Advances in Linear Algebra & Matrix Theory, 2015, 5, 150-155
Published Online December 2015 in SciRes. http://www.scirp.org/journal/alamt
http://dx.doi.org/10.4236/alamt.2015.54015

Trace of Positive Integer Power of Real 2 × 2 Matrices

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Received 21 September 2015; accepted 4 December 2015; published 7 December 2015

Abstract

The purpose of this paper is to discuss the theorems for the trace of any positive integer power of 2 × 2 real matrix. We obtain a new formula to compute trace of any positive integer power of 2 × 2 real matrix *A*, in the terms of Trace of *A* (Tr*A*) and Determinant of *A* (Det*A*), which are based on definition of trace of matrix and multiplication of the matrixn times, where *n* is positive integer and this formula gives some corollary for $\mathrm{Tr}A^n$ when Tr*A* or Det*A* are zero.

Keywords

Trace, Determinant, Matrix Multiplication

1. Introduction

Traces of powers of matrices arise in several fields of mathematics, more specifically, Network Analysis, Numbertheory, Dynamical systems, Matrix theory, and Differential equations [1]. When analyzing a complex network, an important problem is to compute the total number of triangles of a connected simple graph. This number is equal to $\mathrm{Tr}(\boldsymbol{A}^3)/6$, where $\boldsymbol{A}$ is the adjacency matrix of the graph [2]. Traces of powers of integer matrices are connected with the Euler congruence [3], an important phenomenon in mathematics, stating that

$$\mathrm{Tr}\left(\boldsymbol{A}^{p^r}\right) \equiv \mathrm{Tr}\left(\boldsymbol{A}^{p^{r-1}}\right)\left(\mathrm{mod}\ p^r\right),$$

for all integer matrices $\boldsymbol{A}$, all primes p, and all $r \in \mathrm{Z}$. The invariants of dynamical systems are described in terms of the traces of powers of integer matrices, for example in studying the Lefschetz numbers [3]. There are many applications in matrix theory and numerical linear algebra. For example, in order to obtain approximations of the

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How to cite this paper: Pahade, J. and Jha, M. (2015) Trace of Positive Integer Power of Real 2 × 2 Matrices. *Advances in Linear Algebra & Matrix Theory*, **5**, 150-155. http://dx.doi.org/10.4236/alamt.2015.54015

smallest and the largest eigenvalues of a symmetric matrix $\boldsymbol{A}$, a procedure based on estimates of the trace of $\boldsymbol{A}^n$ and $\boldsymbol{A}^{-n}$, $n \in Z$, is proposed in [4].

Trace of a $n \times n$ matrix $\boldsymbol{A} = \left[a_{ij}\right]$ is defined to be the sum of the elements on the main diagonal of $\boldsymbol{A}$, *i.e.*

$$\mathrm{Tr}\boldsymbol{A} = a_{11} + a_{22} + \cdots + a_{nn} = \sum_{i=1}^{n} a_{ii}$$

The computation of the trace of matrix powers has received much attention. In [5], an algorithm for computing $\mathrm{Tr}\left(\boldsymbol{A}^k\right), k \in Z$ is proposed, when $\boldsymbol{A}$ is a lower Hessenberg matrix with a unit codiagonal. In [6], a symbolic calculation of the trace of powers of tridiagonal matrices is presented. Let $\boldsymbol{A}$ be a symmetric positive definite matrix, and let $\{\lambda_k\}$ denote its eigenvalues. For $q \in R$, $\boldsymbol{A}^q$ is also symmetric positive definite, and it holds [7].

$$\mathrm{Tr}\boldsymbol{A}^q = \sum_k \lambda_k^q \tag{1.1}$$

This formula is restricted to the matrix $\boldsymbol{A}$. Also we have other formulae [8] to compute the trace of matrix power such that

$$\mathrm{Tr}\boldsymbol{A}^n = \sum_k \lambda_k^n \tag{1.2}$$

But for many cases, this formula is time consuming. For example

Consider a matrix $\boldsymbol{A} = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$ and let we are to find $\mathrm{Tr}\boldsymbol{A}^5$. Eigenvalues of $\boldsymbol{A}$ are $\frac{3 \pm i\sqrt{7}}{2}$, then by (1.2), $\mathrm{Tr}\boldsymbol{A}^5 = \left(\frac{3+i\sqrt{7}}{2}\right)^5 + \left(\frac{3-i\sqrt{7}}{2}\right)^5$.

Computation of this value is time consuming. Therefore, other formulae to compute trace of matrix power are needed. Now we give new theorems and corollaries to compute trace of matrix power. Our estimation for the trace of $\boldsymbol{A}^n$ is based on the multiplication of matrix.

2. Main Result

Theorem 1. For even positive integer n and 2 × 2 real matrix $\boldsymbol{A}$,

$$\mathrm{Tr}\boldsymbol{A}^n = \sum_{r=0}^{n/2} \frac{(-1)^r}{r!} n\left[n-(r+1)\right]\left[n-(r+2)\right]\cdots\left[\text{up to } r \text{ terms}\right]\cdot(\mathrm{Det}\boldsymbol{A})^r(\mathrm{Tr}\boldsymbol{A})^{n-2r}$$

Proof. Consider a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b, c, d are real.

Then

$$\mathrm{Tr}\boldsymbol{A} = a + d \tag{2.1}$$

and

$$\mathrm{Det}\boldsymbol{A} = ad - bc \tag{2.2}$$

Now

$$\boldsymbol{A}^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & b(a+d) \\ c(a+d) & bc + d^2 \end{bmatrix}$$

Then

$$\begin{aligned} \mathrm{Tr}\boldsymbol{A}^2 &= a^2 + 2bc + d^2 \\ &= a^2 + 2ad + d^2 - 2ad + 2bc \\ &= (a+d)^2 - 2(ad - bc) \\ &= (\mathrm{Tr}\boldsymbol{A})^2 - 2\mathrm{Det}\boldsymbol{A} \end{aligned} \tag{2.3}$$

Now again

$$\boldsymbol{A}^3=\boldsymbol{A}^2\boldsymbol{A}=\begin{bmatrix}a^2+bc & b(a+d)\\ c(a+d) & bc+d^2\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$$\boldsymbol{A}^3=\begin{bmatrix}a^3+abc+bc(a+d) & a^2b+b^2c+bd(a+d)\\ ac(a+d)+bc^2+cd^2 & bc(a+d)+bcd+d^3\end{bmatrix} \tag{2.4}$$

Then

$$\begin{aligned}\mathrm{Tr}\boldsymbol{A}^3&=a^3+d^3+3bc(a+d)\\ &=a^3+d^3+3ad(a+d)-3ad(a+d)+3bc(a+d)\\ &=(a+d)^3-3(a+d)(ad-bc)\\ &=(\mathrm{Tr}\boldsymbol{A})^3-3(\mathrm{Det}\boldsymbol{A})(\mathrm{Tr}\boldsymbol{A})\end{aligned}$$

$$\mathrm{Tr}\boldsymbol{A}^3=(\mathrm{Tr}\boldsymbol{A})^3-3(\mathrm{Det}\boldsymbol{A})(\mathrm{Tr}\boldsymbol{A}) \tag{2.5}$$

Now replace $\boldsymbol{A}$ by $\boldsymbol{A}^2$ in (2.3), we have

$$\begin{aligned}\mathrm{Tr}\boldsymbol{A}^4&=\left[\mathrm{Tr}\boldsymbol{A}^2\right]^2-2\mathrm{Det}\boldsymbol{A}^2\\ &=\left[(\mathrm{Tr}\boldsymbol{A})^2-2\mathrm{Det}\boldsymbol{A}\right]^2-2(\mathrm{Det}\boldsymbol{A})^2\ \left[\mathrm{Det}\boldsymbol{AB}=(\mathrm{Det}\boldsymbol{A})(\mathrm{Det}\boldsymbol{B})\right]\end{aligned}$$

$$\mathrm{Tr}\boldsymbol{A}^4=(\mathrm{Tr}\boldsymbol{A})^4-4\mathrm{Det}\boldsymbol{A}(\mathrm{Tr}\boldsymbol{A})^2+2(\mathrm{Det}\boldsymbol{A})^2 \tag{2.6}$$

Again replace $\boldsymbol{A}$ by $\boldsymbol{A}^2$ in (2.5), we have

$$\begin{aligned}\mathrm{Tr}\boldsymbol{A}^6&=\left[\mathrm{Tr}\boldsymbol{A}^2\right]^3-3\mathrm{Det}\boldsymbol{A}^2\left(\mathrm{Tr}\boldsymbol{A}^2\right)\\ &=\left[(\mathrm{Tr}\boldsymbol{A})^2-2\mathrm{Det}\boldsymbol{A}\right]^3-3(\mathrm{Det}\boldsymbol{A})^2\left[(\mathrm{Tr}\boldsymbol{A})^2-2\mathrm{Det}\boldsymbol{A}\right]\\ &=(\mathrm{Tr}\boldsymbol{A})^6-6(\mathrm{Tr}\boldsymbol{A})^2\,\mathrm{Det}\boldsymbol{A}\left[(\mathrm{Tr}\boldsymbol{A})^2-2\mathrm{Det}\boldsymbol{A}\right]-8(\mathrm{Det}\boldsymbol{A})^3\\ &\quad-3(\mathrm{Det}\boldsymbol{A})^2(\mathrm{Tr}\boldsymbol{A})^2+6(\mathrm{Det}\boldsymbol{A})^3\end{aligned}$$

$$\mathrm{Tr}\boldsymbol{A}^6=(\mathrm{Tr}\boldsymbol{A})^6-6\mathrm{Det}\boldsymbol{A}(\mathrm{Tr}\boldsymbol{A})^4+9(\mathrm{Det}\boldsymbol{A})^2(\mathrm{Tr}\boldsymbol{A})^2-2(\mathrm{Det}\boldsymbol{A})^3 \tag{2.7}$$

Now again replace $\boldsymbol{A}$ by $\boldsymbol{A}^2$ in (2.6), we have

$$\begin{aligned}\mathrm{Tr}\boldsymbol{A}^8&=\left[\mathrm{Tr}\boldsymbol{A}^2\right]^4-4\mathrm{Det}\boldsymbol{A}^2\left[\mathrm{Tr}\boldsymbol{A}^2\right]^2+2\left[\mathrm{Det}\boldsymbol{A}^2\right]^2\\ &=\left[(\mathrm{Tr}\boldsymbol{A})^2-2\mathrm{Det}\boldsymbol{A}\right]^4-4(\mathrm{Det}\boldsymbol{A})^2\left[(\mathrm{Tr}\boldsymbol{A})^2-2\mathrm{Det}\boldsymbol{A}\right]^2+2(\mathrm{Det}\boldsymbol{A})^4\\ &=(\mathrm{Tr}\boldsymbol{A})^8-8(\mathrm{Det}\boldsymbol{A})(\mathrm{Tr}\boldsymbol{A})^2\left[(\mathrm{Tr}\boldsymbol{A})^4+4(\mathrm{Det}\boldsymbol{A})^2\right]+16(\mathrm{Det}\boldsymbol{A})^4+24(\mathrm{Tr}\boldsymbol{A})^4(\mathrm{Det}\boldsymbol{A})^2\\ &\quad-4(\mathrm{Det}\boldsymbol{A})^2\left[(\mathrm{Tr}\boldsymbol{A})^4-4(\mathrm{Tr}\boldsymbol{A})^2(\mathrm{Det}\boldsymbol{A})+4(\mathrm{Det}\boldsymbol{A})^2\right]+2(\mathrm{Det}\boldsymbol{A})^4\end{aligned}$$

$$\mathrm{Tr}\boldsymbol{A}^8=(\mathrm{Tr}\boldsymbol{A})^8-8(\mathrm{Det}\boldsymbol{A})(\mathrm{Tr}\boldsymbol{A})^6+20(\mathrm{Det}\boldsymbol{A})^2(\mathrm{Tr}\boldsymbol{A})^4-16(\mathrm{Det}\boldsymbol{A})^3(\mathrm{Tr}\boldsymbol{A})^2+2(\mathrm{Det}\boldsymbol{A})^4 \tag{2.8}$$

Now we observe from (2.3), (2.6), (2.7) and (2.8) that

$$\mathrm{Tr}\boldsymbol{A}^2=\frac{(-1)^0}{0!}(\mathrm{Det}\boldsymbol{A})^0(\mathrm{Tr}\boldsymbol{A})^{2-2\times0}+\frac{(-1)^1}{1!}2(\mathrm{Det}\boldsymbol{A})^1(\mathrm{Tr}\boldsymbol{A})^{2-2\times1}$$

$$\mathrm{Tr}\boldsymbol{A}^4=\frac{(-1)^0}{0!}(\mathrm{Det}\boldsymbol{A})^0(\mathrm{Tr}\boldsymbol{A})^{4-2\times0}+\frac{(-1)^1}{1!}4(\mathrm{Det}\boldsymbol{A})^1(\mathrm{Tr}\boldsymbol{A})^{4-2\times1}+\frac{(-1)^2}{2!}4(4-3)(\mathrm{Det}\boldsymbol{A})^2(\mathrm{Tr}\boldsymbol{A})^{4-2\times2}$$

$$\operatorname{Tr}\boldsymbol{A}^6=\frac{(-1)^0}{0!}(\operatorname{Det}\boldsymbol{A})^0(\operatorname{Tr}\boldsymbol{A})^{6-2\times 0}+\frac{(-1)^1}{1!}6(\operatorname{Det}\boldsymbol{A})^1(\operatorname{Tr}\boldsymbol{A})^{6-2\times 1}+\frac{(-1)^2}{2!}6(6-3)(\operatorname{Det}\boldsymbol{A})^2(\operatorname{Tr}\boldsymbol{A})^{6-2\times 2}$$
$$+\frac{(-1)^3}{3!}6(6-4)(6-5)(\operatorname{Det}\boldsymbol{A})^3(\operatorname{Tr}\boldsymbol{A})^{6-2\times 3}$$

$$\operatorname{Tr}\boldsymbol{A}^8=\frac{(-1)^0}{0!}(\operatorname{Det}\boldsymbol{A})^0(\operatorname{Tr}\boldsymbol{A})^{8-2\times 0}+\frac{(-1)^1}{1!}8(\operatorname{Det}\boldsymbol{A})^1(\operatorname{Tr}\boldsymbol{A})^{8-2\times 1}+\frac{(-1)^2}{2!}8(8-3)(\operatorname{Det}\boldsymbol{A})^2(\operatorname{Tr}\boldsymbol{A})^{8-2\times 2}$$
$$+\frac{(-1)^3}{3!}8(8-4)(8-5)(\operatorname{Det}\boldsymbol{A})^3(\operatorname{Tr}\boldsymbol{A})^{8-2\times 3}+\frac{(-1)^4}{4!}8(6-5)(8-6)(8-7)(\operatorname{Det}\boldsymbol{A})^3(\operatorname{Tr}\boldsymbol{A})^{8-2\times 4}$$

Continuing this process up to n terms we get

$$\operatorname{Tr}\boldsymbol{A}^n=\frac{(-1)^0}{0!}(\operatorname{Det}\boldsymbol{A})^0(\operatorname{Tr}\boldsymbol{A})^{n-2\times 0}+\frac{(-1)^1}{1!}n(\operatorname{Det}\boldsymbol{A})^1(\operatorname{Tr}\boldsymbol{A})^{n-2\times 1}+\frac{(-1)^2}{2!}n(n-3)(\operatorname{Det}\boldsymbol{A})^2(\operatorname{Tr}\boldsymbol{A})^{n-2\times 2}$$
$$+\cdots+\frac{(-1)^r}{r!}n\left[n-(r+1)\right]\left[n-(r+2)\right]\cdots(\text{up to } r \text{ terms})\cdot(\operatorname{Det}\boldsymbol{A})^r(\operatorname{Tr}\boldsymbol{A})^{n-2\times r} \tag{2.9}$$
$$+\cdots+\frac{(-1)^{n/2}}{n/2!}n\left[n(n/2+1)\right]\left[n-(n/2+2)\right]\cdots(\text{up to } n/2 \text{ terms})\cdot(\operatorname{Det}\boldsymbol{A})^{n/2}(\operatorname{Tr}\boldsymbol{A})^{n-2\times n/2}$$

Finally from above, we get

$$\operatorname{Tr}\boldsymbol{A}^n=\sum_{r=0}^{n/2}\frac{(-1)^r}{r!}n\left[n-(r+1)\right]\left[n-(r+2)\right]\cdots\left[\text{up to } r \text{ terms}\right]\cdot(\operatorname{Det}\boldsymbol{A})^r(\operatorname{Tr}\boldsymbol{A})^{n-2r} \tag{2.10}$$

Hence the proof is completed.

Theorem 2. For odd positive integer n and 2×2 real matrix $\boldsymbol{A}$,

$$\operatorname{Tr}\boldsymbol{A}^n=\sum_{r=0}^{(n-1)/2}\frac{(-1)^r}{r!}n\left[n-(r+1)\right]\left[n-(r+2)\right]\cdots\left[\text{up to } r \text{ terms}\right]\cdot(\operatorname{Det}\boldsymbol{A})^r(\operatorname{Tr}\boldsymbol{A})^{n-2r}$$

Proof. Consider a matrix $\boldsymbol{A}$ as in theorem 1, we have from (1.4) and (1.6).

$$\boldsymbol{A}^5=\boldsymbol{A}^3\boldsymbol{A}^2=\begin{bmatrix} a^3+abc+bc(a+d) & a^2b+b^2c+bd(a+d) \\ ac(a+d)+bc^2+cd^2 & bc(a+d)+bcd+d^3 \end{bmatrix}\begin{bmatrix} a^2+bc & b(a+d) \\ c(a+d) & bc+d^2 \end{bmatrix}$$

$$\begin{aligned}
\operatorname{Tr}\boldsymbol{A}^5 &= (a^2+bc)\left[a^3+abc+bc(a+d)\right]+c(a+d)\left[a^2b+b^2c+bd(a+d)\right] \\
&\quad +b(a+d)\left[ac(a+d)+bc^2+cd^2\right]+(bc+d^2)\left[bc(a+d)+bcd+d^3\right] \\
&= a^5+d^5+2bc(a^3+d^3)+5b^2c^2(a+d)+2bc(a+d)(a^2+d^2)+bc(a+d)^3 \\
&= a^5+d^5+5bc(a+d)^3-10abcd(a+d)+5b^2c^2(a+d) \\
&= a^5+d^5+5ad(a+d)^3-5a^2d^2(a+d)-5ad(a+d)^3+5a^2d^2(a+d) \\
&\quad +5bc(a+d)^3-10abcd(a+d)+5b^2c^2(a+d) \\
&= (a+d)^5-5(ad-bc)(a+d)^3+5ad(ad-bc)^2(a+d)
\end{aligned}$$

$$\operatorname{Tr}\boldsymbol{A}^5=(\operatorname{Tr}\boldsymbol{A})^5-5\operatorname{Det}\boldsymbol{A}(\operatorname{Tr}\boldsymbol{A})^3+5(\operatorname{Det}\boldsymbol{A})^2(\operatorname{Tr}\boldsymbol{A}) \tag{2.11}$$

Now we observe from (2.5) and (2.11) that

$$\operatorname{Tr}\boldsymbol{A}^3=\frac{(-1)^0}{0!}(\operatorname{Det}\boldsymbol{A})^0(\operatorname{Tr}\boldsymbol{A})^{3-2\times 0}+\frac{(-1)^1}{1!}3(\operatorname{Det}\boldsymbol{A})^1(\operatorname{Tr}\boldsymbol{A})^{3-2\times 1}$$

$$\mathrm{Tr}\boldsymbol{A}^5=\frac{(-1)^0}{0!}(\mathrm{Det}\boldsymbol{A})^0(\mathrm{Tr}\boldsymbol{A})^{5-2\times 0}+\frac{(-1)^1}{1!}5(\mathrm{Det}\boldsymbol{A})^1(\mathrm{Tr}\boldsymbol{A})^{5-2\times 1}+\frac{(-1)^2}{2!}5(5-3)(\mathrm{Det}\boldsymbol{A})^2(\mathrm{Tr}\boldsymbol{A})^{5-2\times 2}$$

Now we continuing this as in **Theorem 1**, we get Tr$\boldsymbol{A}^n$ same as **Theorem 1**. But here r varies up to $(n-1)/2$. Hence the theorem follows.

Corollary 1: For any positive integer n and 2 × 2 real singular matrix $\boldsymbol{A}$, $\mathrm{Tr}\boldsymbol{A}^n=(\mathrm{Tr}\boldsymbol{A})^n$.

Proof: For singular matrix $\boldsymbol{A}$, Det$\boldsymbol{A}$ = 0. Hence proof follows from **Theorem 1** and **Theorem 2**.

Corollary 2: For 2 × 2 real matrix $\boldsymbol{A}$ with Tr$\boldsymbol{A}$ = 0.

1) $\mathrm{Tr}\boldsymbol{A}^n=2(-\mathrm{Det}\boldsymbol{A})^{n/2}$ when n is even and;

2) $\mathrm{Tr}\boldsymbol{A}^n=0$ when n is odd.

Proof. Proof follows from theorem 1 and theorem 2.

Corollary 3: For 2 × 2 real matrix $\boldsymbol{A}$ with Tr$\boldsymbol{A}$ = 0 and Det$\boldsymbol{A}$ = 0.

$\mathrm{Tr}\boldsymbol{A}^n=0$ where n is any positive integer.

Proof. Proof follows from **Corollary 2**.

Example 1. Consider a matrix $\boldsymbol{A}=\begin{bmatrix}1 & -1\\ 2 & 2\end{bmatrix}$ and let we are to find Tr$\boldsymbol{A}^5$.

Here $\mathrm{Det}\boldsymbol{A}=2+2=4$ and $\mathrm{Tr}\boldsymbol{A}=1+2=3$. then by **Theorem 2**, we have

$$\mathrm{Tr}\boldsymbol{A}^5=(\mathrm{Tr}\boldsymbol{A})^5-5\mathrm{Det}\boldsymbol{A}(\mathrm{Tr}\boldsymbol{A})^3+5(\mathrm{Det}\boldsymbol{A})^2(\mathrm{Tr}\boldsymbol{A})=(3)^5-5(4)(3)^3+5(4)^2(3)=-57$$

Example 2. Consider a matrix $\boldsymbol{A}=\begin{bmatrix}3 & -5\\ 1 & -2\end{bmatrix}$ and let we are to find Tr$\boldsymbol{A}^{10}$.

Here $\mathrm{Det}\boldsymbol{A}=-6+5=-1$ and $\mathrm{Tr}\boldsymbol{A}=3-2=1$. then by **Theorem 1**, we have

$$\begin{aligned}\mathrm{Tr}\boldsymbol{A}^{10}&=(\mathrm{Tr}\boldsymbol{A})^{10}-10\mathrm{Det}\boldsymbol{A}(\mathrm{Tr}\boldsymbol{A})^8+35(\mathrm{Det}\boldsymbol{A})^2(\mathrm{Tr}\boldsymbol{A})^6-50(\mathrm{Det}\boldsymbol{A})^3(\mathrm{Tr}\boldsymbol{A})^4\\&\quad+25(\mathrm{Det}\boldsymbol{A})^4(\mathrm{Tr}\boldsymbol{A})^2-2(\mathrm{Det}\boldsymbol{A})^5\\&=1+10+35+50+25+2=123.\end{aligned}$$

Example 3. Consider a matrix $\boldsymbol{A}=\begin{bmatrix}20 & 199\\ -2 & -20\end{bmatrix}$ and let we are to find Tr$\boldsymbol{A}^{2015}$.

Here Tr$\boldsymbol{A}$ = 0, Det$\boldsymbol{A}$ = −2 and n = 2015, which is odd, hence by corollary 2, we get Tr$\boldsymbol{A}^{2015}$ = 0.

Example 4. Consider a matrix $\boldsymbol{A}=\begin{bmatrix}15 & 21\\ -10 & -14\end{bmatrix}$ and let we are to find Tr$\boldsymbol{A}^{100}$.

Here $\boldsymbol{A}$ is a singular matrix with Trace 1, and then by **Corollary 1**, we have

$$\mathrm{Tr}\boldsymbol{A}^{100}=(\mathrm{Tr}\boldsymbol{A})^{100}=(1)^{100}=1.$$

Conclusion and Future Work

After to discuss **Theorems 1** and **2**, **Corollaries 1**, **2** and **3**, we are able to find trace of any integer power of a 2 × 2 real matrix. In future, we can be developed similar results for 3 × 3 real matrices.

Acknowledgements

We would like to hardly thankful with great attitude to Director, Maulana Azad National Institute of Technology, Bhopal for financial support and we also thankful to HOD, Department of Mathematics of this institute for giving me opportunity to expose my research in scientific world.

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