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Pseudo Laguerre Matrix Polynomials, Operational Identities and Quasi-Monomiality

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Abstract

The main purpose of this paper is to introduce the matrix extension of the pseudo Laguerre matrix polynomials and to explore the formal properties of the operational rules and the principle of quasi-monomiality to derive a number of properties for pseudo Laguerre matrix polynomials.

Keywords

Pseudo Laguerre Matrix Polynomials, Lowering Operators, Raising Operators, Quasi-Monomiality, Operational Rules

1. Preliminaries and Definitions

In the last two decade, matrix polynomials have become more important and some results in the theory of classical orthogonal polynomials have been extended to orthogonal matrix polynomials see for instance [1]-[7]. Orthogonal matrix polynomials are important from both the theoretical and practical points of view, they appear in connection with representation theory, matrix expansion problems, prediction theory and in the matrix quadrature integration problems, see for example [5] [8] [9]. Numerous problems of chemistry, physics and mechanics are related to second order matrix differential equation. Moreover, some properties of the Hermite and Laguerre matrix polynomials and a generalized form of the Hermite matrix polynomials have been introduced and studied in [4] [9]-[19]. Other classical orthogonal polynomials as Gegenbauer, Chebyshev, Jacobi and Konhauser polynomials have been extended to orthogonal matrix polynomials, and some results have been investigated, see for example [9] [18] [19] [20] [21]. We say that a matrix A in $\mathbb{C}^{N\times N}$ is a positive stable if $\Re(\lambda)>0$ for all $\lambda\in\sigma(A)$, where $\sigma(A)$ is the set of the eigenvalues of A. If

$A_0, A_1, \cdots, A_n, \cdots$ are elements of $\mathbb{C}^{N\times N}$ and $A_n \neq 0$, then we call

$$P(x) = A_n x^n + A_{n-1}x^{n-1} + A_{n-2}x^{n-2} + \cdots + A_1 x + A_0,$$

a matrix polynomial of degree n in x. If $A + nI$ is invertible for every integer $n \geq 0$ then

$$(A)_n = A(A+I)(A+2I)\cdots(A+(n-1)I); n \geq 0, (A)_0 = I.$$

Thus we have

$$(A)_n = \Gamma^{-1}(A)\Gamma(A+nI). \tag{1.1}$$

For any matrix A in $\mathbb{C}^{N\times N}$, we have the following relation [22]

$$(1-x)^{-A} = \sum_{n=0}^{\infty} \frac{(A)_n x^n}{n!}, |x| < 1. \tag{1.2}$$

Next, we recall that the Konhauser matrix polynomials are defined in [21] as

$$Z_n^{(A,\lambda)}(x;k) = \frac{\Gamma(A+(kn+1)I)}{n!} \sum_{s=0}^{n} (-1)^s \binom{n}{s} \Gamma^{-1}(A+(ks+1)I)(\lambda x)^{ks}. \tag{1.3}$$

In [23] Dattoli *et al.* introduced the two variable pseudo Laguerre polynomials $L_n(x,y;k,j)$ in the form:

$$L_n(x,y;k,j) = n! \sum_{s=0}^{n} \frac{(-1)^s y^{n-s} x^{ks+j}}{s!(n-s)!(ks+j)!}. \tag{1.4}$$

In this work, we construct a matrix version of the pseudo Laguerre matrix polynomials given by (1.4) as follows:

Definition 1.1. Let A be a matrix in $\mathbb{C}^{N\times N}$ satisfying the condition $\Re(\lambda) > -1$ for every $\lambda \in \sigma(A)$, $k \in \mathbb{Z}^+$ and $|t| < 1$. We define the pseudo-Laguerre matrix polynomials by the series

$$L_n(x,y;k,A) = n! \sum_{s=0}^{n} \frac{(-1)^s y^{n-s} x^{A+ksI}}{s!(n-s)!} \Gamma^{-1}(A+(ks+1)I). \tag{1.5}$$

The relevant generating function for the polynomials $L_n(x,y;k,A)$ can be obtained by the method suggested in [23], thus getting

Theorem 1.1. Let A be a matrix in $\mathbb{C}^{N\times N}$ satisfying the condition $\Re(\lambda) > -1$ for every $\lambda \in \sigma(A)$, $k \in \mathbb{Z}^+$, and $|t| < 1$. Then

$$\sum_{n=0}^{\infty} L_n(x,y;k,A) \frac{t^n}{n!} = x^A e^{yt} C_A(x^k t;k), \tag{1.6}$$

where

$$C_A(x;k) = \sum_{s=0}^{\infty} \frac{(-1)^s x^s}{s!} \Gamma^{-1}(A+(ks+1)I). \tag{1.7}$$

being the matrix version of the Tricomi function defined in (see [4]).

Proof. If we use the series (1.7) in right-hand side of (1.6), we get

$$x^A e^{yt} C_A(x^k t;k) = \sum_{n,s=0}^{\infty} \frac{(-1)^s x^{ks+A} y^n}{s!} \Gamma^{-1}(A+(ks+1)I) \frac{t^{n+s}}{n!}.$$

Now, by letting $n \to n-s$, we obtain the left-hand side of the assertion (1.6).□

We must emphasize that the matrix polynomials in (1.6) are a generalized form of Konhauser matrix polynomials defined by (1.3) and indeed we have

$$Z_n^{(A,\lambda)}(y;k) = \frac{(\lambda y)^{-A}\Gamma(A+(kn+1)I)}{n!} L_n(\lambda y, 1; k, A).$$

For the purpose of this work we introduce the following matrix version of Kampé de Fériet double hypergeometric series $F_{l:m;n}^{p:q;k}[x,y]$ and matrix version of the generalized hypergeometric function ${}_pF_q$ [24] as follows:

$$F_{l:m;n}^{p:q;k}\left[\begin{matrix}(A_p):(B_q);(C_k);\\(D_l):(E_m);(F_n);\end{matrix} x,y\right] = \sum_{r,s=0}^{\infty} \frac{\prod_{j=1}^{p}(A_j)_{r+s}\prod_{j=1}^{q}(B_j)_r\prod_{j=1}^{k}(C_j)_s}{r!s!}$$

$$\left[\prod_{j=1}^{l}(D_j)_{r+s}\right]^{-1}\left[\prod_{j=1}^{m}(E_j)_r\right]^{-1}\left[\prod_{j=1}^{n}(F_j)_s\right]^{-1} x^r y^s \tag{1.8}$$

and

$${}_pF_q\left[A_1,\cdots,A_p;B_1,\cdots,B_q;x\right] = \sum_{n=0}^{\infty}\frac{(A_1)_n\cdots(A_p)_n(B_1)_n\cdots(B_q)_n}{n!}x^n. \tag{1.9}$$

In view of the definition (1.9) and the definition of the matrix version of the Gauss multiplication theorem

$$(A)_{kn} = k^{kn}\left(\frac{A+jI-I}{k}\right)_n,$$

it is not difficult to show that

$$L_n(x,y;k,A) = x^A y^n \times {}_1F_k\left[-n;\Delta(k;A+I);x\right],$$

where throughout this work $\Delta(m;A+I)$ denotes the array of m parameters $\frac{A}{m},\frac{A+I}{m},\cdots,\frac{A+(m-1)I}{m}, m>0$. For an arbitrary matrix $A\in\mathbb{C}^{N\times N}$ the following two formulas are well-known consequences of the derivative operator $\hat{D}_x$ and the integral $\hat{D}_x^{-1}$ [18]

$$\hat{D}_x^n x^A = \Gamma(A+I)\Gamma^{-1}(A-(n+1)I)x^{A-nI}, \tag{1.10}$$

$$\hat{D}_x^{-n} x^A = \Gamma(A+I)\Gamma^{-1}(A+(n+1)I)x^{A+nI}, \tag{1.11}$$

where $\hat{D}_x = \frac{\partial}{\partial x}$ and $n\in\mathbb{N}\cup\{0\}$.

Note that, in this work we apply the concept of the right-Riemann-Liouville fractional calculus to obtain operational identities and relations. Motivated by the works mentioned above, we aim in this work to present systematic investigation of the matrix version of the pseudo Laguerre polynomials given by (1.5) and exploit methods of operational nature and the monomiality principle to derive a number of operational representations, operators and generating functions constructed matrix polynomials in (1.5).

2. Operational Identities and Quasi-Monomiality

First of all, we establish the following operational representations for pseudo Laguerre matrix polynomials $L_n(x,y;k,A)$.

Theorem 1.1. Let A be a matrix in $\mathbb{C}^{N\times N}$ satisfying the condition $\Re(\lambda)>-1$ for every $\lambda\in\sigma(A)$ and $k\in\mathbb{Z}^+$. Then

$$L_n(x,y;k,A)=\Gamma^{-1}(A+I)\mathrm{e}^{-\hat{D}_x^{-k}\hat{D}_y}\left\{x^A y^n\right\}. \tag{2.1}$$

Proof. In view of (1.10) and (1.11), we have

$$\frac{\partial^s}{\partial y^s}y^n=\frac{n!}{(n-s)!}y^{n-s}, \tag{2.2}$$

and

$$\Gamma^{-1}(A+I)\hat{D}_x^{-ks}x^A=\Gamma^{-1}(A+(ks+1)I)x^{A+ks}, \tag{2.3}$$

The desired result now follows by applying the identities (2.2) and (2.3) to the definition (1.5).□

Theorem 2.2. Let A be a matrix in $\mathbb{C}^{N\times N}$ satisfying the condition $\Re(\lambda)>-1$ for every $\lambda\in\sigma(A)$ and $k\in\mathbb{Z}^+$. Then

$$L_n(x,y;k,A)=\Gamma^{-1}(A+I)\left(y-\hat{D}_x^{-k}\right)^n\left\{x^A\right\}. \tag{2.4}$$

Proof. The result follows directly from the formula

$$(x-y)^n=\sum_{s=0}^{\infty}\frac{(-n)_s}{s!}x^{n-s}y^s,$$

the assertion (2.3) and the definition (1.5).□

The use of the monomiality principle has offered a powerful tool for studying the properties of families of special functions and polynomials. We know that according to the monomiality principle [23] [25], a polynomial set $\{p_n(x)\}, n\in\mathbb{N}$ is quasi-monomial, if there exist two operators $\hat{M}$ and $\hat{P}$, called multiplicative and derivative operators respectively, which when acting on the polynomials $\{p_n(x)\}$ yield [25]

$$\hat{M}\{p_n(x)\}=p_{n+1}(x),$$
$$\hat{P}\{p_n(x)\}=np_{n-1}(x).$$

The operators $\hat{M}$ and $\hat{P}$ satisfy the commutation relation:

$$\left[\hat{M},\hat{P}\right]=\hat{1},$$

and thus display a Weyl group structure. If $\hat{M}$ and $\hat{P}$ have differential realization, then the differential equations satisfied by $p_n(x)$ are

$$\hat{M}\hat{P}\{p_n(x)\}=np_n(x),$$
$$\hat{P}\hat{M}\{p_n(x)\}=(n+1)p_n(x).$$

In this regard, the matrix polynomial set $L_n(x,y;k,A)$ is quasi-monomial under the action of the multiplicative operator

$$\hat{M} = y + \frac{x^{A+I}}{k}\frac{\partial}{\partial x}x^{-A}D_y^{-1}, \tag{2.5}$$

and the derivatives operators

$$\hat{P}_1 = \frac{-1}{k}\frac{\partial^k}{\partial x^k}x^{A+I}\frac{\partial}{\partial x}x^{-A}, \tag{2.6}$$

$$\hat{P}_2 = \frac{\partial}{\partial y}. \tag{2.7}$$

According to the quasi-monomiality properties, we have

$$\hat{M}\left\{L_n\left(x,y;k,A\right)\right\} = L_{n+1}\left(x,y;k,A\right) \tag{2.8}$$

$$\hat{P}_1\left\{L_n\left(x,y;k,A\right)\right\} = nL_{n-1}\left(x,y;k,A\right)$$

$$\hat{P}_2\left\{L_n\left(x,y;k,A\right)\right\} = nL_{n-1}\left(x,y;k,A\right).$$

Therefore, the identities

$$\hat{M}\hat{P}_1\left\{L_n\left(x,y;k,A\right)\right\} = \left(n+1\right)L_n\left(x,y;k,A\right)$$

$$\hat{M}\hat{P}_2\left\{L_n\left(x,y;k,A\right)\right\} = \left(n+1\right)L_n\left(x,y;k,A\right)$$

in differential forms give us

$$\left[\hat{D}_x^k x^{A+I}\frac{\partial}{\partial x}x^{-A}\left(I - A + y\frac{\partial}{\partial y}I\right) + x\hat{D}_x^{k+1}x^{A+I}\frac{\partial}{\partial x}x^{-A} - kn\frac{\partial}{\partial y}I\right]L_n\left(x,y;k,A\right) = 0$$

$$\left[y\frac{\partial}{\partial y}I + \frac{x^{A+I}}{k}\frac{\partial}{\partial x}x^{-A}I - nI\right]L_n\left(x,y;k,A\right) = 0.$$

Moreover, regarding the Lie bracket $[,]$ defined by $[A,B] = AB - BA$, we led to

$$\left[\hat{P}_1,\hat{M}\right]L_n\left(x,y;k,A\right) = L_n\left(x,y;k,A\right),$$

$$\left[\hat{P}_2,\hat{M}\right]L_n\left(x,y;k,A\right) = L_n\left(x,y;k,A\right).$$

From the lowering operators $\hat{P}_1$ and $\hat{P}_2$ in (2.6) and (2.7), we can define operators playing the role of the inverse operators $\hat{P}_1^{-1}$ and $\hat{P}_2^{-1}$ (see [[8], Equation (15)]). Thus, we get

$$\hat{P}_1^{-1} = -kx^A\hat{D}_x^{-1}x^{-(A+I)}\hat{D}_x^{-k}, \tag{2.9}$$

$$\hat{P}_2^{-1} = \hat{D}_y^{-1}, \tag{2.10}$$

and they satisfy

$$\hat{P}_1^{-1}\left\{L_n\left(x,y;k,A\right)\right\} = \hat{P}_2^{-1}\left\{L_n\left(x,y;k,A\right)\right\} = \frac{L_{n+1}\left(x,y;k,A\right)}{\left(n+1\right)}. \tag{2.11}$$

Clearly, we have

$$\hat{P}_1\hat{P}_1^{-1}\left\{L_n\left(x,y;k,A\right)\right\} = \hat{P}_2\hat{P}_2^{-1}\left\{L_n\left(x,y;k,A\right)\right\} = L_n\left(x,y;k,A\right).$$

Further, from (2.9)-(2.11), we can infer that $L_n\left(x,y;k,A\right)$ are the natural solution of the following equation

$$\left[\hat{D}_y^{-1}+kx^{A}\hat{D}_x^{-1}x^{-(A+I)}\hat{D}_x^{-k}\right]L_n\left(x,y;k,A\right)=0.$$

Moreover, from (2.5) in conjunction with (2.8), we get

$$L_{n+1}\left(x,y;k,A\right)=yL_n\left(x,y;k,A\right)+\frac{x^{A+I}}{k}\frac{\partial}{\partial x}x^{-A}\hat{D}_y^{-1}L_n\left(x,y;k,A\right),$$

which yields the following recurrence relation

$$L_{n+1}\left(x,y;k,A\right)-yL_n\left(x,y;k,A\right)-L_n\left(x,y;k,A+kI\right)=0.$$

Finally, let

$$\hat{N}=y-\hat{D}_x^{-k},$$

then upon using (2.4) one obtains by routine calculations

$$\Gamma^{-1}\left(A+I\right)\left[y-\hat{N}\right]^n\left\{x^A\right\}$$
$$=\sum_{s=0}^{\infty}(-1)^s\binom{n}{s}L_n\left(x,y;k,A\right)y^{n-s}=\Gamma\left(A+knI+I\right)x^{A+knI}.$$

3. Generating Functions and Expansions

First, in the identity (2.1) multiply throughout by $\frac{t^n}{n!}$, sum and then employ the formulae (1.10) and (1.11) and the result

$$\mathrm{e}^x=\sum_{n=0}^{\infty}\frac{x^n}{n!},$$

to get

$$\mathrm{e}^{yt}x^A{}_oF_k\left[-;\frac{A+I}{k},\frac{A+2I}{k},\cdots,\frac{A+(k-1)I}{k};-(kx)^k\right]$$
$$=\Gamma\left(A+I\right)\sum_{n=0}^{\infty}L_n\left(x,y;k,A\right)\frac{t^n}{n!}.$$

Next, let us consider the generating relation

$$f^{(A,B)}\left(x,y,z,w;k,l\,|\,t\right)=\sum_{n=0}^{\infty}L_n\left(x,y;k,A\right)\times L_n\left(z,w;l,B\right)\frac{t^n}{n!},$$

which according to operational identity (2.4)and the formulae (1.10) and (1.11) yields the following bilinear generating function

$$\mathrm{e}^{(y+w)t}x^Az^B{}_oF_k\left[-;\frac{A+I}{k},\frac{A+2I}{k},\cdots,\frac{A+(k-1)I}{k};-(kx)^k\right]$$
$$\times{}_oF_k\left[-;\frac{B+I}{k},\frac{B+2I}{k},\cdots,\frac{B+(k-1)I}{k};-t(lz)^k\right]$$
$$=\Gamma\left(A+I\right)\Gamma\left(B+I\right)\sum_{n=0}^{\infty}L_n\left(x,y;k,A\right)\times L_n\left(z,w;l,B\right)\frac{t^n}{n!}.$$

In [14], the following definition of Laguerre matrix polynomials has been introduced:

$$L_n^{(A,\lambda)}(x)=\sum_{s=0}^{\infty}\frac{(-1)^s}{s!(n-s)!}(A+I)_n\left[(A+I)_s\right]^{-1}(\lambda x)^s,\lambda\geq 0,$$

where A be a matrix in $\mathbb{C}^{N\times N}$, $\alpha\in\sigma(A)$ and $(-\alpha)$ is not an eigenvalue of A for every integer $\alpha>0$ and λ be a complex number whose real part is positive. Such matrix polynomials have the following operational representation [14]:

$$L_n^{(B,\lambda)}(z)=\frac{\lambda^{-B}(B+I)_n}{n!}\left(1-z\hat{D}_\lambda^{-1}\right)^n\left\{\lambda^B\right\}. \tag{3.1}$$

Let us consider the generating relation

$$f^{(A,B)}(x,y,z,\lambda\,|\,t)=\sum_{n=0}^{\infty}L_n(x,y;k,A)\times L_n^{(B,\lambda)}(z)t^n. \tag{3.2}$$

Now, directly from (2.4) and (3.1) by employing the previously outlined method leading to the bilinear generating function, we obtain from (3.2) the following bilateral generating function

$$\sum_{m=0}^{\infty}\frac{(B+I)_m}{m!}y^m x^A\lambda^B\,{}_1F_k\left[-m;\frac{A+I}{k},\frac{A+2I}{k},\cdots,\frac{A+(k-1)I}{k};\frac{(kx)^k}{y}\right]$$
$$\times{}_2F_0\left[-B,mI;-;\frac{-z}{\lambda}\right]=\sum_{n=0}^{\infty}L_n(x,y;k,A)\times L_n^{(B,\lambda)}(z)t^n.$$

Similarly, from the operational representation of the two variable Hermite matrix polynomials $H_n(x,y,A)$ (see [10])

$$H_n(x,y,A)=\exp\left(-y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right)\left(x\sqrt{(2A)}\right)^n,$$

and (2.4), we can easily derive the following bilateral generating function

$$\sum_{s=0}^{\infty}\frac{\left(-zyt\sqrt{(2B)}\right)^s}{s!}{}_1F_k\left[-s;\frac{A+I}{k},\cdots,\frac{A+(k-1)I}{k};\frac{(kx)^k}{y}\right]$$
$$\times{}_2F_0\left[\frac{-s}{2},\frac{-s+1}{2};--;\frac{-4w(2B)^{-1}}{z^2}\right]$$
$$=\Gamma(A+I)\sum_{n=0}^{\infty}L_n(x,y;k,A)\times H_n(z,w,B)\frac{t^n}{n!}.$$

Theorem 3.1. Let A and B be a matrices in $\mathbb{C}^{N\times N}$ satisfying the conditions $\Re(\mu)>-1$ for every $\mu\in\sigma(A)$ or $\mu\in\sigma(B)$, $AB=BA$, and $k\in\mathbb{Z}^+$. Then

$$L_n(x,y;k,A)\times L_m(x,y;k,A)$$
$$=\left[\Gamma^{-1}(A+I)\right]^2x^{2A}y^{m+n}\,{}_1F_k\left[-m-n;\frac{A+I}{k},\cdots,\frac{A+kI}{k};\frac{x^k}{k^k y}\right], \tag{3.3}$$

where ${}_1F_k$ is defined by (1.9).

$$\begin{aligned} &L_n(x,y;k,A)\times L_n(z,w;l,B) \\ &=\Gamma^{-1}(A+I)\Gamma^{-1}(B+I)\sum_{s=0}^{\infty}\frac{(-1)^s(-nI)_s(yw)^{n-s}x^{A+ksI}z^{B+lsI}}{s!k^{ks}l^{ls}} \\ &\times\left[\prod_{j=1}^{k}\left(\frac{A+jI}{k}\right)_s\prod_{j=1}^{l}\left(\frac{B+jI}{l}\right)_s\right]^{-1} \\ &\times F^{1:0;0}_{0:k;l}\left[\begin{matrix}-n+s:---------;---------;\\ ----:\Delta(k;A+I)+sI;\Delta(l;B+I)+sI;\end{matrix}\ \frac{x^k}{yk^k},\frac{z^l}{wl^l}\right]. \end{aligned} \tag{3.4}$$

where $F^{1:0;0}_{0:k;l}$ is defined by (1.8).

Proof. According to the operational representation (2.4), we have

$$L_n(x,y;k,A)\times L_n(z,w;l,B)=\left[\Gamma^{-1}(A+I)\right]^2\left(y-\hat{D}_x^{-k}\right)^{m+n}\left\{x^{2A}\right\}$$

which in view of (1.2), the operator in (1.11) and the definition of Pochhammer symbol (1.2), yields the right-hand side of Equation (3.4). Similarly, one can prove the result (3.3).□

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