left(\frac{C_{55}}{C_{66}} + m_H \cos^2\phi\right), \quad (19)$$
$$M_{56} = C_{66}\left(\frac{C_{56}}{C_{66}} + m_H \cos\phi \sin\phi\right)$$

where $m_H = \frac{\mu_e H_0^2}{C_{66}}$ is monoclinic-magnetoelastic coupling parameter.

Hence, the equation of motion for the propagation of shear wave in monoclinic magnetoelastic crustal layer is

$$M_{66}\frac{\partial^2 u}{\partial y^2}+2M_{56}\frac{\partial^2 u}{\partial y\partial z}+M_{55}\frac{\partial^2 u}{\partial z^2}=\rho\frac{\partial^2 u}{\partial t^2}. \quad (20)$$

Now we consider

$$u_1(y,z,t)=U_1(y)e^{ik(z-ct)} \quad (21)$$

where k is wave number and c is wave velocity.

Substituting Equation (21) in Equation (20), we get

$$M_{66}\frac{d^2U_1}{dy^2}+2ikM_{56}\frac{dU_1}{dy}+k^2\left(\rho c^2-M_{55}\right)U_1=0. \quad (22)$$

Solution of Equation (22) is

$$U_1(y)=e^{-\frac{\eta y}{2}}\left(A\cos Py+B\sin Py\right) \quad (23)$$

where

$$\eta=\frac{2ikM_{56}}{M_{66}},\ P=\sqrt{\left\{\frac{\rho\omega^2}{M_{66}}-\frac{M_{55}k^2}{M_{66}}+\left(\frac{kM_{56}}{M_{66}}\right)^2\right\}}$$ and A, B are constants.

Since the upper surface is stress free therefore,

$M_{66}\frac{\partial u_1}{\partial y}+M_{56}\frac{\partial u_1}{\partial z}=0$ at $y=-H$, gives

$$U_1(y)=b_1e^{-\frac{\eta y}{2}}\left[M_{66}\left(-\frac{\eta}{2}\right)\sin P(y+H)-M_{66}P\cos P(y+H)+ikM_{56}\sin P(y+H)\right] \quad (24)$$

In lower inhomogeneous half-space the displacement $u_2(y,z,t)$ satisfy the differential equation

$$\frac{\partial}{\partial z}\left[\mu_0(1-\varepsilon\cos sy)\frac{\partial u_2}{\partial z}\right]+\frac{\partial}{\partial y}\left[\mu_0(1-\varepsilon\cos sy)\frac{\partial u_2}{\partial y}\right]=\rho_0(1-\varepsilon\cos sy)\frac{\partial^2 u_2}{\partial t^2} \quad (25)$$

we considered

$$u_2(y,z,t)=U_2(y)e^{ik(z-ct)} \quad (26)$$

Using (25) and (26), the equation of motion for lower inhomogeneous medium may be written as

$$\begin{aligned}&e^{-isy}\left[-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2U_2(y)+\frac{\varepsilon}{2}k^2U_2(y)-\frac{\varepsilon}{2}\frac{d^2U_2}{dy^2}+\frac{\varepsilon is}{2}\frac{dU_2}{dy}\right]+\frac{d^2U_2}{dy^2}\\&+\left(\frac{\rho_0}{\mu_0}k^2c^2-k^2\right)U_2(y)+e^{isy}\left[-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2U_2(y)+\frac{\varepsilon}{2}k^2U_2(y)-\frac{\varepsilon}{2}\frac{d^2U_2}{dy^2}-\frac{\varepsilon is}{2}\frac{dU_2}{dy}\right]=0.\end{aligned} \quad (27)$$

This is Hill's differential equation, which we will solve by the method given by Valeev [15]. We apply Laplace transform with respect to *y*, *i.e.* we multiply (27) by e^{-py} and integrate with respect to *y* from 0 to ∞, we get

$$\begin{aligned}&\int_0^\infty e^{-(p+is)y}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2U_2(y)+\frac{\varepsilon}{2}k^2U_2(y)-\frac{\varepsilon}{2}\frac{d^2U_2}{dy^2}+\frac{\varepsilon is}{2}\frac{dU_2}{dy}\right\}dy\\&+\int_0^\infty e^{-(p-is)y}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2U_2(y)+\frac{\varepsilon}{2}k^2U_2(y)-\frac{\varepsilon}{2}\frac{d^2U_2}{dy^2}-\frac{\varepsilon is}{2}\frac{dU_2}{dy}\right\}dy\\&+\int_0^\infty e^{-py}\left\{\frac{d^2U_2}{dy^2}+\left(\frac{\rho_0}{\mu_0}k^2c^2-k^2\right)U_2(y)\right\}dy=0.\end{aligned} \quad (28)$$

Now the boundary conditions are

(i) $u_1=u_2$ at $y=0$

(ii) $M_{66}\frac{\partial u_1}{\partial y}+M_{56}\frac{\partial u_1}{\partial z}=\mu_2\frac{\partial u_2}{\partial y}$ at $y=0$ (29)

(iii) $M_{66}\frac{\partial u_1}{\partial y}+M_{56}\frac{\partial u_1}{\partial z}=0$ at $y=-H$.

From (i) of Equation (29) we get

$$U_2(0)=b_1\left[\left\{ikM_{56}-\frac{\eta M_{66}}{2}\right\}\sin PH-M_{66}P\cos PH\right] \quad (30)$$

Now we consider

$$q(0)=\left(\frac{dU_2}{dy}\right)_{y=0}\ \text{and}\ \mu_2^{(0)}=\mu_0(1-\varepsilon).$$

Then (ii) of Equation (29) gives

$$q(0)=\frac{b_1}{\mu_2^{(0)}}\left[M_{66}\left\{\left(-\frac{\eta}{2}\right)\left\{\left(ikM_{56}-\frac{\eta M_{66}}{2}\right)\sin PH-PM_{66}\cos PH\right\}\right.\right.$$
$$\left.\left.+P\left\{\left(ikM_{56}-\frac{\eta M_{66}}{2}\right)\cos PH+PM_{66}\sin PH\right\}\right\}+ikM_{56}\left\{\left(ikM_{56}-\frac{\eta M_{66}}{2}\right)\sin PH-PM_{66}\cos PH\right\}\right] \quad (31)$$

Now defining the Laplace transform of $U_2(y)$ as

$$F(p)=\int_0^\infty e^{-py}U_2(y)\,dy\,. \quad (32)$$

Using the boundary conditions we obtain the following system of equations

$$\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}(p+is)^2+\frac{\varepsilon is}{2}(p+is)\right\}F(p+is)$$
$$+\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}(p-is)^2-\frac{\varepsilon is}{2}(p-is)\right\}F(p-is) \quad (33)$$
$$+\left(p^2-w^2\right)F(p)=p\Im_1+\Im_2$$

where

$$\Im_1=(1-\varepsilon)V_2(0)\,,\ \Im_2=(1-\varepsilon)q(0)$$
$$\text{and } w^2=k^2\left(1-\frac{\rho_0}{\mu_0}c^2\right) \quad (34)$$

To find $F(p)$ from Equation (33), we replace p by $(p+ism)$ and divide throughout by $(ism)^n$, $(m\neq 0)$. We then obtain the following infinite system of linear algebraic equations in the quantities $F(p+ism)$, $(m=0,\pm1,\pm2,\cdots)$

$$(ism)^{-n}\left[-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}\{p+is(m+1)\}^2+\frac{\varepsilon is}{2}\{p+is(m+1)\}\right]F\{p+is(m+1)\}$$
$$+(ism)^{-n}\left\{(p+ism)^2-w^2\right\}F(p+ism)$$
$$+(ism)^{-n}\left[-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}\{p+is(m-1)\}^2-\frac{\varepsilon is}{2}\{p+is(m-1)\}\right]F\{p+is(m-1)\} \quad (35)$$
$$=(ism)^{-n}\left\{\Im_1(p+ism)+\Im_2\right\}$$

where p may be considered as a parameter in the coefficients. It should be noted that in order not to have to consider the special case $m=0$ separately, we include Equation (33) in (35) by agreeing to regard $(ism)^{-n}=1$ when $m=0$. Solving the system of difference Equation (35), we obtain $F(p)$ as the ratio of two infinite determinants, viz.

$$F(p)=\frac{\Delta_1}{\Delta_2} \quad (36)$$

where

$$\Delta_1=\begin{vmatrix}\cdots & \cdots & \cdots & \cdots & \cdots\\ \cdots & (is)^{-n}\left\{(p+is)^2-w^2\right\} & (is)^{-n}\left\{(p+is)\Im_1+\Im_2\right\} & 0 & \cdots\\ \cdots & \begin{matrix}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2\right.\\ \left.-\frac{\varepsilon}{2}(p+is)^2+\frac{\varepsilon is}{2}(p+is)\right\}\end{matrix} & \left(\Im_1P+\Im_2\right) & \begin{matrix}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2\right.\\ \left.-\frac{\varepsilon}{2}(p-is)^2-\frac{\varepsilon is}{2}(p-is)\right\}\end{matrix} & \cdots\\ \cdots & 0 & (-is)^{-n}\left\{(p-is)\Im_1+\Im_2\right\} & (-is)^{-n}\left\{(p-is)^2-w^2\right\} & \cdots\\ \cdots & \cdots & \cdots & \cdots & \cdots\end{vmatrix}$$

and

$$\Delta_2=\begin{vmatrix}\cdots & \cdots & \cdots & \cdots & \cdots\\ \cdots & (is)^{-n}\left\{(p+is)^2-w^2\right\} & (is)^{-n}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}p^2-\frac{\varepsilon is}{2}p\right\} & 0 & \cdots\\ \cdots & \begin{array}{l}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2\right.\\ \left.-\frac{\varepsilon}{2}(p+is)^2+\frac{\varepsilon is}{2}(p+is)\right\}\end{array} & p^2-w^2 & \begin{array}{l}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2\right.\\ \left.-\frac{\varepsilon}{2}(p-is)^2-\frac{\varepsilon is}{2}(p-is)\right\}\end{array} & \cdots\\ \cdots & 0 & (-is)^{-n}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}p^2+\frac{\varepsilon is}{2}p\right\} & (-is)^{-n}\left\{(p-is)^2-w^2\right\} & \cdots\\ \cdots & \cdots & \cdots & \cdots & \cdots\end{vmatrix}$$

The first approximation of Equation (36) is

$$F(P)=\frac{p\Im_1+\Im_2}{p^2-w^2}\approx\frac{p(1-\varepsilon)B_1}{p^2-w^2}-\frac{(1-\varepsilon)B_2}{\mu_2^{(0)}\left(p^2-w^2\right)} \tag{37}$$

where

$$B_1=b_1\left[\left\{ikM_{56}-\frac{\eta M_{66}}{2}\right\}\sin PH-\left(PM_{66}\right)\cos PH\right]$$

and

$$B_2=b_1\left[M_{66}\left\{\left(-\frac{\eta}{2}\right)\right\}\left\{\left(ikM_{56}-\frac{\eta M_{66}}{2}\right)\sin PH-PM_{66}\cos PH\right\}+P\left\{\left(ikM_{56}-\frac{\eta M_{66}}{2}\right)\cos PH+PM_{66}\sin PH\right\}\right\}$$
$$+ikM_{56}\left\{\left(ikM_{56}-\frac{\eta M_{66}}{2}\right)\sin PH-PM_{66}\cos PH\right\}\Bigg].$$

The second approximation of Equation (36) is

$$F(p)=\frac{\Delta_3}{\Delta_4} \tag{38}$$

where

$$\Delta_3=\begin{vmatrix}(is)^{-n}\left\{(p+is)^2-w^2\right\} & (is)^{-n}\left\{(p+is)\Im_1+\Im_2\right\} & 0\\ \begin{array}{l}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2\right.\\ \left.-\frac{\varepsilon}{2}(p+is)^2+\frac{\varepsilon is}{2}(p+is)\right\}\end{array} & \left(\Im_1P+\Im_2\right) & \begin{array}{l}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2\right.\\ \left.-\frac{\varepsilon}{2}(p-is)^2-\frac{\varepsilon is}{2}(p-is)\right\}\end{array}\\ 0 & (-is)^{-n}\left\{(p-is)\Im_1+\Im_2\right\} & (-is)^{-n}\left\{(p-is)^2-w^2\right\}\end{vmatrix}\quad\text{and}$$

$$\Delta_4=\begin{vmatrix}(is)^{-n}\left\{(p+is)^2-w^2\right\} & (is)^{-n}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}p^2-\frac{\varepsilon is}{2}p\right\} & 0\\ \begin{array}{l}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2\right.\\ \left.-\frac{\varepsilon}{2}(p+is)^2+\frac{\varepsilon is}{2}(p+is)\right\}\end{array} & p^2-w^2 & \begin{array}{l}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2\right.\\ \left.-\frac{\varepsilon}{2}(p-is)^2-\frac{\varepsilon is}{2}(p-is)\right\}\end{array}\\ 0 & (-is)^{-n}\left\{-\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}p^2+\frac{\varepsilon is}{2}p\right\} & (-is)^{-n}\left\{(p-is)^2-w^2\right\}\end{vmatrix}$$

Neglecting the terms containing ε^2 and higher powers, we get

$$s^{2n}\Delta_3 = \left(p\Im_1+\Im_2\right)\left\{\left(p-is\right)^2-w^2\right\}\left\{\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2-\frac{\varepsilon}{2}k^2+\frac{\varepsilon}{2}\left(p+is\right)^2-\frac{\varepsilon is}{2}\left(p+is\right)\right\}$$
$$+\left(p\Im_1+\Im_2\right)\left\{\left(p+is\right)^2-w^2\right\}\left\{\left(p-is\right)^2-w^2\right\}$$
$$+\left\{\left(p-is\right)\Im_1+\Im_2\right\}\left\{\left(p+is\right)^2-w^2\right\}\left\{\frac{\varepsilon}{2}\frac{\rho_0}{\mu_0}k^2c^2-\frac{\varepsilon}{2}k^2+\frac{\varepsilon}{2}\left(p-is\right)^2+\frac{\varepsilon is}{2}\left(p-is\right)\right\}$$

and

$$s^{2n}\Delta_4 = \left\{p^2-w^2\right\}\left\{\left(p-is\right)^2-w^2\right\}\left\{\left(p+is\right)^2-w^2\right\}.$$

Hence Equation (38) becomes

$$F\left(P\right)=\frac{p\Im_1+\Im_2}{p^2-w^2}+\frac{\varepsilon}{2}\frac{V_2\left(0\right)\left(p+is\right)+q\left(0\right)}{\left(p^2-w^2\right)\left\{\left(p+is\right)^2-w^2\right\}}\left\{\frac{\rho_0}{\mu_0}k^2c^2-k^2+\left(p+is\right)^2-is\left(p+is\right)\right\}$$
$$+\frac{\varepsilon}{2}\frac{V_2\left(0\right)\left(p-is\right)+q\left(0\right)}{\left(p^2-w^2\right)\left\{\left(p-is\right)^2-w^2\right\}}\left\{\frac{\rho_0}{\mu_0}k^2c^2-k^2+\left(p-is\right)^2+is\left(p-is\right)\right\} \quad (39)$$

Then $U_2\left(y\right)$ will be obtained from the following inversion formula:

$$U_2\left(y\right)=\frac{1}{2\pi}\int_{\gamma-i\infty}^{\gamma+i\infty}F\left(p\right)e^{py}dp \quad (40)$$

The residues R_1, R_2, R_3 at the poles $p=w$, $p=w+is$, $p=w-is$ are given respectively by

$$R_1=\left\{\frac{q\left(0\right)+wV_2\left(0\right)}{2w}\right\}\left\{\left(1-\varepsilon\right)+\frac{\varepsilon w^2}{s^2+4w^2}\right\}$$
$$-\frac{\varepsilon D_1}{s^2+4w^2}\left\{\frac{q\left(0\right)+wV_2\left(0\right)}{2w}\right\}+\frac{\varepsilon V_2\left(0\right)}{2}\left(\frac{s^2+2w^2}{s^2+4w^2}\right) \quad (41)$$

$$R_2=\frac{-i\varepsilon}{4s}\frac{\left\{q\left(0\right)+wV_2\left(0\right)\right\}\left\{D_1+w^2+isw\right\}}{w\left(2w+is\right)}e^{\left(is+w\right)y} \quad (42)$$

and

$$R_3=\frac{\varepsilon}{4s}\frac{\left\{q\left(0\right)+wV_2\left(0\right)\right\}\left\{D_1+w^2-isw\right\}}{w\left(2w-is\right)}e^{\left(-is+w\right)y} \quad (43)$$

where

$$D_1=\frac{\rho_0}{\mu_0}k^2c^2-k^2$$

Equations (41), (42) and (43) show the conditions for a large amount of energy to be confined near the surface are

$$wV_2\left(0\right)+q\left(0\right)=0 \quad (44)$$

$$2w^2+s^2=0 \quad (45)$$

and

$$q\left(0\right)-wV_2\left(0\right)=0\,. \quad (46)$$

Equations (44) and (46) give

$$q\left(0\right)=\pm wV_2\left(0\right) \quad (47)$$

which finally gives the dispersion relation as

$$\tan PH=\pm\frac{w\mu_0\left(1-\varepsilon\right)}{PM_{66}}. \quad (48)$$

We will consider only the positive sign for further discussion.

Now Equation (45) gives

$$\omega=kc=\sqrt{\frac{\mu_0}{2\rho_0}\left(2k^2+s^2\right)}$$

and hence we get the expression for group velocity as

$$G=\frac{\partial\omega}{\partial k}=\beta_2\frac{\sqrt{2}\,k}{\sqrt{\left(2k^2+s^2\right)}}. \quad (49)$$

It follows from Equation (49) that $G<\beta_2$, *i.e.* the group velocity is less than the shear wave velocity in the upper mantle.

3. Particular cases

Case 1

When $\varepsilon=0$ the Equation (48) reduces to

$$\tan PH=\frac{w\mu_0}{PM_{66}} \quad (50)$$

which is the dispersion relation for the case when monoclinic magnetoelastic layer lying over an isotropic half-space.

Case 2

When $C_{55} = C_{66} = \mu_1,\ C_{56} = 0$ the Equation (48) reduces to

$$\tan\left(kH\sqrt{\frac{(c/\beta_1)^2}{1+\varepsilon_H \sin^2\phi}-1}\right) = \frac{\mu_0(1-\varepsilon)\sqrt{\left(1-c^2/\beta_2^2\right)}}{\mu_1\left(1+\varepsilon_H \sin^2\phi\right)\sqrt{\frac{(c/\beta_1)^2}{1+\varepsilon_H \sin^2\phi}-1}} \quad (51)$$

where $\varepsilon_H = \dfrac{\mu_e H_0^2}{\mu_1}$.

Case 3

When $C_{55} = C_{66} = \mu_1, \quad C_{56} = 0, \quad m_H = 0$ the Equation (48) reduces to

$$\tan\left(kH\sqrt{\frac{c^2}{\beta_1^2}-1}\right) = \frac{\mu_0(1-\varepsilon)\sqrt{\left(1-c^2/\beta_2^2\right)}}{\mu_1\sqrt{\left(c^2/\beta_1^2-1\right)}} \quad (52)$$

which is the result obtained by Chattopadhyay *et al.* [14] for isotropic layer lying over an inhomogeneous isotropic half space.

Case 4

When $C_{55} = C_{66} = \mu_1, \quad C_{56} = 0, \quad m_H = 0$ and $\varepsilon = 0$ the Equation (48) reduces to

$$\tan\left(kH\sqrt{\left(\frac{c^2}{\beta_1^2}-1\right)}\right) = \frac{\mu_0\sqrt{\left(1-\frac{c^2}{\beta_2^2}\right)}}{\mu_1\sqrt{\left(\frac{c^2}{\beta_1^2}-1\right)}} \quad (53)$$

which is the usual dispersion equation for Love wave with $\beta_1 < c < \beta_2$ (Chattopadhyay [10]).

4. Numerical Examples and Discussion

For the case of monoclinic magnetoelastic layer lying over a non homogeneous isotropic half-space, we select the following data:

1) For monoclinic magnetoelastic layer (Tiersten [16])

$$C_{55} = 94.0\times10^9\,\text{N/m}^2,\ C_{66} = 93.0\times10^9\,\text{N/m}^2,$$
$$C_{56} = -11.0\times10^9\,\text{N/m}^2,\ \rho = 7450\,\text{Kg/m}^3.$$

2) For non homogeneous isotropic half-space (Gubbins [17])

$$\mu_0 = 78.4\times10^9\,\text{N/m}^2,\ \rho_0 = 3535\,\text{Kg/m}^3.$$

Moreover, the following data are used

$$m_H = 0,\ 0.8;\ \varepsilon = 0, 0.2, 0.4$$

Figure 2 and **Figure 3** represent the variation in dimensionless phase velocity c/β_1 against dimensionless wave number kH in monoclinic magnetoelastic layer, for the case when magnetic field is absent and present respectively. By the comparative study of these two graphs we can conclude that presence of magnetic field increases the phase velocity, whereas increment in non-homogeneity parameter ε decreases the phase velocity. **Figure 3** and **Figure 4** represent the variation in dimensionless phase velocity c/β_1 against dimensionless wave number kH in isotropic magnetoelastic

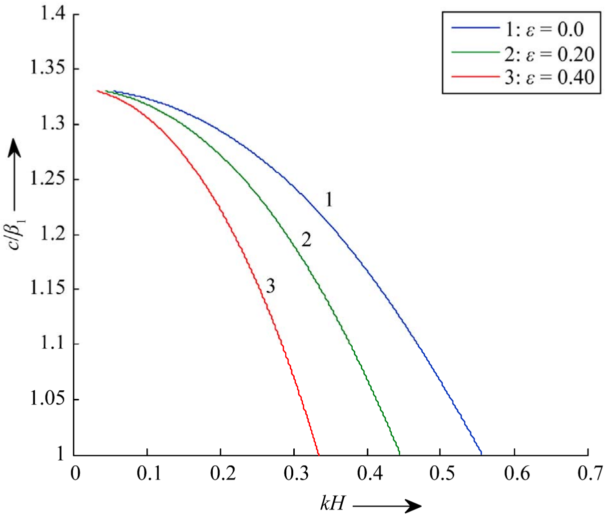

Figure 2. Variation of dimensionless phase velocity against dimensionless wave number in an monoclinic magnetoelastic layer over an inhomogeneous isotropic semi-infinite medium when $m_H = 0.0$.

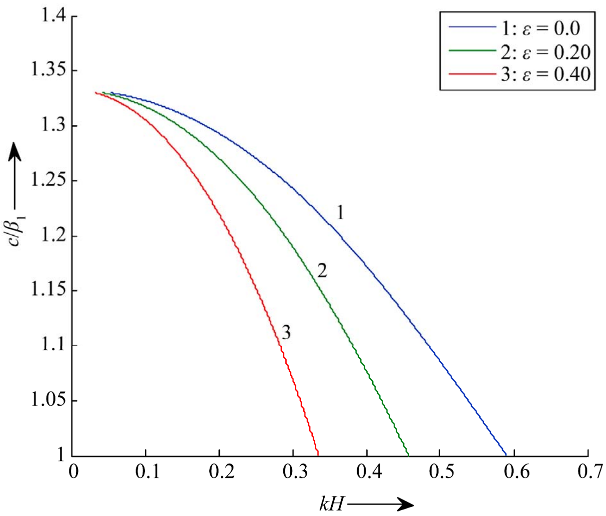

Figure 3. Variation of dimensionless phase velocity against dimensionless wave number in an monoclinic magnetoelastic layer over an inhomogeneous isotropic semi-infinite medium when $m_H = 0.8$.

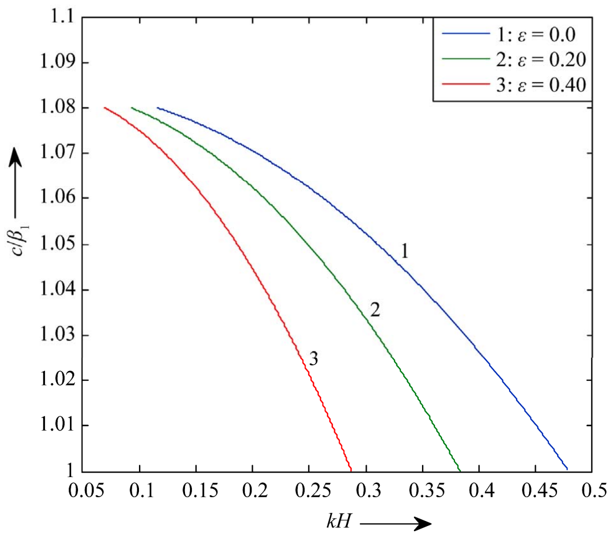

Figure 4. Variation of dimensionless phase velocity against dimensionless wave number in an isotropic magnetoelastic layer over an inhomogeneous isotropic semi-infinite medium when $\varepsilon_H = 0.0$.

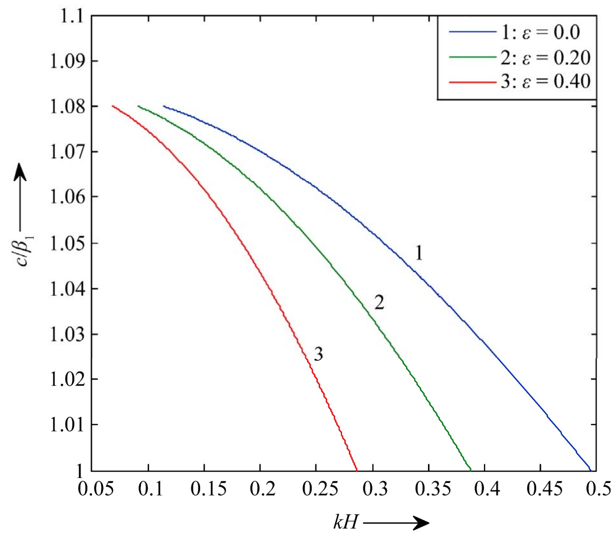

Figure 5. Variation of dimensionless phase velocity against dimensionless wave number in an isotropic magnetoelastic layer over an inhomogeneous isotropic semi-infinite medium when $\varepsilon_H = 0.8$.

layer, for the case when magnetic field is absent and present respectively. The presence of magnetic field increases the phase velocity, whereas increase in the non-homogeneity parameter ε decreases the phase velocity. It is observed from **Figures 2-4** that presence of monoclinic medium favours more to the phase velocity as compared to simply isotropic one. The magnetic field and inhomogeneity parameter show similar type of tendency for both the isotropic and monoclinic medium.

Figure 6 shows the variation in dimensionless group velocity G/β_2 with respect to scaled wave number k/s. This graph explains that group velocity increases with scaled wave number and approaches to shear wave velocity asymptotically. Keeping in the mind the dependence of group velocity G on wave number k and depth parameter s, surface plot of group velocity against varying k and s has been drawn in **Figure 7**.

For the case of magnetoelastic isotropic layer lying over a non homogeneous isotropic half-space, we select the following data:

3) For isotropic magnetoelastic layer (Gubbins [17])

$$\mu_1 = 63.4\times10^9\,\text{N/m}^2,\ \rho_1 = 3364\,\text{Kg/m}^3.$$

4) For non homogeneous isotropic half-space (Gubbins [17])

$$\mu_0 = 78.4\times10^9\,\text{N/m}^2,\ \rho_0 = 3535\,\text{Kg/m}^3.$$

Moreover, the following data are used

$$\varepsilon_H = \frac{\mu_e H_0^2}{\mu_1} = 0,\ 0.8;\ \varepsilon = 0, 0.2, 0.4$$

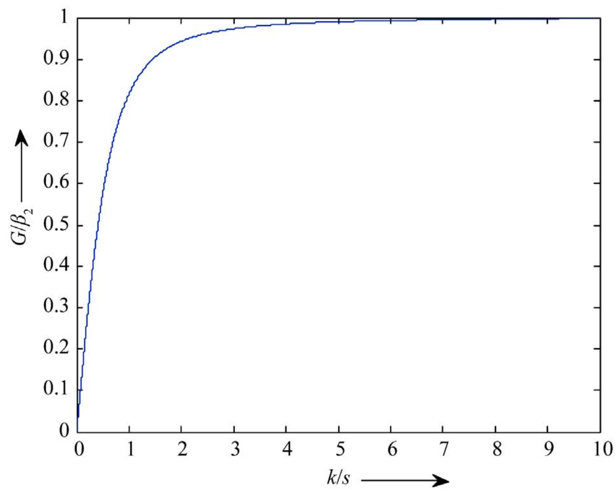

Figure 6. Variation of dimensionless group velocity (G) against scaled wave number k/s.

5. Conclusions

Dispersion equation for the propagation of G-type seismic wave in monoclinic magnetoelastic layer lying over an inhomogeneous isotropic half space is obtained, using the transform technique and Valeev's method [15]. Condition for maximum energy to be confined near the surface and expression for group velocity are found. Phase and group velocity curves against wave number are plotted taking variation in inhomogeneity parameter. It is observed that the presence of non-homogeneity decreases the phase velocity whereas the presence of magnetic field increases the phase velocity, in both the cases when there

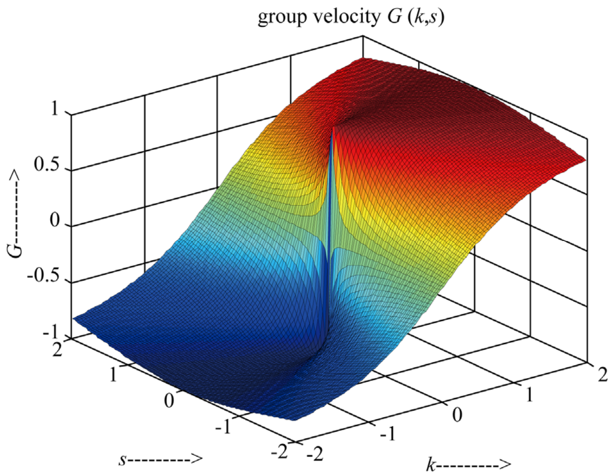

Figure 7. Variation of group velocity (G) with respect to parameter k and s.

is isotropic magnetoelastic layer or monoclinic magnetoelastic layer. It is also observed from the comparative study that presence of monoclinic medium favours more to the phase velocity as compared to simply isotropic one. Group velocity is found lower than the shear wave velocity in the upper mantle. The present study has its application especially to the problem of waves and vibrations where the wave signals have to travel through different layers of different material properties. This study may be helpful to understand the cause of damages during large earthquakes; also it may be useful to predict the nature of long period Love waves. These results can also be utilized in the interpretation and analysis of data of geophysical studies. The findings will be useful in forecasting formation details at greater depth through signal processing and seismic data analysis. The present study may be effectively utilized to generate initial data prior to exploitation of the formation. This study may be useful to geophysicist and metallurgist for analysis of rock and material structures through Non-Destructive Testing (NDT).

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